ENERGY FLOW BETWEEN JETS IN THE K_T ALGORITHM.

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We consider the impact of the k_t algorithm on energy flow into gaps between jets in any QCD hard process. While we confirm the observation that the k_t clustering procedure considerably reduces the impact of non-global logarithms, we unearth yet new sources of logarithmic enhancement, that stem from using the k_t algorithm to define the final state. We comment on the nature of the logarithms we find and discuss their all-orders treatment.

1. Introduction

The transverse energy (E_t) flow into gaps between hard jets is an observable that can offer important insights into different aspects of QCD. This includes information on the strong coupling, understanding of "all-order" behaviour as manifested in resummed predictions, non-perturbative power corrections and the underlying event at hadron colliders.

In order however to obtain information as accurately as possible from such an observable, one might expect that the minimum requirement is a solid (and correct) perturbative estimate, at least to the accuracy claimed. Failure to provide a correct estimate leads to attributing a potentially significant chunk of the model independent perturbative answer to model dependent pieces such as the underlying event or power corrections. How significant such a misattribution is, will naturally vary on a case-by–case basis but it suffices to say that the overall picture emerging from most studies of this kind, would be incomplete. This unfortunately is in fact the current situation, as described below.

*Work done in collaboration with Andrea Banfi.

2. Non global logs and the k_t definition

We wish to consider the distribution in the E_t flow into a gap Ω, $1/\sigma d\sigma/dQ_\Omega$. We define the gap transverse energy as

$$Q_\Omega = \sum_i E_{t,i} \tag{1}$$

where the sum runs over either hadrons in Ω or as is more commonly the case, over soft jets in Ω. These are obtained after a jet algorithm has been employed to cluster the final state into jets. This leaves aside from the high E_t hard jets outside Ω, soft jets that can populate the gap region.

The main problem in obtaining a resummed perturbative prediction for this observable is its non-global nature, that has itself been pointed out only relatively recently [1,2]. Thus while for several observables such as many event shapes, one can obtain a next-to–leading log prediction by considering a veto on real emissions attached just to the hard emitting partons, this is not the case here even at leading logarithmic accuracy. For non global observables like the gap energy distribution the leading single-logarithmic resummed result can be expressed as (we consider first the definition involving a sum over hadrons in the gap)

$$\frac{1}{\sigma}\frac{d\sigma}{dQ_\Omega} \approx \frac{1}{\sigma}\frac{d}{dQ_\Omega}\left(e^{-R(Q/Q_\Omega)} S\left(Q/Q_\Omega\right)\right), \tag{2}$$

where the factor e^{-R} represents uncancelled virtual emissions attached just to the hard jets, integrated over the gap region. The factor S is the non-global term, where one has to consider a soft gluon emitted in Ω as being coherently emitted from an arbitrarily complex ensemble consisting not merely of the hard jets but additionally any number of soft gluons outside Ω. Like the term e^{-R}, the factor S also resums a class of single-logarithms, $\alpha_s^n \ln^n Q/Q_\Omega$. Till date the calculation of the non-global term S has only been performed in the large N_c approximation, making the non-global piece the dominant source of perturbative uncertainty at low Q_Ω.

A partial solution to the problem was proposed by Appleby and Seymour [3], who pointed out that in several experimental studies one actually employs the definition based on summing over soft jets given by k_t clustering. They assumed that the factor e^{-R} was left intact by the clustering procedure, since it can be considered as exponentiating a result, R, obtained by considering just a single emission and its virtual counterpart. On the other hand the non-global piece involves multiple emission and has thus to be recomputed for the case of clustering. In particular gluons that fly into the gap can be pulled out of the gap by harder gluons outside which reduces the non-global component significantly, but does not eliminate it altogether. Thus the Appleby Seymour result assumed the form

$$\frac{1}{\sigma}\frac{d\sigma}{dQ_\Omega} \approx \frac{1}{\sigma}\frac{d}{dQ_\Omega}\left(e^{-R(Q/Q_\Omega)}S_{\mathrm{kt}}\left(Q/Q_\Omega\right)\right), \tag{3}$$

where S_{kt} is the non-global contribution recomputed with k_t clustering. This new non-global correction was found to be less than 20 % of the unclustered result.

3. Additional real-virtual mismatch induced by k_t clustering

Now we reconsider the result (3) and show that it is incorrect in the sense that it does not capture all the relevant single-logarithms even leaving aside those suppressed by $1/N_c^2$ [4].

Let us concentrate on the factor e^{-R} where for the simple case of e^+e^- annihilation $R \sim C_F\alpha_s(Q)\ln Q/Q_\Omega$. This term represents purely virtual emissions above the scale Q_Ω, integrated in a phase space corresponding to the gap region. Real emissions below the scale Q_Ω have been assumed to totally cancel while those above Q_Ω are vetoed.

In fact real emissions attached to the hard jets (thus not pertaining to the non-global term) *do not completely cancel away*, due to the use of clustering. Consider two energy-ordered real emissions k_1 and k_2 for which the probability of independent emission from a hard dipole ab, can be written as :

$$\mathcal{P}^{\mathrm{real,real}} = C_F^2\alpha_s^2 W_{ab}(k_1)W_{ab}(k_2), \tag{4}$$

where the W_{ab} are eikonal emission factors for gluons k_1 and k_2 from the ab dipole. Likewise if the emitting (more energetic) gluon k_1 is virtual we have the one-real one–virtual independent emission probability.

$$\mathcal{P}^{\mathrm{real,virtual}} = -C_F^2\alpha_s^2 W_{ab}(k_1)W_{ab}(k_2). \tag{5}$$

For the pure virtual piece e^{-R} to be built up, one assumes these contributions to cancel, which is the case without clustering.

In the case one uses clustering on the final state, consider a situation when the softer gluon k_2 is in Ω and k_1 outside. If the distance bewteen the gluons $(\Delta\eta_{12})^2 + (\Delta\phi_{12})^2 < R^2$, , where R is the jet radius, the gluon k_2 is clustered out of the gap and hence in this region the double real contribution to the energy distribution in Ω is zero. However the mixed real-virtual term persists in this region, making a finite contribution to the distribution, since k_2 cannot be clustered away by a virtual gluon outside the gap. Thus instead of a cancellation we are left with a contribution that at order α_s^2 has the colour factor C_F^2. This is clearly distinct from the non-global term at this order which has colour factor $C_F C_A$ and is not accounted for by expanding e^{-R} either, confirming that it is a piece left out previously.

Specialising to the case of Ω being a rapidity slice of width $\Delta\eta \geq R$ we obtain the following additional single-logarithmic contribution to the integrated quantity $\int_0^{Q_\Omega} \frac{1}{\sigma}\frac{d\sigma}{dE_t} dE_t$:

$$C_2^{\text{primary}} = \frac{16}{\pi} C_F^2 L^2 R^3, \tag{6}$$

where $L = \ln Q/Q_\Omega$ and "primary" refers to the fact that we have attached the gluons only to the primary hard partons produced in the process. We have confirmed the result above, valid for the simple case of $e^+e^- \to 2$ jets with exact fixed-order computations.

4. All orders contribution

We have been able to numerically resum the terms we describe above, to all orders for the simple $e^+e^- \to 2$ jets and DIS (1+1) jets cases. The effect we find is moderate over most of the phenomenological region of interest, changing the previous results by a maximum of 30%. However for the more complex cases of dijet photoproduction and hadron–hadron energy flow variables, further work is needed to estimate this effect at all orders. In these cases additional insight is also required to understand the potential role of superleading logarithms [5]. A satisfactory understanding of the energy flow even to leading logarithmic accuracy, is thus some way off.

References

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