HOW TO EXTRACT ΔG FROM MEASUREMENTS OF $A_{\rm LL}$?

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We discuss some of the theoretical issues critical to a future global analysis of spin-dependent parton densities, in particular, the question of how to make use of hadronic cross section data reliably.

Extracting information about parton densities from hadronic cross sections is an intricate problem, even more so if one is investigating the spin structure of nucleons. If applicable, the factorization theorem allows us to study the non-perturbative hadronic structure with the help of perturbatively calculable partonic scattering cross sections. To control and quantify uncertainties inherent to any theoretical calculation one has to go beyond the lowest order approximation of QCD. Most cross sections relevant to the RHIC spin program are available now at next-to-leading order (NLO) accuracy. However, the numerical expressions are often too time-consuming to be of use in a "global analysis" which usually requires thousands of evaluations of the cross section for any given data point to determine the set of parameters used to describe the parton distributions $\Delta f(x,\mu)$ at some initial momentum scale μ. Therefore the prospects of learning from data depend on our ability to efficiently evaluate, e.g., the cross section

$$d\Delta\sigma = \sum_{abc} \Delta f_a \otimes \Delta f_b \otimes d\Delta\hat{\sigma}_{ab\to cX} \otimes D_c^{\pi} \tag{1}$$

for $pp \to \pi X$ at NLO with $\otimes$ denoting a convolution.

It is tempting to use approximations for (1), e.g., to assume that NLO corrections drop out in experimentally relevant spin asymmetries $A_{\rm LL} \equiv d\Delta\sigma/d\sigma$ or that they are constant in the kinematical regime of interest. NLO calculations have revealed that such assumptions are off target for most processes in polarized hadron-hadron and lepton-hadron collisions,

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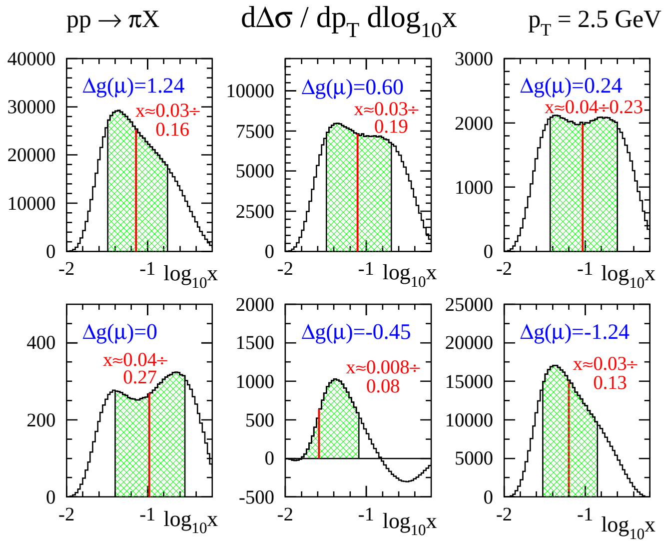

Figure 1. $d\Delta\sigma/dp_T d\log_{10}(x)$ at NLO for $p_T = 2.5\,\mathrm{GeV}$ and six different values for $\Delta g(\mu)$. The shaded areas denote the x-range dominantly contributing to $d\Delta\sigma$.

see, e.g., [1,2]. In general, without knowing the polarized parton densities, in particular the elusive Δg, it is virtually impossible to come up with a sensible estimate of the relevance of NLO corrections. The situation is very much different in the unpolarized case. Here, inclusive DIS data already constrain the quark *and* gluon distributions pretty well and hadronic cross sections are only required for fine-tuning. As a consequence, the theory answer for a certain cross section is changing in a very predictable way when going from the lowest order to the NLO approximation.

The complications in pinning down Δg from hadronic data are exemplified further by studying the range of momentum fractions x predominantly probed in a measurement of $pp \to \pi X$ at RHIC. Figure 1 shows the polarized cross section $d\Delta\sigma/dp_T d\log_{10}(x)$ at NLO for $p_T = 2.5\,\mathrm{GeV}$, $\sqrt{S} = 200\,\mathrm{GeV}$, and six different assumptions about the first moment of $\Delta g(\mu)$ at the input scale of the GRSV analysis[4]. The panel with $\Delta g(\mu) = 0.24$ refers to the "standard" set of GRSV[4].

Due to the convolutions in (1) the x-distributions spread out significantly. The corresponding unpolarized cross section $d\sigma$ peaks approximately at $x \simeq 2p_T/\sqrt{S} = 0.025$, similar to what happens for $d\Delta\sigma$ if the gluon polarization is large. For smaller, positive or negative Δg, the behavior of $d\Delta\sigma$ is, however, much more complicated than in the unpolarized

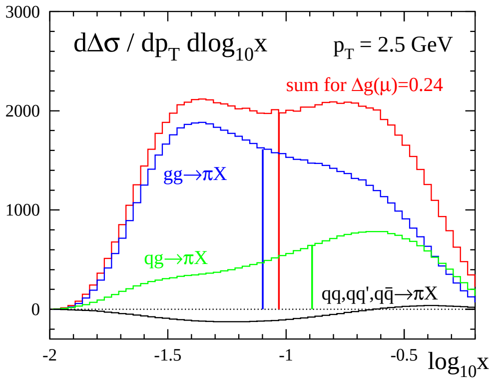

Figure 2. As in Fig. 1 for the panel based on the GRSV "standard" Δg but now including a breakdown into contributions from different partonic channels.

case. While gluon-gluon initiated subprocesses give a positive contribution to $d\Delta\sigma$, the sign of quark-gluon channels is correlated with the sign of Δg. Since both contributions peak at significantly different values of x, this can lead to a much spread out x-distribution as is illustrated in Fig. 2 in case of the GRSV "standard" gluon. Even a node in $d\Delta\sigma$ is possible, such that the notion of an "average x probed" becomes meaningless. This makes extractions of $\Delta g/g$ based on a Monte-Carlo "signal-to-background" separation in $A_{\rm LL}$ ("purities") problematic. Since the convolutions in (1) do not allow to factor out $\Delta g/g$ from $A_{\rm LL}$ one has to apply some mean-value theorem, i.e., one has to assume some average momentum fraction $\langle x\rangle$ in $A_{\rm LL}$ – a catch-22. We also wish to point out that extracting Δg without refitting the polarized (anti-)quark densities leads to misleading results.

Since these complications are most relevant for realistic, moderate gluon polarizations or if Δg itself develops a node, the goal must be to circumvent all approximations in analyzes of polarized parton densities. In [3] a technique was devised which accomplishes just this. The idea is to express the parton densities $\Delta f_{a,b}$ in (1) by their Mellin inverses. This allows to re-order all time-consuming integrations and store their results in large grids in complex Mellin moment space *prior* to the global analysis. A numerically fast, double inverse Mellin transformation along an appropriate contour links the moments of the Δf's (to be fitted) with these pre-calculated grids.

This technique has to stand the test with all processes relevant to the RHIC spin program: up to 100 evaluations of NLO cross sections per second are sufficient for global analyzes. Figure 3 compares NLO calculations[1,5] based on different constraints on the first moment of Δg with recent PHENIX

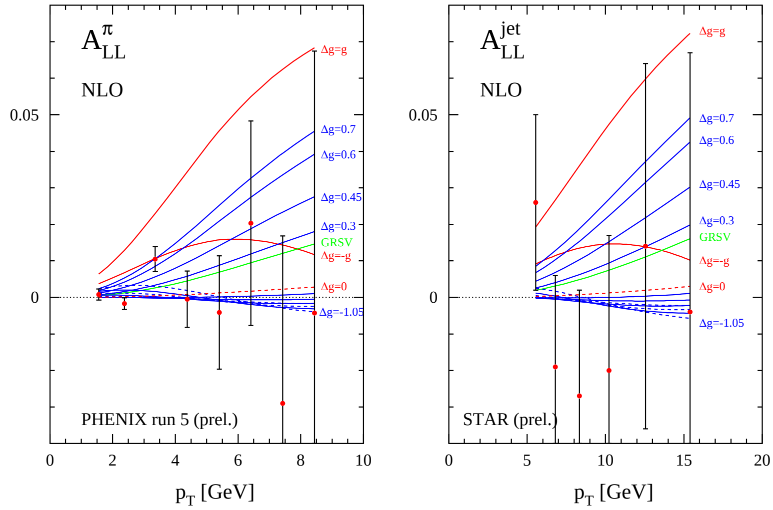

Figure 3. Recent PHENIX and STAR data on A_{LL} compared to NLO calculations based on different constraints on the first moment of Δg.

and STAR data[6] on A_{LL}. Δg's with a large and positive first moment are clearly disfavored. A combined global χ^2-analysis of RHIC data and information from DIS and semi-inclusive DIS based on the "Mellin technique" is currently under way[7]. Other approaches are being pursued in [8,9].

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