

NEURAL NETWORK DETERMINATION OF THE NON-SINGLET QUARK DISTRIBUTION

NNPDF COLLABORATION:

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We summarize the main features of our approach to parton fitting, and we show a preliminary result for the non-singlet structure function. When comparing our result to other PDF sets, we find a better description of large x data and larger error bands in the extrapolation regions.

1. The NNPDF approach

The standard approach to PDF fitting has two main shortcomings. The first is the difficulty in propagating the error from data to the parametrization, and then from the parametrization to any observable that it is evaluated with it: this is easy to do only in a linearized approximation, which is not always adequate. The second is the difficulty in assessing the bias associated to the choice of functional form, which is done on the basis of theoretical prejudice. The latter is especially delicate, because a functional form parametrized by a small number of parameters must be chosen in order for the fits to converge, but this is then inevitably a source of bias: a bias free fit would never converge.

We have proposed a new approach to this problem ^{1,2}, which is based on the use of neural networks combined with the Monte Carlo method. The Monte Carlo approach addresses the first difficulty of the standard approach. Instead of propagating the experimental error on the parameters of the parton distributions, we generate replicas of the true experimental data, which fluctuate about the central experimental values in a way that reproduces the data uncertainty. If the number of replicas is sufficiently large, averaging over the replicas we can reconstruct the data we started from with their errors and correlations. Instead of producing a single set of parton distributions, we then produce as many replicas of the parton distributions as we generated replicas of the original data. The fluctuation of these replicas then automatically propagates the fluctuations of the data we started from, and averaging over them we can reconstruct the value and uncertainty on the parton distributions, and indeed of any physical observable which depends on them.

In order to avoid any assumption on the shape of the PDF at the initial scale, for each replica we use a redundant parametrization provided by a neural network. Neural networks are a class of algorithms designed in order to extract information from noisy or incomplete data, without having to make assumptions on the underlying law which is obeyed by the data. The only assumption is a certain degree of smoothness of the function which describes the data. Neural networks are non-linear functions defined recursively as layers of nodes which receive inputs from others nodes, and give an output which is fed to nodes of the next layer. As an example, in a simple case with one input $\xi_1^{(1)}$, two hidden neurons and one output $\xi_1^{(3)}$, (1-2-1), we have

$$\xi_1^{(3)} = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - \xi_1^{(1)}} \omega_{11}^{(1)}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - \xi_1^{(1)}} \omega_{21}^{(1)}}}}$$

where $\omega^{(i)}$ (weights) and $\theta^{(i)}$ (thresholds) are the parameters of the i -th layer. Data are fitted evolving the PDF from the initial scale to the scale of data, and comparing a physical observable thus computed to the data in order to tune the best-fit form parameters of the input PDF, now given by a neural network.

When a large number of parameters is fitted and when correlations between them are large, as it is the case with a redundant parametrization, the usual minimization techniques are not optimal. We have thus implemented a Genetic Algorithm technique ³, based on mutation and selection

of copies of a given parameters set. The main advantage of the genetic minimization is that it works on a population of solutions, rather than tracing the progress of one point through parameter space. Thus, many regions of parameter space are explored simultaneously, thereby lowering the possibility of getting trapped in local minima.

The feature of neural networks which solves the problems of the bias imposed by a choice of functional form is the fact that the minimization of a very redundant neural network can be performed, and stopped when a suitable criterium is met, but before the minimum χ^2 is reached. This is to be contrasted with standard fits, where one reaches the lowest χ^2 compatible with the given functional form, and eventually if one increases the size of the fitting function no stable fit can be obtained. The stopping criterium is the following. For each replica we separate randomly data into two sets: one of them is fitted and the other one is predicted. Since both sets represents the same physical quantity, the accuracy on both must be same. When χ^2 on the trained set goes on improving, while on the predicted one starts growing or oscillating, we stop the minimization. From then on the fit would only be learning the noise of the fitted set.

2. Results

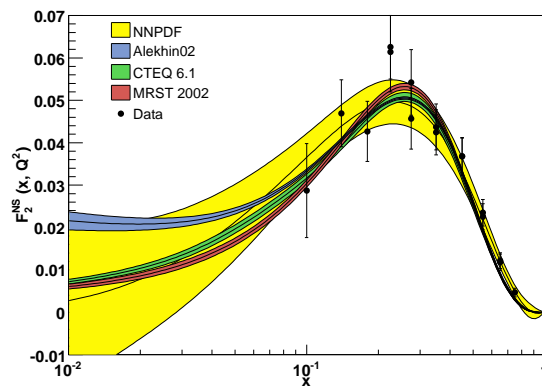


Figure 1. NNPDF best-fit of the non-singlet structure function at $31\text{GeV}^2 < Q^2 < 33\text{GeV}^2$ compared to other PDF sets. Our $1\text{-}\sigma$ error band has been evaluated with 1000 replicas; the initial scale PDF is a (2-5-3-1) neural network.

A full determination of $F_2^{NS}(x, Q^2) = F_2^p(x, Q^2) - F_2^d(x, Q^2)$ has been

performed with this method. We show our best-fit for a given Q^2 bin compared to the results obtained by different PDF sets ^{4,5,6}. From Fig. 1 we can see that the experimental points have large errors due to the fact we are taking a difference between two measurements, while the predictions given by the PDF sets have smaller errors, since they combine different measurements for the same points, and since due to evolution points with larger/smaller x and Q^2 carry the same amount of information of the ones shown in the plot. In the extrapolation region at small x the different behavior between the Alekhin's fit and the other sets is due to the fact that Alekhin does not assume any Regge-like constraint. If no assumption is made on the shape of the PDF, we obtain a result which agrees both with data and with the other sets within errors, describes better the large x range and predicts a larger uncertainty where there are no data. One may argue that our error band is wider than the other PDF sets since these are obtained by fitting much more data than us. However, since this flavour combination is only constrained by the non-singlet data which we also use, the small error bands obtained with the standard approach are more likely due to the way errors are propagated and to choice of a particular functional form for the initial PDF. Other examples of the underestimation of errors have been shown in ⁷, where a larger error for the Gottfried sum rule is obtained once the propagation of errors is performed without a linearized approximation, and in ⁸, where a larger error on α_s is obtained once no assumption on the PDF shape is made to fit data.

Further details on our technique and more results will be given in a forthcoming paper.

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