RECENT DEVELOPMENTS IN PERTURBATIVE QCD

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A brief overview of some recently active topics in perturbative QCD, including: string-inspired recursion techniques at tree level; recursion approaches and automation of standard techniques for 1-loop calculations; the status of NNLO jet calculations; and non-trivial structures that appear in higher-order calculations.

1. Introduction

As the startup of LHC approaches, much current work in QCD is directed towards developing techniques for improving the flexibility and accuracy of perturbative calculations.

Flexibility (section 2) is crucial because of the vast range of multi-jet final states that will be studied in LHC new-particle searches. At tree level, numerical recursion techniques have long been used to build multi-leg amplitudes from amplitudes with fewer legs — recent developments inspired by string theory have led to analytically more powerful recursions, giving many new compact results for tree-level amplitudes. This new understanding is also being applied to 1-loop amplitudes, often the missing ingredient for quantitatively reliable (NLO) multi-jet predictions. In parallel, more traditional 1-loop techniques are being subjected to automation, and here too major progress has recently been made.

Accuracy (section 3), in the sense of NNLO jet calculations, is looking like it might be within reach in the coming year. This is welcome since for over a decade LEP and HERA have been delivering final-state measurements with precisions several times better than the NLO theory uncertainties, and the latter limit our ability to extract fundamental parameters of QCD such as α_s and parton distributions. NNLO results also provide clues as to the general structures of high orders in QCD. This is both of fundamental interest and potentially useful in predicting large parts of yet higher orders.

2. Multi-jets

2.1. *Tree level*

The multiplicity of Feynman graphs grows factorially with the number of legs of a process, hampering the usefulness of traditional techniques for calculating multi-leg amplitudes. An important discovery of the 1980's was the Berends-Giele recursion,[1] allowing amplitudes to be constructed by assembling smaller, off-shell sub-amplitudes, fig.1a. This was suited to recursive numerical evaluations and helped prove analytical all-order results,[3] thanks to simplifications that occur independently in each sub-amplitude. Berends-Giele recursion joins amplitudes via three and four-gluon vertices. Recently two new recursions were discovered, CSW[4] and BCF[2] (fig.1b) which join amplitudes via a scalar propagator. In the latter case the amplitudes are made on-shell using analytic continuation of reference momenta (legs 1,2 in figure). The simpler structure of CSW and BCF recursion makes it easier to identify simplifications at each order, leading to many new all-order analytical results, including applications to processes with heavy quarks and electroweak bosons (for a review, see ref. 5). Though originally inspired from string theory, the new recursion relations have been proved based on general field theoretical arguments[2,6] (exploiting the rationality of tree-level amplitudes) and can also be related directly to Feynman diagrams.[7]

The above developments represent an impressive improvement in our understanding of field theory. Nevertheless one should bear in mind that for practical numerical implementations of tree level calculations, existing methods[1,8] still remain competitive or superior.[9]

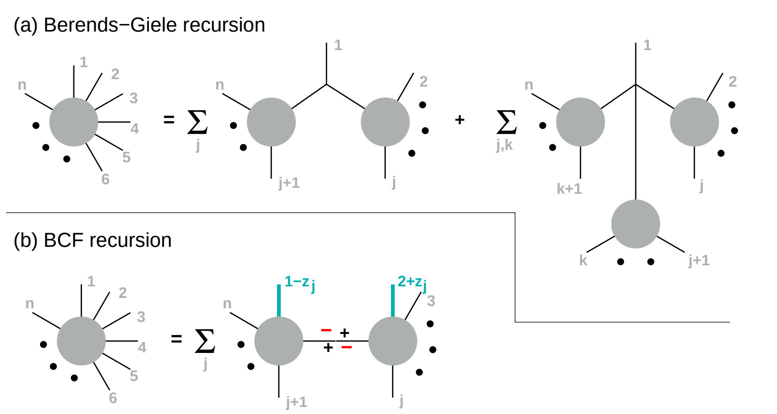

Figure 1. Graphical comparison of Berends-Giele[1] and BCF[2] recursion relations.

2.2. *NLO*

While many NLO calculations already exist,[10] there is a recognized phenomenological need for further multi-leg NLO calculations, in particular to simulate backgrounds to new physics signals (for a full discussion of recent developments, see ref.11). The ingredients in n-leg NLO calculations are $n + 1$ leg tree amplitudes, n-leg 1-loop amplitudes and a procedure for combining the pieces. The hardest part is the 1-loop calculation, for which several 5-leg results exist and some first 6-leg results are starting to appear.

The string theory inspired approach to tree-level diagrams does not trivially generalise to loop level, in part because of the more complicated analytical structure of loop diagrams (cuts as well as poles). Nevertheless the string-inspired approach has led to much new work on loops as well, notably using the "sewing together" of tree diagrams. This works most easily for supersymmetric loop amplitudes, where cancellations between scalar, fermionic and vector particles in the loop lead to simpler structures in the final answer. The plain QCD result is then obtained by combining answers with $\mathcal{N} = 4$ SUSY, $\mathcal{N} = 1$ SUSY and a scalar particle in the loop, the latter being the most difficult (split into "cut-constructable" (c, d, e) and purely rational (R) pieces). Considerable progress has been made, as illustrated in table 1 which shows the contributions to the analytical evaluation of the six-gluon 1-loop amplitude, for all independent helicity configurations. For MHV configurations (two $-$ helicities, all others $+$) and split NMHV and NNMHV configurations (3 or 4 adjacent $-$ helicities, all others $+$), general multiplicity results have very recently become available.[12,13]

An alternative approach automates traditional methods, *i.e.* Feynman diagram generation, and the recursive reduction of the resulting loop integrals to a set of known basis integrals. It has the advantage of being easier to generalise to processes with external particles other than gluons, but

Table 1. The analytically derived helicity components of the 1-loop 6-gluon amplitude (adapted from ref. 14).

	$\mathcal{N} = 4$	$\mathcal{N} = 1$	$S\,(c, d, e)$	$S\,(R)$
$A(--++++)$	[15]	[16]	[16]	[17]
$A(-+-+++)$	[15]	[16]	[18]	[13, 19]
$A(-++-++)$	[15]	[16]	[18]	[13, 19]
$A(---+++)$	[16]	[20]	[21]	[12]
$A(--+-++)$	[16]	[22, 23]	[24]	[19]
$A(-+-+-+)$	[16]	[22, 23]	[24]	[19]

suffers from the large number of Feynman diagrams, each term of which is broken up into many further terms by the (sometimes numerical) recursion. Sometimes the recursion introduces numerical instabilities and alternative strategies are then required.[25,26] A notable result with such methods was the first full evaluation of the 6-gluon 1-loop amplitude for arbitrary helicity configurations,[27] and work is in progress for the 6-quark 1-loop amplitude.[28] Full $2 \to 4$ NLO jet predictions are however still some way off.

Related automated methods have been successful also in electroweak calculations, with recent full results for $e^+e^- \to 4$ fermions[29] and $e^+e^- \to HH\nu\bar{\nu}$,[30] and progress made there will hopefully in part carry over to QCD. Also, traditional techniques can simplify considerably[19] when extracting just the scalar rational components of the decomposition in table 1 (*i.e.* the parts hardest to obtain in the string-related approaches).

3. Precision QCD

3.1. *NNLO jets*

Various results exist at NNLO for processes with two QCD partons at Born level and one or two non-QCD particles. The current challenge is to address processes with three or more QCD legs at Born level, in particular $e^+e^- \to 3$ jets. All tree-level, 1 and 2-loop amplitudes are known — the difficulty is in cancelling divergences between them for a general jet observable.

Two approaches exist. Subtraction (as at NLO[31]) identifies a function with the same divergences as the real amplitudes, but that is sufficiently simple that it can be integrated analytically — one then subtracts the unintegrated form from the real amplitudes and adds the integrated form to the virtual amplitudes, cancelling all divergences. Finding the subtraction functions requires deep understanding of the QCD divergences and ingenuity so as to make the result integrable. A full scheme at NNLO for processes with just final-state particles has been proposed[32] and as a proof of concept used to calculate to the α_s^3/N_c^3 contribution to the mean thrust in e^+e^-.

An alternative approach, sector decomposition,[33] rewrites phase space to as to isolate single divergences and then effectively introduces plus-prescriptions (as in splitting functions) so as to allow separate extraction of different powers of the dimension regularisation ϵ. This is less dependent on the specific structure of QCD divergences, but becomes more complicated as the number of QCD particles increases. It has been successfully used for hadron-hadron processes with two Born QCD particles,[34] and for a part of the NNLO $e^+e^- \to 3$ jets cross section.[35]

Given the above progress one can perhaps expect first full NNLO predictions for $e^+e^- \to 3$ jets in the coming year, hopefully with a major impact on measurements of the coupling and studies of analytical hadronisation models. Extensions to DIS $2+1$ jet events and hadron-collider dijets will probably take somewhat longer. Note that for jets at hadron colliders, an issue remains with the experimental jet definitions. Because the standard midpoint cone (ILCA[36]) has the drawback that it can leave large energy deposits unclustered,[37] an extra 'search-cone' step that has been proposed[37] and used.[38] However this turns out to be infrared (IR) unsafe as the seed threshold is taken to zero,[39] compromising theory-data comparisons. A positive development is that hadron-collider measurements with the more physically motivated (and IR safe) k_t algorithm have been shown to be feasible now by both Tevatron collaborations,[40,41] and the long-standing speed issue for the k_t algorithm at high-multiplicity has also been resolved.[42]

3.2. *Structure of perturbation theory*

Two years have passed since Moch, Vermaseren and Vogt's (MVV) seminal calculation of the NNLO splitting functions.[43] With related technology, the same authors have obtained the third order coefficient functions,[44] threshold resummation coefficients,[45] and quark and gluon form factors.[46] These results have served as ingredients to calculations of 3-loop $\mathcal{N} = 4$ SUSY splitting functions,[47] Drell-Yan and Higgs threshold resummations,[48] and 3-loop non-singlet time-like splitting functions.[49]

Various unexpected structures appear in the above results. E.g. writing

$$P_{ij}(x) = \frac{A}{(1-x)_+} + B\delta(1-x) + C\ln(1-x) + \mathcal{O}(1) \qquad (1)$$

with $A = \sum_n A_n(\alpha_s/4\pi)^n$, etc., it was noted at NLO[50] that $C_2 = A_1^2$. At NNLO, MVV observed $C_3 = 2A_1A_2$. If one postulates splitting functions to be universal[51] (identical for time and space-like evolution) when expressed for a modified evolution variable $z^\sigma Q^2$ ($\sigma = \pm 1$ for the $^{\text{time}}_{\text{space}}$-like case)[a] and furthermore assumes the universal splitting function to be classical at large x (having $C \equiv 0$), then for normal space-like splitting functions one predicts that $C = A^2$ at all orders,[52] precisely as found at NLO and NNLO.

The idea of a universal splitting function is given further credibility by an analysis[49] which uses the usual[50] analytical continuation $x \to 1/x$ to go from the space-like to the time-like non-singlet (NS) case and finds

[a] Specifically $\partial_{\ln Q^2} D = \int \frac{dz}{z}\mathcal{P}(z,Q^2)D(\frac{x}{z}, z^\sigma Q^2)$, with D a parton distribution or fragmentation function.

it to be identical to the time-like result found assuming universality with the $z^\sigma Q^2$ evolution variable. Note that universality predicts the 3-loop $P_{NS}^{\sigma=+1} - P_{NS}^{\sigma=-1}$ difference using only 2-loop information. Given that the full 3-loop $P_{NS}^{\sigma=+1}$ and $P_{NS}^{\sigma=-1}$ are themselves also related by $x \to 1/x$, this implies the existence of non-trivial (and yet to be understood) properties of the analytic structure of the splitting functions. The universality also suggests an explanation for the till-now mysterious absence of 2 and 3-loop leading log x terms in the space-like splitting functions, as being closely related to exact angular order in fragmentation.[53] Despite these successes the universality hypothesis requires further development notably as concerns the treatment of the singlet sector and the factorisation scheme.

Other intriguing perturbative structures that have also been found recently include the following: in $\mathcal{N} = 4$ SUSY QCD there is increasing evidence that n-loop m-leg amplitudes are related to the n^{th} power of the 1-loop m-leg amplitude[54] (new numerical methods[55] for loop calculations providing powerful cross checks); in large-angle soft-gluon resummation for $2 \to 2$ scattering, there is a mysterious symmetry[56] when exchanging the kinematic quantity $(\ln s^2/ut - 2\pi)/(\ln u/t)$ and the number of colours, N_c.

4. Other results

Owing to limitations of space, many active topics have been omitted. Some (small-x saturation, generalised parton distributions) are reviewed in these proceedings.[57] A more extensive bibliography is to be found in ref. 58. For others new developments, the reader is referred to the literature, notably for 4-loop decoupling relations for α_s;[59] jet definitions that preserve the IR safety of flavour;[60] the release of the first `C++ ThePEG`-based hadron-collider Monte Carlo (MC) generator;[61] progress in practical and conceptual aspects of matching MC and NLO;[62] reweighting to match MC with NNLL and NNLO;[63] and soft large-angle resummations, both in terms of phenomenology,[64] understanding of treatment of jet-algorithms for non-global resummation,[65] two-loop soft colour evolution matrices[66] and other recent NNLO resummation results,[67] and an intriguing (but still to be confirmed) suggestion of a breakdown of coherence at high orders.[68]

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