SOFT-GLUON CORRECTIONS IN HARD-SCATTERING PROCESSES THROUGH NNNLO

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I discuss soft-gluon corrections in hard-scattering processes and their resummation. I present master formulas for the expansion of the resummed cross section through NNNLO and discuss the significance of these corrections in a number of processes.

1. Introduction

The cross section for the production of a final state F in collisions of hadrons h_1 and h_2, $h_1 + h_2 \to F(p) + X$, can be written in factorized form as $\sigma = \sum_f \int [\prod_i dx_i \, \phi_{f/h_i}(x_i, \mu_F)] \, \hat{\sigma}(s, t_i, \mu_F, \mu_R)$ where ϕ are the parton densities and $\hat{\sigma}$ is the perturbatively calculable partonic hard-scattering cross section. Near threshold for the production of F there is restricted phase space for real gluon emission and thus incomplete cancellation of infrared divergences between real and virtual graphs resulting in the appearance of large logarithms in the perturbative series. These soft and collinear logarithmic corrections take the form of plus distributions. For the partonic reaction $f_1(p_1)+f_2(p_2) \to F(p)+X$ we define $s=(p_1+p_2)^2$, $t_1=(p_1-p)^2$, $t_2=(p_2-p)^2$, and $s_4=s+t_1+t_2-\sum m^2$. At threshold $s_4 \to 0$. The plus didtributions are of the form $D_l(s_4) \equiv [\ln^l(s_4/M^2)/s_4]_+$ with $l \le 2n-1$ for the n-th order corrections in α_s.

If we define moments of the cross section $\hat{\sigma}(N)=\int_0^\infty ds_4 e^{-Ns_4/M^2} \hat{\sigma}(s_4)$ then the soft corrections become logarithms of the moment variable N: $D_l(s_4) \equiv \left[\frac{\ln^l(s_4/M^2)}{s_4}\right]_+ \to \frac{(-1)^{l+1}}{l+1} \ln^{l+1} N + \cdots$. We can formally resum these logarithms to all orders in α_s by factorizing the soft gluons from the hard scattering.[1,2] To obtain physical cross sections we need to invert the moment-space resummed cross section back to momentum space. Resummation prescriptions are needed to deal with the Landau singularity. Theoretical ambiguities are involved, and differences between prescriptions can be numerically bigger than higher-order terms.

Alternatively, we can expand the resummed cross section to finite order.[3] No prescription is then necessary and no further approximation is imposed on the kinematics. In the expansion at next-to-leading order (NLO) in α_s, we encounter $D_1(s_4)$ and $D_0(s_4)$ terms. At next-to-next-to-leading order (NNLO) we have $D_3(s_4)$ through $D_0(s_4)$ terms. At next-to-next-to-next-to-leading order (NNNLO) we find $D_5(s_4)$ through $D_0(s_4)$ terms. The highest-power logarithms at each order are the leading logarithms (LL), the second highest are the next-to-leading logarithms (NLL), etc.

The threshold resummation formalism has been applied by now to many processes including heavy quark hadroproduction,[4] jet production,[5] and electroweak processes.[6] The numerical results invariably show that the soft corrections are a good approximation of the full NLO result, that higher-order corrections are sizable, and that the scale dependence is decreased dramatically when these corrections are included.

2. Threshold resummation

A unified formula for the resummed cross section for arbitrary processes is

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E^{f_i}(N_i)\right] \exp\left[\sum_i 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\, \gamma_{i/i}\left(\alpha_s(\mu)\right)\right]$$
$$\times \exp\left[\sum_j E'^{f_j}(N_j)\right] \exp\left[2\, d_{\alpha_s} \int_{\mu_R}^{\sqrt{s}} \frac{d\mu}{\mu}\, \beta\left(\alpha_s(\mu)\right)\right] \mathrm{Tr}\left\{H^{f_i f_j}\left(\alpha_s(\mu_R)\right)\right.$$
$$\left.\times \exp\left[\int_{\sqrt{s}}^{\frac{\sqrt{s}}{\tilde{N}_j}} \frac{d\mu}{\mu}\, \Gamma_S^{\dagger\, f_i f_j}(\mu)\right] \tilde{S}^{f_i f_j}\left(\frac{\sqrt{s}}{\tilde{N}_j}\right) \exp\left[\int_{\sqrt{s}}^{\frac{\sqrt{s}}{\tilde{N}_j}} \frac{d\mu}{\mu}\, \Gamma_S^{f_i f_j}(\mu)\right]\right\}. \quad (1)$$

The sum over i (j) is over incoming (outgoing) partons. The exponents E^{f_i} and E'^{f_j} resum collinear contributions from the incoming and outgoing partons in the hard scattering and are given explicitly in Ref. [3]. $H^{f_i f_j}$ are hard scattering matrices in color space while $S^{f_i f_j}$ are soft matrices that desribe noncollinear soft-gluon emission and whose evolution is given by the soft anomalous dimension matrices $\Gamma_S^{f_i f_j}$.[1]

3. NNNLO master formulas

Expanding the resummed cross section through NNNLO and inverting back to momentum space, we derive master formulas[3] for the soft-gluon corrections for arbitrary processes.

The master formula for the NLO corrections is

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu_R^2)}{\pi} \{c_3\, D_1(s_4) + c_2\, D_0(s_4) + c_1\, \delta(s_4)\} + \frac{\alpha_s^{d_{\alpha_s}+1}(\mu_R^2)}{\pi} [A^c\, D_0(s_4) + T_1^c\, \delta(s_4)] \tag{2}$$

where σ^B is the Born term, $c_3 = \sum_i 2\, C_i - \sum_j C_j$, with $C_q = C_F$ for quarks and $C_g = C_A$ for gluons, and c_2 is defined by $c_2 = c_2^\mu + T_2$, with $c_2^\mu = -\sum_i C_i \ln(\mu_F^2/M^2)$ denoting the terms involving logarithms of the scale, and $T_2 = -\sum_i \left[C_i + 2\, C_i \ln\left(\frac{-t_i}{M^2}\right) + C_i \ln\left(\frac{M^2}{s}\right)\right] - \sum_j \left[B_j^{(1)} + C_j + C_j \ln\left(\frac{M^2}{s}\right)\right]$ denoting the scale-independent terms. Here μ_F (μ_R) is the factorization (renormalization) scale, M is a hard scale relevant to the process under study, and $B_j^{(1)}$ equals $3C_F/4$ for quarks and $\beta_0/4$ for gluons. The function A^c is process-dependent and depends on the color structure of the hard-scattering. It is defined by $A^c = \mathrm{tr}\left(H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma_S^{(1)}\right)$. With regard to the $\delta(s_4)$ terms, we split them into a term c_1, that is proportional to the Born cross section, and a term T_1^c that is not.

The master formula for the NNLO corrections is

$$\begin{aligned}\hat{\sigma}^{(2)} &= \sigma^B \frac{\alpha_s^2}{\pi^2} \frac{1}{2} c_3^2 D_3(s_4) \\ &+ \sigma^B \frac{\alpha_s^2}{\pi^2} \left\{\frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + \sum_j C_j \frac{\beta_0}{8}\right\} D_2(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}}{\pi^2} \frac{3}{2} c_3 A^c D_2(s_4) \\ &+ \sigma^B \frac{\alpha_s^2}{\pi^2} C_{D_1}^{(2)} D_1(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}}{\pi^2} \left\{\left(2c_2 - \frac{\beta_0}{2}\right) A^c + c_3 T_1^c + F^c\right\} D_1(s_4) \\ &+ \cdots \end{aligned} \tag{3}$$

where

$$C_{D_1}^{(2)} = c_3\, c_1 + c_2^2 - \zeta_2\, c_3^2 - \frac{\beta_0}{2}\, T_2 + \frac{\beta_0}{4}\, c_3 \ln\left(\frac{\mu_R^2}{M^2}\right) + c_3\, \frac{K}{2} - \sum_j \frac{\beta_0}{4}\, B_j^{(1)}, \tag{4}$$

with $K = C_A(67/18 - \pi^2/6) - 5n_f/9$, and

$$F^c = \mathrm{tr}\left[H^{(0)} \left(\Gamma_S^{(1)\,\dagger}\right)^2 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma_S^{(1)}\right)^2 + 2 H^{(0)} \Gamma_S^{(1)\,\dagger} S^{(0)} \Gamma_S^{(1)}\right]. \tag{5}$$

The master formula for the NNNLO corrections is

$$\hat{\sigma}^{(3)} = \sigma^B \frac{\alpha_s^3}{\pi^3} \frac{1}{8} c_3^3 D_5(s_4)$$
$$+ \sigma^B \frac{\alpha_s^3}{\pi^3} \left\{ \frac{5}{8} c_3^2 c_2 - \frac{5}{2} c_3 X_3 \right\} D_4(s_4) + \frac{\alpha_s^{d_{\alpha_s}+3}}{\pi^3} \frac{5}{8} c_3^2 A^c D_4(s_4)$$
$$+ \sigma^B \frac{\alpha_s^3}{\pi^3} \left\{ c_3 c_2^2 + \frac{c_3^2}{2} c_1 - \zeta_2 c_3^3 + (\beta_0 - 4c_2) X_3 + 2 c_3 X_2 - \sum_j C_j \frac{\beta_0^2}{48} \right\} D_3(s_4)$$
$$+ \frac{\alpha_s^{d_{\alpha_s}+3}}{\pi^3} \left\{ \frac{1}{2} c_3^2 T_1^c + \left[2 c_3 c_2 - \frac{\beta_0}{2} c_3 - 4 X_3 \right] A^c + c_3 F^c \right\} D_3(s_4) + \cdots \qquad (6)$$

where $X_3 = \beta_0 c_3/12 - \sum_j C_j \beta_0/24$ and $X_2 = -(\beta_0/4) T_2 + (\beta_0/8) c_3 \ln(\mu_R^2/M^2) + c_3 K/4 - \sum_j \beta_0 B_j^{(1)}/8$.

The formalism has been applied recently to top quark production at the Tevatron.[3,4] The corrections are non-negligible and serve to substantially reduce the scale dependence of the cross section. The theoretical results are in excellent agreement with data from the CDF[7] and D0[8] experiments.

References

1. N. Kidonakis and G. Sterman, *Phys. Lett.* **B387**, 867 (1996); *Nucl. Phys.* **B505**, 321 (1997); N. Kidonakis, *Int. J. Mod. Phys.* **A15**, 1245 (2000); in *DIS 2003*, hep-ph/0307145.
2. E. Laenen, G. Oderda, and G. Sterman, *Phys. Lett.* **B438**, 173 (1998).
3. N. Kidonakis, *Int. J. Mod. Phys.* **A19**, 1793 (2004); *Mod. Phys. Lett.* **A19**, 405 (2004); *Phys. Rev.* **D73**, 034001 (2006); in *DIS 2005*, hep-ph/0506299; *PoS (HEP 2005)* **055**, hep-ph/0512017.
4. N. Kidonakis, *Phys. Rev.* **D64**, 014009 (2001); N. Kidonakis and R. Vogt, *Phys. Rev.* **D68**, 114014 (2003); *Eur. Phys. J.* **C36**, 201 (2004).
5. N. Kidonakis and J. F. Owens, *Phys. Rev.* **D63**, 054019 (2001).
6. *Les Houches 2003*, hep-ph/0406152; N. Kidonakis and A. Belyaev, *JHEP* **12**, 004 (2003); N. Kidonakis and A. Sabio Vera, *JHEP* **02**, 027 (2004); R.J. Gonsalves, N. Kidonakis, and A. Sabio Vera, *Phys. Rev. Lett.* **95**, 222001 (2005); N. Kidonakis, R.J. Gonsalves, and A. Sabio Vera, in these proceedings, hep-ph/0606145; N. Kidonakis, *JHEP* **05**, 011 (2005).
7. CDF Coll., *Phys. Rev.* **D72**, 032002 (2005); *Phys. Rev. Lett.* **96**, 202002 (2006).
8. D0 Coll., *Phys. Lett.* **B626**, 55 (2005); hep-ex/0604020.