INFRARED SAFE DEFINITION OF JET FLAVOUR

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Though it is widely taken for granted that it makes sense to separately discuss quark and gluon jets, normal jet algorithms lead to a net parton-level jet flavour that is infrared (IR) unsafe. This writeup illustrates the problem and explains how the k_t algorithm can be modified to provide an IR safe parton-level flavour. Jet-flavour algorithms are of use in theoretical calculations that involve a projection of higher-order contributions onto a flavour-channel of a lower order, and also offer the prospect of large improvements in the accuracy of heavy-quark jet predictions.

1. Introduction

Over 350 articles on SPIRES refer in their title to “quark-jet(s)” or “gluon-jet(s)”. This presupposes that such a distinction can be made sensibly.

It is well known that there is no unique way of defining jets — *e.g.* the mapping of $n+1$ partons onto n-jets is ambiguous when all $n+1$ particles are hard and widely separated in angle. This ambiguity persists when trying to identify n *flavoured* jets from $n+1$ partons. But one might hope that, identifying the flavour of a jet as the sum of flavours of its constituents, then that flavour will be meaningful, i.e. infrared (IR) safe, just like the energies and angles of the jets.

When mapping $n+1$ partons onto n jets, IR safety of the flavour holds trivially. With $n+2$ or more partons, there can be an extra large-angle soft $q\bar{q}$ pair stemming from the branching of a soft gluon (fig. 1), such that the quark is clustered into one jet and the anti-quark into another. Both those jets have their flavours ‘contaminated’. Because of the soft divergence for the gluon that branched to the large-angle $q\bar{q}$ pair, a perturbative calculation of the jet flavours leads to an IR divergent result for cone, k_t and Cambridge type jet algorithms.[1]

The jet algorithm that offers the most scope for resolving this problem is the k_t algorithm. It repeatedly recombines the pair of objects that are closest according to a distance measure $y_{ij}^{kt} = \frac{2\min(E_i^2,E_j^2)}{Q^2}(1-\cos\theta_{ij})$, where

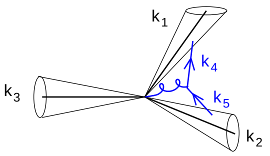

Figure 1. 5 parton configuration clustered to 3 jets, where a large-angle soft $q\bar{q}$ pair (k_4, k_5) contaminates the flavour of two of the jets.

E_i is the energy of particle i, θ_{ij} is the angle between particles i and j and Q is the centre of mass energy (for e^+e^- collisions). This choice of distance measure can be justified because the emission of a gluon has two divergences (soft and collinear): $[dk_j]|M^2_{g\to g_i g_j}(k_j)| \simeq \frac{\alpha_s C_A}{\pi}\frac{dE_j}{\min(E_i,E_j)}\frac{d\theta^2_{ij}}{\theta^2_{ij}}$ $(E_j \ll E_i\,, \theta_{ij} \ll 1)$. Quark production in contrast has only a collinear divergence: $[dk_j]|M^2_{g\to q_i\bar{q}_j}(k_j)| \simeq \frac{\alpha_s T_R}{2\pi}\frac{dE_j}{\max(E_i,E_j)}\frac{d\theta^2_{ij}}{\theta^2_{ij}}$ (note the "max" in the denominator). When one is interested mainly in the *kinematics* of jets this is largely irrelevant, since most of the branchings in an event produce gluons. However when investigating flavour, a problem arises because the y^{kt}_{ij} makes it easy for a soft quark to recombine with a hard particle, even though there is no corresponding divergence for producing that soft quark.

A simple solution to the problem is to modify the jet distance measure to reflect the structure of divergences. We introduce a new "flavour distance",

$$y^F_{ij} = \frac{2(1-\cos\theta_{ij})}{Q^2} \times \begin{cases} \max(E_i^2, E_j^2)\,, & \text{softer of } i,j \text{ is flavoured,} \\ \min(E_i^2, E_j^2)\,, & \text{softer of } i,j \text{ is flavourless.} \end{cases} \qquad (1)$$

Note that this requires information on the flavour of each object. Figure 2 illustrates the y_{ij} distances between various partons in an event with the two distance measures and in particular shows how a soft quark has a large y^F_{ij} with all hard particles in the event. Thus it will first recombine with the antiquark of similar softness, producing a gluon-flavoured object which can recombine with hard objects without changing their flavour. So a modification of the distance measure to better reflect the divergences in the theory allows one to obtain an infrared-safe definition of jet flavour.[2]

The IR safety can be concretely illustrated by taking a fixed-order parton-level e^+e^- event, clustering it to two jets and examining the cross section for cases where the flavour of the two jets is not simply that of

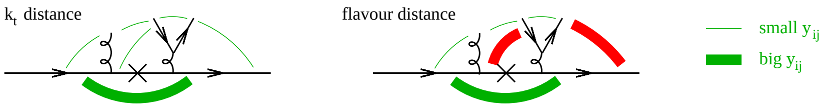

Figure 2. Representation of y_{ij} distances between particles in an e^+e^- event with k_t and flavour distance measures. Thick lines indicate a large y_{ij}, thin lines a small y_{ij}.

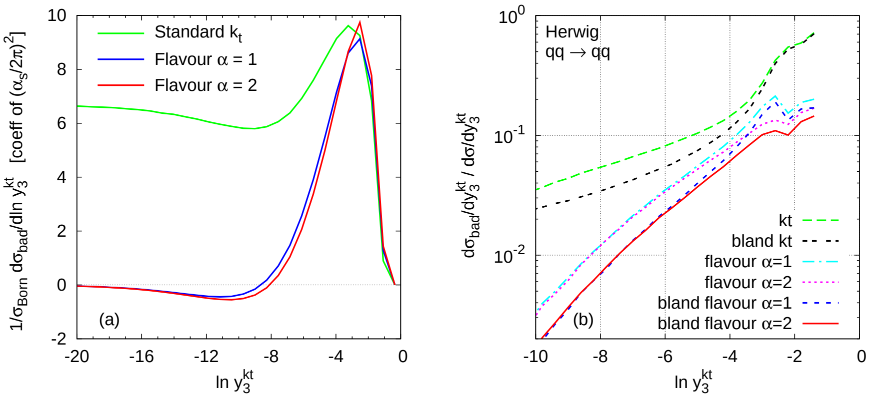

Figure 3. Proportions of events whose flavour is misidentified after clustering, as a function of the softness of the event: (a) in e^+e^-, coefficient of $\mathcal{O}\left(\alpha_s^2\right)$, calculated with EVENT2,[3] (b) in $qq \to qq$ LHC events (with ∼ 1 TeV jets) calculated using Herwig.[4]

q, $\bar{q}$. Plotted as a function of y_3^{kt} (the y_{cut} resolution threshold in the k_t algorithm above which the event is clustered to two jets), the cross section should vanish as $y_3^{kt} \to 0$, *i.e.* in the soft/collinear limit. At order α_s^2, fig. 3a, this is seen to happen for the flavour algorithms (which actually form a class defined by $\max(E_i^2, E_j^2) \to [\max(E_i, E_j)]^\alpha \cdot [\min(E_i, E_j)]^{2-\alpha}$, $0 < \alpha \leq 2$), but not the k_t algorithm.

So far we have examined the flavour algorithm just for e^+e^- collisions. In hadron-hadron collisions (and some DIS contexts) a longitudinally invariant algorithm is used, in which the distance measure d_{ij}^{kt} is obtained by replacing $Q^2 \to 1$, $E_i \to k_{ti}$ and $2(1 - \cos\theta_{ij}) \to \Delta\eta_{ij}^2 + \Delta\phi_{ij}^2$, and by introducing an additional beam distance measure $d_{iB} = k_{ti}^2$. For the flavour algorithm, the replacements are identical, while the beam distance measure for flavoured objects becomes $d_{iB}^{(F)} = \max(k_{ti}^2, k_{tB}^2(\eta_i))$, where $k_{tB}^2(\eta)$ is a beam hardness as a function of rapidity, defined in reference [2]. It is more complex to illustrate the IR safety in hadron-hadron collisions than in e^+e^- because hadron-collider fixed-order (NLO) programs do not currently provide any information on parton flavour. So instead we take HERWIG parton shower events, and look (fig. 3b) at the proportion of events in which the reconstructed jet flavours fail to correspond to the original $2 \to 2$ event jet flavours before showering, again as a function of the event softness. All algorithms show a failure rate that vanishes as $y_3 \to 0$ (in that limit all parton-showering is forbidden), however the faster vanishing for the flavour algorithms is a sign of their IR safety. One can also impose *blandness* of recombinations,[5] *i.e.* forbid multi-flavoured recombinations (*e.g.* uu, $u\bar{d}$).

This improves the overall normalisation.

There are various applications of jet flavour algorithms. Certain theory calculations (*e.g.* CKKW matching,[5] hadron-collider resummations [6]) need to project fixed-order configurations onto a lower-order flavour channel so as to match with a parton-shower or resummation based on the lower order.

At hadron level the use of the flavour algorithm is hampered by the need to know the flavour of every object, making it inappropriate for experimental discrimination of quark versus gluon jets. However it *is* feasible experimentally [7] to identify all heavy-flavour hadrons in an event. Therefore one can use the flavour algorithm to identify heavy-quark jets, treating only heavy-flavour objects as flavoured. The IR safety of the algorithm means that the heavy-quark jet distribution will be free of any logs of P_t/m_H, except those in the PDF (P_t is the jet transverse momentum and m_H the heavy-quark mass). Furthermore, neglecting terms $\sim m_H^2/P_t^2$, it can be calculated with a normal light-flavour NLO program [8] (extended to provide access to flavour) rather than a dedicated heavy-flavour program.[9]

The fact that the new algorithms for heavy-quark jets lead to far fewer logs of P_t/m_H than currently used definitions,[10] and that the remaining logs can be resummed in the PDFs, suggests that heavy-flavour jet distributions can be predicted better for the new algorithms, probably reducing the $40-60\%$ uncertainty typical of current heavy flavour NLO calculations down to the $10-20\%$ typical of inclusive light-jet distributions at NLO.

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