# THEORY OF DIFFRACTIVE STRUCTURE FUNCTIONS

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We review the perturbative QCD description of diffractive deep-inelastic scattering. Using the same data, we compare the diffractive parton distributions obtained in this way with the H1 2006 distributions.

Diffractive deep-inelastic scattering (DDIS), $\gamma^* p \to X + p$, is characterised by a large rapidity gap (LRG) between the cluster $X$ of outgoing hadrons and the slightly deflected proton, understood to be due to 'Pomeron' exchange. Let the momenta of the incoming proton, the outgoing proton, and the photon be labelled $p$, $p'$, and $q$ respectively; see Fig. 1(a). Then the basic kinematic variables in DDIS are the photon virtuality $Q^2 = -q^2$, the Bjorken-$x$ variable $x_B = Q^2/(2p \cdot q)$, the squared momentum transfer $t = (p - p')^2$, the fraction of the proton's light-cone momentum transferred through the rapidity gap, $x_{\mathbb{P}} = 1 - p'^+/p^+$, and the fraction of the Pomeron's light-cone momentum carried by the struck quark, $\beta = x_B/x_{\mathbb{P}}$.

The virtualities of the $t$-channel partons in Fig. 1(a) are strongly ordered as required by DGLAP evolution: $\mu_0^2 \ll \ldots \ll \mu^2 \ll \ldots \ll Q^2$. The scale $\mu^2$ at which the Pomeron-to-parton splitting occurs can vary between $\mu_0^2 \sim 1$ GeV$^2$ and the factorisation scale $Q^2$. For $\mu^2 < \mu_0^2$, the representation of the Pomeron as a perturbative parton ladder is no longer valid and instead, in the lack of a precise theory of non-perturbative QCD, we appeal to Regge theory where the 'soft' Pomeron is a Regge pole with intercept $\alpha_{\mathbb{P}}(0) \simeq 1.08$; see Fig. 1(b). In addition to the *resolved* Pomeron contributions of Fig. 1(a,b), we must also account for the *direct* interaction of the

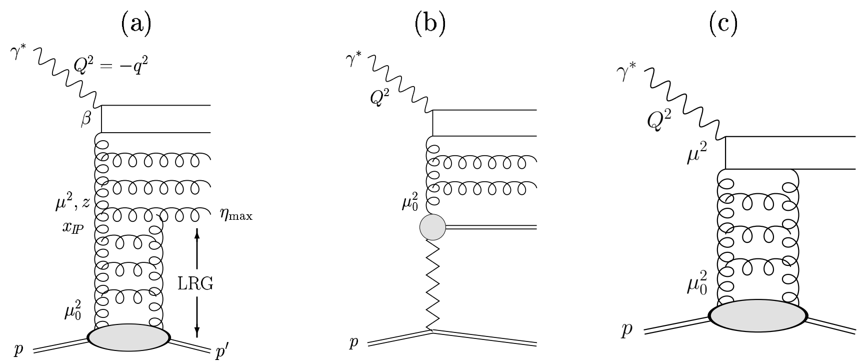


Figure 1. (a) Perturbative 'resolved' Pomeron contribution. (b) Non-perturbative 'resolved' Pomeron contribution. (c) Perturbative 'direct' Pomeron contribution.

perturbative Pomeron in the hard subprocess, Fig. 1(c), where there is no DGLAP evolution in the upper part of the diagram.

The diffractive structure function $F_2^{\mathrm{D}(3)}$ is then given, using collinear factorisation, by [1, 2]

$$F_2^{\mathrm{D}(3)}(x_{\mathbb{P}}, \beta, Q^2) = \sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}} \;+ \sum_{\mathbb{P}=G,S,GS} C_{2,\mathbb{P}}, \qquad (1)$$

where the first *resolved* Pomeron term corresponds to Fig. 1(a,b) while the second *direct* Pomeron term corresponds to Fig. 1(c). A similar factorisation holds for final-state observables in DDIS, such as jet or heavy quark production. The diffractive parton distribution functions (DPDFs) $a^{\mathrm{D}} = zq^{\mathrm{D}}$ or $zg^{\mathrm{D}}$ satisfy the inhomogeneous evolution equation [1, 2],

$$\frac{\partial a^{\mathrm{D}}(x_{\mathbb{P}}, z, Q^2)}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}} \;+ \sum_{\mathbb{P}=G,S,GS} P_{a\mathbb{P}}(z)\, f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2). \qquad (2)$$

Here, the first term, involving the usual parton-to-parton splitting functions, $P_{aa'}$, arises from the DGLAP evolution in the upper parts of Fig. 1(a,b). The second (inhomogeneous) term, involving the Pomeron-to-parton splitting functions, $P_{a\mathbb{P}}$, arises from the transition from the two $t$-channel partons (that is, the Pomeron) to a single $t$-channel parton in Fig. 1(a). The notation $\mathbb{P} = G, S, GS$ denotes whether these two $t$-channel partons are gluons ($\mathbb{P} = G$) or sea-quarks ($\mathbb{P} = S$), while the interference term is denoted by $\mathbb{P} = GS$. The LO Pomeron-to-parton splitting functions, $P_{a\mathbb{P}}$, were calculated in Ref. [1], where the perturbative Pomeron flux factors, $f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2)$, are also given. Therefore, we see that the diffractive structure function is analogous to the photon structure function, where there are both resolved and direct components and the photon PDFs also satisfy an inhomogeneous evolution equation.

Shortly after this workshop took place, the long-awaited H1 leading-proton [3] and LRG [4] data were released. We will concentrate on the latter data set for the remainder of this article, where we will investigate the effect of neglecting the perturbative Pomeron terms in (1) and (2), as assumed in the H1 2006 DPDF analysis [4]. We will make two types of fits, labelled "Regge" and "pQCD", where the "Regge" fits neglect the perturbative Pomeron terms while the "pQCD" fits include them. Note that the "Regge" fits only neglect the perturbative Pomeron-to-parton splitting from scales $\mu^2 > Q^2_{\rm min}$, provided that the input scale $Q_0^2 \leq Q^2_{\rm min}$ where $Q^2_{\rm min}$ is the minimum $Q^2$ of the data fitted; the contributions from $\mu^2 \leq Q^2_{\rm min}$ are included in the input DPDFs. We will take $Q^2_{\rm min} = 8.5$ GeV$^2$ in order to allow a direct comparison with the H1 2006 analysis [4].

We take the input DPDFs at a starting scale $Q_0^2 = 2$ GeV$^2$ in the form

$$z\Sigma^{\rm D}(x_{\mathbb{P}}, z, Q_0^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\; A_q\, z^{B_q}(1-z)^{C_q}, \qquad (3)$$

$$zg^{\rm D}(x_{\mathbb{P}}, z, Q_0^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\; A_g\, z^{B_g}(1-z)^{C_g}, \qquad (4)$$

where the Pomeron flux factor is taken from Regge phenomenology [4],

$$f_{\mathbb{P}}(x_{\mathbb{P}}) = A_{\mathbb{P}} \int_{t_{\rm cut}}^{t_{\rm min}} {\rm d}t \quad {\rm e}^{B_{\mathbb{P}}\, t}\; x_{\mathbb{P}}^{1-2\alpha_{\mathbb{P}}(t)}. \qquad (5)$$

Here, $\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}\, t$, where we take $\alpha_{\mathbb{P}}(0)$ as a free parameter and fix $B_{\mathbb{P}} = 5.5$ GeV$^{-2}$ and $\alpha'_{\mathbb{P}} = 0.06$ GeV$^{-2}$ [3]. The secondary Reggeon contributions are included in a similar way to the H1 2006 analysis [4].

Note that the inhomogeneous term in (2) will break the $x_{\mathbb{P}}$ factorisation assumed at the input scale $Q_0^2$ when evolving upwards in $Q^2$. The recent H1 analysis [4] claims that the LRG data are well described assuming the Pomeron to be a single Regge pole with effective intercept $\alpha_{\mathbb{P}}(0) \simeq 1.12$. It is instructive to look at the $Q^2$ dependence of the effective intercept $\alpha_{\mathbb{P}}(0)$ for fixed $\beta$, shown in Fig. 2(a). For $\beta = 0.4$ and 0.65, there is a clear rise with $Q^2$ of the effective $\alpha_{\mathbb{P}}(0)$ implying breaking of $x_{\mathbb{P}}$ factorisation, and so the constant value of $\alpha_{\mathbb{P}}(0) \simeq 1.12$ obtained by H1 is only some 'average' value. However, the breaking of $x_{\mathbb{P}}$ factorisation arising from the inhomogeneous term in the evolution equation seems to be small; see the predictions from the "pQCD" fit in Fig. 2(a).

In Fig. 2(b) we compare the "Regge" and "pQCD" DPDFs to the H1 2006 Fit A [4]. There are only very small differences between the "Regge" fit and the H1 2006 Fit A due to minor details in the two fits. H1 have

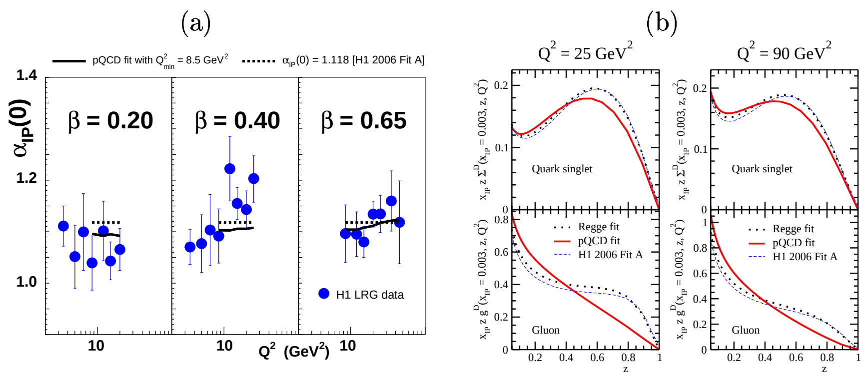


Figure 2. (a) The effective Pomeron intercept, $\alpha_{\mathbb{P}}(0)$, determined by fitting the reduced diffractive cross section $\sigma_r^{\mathrm{D}(3)} \propto f_{\mathbb{P}}(x_{\mathbb{P}})$ in each $(\beta, Q^2)$ bin containing four or more data points with $x_{\mathbb{P}} \leq 0.01$, $y \leq 0.45$ and $M_X \geq 2$ GeV. (b) The "Regge" and "pQCD" DPDFs with $Q^2_{\mathrm{min}} = 8.5$ GeV$^2$ compared to the H1 2006 Fit A.

recently made a combined fit of the inclusive DDIS data with diffractive dijet data [5]. This gives a smaller gluon density at moderate to high $z$ compared to the H1 2006 Fit A. Moreover, the $\chi^2$ for the 190 inclusive DDIS points increases from 158 (H1 Fit A) to 169 (H1 combined fit) on inclusion of the dijet data. Our "pQCD" fit has a smaller gluon distribution at moderate to high $z$ than the H1 2006 Fit A, suggesting that the inclusion of the perturbative Pomeron terms alleviates the tension between the inclusive DDIS and dijet data. Therefore, including dijet data in the fit is not necessarily a good idea if the theory used is unreliable.

In a forthcoming paper we will perform a more detailed analysis of diffractive structure functions, in which we investigate the theoretical uncertainties and include a wider variety of data in the fit.

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