HEAVY FLAVOUR PHYSICS – FFNS AND VFNS

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I present the new MRST parton distributions in the fixed-flavour number scheme, but emphasize the importance of using a general mass variable-flavour number scheme if possible. I highlight the developments for the NNLO variable-flavour number scheme.

It is essential to treat the charm ($m_c \sim 1.5$GeV) and bottom ($m_b \sim 4.3$GeV) quarks correctly in global fits to determine parton distributions. There are two distinct regimes for these heavy quarks. For low scales $Q^2 \sim m_H^2$ it is simplest to assume that the massive quarks are not partons but are all created in the final state. Hence, their production may be described using the Fixed Flavour Number Scheme (FFNS), where only the light quarks are constituents of the proton.

$$F(x, Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2). \qquad (1)$$

This approach does not sum $\alpha_S^n \ln^n Q^2/m_H^2$ terms in the perturbative expansion. This summation can be achieved by the definition of heavy-flavour parton distributions and the solution of evolution equations for these partons, which is generally a preferable approach. However, FFNS partons are sometimes needed because the hard cross-sections for some processes are only calculated with all heavy flavour generated in the final state, e.g. HQVDIS for differential heavy flavour production in DIS[1], MC@NLO for heavy flavours[2], HERWIG for heavy flavour production[3], *etc.*

However, the FFNS analysis must be done properly. The NLO ($O(\alpha_S^2)$) coefficient functions for heavy flavour in DIS are calculated[4] in a scheme where the coupling α_S is fixed at $n_f = 3$. Hence, the partons have to be defined in the same way. This is very often done incorrectly, and the variable-flavour coupling constant, which runs less quickly, is used instead[a]. Compared to the variable-flavour α_S, the $n_f = 3$ version is either $\sim 12\%$ smaller at $\mu^2 = M_Z^2$ if they are the same below m_c^2, or if they are identical at

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M_Z^2 the fixed flavour coupling is hugely bigger at low μ^2. One cannot really determine $\alpha_S(M_Z^2)$ from a FFNS fit – the $n_f = 3$ definition of $\alpha_S(M_Z^2)$ is not the same quantity as the more usual $n_f = 5$ definition of $\alpha_S(M_Z^2)$. The error made in using the wrong coupling is quite significant. If the coupling is too big the evolution is too quick. A comparison of incorrect and correct gluons obtained from the same input distributions is shown for $Q^2 = 100\text{GeV}^2$ in Fig. 1. The error can be bigger than the uncertainty.

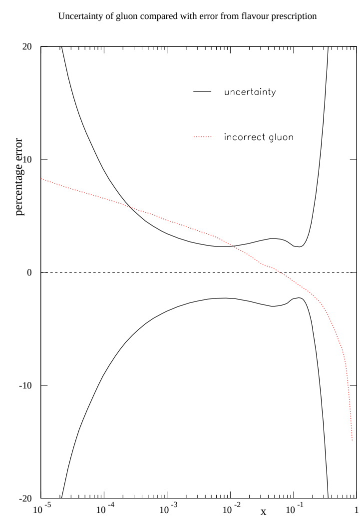

Figure 1. Comparison of the gluon evolved with the correct definition of $\alpha_S(Q^2)$ and with the the incorrect definition.

MRST now generate FFNS partons[5] by evolving from the usual (MRST04) partons[6] at $Q_0^2 = 1\text{GeV}^2$ but keeping $n_f = 3$ fixed everywhere. The charm contribution is rather smaller than in the VFNS due to a lack of summation of logs. Also, using of the correct α_S procedure leads to a much smaller $F_2^c(x, Q^2)$ than the incorrect procedure – α_S in the cross-section and the small-x gluon are smaller due to the reduction in the coupling in the evolution. It is difficult to do a global fit in FFNS since little else other than neutral current DIS is calculated in this scheme. The approximate global fit is poor for HERA $F_2(x, Q^2)$ with $\chi^2 = 80$ worse than usual.

Furthermore, FFNS is not defined at NNLO because $\alpha_S^3 C_{2,Hg}^{FF,3}$ is unknown. The NLO coefficient function $C_{2,Hg}^{FF,2}$ contains no information on P_{qg}^2 and so $\alpha_S^2 C_{2,Hg}^{FF,2} \otimes g^{n_f}$ cannot represent the NNLO evolution of $F_2^H(x, Q^2)$. This is important because the unknown $\alpha_S^3 C_{2,Hg}^{FF,3}$ is not just $O(\alpha_S^3)$ but is $O(\alpha_S^3 \ln^3(Q^2/m_H^2))$, and hence not necessarily suppressed.

Hence, it is usually preferable at high scales to recognize that for $Q^2 \gg m_H^2$ heavy quarks should behave like the strange quarks and to sum $\ln(Q^2/m_H^2)$ terms via evolution. When changing the flavour number the partons in different number regions are related to each other perturbatively: $f_k^{n_f+1}(Q^2) = A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$. The perturbative matrix elements $A_{jk}(Q^2/m_H^2)$[7] containing $\ln(Q^2/m_H^2)$ terms relate $f_k^{n_f}(Q^2)$ and $f_k^{n_f+1}(Q^2)$ leading to the correct evolution for both. The simplest means of incorporating these partons is the Zero Mass Variable Flavour Number Scheme (ZMVFNS). This ignores $O(m_H^2/Q^2)$ corrections, i.e.

$$F(x, Q^2) = C_j^{ZMVF} \otimes f_j^{n_f+1}(Q^2), \tag{2}$$

which is clearly inaccurate for $Q^2 \sim m_H^2$.

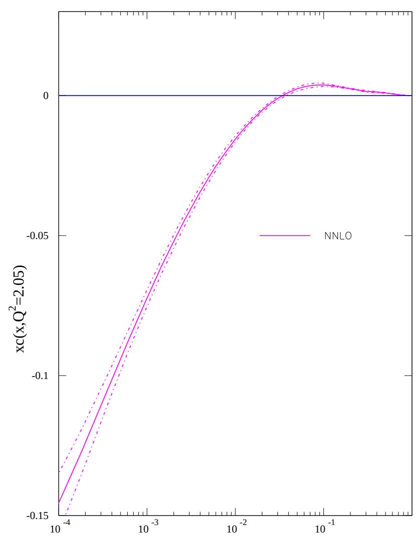

Figure 2. The charm parton distribution at the input scale $Q^2 = m_c^2$

In order to obtain heavy quark parton distributions and include all $O(m_H^2/Q^2)$ effects correctly we need a general Variable-Flavour Number Scheme (VFNS)[8,9], interpolating between the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$. The VFNS can be defined by demanding equivalence of the n_f (FFNS) and $n_f + 1$-flavour descriptions at all orders,

$$F^H(x, Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) = C_j^{VF}(Q^2/m_H^2) \otimes f_j^{n_f+1}(Q^2) \tag{3}$$
$$\equiv C_j^{VF}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2).$$

Hence, the VFNS coefficient functions satisfy

$$C_k^{FF}(Q^2/m_H^2) = C_j^{VF}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2). \tag{4}$$

This equation forms the basis for a definition of a VFNS. However, there are many details to sort out and choices to be specified. Moreover, at NNLO particular care must be taken since the partons themselves become discontinuous at the transition point $\mu^2 = m_H^2$. Indeed at small x they turn on with a negative value as shown in Fig. 2. A detailed NNLO VFNS has been presented previously[10], which is based upon both the ACOT(χ) scheme[11] and the Thorne-Roberts VFNS[12]. It has been shown to match HERA data well[13] and is now being used in a global fit[14]. There is one notable feature of the NNLO heavy flavour structure function, i.e. it tends to be flatter with Q^2 than that at NLO. This is because the NNLO $F_2^c(x, Q^2)$ starts from a higher value at low Q^2, and at high Q^2 is dominated by $(c + \bar{c})(x, Q^2)$, which has started evolving from negative value at $Q^2 = m_c^2$ and remains lower than at NLO for similar evolution. This has an important effect on the gluon distribution when going from NLO to NNLO.

Hence, we now have a full definition of a NNLO VFNS that can be used in global fits. The partons evolve entirely as in the massless $\overline{\text{MS}}$ scheme, and all mass effects are included correctly in the coefficient functions. At NNLO this can be constructed so that physical quantities are continuous, even though the partons are discontinuous at transition points. The correct heavy-flavour prescription works well, and its implementation is important for quantitative parton analyses.

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