# MEASUREMENT OF $F_L(x,Q^2)$ AT HERA

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I present the case for why a direct measurement of the longitudinal structure Function $F_L(x,Q^2)$ is extremely important for our theoretical understanding of the physics of hadrons and QCD.

A real direct measurement of $F_L(x,Q^2)$ at HERA would be an important test of the success of different theories in QCD. In particular, it would give an independent test of the gluon distribution at low $x$ to go along with that determined from $dF_2(x,Q^2)/d\ln Q^2$. At present the fits to $F_2(x,Q^2)$ at low $x$ are reasonably good but the gluon is free to vary to make them as good as possible. We need a cross-check.

We already know a little about $F_L(x,Q^2)$ at small $x$. The total cross-section for deep inelastic scattering at HERA is proportional to $\tilde{\sigma}(x,Q^2) = F_2(x,Q^2) - y^2/(1+(1-y)^2)F_L(x,Q^2)$, where $y = Q^2/x\,s$, the fraction of the electron energy transferred in the scattering. Since both $F_L(x,Q^2)$ and $y$ are usually small we usually interpret the measurement of $\tilde{\sigma}(x,Q^2)$ as a measurement of $F_2(x,Q^2)$. However, we do have consistency checks on the relationship between $F_2(x,Q^2)$ and $F_L(x,Q^2)$ at high $y$, where both contribute. Indeed, one can use these data to determine $F_L(x,Q^2)$ by extrapolating in $y$ using either NLO perturbative QCD or $(d\sigma/d\ln y)_{Q^2}$ whilst making assumptions about $(dF_2(x,Q^2)/d\ln y)_{Q^2}$[1]. As a measurement of $F_L(x,Q^2)$ this has model-dependent uncertainties that are difficult to quantify fully[2]. However, the fit to the high-$y$ data could show up major flaws in a given theory, e.g NLO QCD.

The consistency check at high $y$ works well for the H1 NLO fit, and some others, but not for the MRST NLO fit, as seen in Fig. 1. This is because the MRST fit has more constraints on the gluon over a wider range of $x$. However, at low $x$, where the high-$y$ data exist, standard perturbation theory is not necessarily reliable because the splitting and coefficient functions pick up an extra power of $\ln(1/x)$ at each order, and hence enhancements are possible. At NNLO the splitting functions have now been calculated[3] and so too recently have the NNLO coefficient

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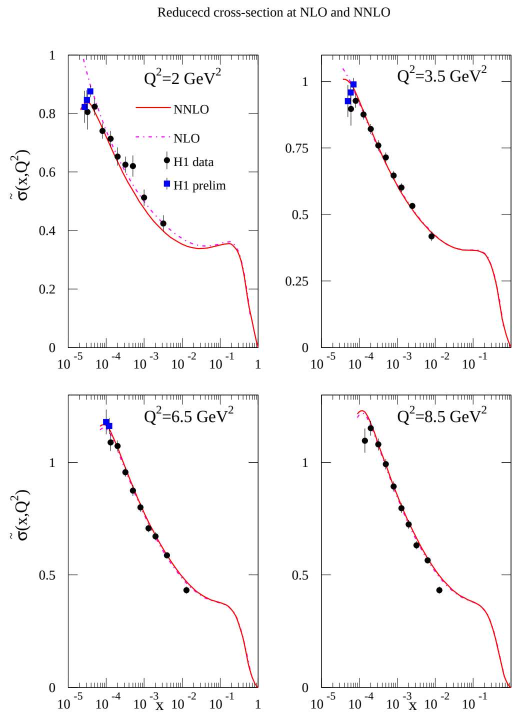


Figure 1. A comparison of the NLO and NNLO results in the MRST fits to the high-$y$ data on $\tilde{\sigma}(x, Q^2)$

functions[4] for $F_L(x, Q^2)$. There is an additional, positive small-$x$ contribution in $P_{qg}$ leading to a smaller small-$x$ gluon. However, $C^3_{Lg}(x) = n_f(\alpha_S/(4\pi))^3(409.5\ln(1/x)/x - 2044.7/x + \cdots)$, so there is clearly a significant positive contribution at small $x$. This more than compensates for the decrease in the small-$x$ gluon, and the NNLO contribution to $F_L(x, Q^2)$ solves the previous high-$y$ problem with $\tilde{\sigma}(x, Q^2)$, as seen in Fig. 1.

However, this suggests that higher orders still, or higher twist, might also be quite large. A fit that performs a resummation of leading $\ln(1/x)$ and of $\beta_0$ terms leads to a better fit to small-$x$ data than a conventional perturbative fit[5] and also seems to stabilize $F_L(x, Q^2)$ and small $x$ and $Q^2$. Similarly, a dipole-motivated fit contains terms in $\ln(1/x)$ and higher twists, and guarantees sensible behaviour for $F_L(x, Q^2)$ at low $Q^2$ from the form of the wavefunction. The $Q^2$ evolution for various predictions for $F_L(x, Q^2)$ at $x = 10^{-4}$ is shown in Fig. 2. It implies that a measurement of $F_L(x, Q^2)$ over as wide a range of $x$ and $Q^2$ as possible would be very useful.

HERA now propose running at lower beam energy before finishing in order to make a direct measurement of $F_L(x, Q^2)$. They intend to mea-

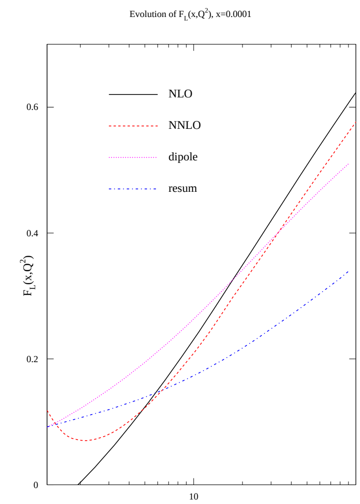


Figure 2. Evolution of various predictions for $F_L(x,Q^2)$ at $x = 0.0001$.

sure data from $Q^2 = 5 - 40\text{GeV}^2$ and $x = 0.0001 - 0.003$ with a typical error of at best $12 - 15\%$[6]. How important would this be in distinguishing between different theoretical approaches to structure functions? To test I have generated a set of data based on the central dipole prediction but with a random scatter such that $\chi^2 = 20/18$ for the dipole prediction. A comparison to other predictions is shown in Fig. 3. Clearly, there is some reasonable differentiating power. But these are central predictions.

I have also performed studies[8] of fits at NLO and at NNLO as the weight of these $F_L(x,Q^2)$ pseudo-data is increased in the fit. In each case the best fit results in $\chi^2 \sim 27/18$ for the $F_L(x,Q^2)$ *data* but corresponds to an unacceptable global fit. As we start to approach an acceptable global fit we get $\chi^2 \sim 30/18$ for $F_L(x,Q^2)$ *data*. At both NLO and NNLO the fit to $F_L(x,Q^2)$ *data* is never that good because the shape in $Q^2$ is incorrect.

Another possible source of large corrections to $F_L(x,Q^2)$ is higher-twist renormalon corrections[9]. These may be large even in the nonsinglet quark sector, since they do not die away at small $x$ as do those for $F_2(x,Q^2)$ in order to satisfy the Adler sum rule. There is already some evidence from the high-$y$ data on $\tilde{\sigma}(x,Q^2)$ that this contribution is important[10], and a direct measurement could provide better evidence[8].

Hence, a measurement of $F_L(x,Q^2)$ seems to be the best way to determine the gluon distribution at low $x$, particularly at low $Q^2$, and to determine whether fixed order calculations are sufficient, or whether resummations, or other theoretical extensions, may be needed. It is a vital measurement for our understanding of precisely how best to use perturba-

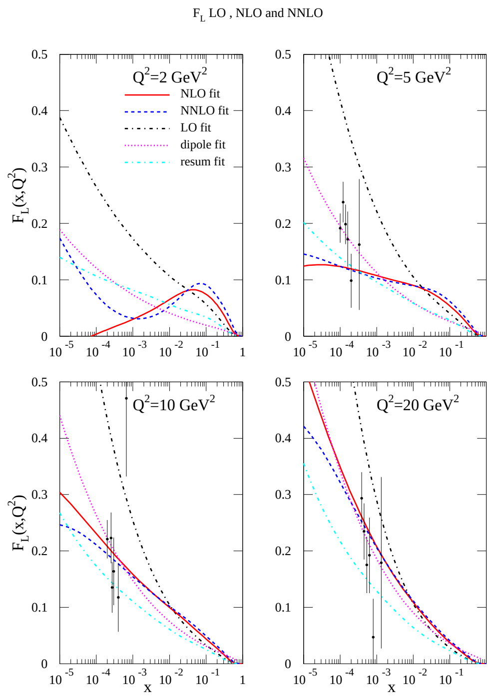


Figure 3. A comparison of various theoretical predictions for $F_L(x, Q^2)$ at HERA compared with the type of accuracy of measurements that could be made.

tive QCD to describe the structure of the proton and also for making really reliable predictions and comparisons at the LHC.

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