COLOR-EVAPORATION MODEL VS. NRQCD IN CHARMONIUM PRODUCTION*

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We deduce relationships that are implied by the color-evaporation model (CEM) between the nonperturbative NRQCD matrix elements that appear in the factorization formula for quarkonium production. These relationships are at odds with the phenomenological values of the matrix elements that have been extracted from the Tevatron data for charmonium production at large transverse momentum. A direct comparison of the CEM and NRQCD factorization predictions with the CDF charmonium production data is discussed.

We derive relationships between the nonrelativistic quantum chromodynamics (NRQCD) nonperturbative factors that follow from the model assumptions of the color-evaporation model (CEM). We find that these relationships are often poorly satisfied by phenomenological values of the NRQCD matrix elements. Furthermore, the relationships sometimes violate the velocity-scaling rules of NRQCD. We conclude that the CEM and NRQCD provide very different pictures of the evolution of a heavy quark-antiquark pair into a quarkonium. This expectation is borne out by direct

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comparison of CEM and NRQCD predictions with the CDF data for for J/ψ, $\psi(2S)$, and χ_c production at order α_s^3.

The NRQCD factorization formula for the inclusive cross section for production of a specific heavy-quarkonium state H is

$$\sigma[AB \to H + X] = \sum_n c_n^{AB} \langle \mathcal{O}_n^H \rangle. \tag{1}$$

Here, A and B are light hadrons, photons, or leptons. The c_n^{AB} are short-distance coefficients that can be calculated in perturbation theory. The matrix elements $\langle \mathcal{O}_n^H \rangle$ are vacuum-expectation values of four-fermion operators in NRQCD [1]. The subscript n represents the angular-momentum quantum numbers (s, l, and j) and the color state (singlet or octet). The matrix elements in Eq. (1) fall into a hierarchy according to their scaling with the velocity v of the heavy quark in the quarkonium rest frame.

The CEM [2,3,4,5] version of the cross section is

$$\sigma_{\text{CEM}}[AB \to H + X] = F_H \int_{4m^2}^{4m_M^2} dm_{Q\bar{Q}}^2 \, \frac{d\sigma}{dm_{Q\bar{Q}}^2}[AB \to Q\bar{Q} + X], \tag{2}$$

where $m_{Q\bar{Q}}$ is the invariant mass of the $Q\bar{Q}$ pair, m is the heavy-quark mass, M is the lowest-mass meson containing Q, $d\sigma/dm_{Q\bar{Q}}^2$ is the inclusive differential cross section for a $Q\bar{Q}$ pair, and the colors and spins of the final-state $Q\bar{Q}$ pair are summed. This is where the central model assumptions of color evaporation and spin randomization manifest themselves.

Under this assumption, the CEM predicts that S-wave and P-wave NRQCD matrix elements are related by [6]

$$\langle \mathcal{O}_n^H \rangle = \frac{3(2j+1)}{(2l+1)(2l+3)} C_n k_{\max}^{2l} \langle \mathcal{O}_1^H({}^1S_0) \rangle, \tag{3}$$

where $C_n = 1$ or $4/3$ if $\mathcal{O}_n^H$ is a color-singlet or color-octet operator, respectively. In general, the matrix elements in Eq. (3) do not respect the velocity-scaling rules of NRQCD. Therefore, the CEM and NRQCD provide very different pictures.

In the production of S-wave charmonium at the Tevatron with transverse momentum $p_T > 5$ GeV, it is known phenomenologically that the most important NRQCD matrix elements for $H = J/\psi$ or $\psi(2S)$ are the color-octet matrix element $\langle \mathcal{O}_8^H({}^3S_1) \rangle$ and a specific linear combination of color-octet matrix elements $M_r^H = (r/m^2)\langle \mathcal{O}_8^H({}^3P_0) \rangle + \langle \mathcal{O}_8^H({}^1S_0) \rangle$, where $r \approx 3$. Let us examine the ratio of these matrix elements

$$R^H = \frac{M_r^H}{\langle \mathcal{O}_8^H({}^3S_1) \rangle}, \tag{4}$$

where H stands for J/ψ or $\psi(2S)$. The relation (3) yields the CEM ratio

$$R^H_{\rm CEM} = \frac{M_r^H}{\langle \mathcal{O}_8^H(^3S_1)\rangle} = \frac{r}{15}\,\frac{k^2_{\rm max}}{m^2} + \frac{1}{3}. \tag{5}$$

The velocity-scaling rules of NRQCD predict that the ratio in Eq. (4) scales as v^0. Since $k_{\rm max}$ scales as mv, the second term in the CEM ratio in Eq. (5) satisfies this scaling relation, but the first term does not.

Let us turn to the case of production of the P-wave charmonium states χ_{cj} $(j = 0, 1, 2)$ at the Tevatron at $p_T > 5$ GeV. It is known phenomenologically that the most important NRQCD matrix elements are the color-singlet matrix elements $\langle \mathcal{O}_1^{\chi_{cj}}(^3P_j)\rangle$ and the color-octet matrix elements $\langle \mathcal{O}_8^{\chi_{cj}}(^3S_1)\rangle$. The matrix elements can be simplified by making use of the heavy-quark spin-symmetry relations $\langle \mathcal{O}_{1,8}^{\chi_{cj}}(^3P_j)\rangle = (2j+1)\langle \mathcal{O}_{1,8}^{\chi_{c0}}(^3P_0)\rangle$, which hold up to corrections of order v^2. Let us define a ratio

$$R^{\chi_c} = \frac{\langle \mathcal{O}_8^{\chi_{c0}}(^3S_1)\rangle}{\langle \mathcal{O}_1^{\chi_{c0}}(^3P_0)\rangle/m^2}. \tag{6}$$

The relation (3) yields the CEM prediction $R^{\chi_c}_{\rm CEM} = 15 C_F m^2/k^2_{\rm max}$. The velocity-scaling rules of NRQCD predict that the ratio R^{χ_c} in Eq. (6) scales as v^0. In contrast, we see that the CEM prediction scales as v^{-2}.

A comparison of the CEM ratios with the phenomenological ratios that have been extracted from the CDF data indicates that the CEM predicts a ratio $M_r^H/\langle \mathcal{O}_8^H(^3S_1)\rangle$ that is too small in J/ψ and $\psi(2S)$ production and a ratio $\langle \mathcal{O}_8^{\chi_{Q0}}(^3S_1)\rangle/\langle \mathcal{O}_1^{\chi_{Q0}}(^3P_0)\rangle$ that is too large in χ_c production. Both of these predictions of the CEM would be expected to lead to cross sections that have too positive a slope, as a function of p_T, relative to the data.

This expectation is borne out by comparisons of the CEM with the CDF data for J/ψ, $\psi(2S)$, and χ_c production [6]. The CEM predictions are from a calculation by Vogt [7] that makes use of the order-α_s^3 cross section for production of a $Q\bar{Q}$ pair [8]. The NRQCD predictions were generated from modified versions of computer codes created by Maltoni, Mangano, and Petrelli [9]. The codes compute the order-α_s^3 quarkonium production cross sections [10] and the standard DGLAP evolution of the fragmentation contribution to the evolution of a $Q\bar{Q}$ pair in a $^3S_1$ color-octet state into a quarkonium. This fragmentation contribution is the dominant contribution at large p_T. Details of these calculations are given in Refs. [6,11]. According to analyses given in Ref. [6], the CEM predictions do not yield satisfactory fits to the J/ψ, $\psi(2S)$, or χ_c data. The NRQCD factorization predictions yield satisfactory fits to the J/ψ and $\psi(2S)$ data, but not to the χ_c data.

k_T smearing provides a phenomenological model for the effects of multiple gluon emission from the initial-state partons in a hard collision. Its effects are to smooth singularities at $p_T = 0$ in fixed-order calculations, to increase the predicted cross section at moderately low p_T (away from the singular region), and to increase the predicted cross section by a smaller amount at high p_T. Hence, the inclusion of k_T smearing would be expected to improve the fits of the CEM predictions to the charmonium data, which it does. Even with k_T smearing, the CEM predictions show substantial disagreement with the data for J/ψ and χ_c production, but agree with the $\psi(2S)$ data, which have larger error bars. The smeared NRQCD factorization predictions are in good agreement with the data in the J/ψ and $\psi(2S)$ cases and in reasonably good agreement in the χ_c case.

In the case of χ_c production, the NRQCD factorization fits are constrained by the relationship of $\langle \mathcal{O}_1^{\chi_{c0}}(^3P_0) \rangle$ to the corresponding decay matrix element. Thus, there is less freedom in that case to tune the matrix elements to obtain a good fit to the data than in the cases of J/ψ and $\psi(2S)$ production. Consequently, χ_c production may provide a more stringent test of NRQCD factorization. The disagreement of the unsmeared NRQCD factorization prediction and the reasonable agreement of the smeared NRQCD factorization prediction with the shape of the χ_c production data suggest that, if the NRQCD factorization picture is valid, then inclusion of the effects of multiple gluon emission is essential in obtaining the correct shape of the cross section.

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