HEAVY QUARK MASS EFFECTS AND HEAVY FLAVOR PARTON DISTRIBUTIONS *

WU-KI TUNG
Michigan State University, E. Lansing, MI 48824, USA
and
University of Washington, Seattle, WA 98195 USA
E-mail: tungw@msu.edu

We describe a new implementation of precise perturbative QCD calculation of deep inelastic scattering structure functions and cross sections, incorporating heavy quark mass effects. For consistency, the calculation is based on the general mass variable-flavor-number formalism of Collins. For accuracy, kinematic effects due to heavy flavor masses are uniformly taken into account, using appropriate rescaling variables for both neutral current (NC) and charged current (CC) processes. For efficiency, the simplified ACOT (SACOT) scheme for calculating the Wilson coefficients of heavy quark initiated subprocesses is systematically adopted. Application of this implementation to the first study of the charm parton degree of freedom in the nucleon is presented.

1. Introduction

With the accumulation of extensive precision deep inelastic scattering cross section measurements of both the neutral current (NC) and charged current (CC) processes from HERA I (and even more precise data from HERA II to come soon), it is necessary to employ reliable theoretical calculations that match the accuracy of the best data in the global analysis. We describe in this work a new implementation of the general PQCD framework of Collins with heavy quark masses that properly treat the kinematics of the final states (with heavy particles) using the ACOT(χ) rescaling procedure, as well as consistently incorporate the simplification of the SACOT scheme for all subprocesses.

---

*Work done in collaboration with H.L. Lai, A. Belyaev, J. Pumplin, D. Stump, and C.-P. Yuan; it is partially supported by grant # phy-0354838 from the US National Science Foundation.

2. New Implementation of the General PQCD Formalism including Heavy Quarks

We summarize here the key ingredients of the new implementation. Details will be published separately. For simplicity, we shall focus on the charm quark, and consider the relevant issues relating to the calculation of *structure functions* at an energy scale Q of the order of the charm mass M_c (where mass effects make a difference). The same considerations apply to the other heavy quarks, and to the calculation of *cross sections*.

The Factorization Formula: The PQCD factorization theorem for the structure functions is of the general form:

$$F^{\lambda}(x,Q^2) = \sum_a f^a \otimes \widehat{\omega}_a^{\lambda} = \sum_a \int_{\chi}^{1} \frac{d\xi}{\xi}\; f^a(\xi,\mu)\; \widehat{\omega}_a^{\lambda}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{M_i}{\mu}, \alpha_s(\mu)\right), \tag{1}$$

where the summation is over the active parton flavor label a, $f^a(x,\mu)$ are the parton distributions, and $\widehat{\omega}_a^{\lambda}$ are the hard-scattering amplitudes. The lower limit of the convolution integral χ is usually taken to be equal to $x = Q^2/2q \cdot p$; but this needs to be revised in the presence of heavy quark mass effects, as we shall discuss below. It is conventional to choose $\mu = Q$.

The (Scheme-dependent) Summation over Parton Flavors: The summation over "parton flavor" label a, $\sum_a$, depends on the factorization scheme chosen. In the *fixed flavor number scheme* (FFNS), one sums over $a = g, u, \bar{u}, d, \bar{d}, ...$ up to n_f flavors of quarks, where n_f is fixed at a given value $(3, 4, ...)$. The more general *variable flavor number scheme* (VFNS) is really a *composite scheme*: it consists of a series of FFNS's matched at conveniently chosen *match points* μ_i, one for each of the heavy quark thresholds. Our implementation of the general mass formalism includes both FFNS and VFNS. In practice, however, the FFNS has very limited range of validity; only the VFNS is suitable for general applications.

The Summation over (Physical) Final-state Flavors: Since we are concerned with the total inclusive structure functions, final states of all quark flavors must be included. This implies, there is an implicit sum over final-state quark flavors in the parton-level amplitude $\hat{\omega}^a$, i.e. $\hat{\omega}_a = \sum_b \hat{\omega}_a^b$ in Eq. 1. This sum is conceptually quite different from the sum over initial-state parton flavors: in contrast to the latter, (i) the final-state sum (over b) is over *all flavors* that can be physically produced; and (ii) the final state particle(s) "b" must be put on the mass-shell in order for the kinematics to work out correctly (see next paragraph). The distinction between the two summations is absent in the usual implementation of the conventional (i.e. textbook) zero-mass parton formalism.

Kinematic Constraints and Rescaling: Kinematic effects have a significant impact on the numerical results of the calculation. In DIS, with heavy flavor produced in the final state, the simplest kinematic constraint is $W > \sum M_f$ where W is the CM energy, and the right-hand side is the sum of all heavy particles in the final state, *including the target fragment*. Because of the more restricted phase space available, the parton momentum fraction needs to be shifted to a higher value than that in the zero-mass case by *rescaling*: i.e. the lower limit of the convolution integral ($\chi_c < \xi < 1$) in Eq. 1 should become $\chi_c = x\left(1 + \frac{4M_c^2}{Q^2}\right)$, rather than the Bjorken x.

Hard Scattering Amplitudes and the SACOT Scheme: The calculation of the hard scattering amplitudes beyond LO can be greatly simplified without loss of generality by using the SACOT scheme: (a) keep the full mass dependence of the gluon fusion subprocesses, (b) for NC scattering (γ/Z exchanges), set all quark masses to zero in the quark-initiated subprocesses; and (c) for CC scattering ($W_\pm$ exchange), set the initial-state quark masses to zero, but keep the final-state quark masses on shell.

Choice of Factorization Scale: The total inclusive structure function F_i^{tot} is infra-red safe. For the simple case of just one effective heavy flavor (charm), $F_i^{tot} = F_i^{light} + F_i^c$. If we use the same factorization scale μ for both terms, then the sum is insensitive to the value of μ. Since the right-hand side is dominated by the light-flavor term F_i^{light}, and the natural choice of scale for this term is $\mu = Q$, it is reasonable to use this scale for both terms. With this choice, F_i^c and F_i^{tot} are both continuous across the boundary line separating the 3-flavor region ($\mu < M_c$) from the 4-flavor region ($\mu > M_c$).

We have applied this new implementation of precision PQCD calculation of DIS structure functions and cross sections to a new global analysis of parton distributions, incorporating the complete HERA I data sets, as reported in the SF work group. In the following section, we describe a second application that explores an uncharted frontier of PDF analysis.

3. First Study of the Charm Parton Content of the Nucleon

Conventional global analyses of PDFs assume that charm partons are *radiatively generated* by QCD evolution—essentially by gluon splitting into $c\bar{c}$ pairs. Whereas this assumption does not contradict any known experimental data, it is nonetheless open to question because: (i) phenomenologically, the sensitivity of current (limited) experimental charm production data to this assumption has not been studied; and (ii) theoretically, this ansatz is inherently ill-defined since it depends critically on the scale at which the radiative process starts to take effect, and this scale is not known à priori.

With the more precise implementation of heavy quark mass effects described in the previous section at hand, we perform a new series of global QCD analysis, including an *independent, non-perturbative* charm component of the nucleon, to be determined phenomenologically. The goals are: (i) to do a first exploration of this new degree of freedom and to determine what limits can be placed on it; and (ii) to pave the way to study other heavy quark components, such as the bottom, that are relevant for LHC.

Two distinct scenarios for the input charm distribution at the starting scale $Q = M_c$ are used: (i) a sea-like shape (for convenience, proportional to the strange quark); and (ii) a light-cone model motivated shape ("*intrinsic charm*"), centered at moderate values of x. In each case, we vary the magnitude (normalization) of this input charm component and compare the goodness-of-fit (the overall χ^2) of the results global fits.

The results can be summarized as follows. In both scenarios: (i) the "best fit" (i.e. the one with the lowest overall χ^2) does have a small non-zero input charm component at the scale $Q = M_c$, although the difference between this and a conventional fit with no intrinsic charm is well within the commonly accepted uncertainty range of the global ; and (ii) the overall χ^2 for the global fit rises steadily with increasing input non-perturbative (intrinsic) charm, resulting in an upper limit on the magnitude of the intrinsic charm component at about 1.8% (measured by the the fraction of momentum carried) at the scale $Q = M_c$. Beyond this limit, the quality of the global fits become unacceptable by our usual goodness-of-fit criteria. No specific data sets stand out as being the determining ones.

4. Outlook

Precision global QCD analysis represented by our new implementation of PQCD provides improved determination of PDFs that are vital to pursue the physics goals of the hadron colliders. The study of the charm degrees of freedom briefly described here only marks the beginning of our exploration of the heavy quark sector of the parton distributions of the nucleon. Of much interest is the b-quark content of the nucleon, because of its direct impact on top physics (particularly single-top), Higgs physics and various New Physics scenarios. Much further work is needed.