MULTI-JET PROCESSES IN THE HIGH ENERGY LIMIT OF QCD

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We discuss how the multi-Regge factorisation of QCD amplitudes can be used in the study of multi-jet processes at colliders. We describe how the next-to-leading logarithmic (NLL) BFKL evolution can be combined with energy and momentum conservation. By recalculating the quark contribution to the next-to-leading logarithmic corrections to the BFKL kernel we can study several properties of the NLL corrections. We demonstrate that in the standard analysis, the NLL corrections to a single gluon emission includes contributions from significantly more energetic quark–anti-quark configurations, something that could contribute to the sizable NLL corrections in the standard BFKL analysis.

1. Introduction

One of the many immediate challenges for QCD is to provide a reliable description of the multiple hard jet environment which is to be expected at the LHC. Besides posing a very interesting problem in itself, the QCD dynamics will provide signals similar to that of many sources of physics beyond the standard model, and so is very important to understand in detail. An intriguing alternative to the standard approach of calculating the production rate of a few hard partons by fixed order perturbation theory is to use the framework arising from the multi-Regge form of QCD amplitudes (recently proved at next-to-leading logarithmic accuracy[1]) to calculate the emission of gluons (and quarks at next-to-leading logarithmic accuracy) from the evolution of an effective, Reggeized gluon (Reggeon) propagator. The starting point here is the observation that for e.g. $2\to2, 2\to3, \ldots$ gluon scattering, Feynman diagrams with a t-channel gluon exchange dominate the partonic cross section, in the limit where the rapidity span of the two leading gluons is large. This t-channel gluon is then evolved according to the BFKL equation, and will emit partons accordingly. Starting from the $2\to2$–gluon exchange, the $2\to2+n$ gluon scattering process can be calculated in the limit of large rapidity spans Δy, thanks to the Regge factorisation of the colour octet exchange. Obviously, this means that the formalism is relevant only if there is sufficient energy at colliders to have multiple emissions spanning large ($\geq$ 2) rapidity intervals. In this high energy limit, the partonic cross section for $2 \to 2+n$ gluon scattering $(p_{a'}, p_{b'} \to p_a, \{p_i\}, p_b)$ factorises as

$$\mathrm{d}\hat{\sigma}(p_a, \{p_i\}, p_b) = \Gamma_{a'a} \left(\prod_{i=1}^{n} \frac{e^{\omega(\mathbf{q}_i)(y_{i-1}-y_i)}}{\mathbf{q}_i^2} V^{J_i}(\mathbf{q}_i, \mathbf{q}_{i+1}) \right) \frac{e^{\omega(\mathbf{q}_{n+1})(y_n - y_{n+1})}}{\mathbf{q}_{n+1}^2} \Gamma_{b'b} \tag{1}$$

where $\mathbf{q}_i = -\left(\mathbf{p}_a + \sum_{l=1}^{i-1} \mathbf{p}_l\right)$, p_a, p_b is the momentum of the partons furthest apart in rapidity, and $\Gamma_{a'a}, \Gamma_{b'b}$ are the process dependent impact factors (the momentum dependence has been suppressed in Eq. (1)). $V^{J_i}(\mathbf{q}_i, \mathbf{q}_{i+1})$ denote the effective Lipatov vertices at LL or NLL. It is of course possible to study other processes such as $W + n$ jets, $n \geq 2$ (see Ref. 2) within this framework by substituting the relevant impact factors in Eq. (1). The sum over any number of gluon emissions, with their phase space integrated to infinity, can be found by substituting for all but the impact factors in Eq. (1) the solution $f(\mathbf{k}_a, \mathbf{k}_b, \Delta y)$, $\Delta y = y_0 - y_{n+1}$, to the BFKL equation. This is what traditionally is done in BFKL phenomenology, since it allows for analytic results to be readily obtained. The huge rise in cross sections driven by the leading logarithmic evolution is due in parts to these unconstrained phase space integrations, and it is clear there can be large corrections if the phase space integrals are constrained to the physical phase space.

2. Combining BFKL Evolution with Energy and Momentum Conservation

At leading logarithmic accuracy, the task of combining energy and momentum conservation with BFKL evolution thus becomes a question of integrating Eq. (1) over only the available phase space for a given process. This is equivalent to performing a leading logarithmic approximation to the $2 \to 2 + n$ matrix element, without the further phase space approximation inherent when using the standard solution to the BFKL equation. Technically, this is most conveniently performed by the direct solution to the BFKL evolution[3] — the framework of the BFKL equation provides a convenient prescription for regularising the singularities in Eq. (1), while the direct solution is a numerically efficient and physically intuitive approach to performing the sum over any number of emissions and their phase space integral. Please refer to Ref. 3 for further details. The processes for pure multi-jets, and forward $W + (2 + n)$-jets have been implemented according to this formalism, and the computer code is available at `http://www.hep.phy.cam.ac.uk/~andersen/BFKL`.

2.1. *Next-to-Leading Logarithmic Evolution*

The first step towards combining the BFKL evolution with energy and momentum conservation was taken when the BFKL equation was solved to next-to-leading logarithmic accuracy in an iterative framework[4,5]. However, at next-to-leading logarithmic accuracy it is no longer sufficient to use the regularised versions of the effective vertex and trajectory arising in the iterative approach, as derived from the BFKL kernel[6,7] (which is the case at leading logarithmic accuracy[8,9]). This is because the contributions to the NLL BFKL kernel already includes unconstrained phase space integrals over two-particle states. In order to combine the evolution at next-to-leading logarithmic accuracy with energy and momentum conservation, it is therefore necessary to re-calculate the next-to-leading logarithmic contribution to the BFKL kernel, but leave the phase space integrals within each Lipatov vertex undone, and furthermore perform the regularisation of the amplitudes using phase space slicing.

The contribution to the NLL vertex from quark–anti-quark production is given by

$$\begin{aligned} K_r^{(2),q\bar{q}}(\mathbf{q}_1,\mathbf{q}_2) =& \frac{1}{2\mathbf{q}_1^2\mathbf{q}_2^2}\frac{1}{N_c^2-1}\int \mathrm{d}\kappa\ \mathrm{d}\rho_f\ \delta^{(D)}(q_1-q_2-k_1-k_2) \\ & \sum_{i_1,i_2,f}\left|\gamma_{i_1 i_2}^{q\bar{q}}(q_1,q_2,k_1,k_2)\right|^2 \end{aligned} \tag{2}$$

where the sum is over spin, colour and flavour states of the produced quark–anti-quark pair, $\kappa = (q_1-q_2)^2 = (k_1+k_2)^2$ is the invariant mass, and

$$\mathrm{d}\rho_f = \prod_{n=1,2}\frac{\mathrm{d}^{D-1}k_n}{(2\pi)^{D-1}2E_n}. \tag{3}$$

q_1, q_2 is the momentum of the Reggeons, while k_1, k_2 is the momentum of the produced quark and anti-quark, and the form of the amplitude $\gamma_{i_1 i_2}^{q\bar{q}}(q_1,q_2,k_1,k_2)$ can be obtained either from the effective Feynman rules for the Regge limit of QCD[10] or by considering the high energy limit of the tree level $gg \to ggq\bar{q}$ matrix element[11].

The $1/N_c^2$–suppressed contribution to the square of the amplitude is IR-finite, and so the results of a numerical integration can be directly compared to the results in Ref.12. We find complete agreement[a]. Using the phase space slice regulated integral of Eq. (2) combined with the quark-contribution to the NLL corrections to the one-gluon production vertex, it

[a]The agreement is complete, once a misprint in Eq. (23) of Ref.12 is corrected

then becomes possible to study the final state configuration of the quark–anti-quark contribution to the NLL vertex. For a given $\mathbf{q}_1, \mathbf{q}_2$, the leading logarithmic contribution to the Lipatov vertex comes from the emission of a single gluon of energy $|\mathbf{k}_i|^2 = |\mathbf{q}_1 - \mathbf{q}_2|^2$. However, at NLL there will be a spread in the energy of the quark–anti-quark pair. For $\mathbf{q}_1 = (20, 0)$ GeV and $\mathbf{q}_2 = (0, 20)$ GeV we find that the average value of the energy of the $q\bar{q}$-pair is 40 GeV(see Fig. 1) , and the average rapidity separation between the quark and anti-quark is .56 units of rapidity. The standard calculation of the NLL corrections to the kernel for the emission of a $20\sqrt{2}$ GeV gluon therefore includes corrections from significantly larger energies, and configurations which would usually be described as two separate jets.

This is clearly uncomfortable, and could be contributing to the sizable NLL corrections found in the standard analysis. However, the approach outlined here will allow for such effects to be properly taken into account, by combing energy and momentum conservation with the NLL BFKL evolution of the t–channel gluon. Furthermore, proper jet-definitions can be applied to the study of the multiple hard jets of the processes.

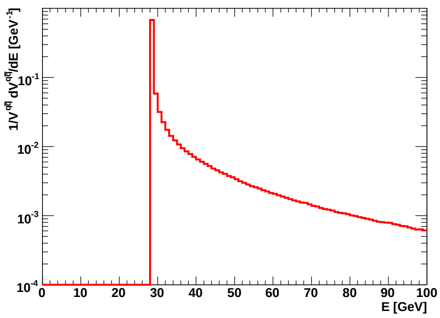

Figure 1. The quark–anti-quark contribution to the NLL vertex as a function of the energy of the $q\bar{q}$–pair for $\mathbf{q}_1 = (20, 0)$ GeV, $\mathbf{q}_2 = (0, 20)$ GeV, which at LL would be ascribed to the emission of a single gluon of $\sqrt{2} \cdot 20 \approx 28$ GeV.

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