

PARTON DISTRIBUTIONS – DIS06.

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I discuss the current status of parton distributions. I outline the wide variety of different parton distributions available, and highlight which are either necessary or suitable for use at present.

There are a large number of different parton distributions. If we consider the different types of particle which are partons we start with the quark model valence partons $u_V(x, Q^2)$ and $d_V(x, Q^2)$. However, these carry only 50% of the proton's momentum: there are also $\bar{u}(x, Q^2)$ and $\bar{d}(x, Q^2)$, which are not the same¹; gluons $g(x, Q^2)$ which carry over 30% of the momentum; $s(x, Q^2)$ and $\bar{s}(x, Q^2)$, with the possibility² that $s(x, Q^2) \neq \bar{s}(x, Q^2)$; $c(x, Q^2)$ and $b(x, Q^2)$, which are perturbatively generated (there could be intrinsic contributions³ at large x); and at some level isospin violation, i.e. $u^p(x, Q^2) \neq d^n(x, Q^2)$, $u^p(x, Q^2) \neq d^n(x, Q^2)$ – automatically the case with QED corrections⁴, which also lead to a $\gamma(x, Q^2)$ distribution. Overall there are 6 – 16 different parton distributions, but some are very small and are often not needed. There is another way of counting, i.e. the different sets of parton distributions from different prescriptions: LO, NLO, or NNLO in α_S ; with resummation corrections or allowances for higher twist; using $\overline{\text{MS}}$, DIS, or potentially other factorization schemes; fixed-flavour (FFNS), zero-mass variable-flavour (ZM-VFNS) and general-mass variable-flavour number scheme GM-VFNS, and even different versions of the last. This freedom, and choices in data sets used, cuts applied, ways to treat errors, *etc.* lead to a staggering array – CTEQ4A1, CTEQ4A2⁵ ... CTEQ5HJ⁶, ... CTEQ6⁷ ..., MRST98⁸ ... MRST03c⁹, ... MRST04QED⁴, MRST04¹⁰ ..., Alekhin00¹¹, Alekhin03¹², GRV98¹³, Fermi02¹⁴ ..., ZEUS¹⁵, ZEUS-ZJ¹⁶, H1¹⁷, Botje¹⁸ and many more. Are all of these different sets really necessary? This is a complicated and controversial question.

One restriction is very easy to impose – many (still used) partons are simply out of date. Unless there is a very good reason, one should not use pre-2000 parton distributions. The available data have improved a great deal since then, particularly the HERA structure functions^{17,19} and Tevatron jets²⁰. Also, some older partons have minor bugs.

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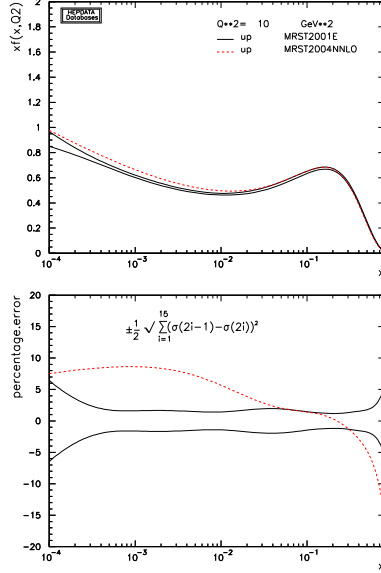


Figure 1. Comparison of the NLO up distribution with the NNLO up distribution.

Consider the order of the partons. LO requires $\alpha_S(M_Z^2) \sim 0.130$, and the χ^2 from a global fit is far inferior to that at NLO and NNLO²¹. The LO partons are qualitatively different from NLO and NNLO partons in some regions, in particular the gluon is much bigger at small x due to important corrections in the splitting functions. This can cause misleading conclusions on the evidence for saturation *etc.*. Such results derived from LO partons should be treated with care. The default has long been NLO, but the NNLO coefficient functions for structure functions have long been known²² and the splitting functions are now complete²³. These improve the quality of the fit slightly²¹ and reduce α_S . A big change in the partons can occur when going from NLO \rightarrow NNLO, as seen in Fig. 1.

To perform an absolutely correct NNLO fit we need both exact NNLO splitting functions and cross-sections. The NNLO Drell-Yan cross-sections have recently been calculated as a function of rapidity²⁴, leading to a decrease in the sea quarks. The one remaining gap is the NNLO corrections to jet production in $pp(\bar{p})$ collisions. However, the NLO corrections themselves are not large – at central rapidities they are $\leq 10\%$, similar to the size of the correlated errors. There are also some NNLO estimates, i.e. the leading threshold corrections, which are expected to be a significant component of the total²⁵ (there are issues concerning the application within a given jet definition). These give a flat, small 3 – 4% correction, which is consistent with what we already see at NLO and is much smaller than the systematic

errors on the data. Hence, it seems perverse to leave the jet data out of a NNLO fit due to the lack of the full NNLO hard cross-section.

For a full NNLO fit we also require a rigorous treatment of heavy quark thresholds, which is now available²⁶. Therefore, an essentially full NNLO determination of partons is possible. Of course, this is the best way to test our understanding of the partons in terms of QCD, but we only know a limited number of cross-sections at NNLO. Processes with two strongly interacting particles are largely completed – DIS coefficient functions, $pp(\bar{p}) \rightarrow \gamma^*, W, Z$ (including the rapidity distribution), H, A^0, WH, ZH . For many other final states the NNLO cross-sections are not known and NLO is still more appropriate. Moreover, resummations may be important even beyond NNLO in some regions, as may higher twist.

There is the issue of factorization schemes. In practice cross-sections are calculated in $\overline{\text{MS}}$ scheme, so we use $\overline{\text{MS}}$ parton distributions. However, DIS-scheme can be more useful for relating partons to real physical results, e.g. it is easier to fit the Tevatron jet data in the DIS scheme²⁷, or viewed differently the large high- x gluon required at NLO and NNLO in $\overline{\text{MS}}$ scheme can be determined from the scheme dependence¹⁰. Schemes other than $\overline{\text{MS}}$ are valuable in this type of context, but at present rarely used.

There are also partons corresponding to different prescriptions for heavy flavours. FFNS is intrinsically inferior to VFNS – it does not sum $\ln Q^2/m_H^2$ terms in the perturbative expansion and at high scales this can lead to inaccuracies. Moreover, it is often necessary to have heavy flavour partons due to the lack of mass effects in the known cross-sections. Nevertheless, FFNS partons are also sometimes needed because some hard cross-sections are only calculated in this scheme^{28,29,30}. However, in this case the treatment must be correct, and is often not so. The NLO ($O(\alpha_S^2)$) coefficient functions for heavy flavour in DIS are calculated³¹ in a renormalization scheme where the coupling α_S is fixed at 3 flavours. The partons have to be defined in the same way, otherwise there is double counting of $\alpha_S^2 \ln^2(Q^2/m_H^2)$, terms which can lead to large errors³². Also, there are no FFNS coefficient functions at NNLO. This absence is particularly important since NNLO FFNS contains terms of the form $\alpha_S^3 \ln^2(Q^2/m_H^2)$.

At the other extreme we have the ZM-VFNS. Here the terminology *scheme* is misleading. It usually means a different way of arranging the complete calculation. In this case there is an overall error of $O(m_H^2/Q^2)$. In my opinion ZM-VFNS is not useful. At high scales we are often in the limit where charm and bottom are effectively massless and a GM-VFNS is identical to the ZM-VFNS. However, the partons are obtained from fitting

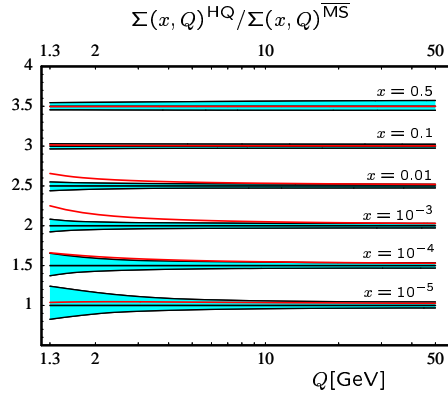


Figure 2. Comparison of the CTEQ6M singlet distribution with uncertainties and the CTEQ6HQ singlet distribution.

to data in regions where $O(m_H^2/Q^2)$ corrections are important, and ignoring these leads to incorrect partons at all Q^2 in the ZM-VFNS. In Fig. 2 we see the difference between the GM-VFNS CTEQ6HQ partons³³ and the ZM-VFNS CTEQ6 partons with their (conservative) uncertainties. At NNLO the partons become discontinuous at the transition points, indeed $c(x, Q^2)$ at m_c^2 is negative, and at this order we certainly need a GM-VFNS. If for some process GM-VFNS coefficient functions are not known, the error of $O(m_H^2/Q^2)$ from using the GM-VFNS partons is no worse than the permanent error from using ZM-VFNS. At worst we can input kinematic constraints into coefficient functions. There are a variety of definitions of a GM-VFNS³⁴, but they generally agree on fundamentals. Each choice is superior to ZM-VFNS and to FFNS. However, most are not defined in detail up to NNLO²⁶, and there are some lingering differences.

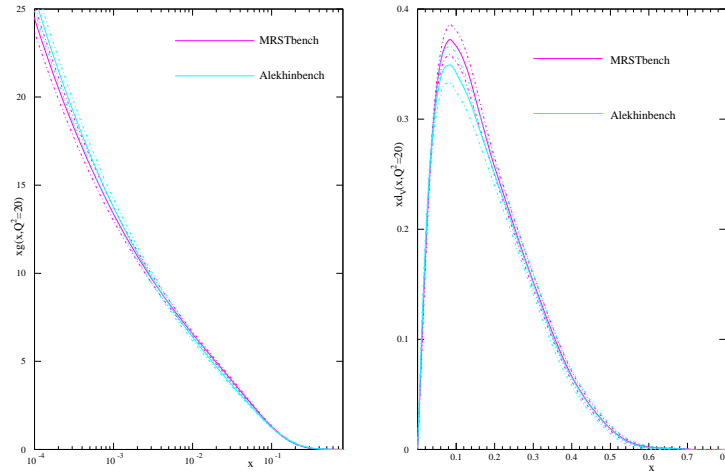


Figure 3. Comparison of the benchmark gluon distributions and d_V distributions.

For a given theoretical prescription we still have a wide choice of partons. It is obvious that some competition is necessary, but not all partons are equal – some are, in some sense, incorrect. There are a variety of reasons for this – bugs in programs, incorrect theoretical approach (e.g. wrong coupling for flavour scheme), approximations to complete theoretical approach, or region of applicability, e.g. MRST03c partons are only suitable within the region of cuts on the data fit. The error is sometimes small, but can be the size of the intrinsic uncertainty or greater. If so, such partons should not be used. Indeed, NNLO is often still in the approximate stage.

There is also the issue of the treatment of experimental errors. As an exercise for the HERA-LHC³⁵ workshop, partons were produced from fits to H1, ZEUS, NMC³⁶ and BCDMS³⁷ structure function data for $Q^2 > 9\text{GeV}^2$ using ZM-VFNS and a common form of parton inputs at $Q_0^2 = 1\text{GeV}^2$ – clearly very conservative. Partons were obtained using the rigorous treatment of all systematic errors (labelled Alekhin) and using the simple quadratures approach (labelled MRST), both using $\Delta\chi^2 = 1$ to define the limits of uncertainty. As seen in Fig. 3 there are small differences in the central values and similar errors, i.e. the two sets are fairly consistent. Even so, the full treatment of systematic errors is presumably better, but perhaps it is not so straightforward. Consider the averaged H1-ZEUS data sets³⁸, where the systematics of one data set can be significantly reduced by fitting to the other set. The averaged data set is much more precise with very small systematic errors. At the HERA-LHC workshop a comparison was made of a fit to both data sets and a fit to the averaged data set³⁹. The partons resulting can differ by more than the uncertainty in each, and the movement of the data relative to the theory was different in each case. Data can move relative to theory due to systematic uncertainties, but in reality this may be due to failures in theory rather than due to the central values for the data being incorrect. The conventional approach to systematic errors assumes we fit data to a perfect theoretical model with some unknown parameters, whereas in fact we are testing QCD at some order, and it may be slightly lacking. It is always best to remember this and try to minimize systematic errors. This makes the averaged H1/ZEUS data set very desirable. It is easier to understand and trust dominant statistical errors.

We should also include as many data as possible in order to determine the partons, e.g. we see in Fig. 4 the reduced uncertainty in the ZEUS partons when including their own jet data¹⁶. The central values can also move, but do not do so much in this case. As a more dramatic example we consider the HERA-LHC benchmark partons and investigate how these

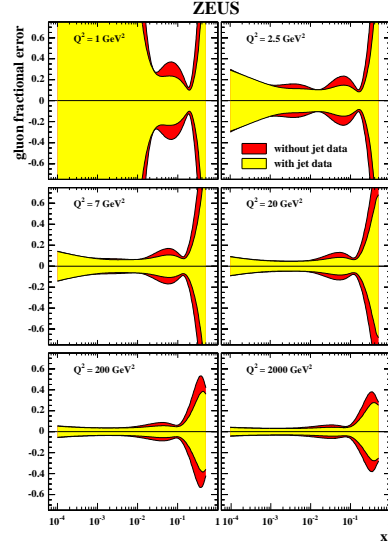


Figure 4. Comparison of the uncertainty on the ZEUS gluon distribution with and without the inclusion of their jet cross-section data.

compare to partons obtained from a global fit (the MRST01 partons⁴⁰), where the uncertainty is determined using $\Delta\chi^2 = 50$. There is an enormous difference in the central values, sometimes many σ , as seen in Fig. 5. The uncertainties are similar using $\Delta\chi^2 = 1$ compared to $\Delta\chi^2 = 50$ with approximately twice the data. Moreover, $\alpha_S(M_Z^2) = 0.1110 \pm 0.0015$ from the benchmark fit compared to $\alpha_S(M_Z^2) = 0.119 \pm 0.002$. Something is clearly seriously wrong in one of these analyses, and I am very confident that it is the benchmark fit. It fails when compared to most data sets not included, and not all can be unreliable. Partons should be constrained by all possible reliable data. The benchmark fit partons are extreme, but some other partons frequently used are similar in terms of the quantity of data fit, but many input implicit constraints from elsewhere. Also, for the global fit $\Delta\chi^2 = 1$ is not reliable. There must be something better than $\Delta\chi^2 = 50(100)$ or the offset method¹⁵, but we are not yet sure what that is. The problems are partially due to the strict incompatibility of different data sets. Systematic errors are difficult to understand and not usually Gaussian in nature. Also our theory is never perfect – it is not simply a matter of tying down unknown constants. There are corrections possible at low Q^2 , small x and high x . Indeed, comparing different sets of partons, one finds that the gluon is still very uncertain at low x and Q^2 , even though all partons are fit to the same small- x HERA data. The additional constraint from a direct measurement of $F_L(x, Q^2)$ would help this situation⁴¹.

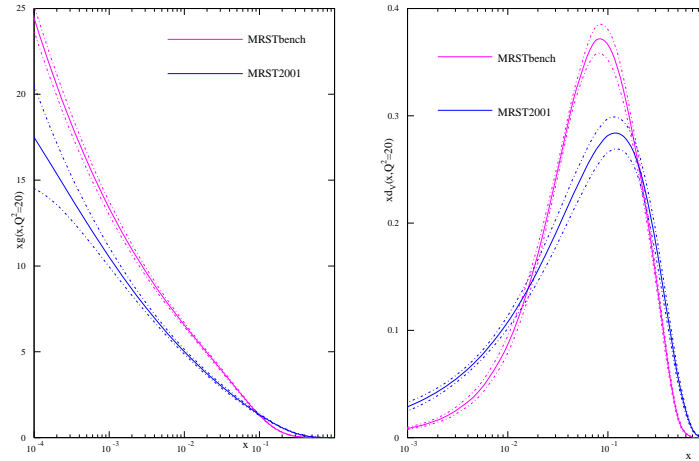


Figure 5. Comparison of the benchmark gluon distribution and d_V distribution with the corresponding MRST2001E partons.

To conclude, there are many types of partons, and although some may be ignored, a variety is needed for the full range of applications and to estimate the uncertainties due to different assumptions (though one should be wary of partons that have only a limited set of constraints from data). We need different prescriptions for heavy flavours (though not ZM-VFNS), different factorization schemes and different orders. As a test of QCD, NNLO is preferable, and we are now obtaining reliable NNLO partons. We sometimes need partons for special occasions, e.g. to investigate the NuTeV $\sin^2 \theta_W$ anomaly⁴². There are many available, with QED corrections⁴, isospin violation⁹, $s(x, Q^2) \neq \bar{s}(x, Q^2)$ ⁴³ etc.. We also need to determine whether resummations at small or large x , higher twist or other theoretical corrections are important in some regions. There is much activity in these areas, and hopefully it will very soon provide concrete results.

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