FAST pQCD CALCULATIONS FOR PDF FITS

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We present a method for very fast repeated computations of higher-order cross sections in hadron-induced processes for arbitrary parton density functions. A full implementation of the method for computations of jet cross sections in Deep-Inelastic Scattering and in Hadron-Hadron Collisions is offered by the "fastNLO" project at `http://hepforge.cedar.ac.uk/fastnlo/`.

The aim of the "fastNLO" project is to include jet data that have either been neglected or included using inadequate approximations in global PDF fits. These data have been excluded because the computing time for jet cross sections is prohibitive using standard calculation techniques. The fastNLO project implements a method that offers exact and very fast pQCD calculations for a large number of jet data sets allowing to take full advantage of their direct sensitivity to the gluon density in the proton in future PDF fits. This includes Tevatron jet data beyond the inclusive jet cross section and also HERA jet data which have been used to determine the proton's gluon density,[1] but which are ignored in current PDF fits.[2,3,4]

Perturbative QCD predictions for observables in hadron-induced processes depend on the strong coupling constant α_s and on the PDFs of the hadron(s). Any cross section in hadron-hadron collisions can be written as the convolution of the strong coupling constant α_s in order n, the perturbative coefficient $c_{n,i}$ for the partonic subprocess i, and the corresponding linear combination of PDFs from the two hadrons F_i which is a function of the momentum fractions $x_{a,b}$ of the two hadrons, carried by the partons

$$\sigma(\mu_r, \mu_f) = \sum_{n,i} c_{n,i}(x_a, x_b, \mu_r, \mu_f) \otimes [\alpha_s^n(\mu_r) \cdot F_i(x_a, x_b, \mu_f)] \ . \qquad (1)$$

The PDFs and α_s also depend on the factorization and the renormalization scales $\mu_{f,r}$, respectively, as does the perturbative prediction for the cross

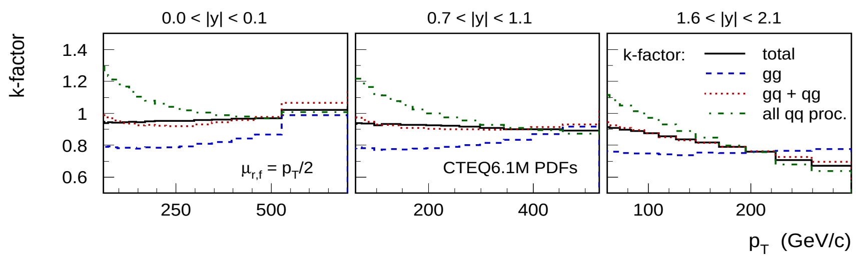

Figure 1. The k-factor for the inclusive $p\bar{p}$ jet cross section at $\sqrt{s} = 1.96\,\text{TeV}$ as a function of p_T at different rapidities y for the total cross section (solid line) and for different partonic subprocesses: gluon-gluon (dashed), gluon-quark (dotted) and the sum of all quark and/or anti-quark induced subprocesses (dashed-dotted).

section in finite order n. An iterative PDF fitting procedure using exact NLO calculations for jet data, based on Monte-Carlo integrations of (1), is too time-consuming. Only an approximation of (1) is, therefore, currently being used in global PDF fits.

The "*k*-factor approximation" as used in [3,4] parameterizes higher-order corrections for each observable bin by a factor $k = \frac{\sigma_{\text{LO}}+\sigma_{\text{NLO}}}{\sigma_{\text{LO}}}$ computed from the contributions with $n = 2$ (σ_{LO}) and $n = 3$ (σ_{NLO}) for a fixed PDF, averaged over all subprocesses i. In the iterative fitting procedure only the LO cross section is computed and multiplied with k to obtain an estimate of the NLO cross section. This procedure does not take into account that different partonic subprocesses can have largely different higher-order corrections. Fig. 1 shows that the k-factors for quark-only and gluon-only induced subprocesses can differ by more than $\pm 20\%$ from the average. The χ^2 is therefore minimized under a wrong assumption of the true PDF dependence of the cross section. Further limitations of this approach are:

- Even the LO Monte-Carlo integration of (1) is a trade-off between speed and precision. Finite statistical errors distort the χ^2 contour during the error analysis, especially for parameters with small errors.
- The procedure can only be used for observables for which LO calculations are fast. Currently, this prevents the global PDF analyses from using Tevatron dijet data and DIS jet data.

In a time when phenomenology is aiming towards NNLO precision,[2,3] the k-factor approximation is clearly not satisfying concerning both its limitation in precision and its restrictions concerning data sets.

A better solution is implemented in the fastNLO project. The basic idea is to transform the convolution in (1) into the factorized expression (3).

Many proposals for this have been made in the past, originally related to solving the DGLAP parton evolution equations [5] and later to computing jet cross sections.[6,7,8,9,10] The fastNLO method is an extension of the concepts developed for DIS jet production [6,9] which have been applied at HERA to determine the gluon density in the proton from DIS jet data.[1] Starting from (1) for the following discussion the renormalization scale is set equal to the factorization scale ($\mu_{r,f} = \mu$). The extension to $\mu_r \neq \mu_f$ is, however, trivial. The x dependence of the PDFs and the scale dependence of α_s^n and the PDFs can be approximated using interpolation between sets of fixed values $x^{(k)}$ and $\mu^{(m)}$ $(k, m = 1, 2, 3, \cdots, k_{\max}, m_{\max})$

$$\alpha_s^n(\mu) \cdot F_i(x_a, x_b, \mu) \;\simeq \qquad \left[\text{“=” is true for } k_{\max}, l_{\max}, m_{\max} \to \infty\right]$$
$$\sum_{k,l,m} \alpha_s^n(\mu^{(m)}) \cdot F_i(x_a^{(k)}, x_b^{(l)}, \mu^{(m)}) \,\cdot\, e^{(k)}(x_a) \cdot e^{(l)}(x_b) \cdot b^{(m)}(\mu)$$

where $e^{(k,l)}(x)$ and $b^{(m)}(\mu)$ are interpolation functions for the x and the μ dependence, respectively. All information of the perturbatively calculable piece (including phase space restrictions, jet algorithm, etc. but excluding α_s and the PDFs) is fully contained in the quantity

$$\tilde{\sigma}_{n,i,k,l,m}(\mu) = c_{n,i}(x_a, x_b, \mu) \otimes \left[e^{(k)}(x_a) \cdot e^{(l)}(x_b) \cdot b^{(m)}(\mu)\right] . \qquad (2)$$

The final prediction for the cross section is then given by the simple product

$$\sigma(\mu) \;\simeq \sum_{n,i,k,l,m} \tilde{\sigma}_{n,i,k,l,m}(\mu) \,\cdot\, \alpha_s^n(\mu^{(m)}) \cdot F_i(x_a^{(k)}, x_b^{(l)}, \mu^{(m)}) \,. \qquad (3)$$

The time-consuming step involving the calculation of the universal (PDF and α_s independent) $\tilde{\sigma}$ is therefore factorized and needs to be done only once. Any further calculation of the pQCD prediction for arbitrary PDFs and α_s values can later be done very fast by computing the simple product in (3). While the extension of the method from one initial-state hadron [9] to two hadrons was conceptually trivial, the case of two hadrons requires additional efforts to improve the efficiency and precision of the interpolation. Both are directly related to the choices of the points $x^{(k)}$, $\mu^{(m)}$ and the interpolation functions $e(x)$, $b(\mu)$. fastNLO achieves a precision of $< 0.1\%$ for $k_{\max}, l_{\max} = 10$ and $m_{\max} \leq 4$. Further details are given in Ref. [11].

The $\tilde{\sigma}$ in (2) are computed using NLOJET++.[12,13] A unique feature in fastNLO is the inclusion of the $O(\alpha_s^4)$ threshold correction terms to the inclusive jet cross section,[14] a first step towards a full NNLO calculation.

fastNLO calculations are available at [15] for a large set of (published and planned) jet cross section measurements at HERA, RHIC, the Tevatron, and the LHC (either online or as computer code for inclusion in PDF fits).

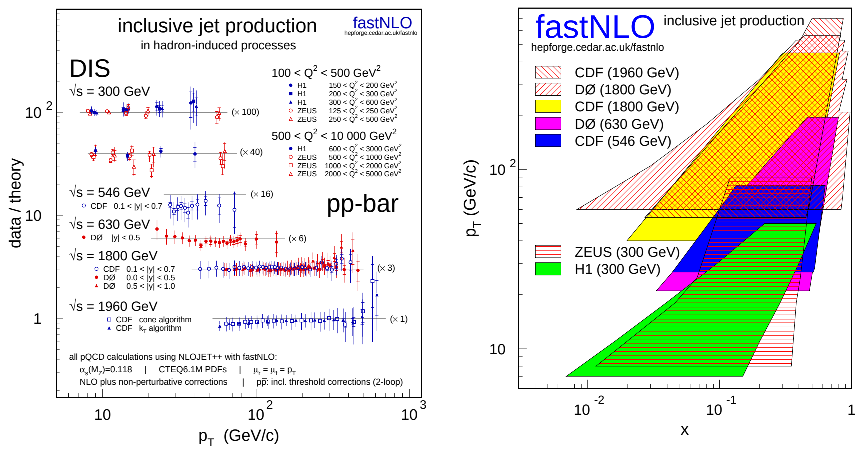

Figure 2. An overview of data over theory ratios for inclusive jet cross sections, measured in different processes at different center-of-mass energies. The data are compared to calculations obtained by fastNLO in NLO precision (for DIS data) and including $O(\alpha_s^4)$ threshold corrections (for $p\bar{p}$ data). In all cases the perturbative predictions have been corrected for non-perturbative effects. The figure on the right shows the$(x,\ p_T)$ phase space accessible by these data sets.

Some results for inclusive jet cross section measurements are shown in Fig. 2 (left) as ratios of data over theory. The phase space in x and p_T covered by these measurements is shown in Fig. 2 (right), demonstrating what can be gained by using fastNLO to include these data sets in future PDF fits.

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