# QCD AND MONTE CARLO EVENT GENERATORS*

ZOLTÁN NAGY
*Institute for Theoretical Physics*
*University of Zürich*
*Winterthurerstrasse 190*
*CH-8057 Zürich, Switzerland*
*E-mail: nagyz@physik.unizh.ch*

DAVISON E. SOPER
*Institute of Theoretical Science*
*University of Oregon,*
*Eugene, OR 97403, USA*
*E-mail: soper@physics.uoregon.edu*

Shower Monte Carlo event generators have played an important role in particle physics. Modern experiments would hardly be possible without them. In this talk I discuss how QCD physics is incorporated into the mathematical structure of these programs and I outline recent developments including matching between events with different numbers of hard jets and the inclusion of next-to-leading order effects.

## 1. A critique of pure perturbation theory

Before beginning a discussion of shower Monte Carlo event generators, let us examine programs that do purely perturbative calculations at next-to-leading order (NLO). Consider the cross section to produce three jets in electron-positron annihilation (using a suitable definition of what one means by a jet). The ratio of this cross section to the total cross section is the three-jet fraction, $f_3$. Now, $f_3$ is an infrared safe observable that is amenable to calculation at NLO accuracy. In such a calculation, the program produces simulated partonic events with three partons and others with four partons. In either case, if the parton momenta meet certain criteria, the event can be classified as a three jet event. Let us look at this calculation[1] and ask

*Presented by D. E. Soper

for each jet in each three jet event what the mass of the jet is. Then we can plot the calculated probability of finding a jet in a given bin of jet mass $M$, $f_3^{-1} df_3/dM$. The result is that $f_3^{-1} df_3/dM$ increases without bound as $M \to 0$. Exactly at $M = 0$ there is a term $A\delta(M)$ where $A$ is negative and infinite. Clearly, this is not a good representation of nature.

This nonsensical result can be contrasted to the result from the standard parton shower Monte Carlo program `Pythia`[2], which produces a sensible result in which the distribution $f_3^{-1} df_3/dM$ peaks at about 8 GeV for $\sqrt{s} = M_Z$.

Clearly it would be best to keep the NLO accuracy for $f_3$ while at the same time fixing the internal structure of the jets to be more like what one gets in `Pythia`. This can be done if one keeps track of what the parton shower algorithm does, expands the parton shower effects perturbatively, and subtracts the NLO contribution of the parton shower from the NLO term in the perturbative calculation. Then one can obtain a result that combines the NLO calculation with `Pythia`.[3] The result for $f_3^{-1} df_3/dM$ closely follows the pure `Pythia` result. The result of this program for just $f_3$ closely follows the pure NLO result.

The program just mentioned is for electron-positron annihilation to make three jets. For hadron-hadron collisions, programs for several important processes are available in the package `MC@NLO`.[4] I will not say more about the technical methods involved in combining "MC" with "NLO," but I will comment briefly on one further development later in this talk. In the rest of this talk, I will mainly concentrate on leading order aspects of Monte Carlo event generators.

## 2. Showers from the inside out

Consider the parton shower picture of hadron-hadron scattering in which there is some sort of hard event, say jet production or squark-pair production. The first thing to understand is that the parton shower description starts from the hard scattering and proceeds toward softer scatterings. For final state partons, one is thus working forwards in time, but for the initial state partons one is working backwards in time. Although the development of the parton shower description of hadron scattering dates from about 1980,[5] it was not until somewhat later that this backwards evolution scheme was developed.[6] (I should mention that the program `Herwig`[7] is organized differently, with splittings at the widest angles done first.)

### 3. Color coherence

Now let us think about soft gluon radiation. I consider three jet production in electron-positron annihilation as an example. At the Born level, one has a $q\,\bar{q}\,g$ final state. The gluon is a color $\mathbf{8}$, but to leading order in an expansion in powers of $1/N_c^2$, where $N_c = 3$ is the number of colors, the gluon can be considered to be a $\mathbf{3}\,\bar{\mathbf{3}}$ state. Then the outgoing quark and the $\bar{\mathbf{3}}$ part of the gluon constitute a $\mathbf{3}\,\bar{\mathbf{3}}$ dipole, while the outgoing antiquark and the $\mathbf{3}$ part of the gluon constitute another $\mathbf{3}\,\bar{\mathbf{3}}$ dipole.

The two dipoles will radiate soft gluons. Given the (approximate) color structure, the two dipoles radiate independently: there is no quantum interference between a gluon radiation from one dipole and radiation from the other dipole. The radiation pattern is depicted in Figure 1. For each dipole, there is soft-collinear radiation that is concentrated in the directions of the two outgoing partons for that dipole. There is also a wider angle component that is, approximately, spread over the angular region between the parton directions. Thus the wide angle dipole has soft radiation spread over a wide angular region while the narrow angle dipole has soft radiation spread over a narrow angular region.

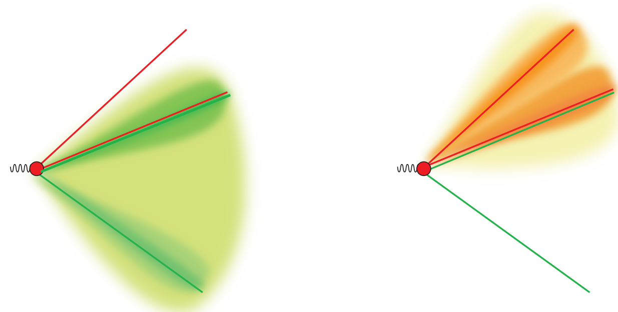

Figure 1. Radiation from the two dipoles in a $q\,\bar{q}\,g$ final state. For each dipole, the radiation is concentrated along the direction of the two outgoing partons and also contains a wider angle component that is spread over an angular range that is roughly that subtended by the two parton momenta.

In a parton shower Monte Carlo program, one can work to leading order in $1/N_c^2$ (as parton shower programs generally do) and make sure that the parton splitting formulas properly take into account the interference between gluons emitted from the two parts of a color dipole. The program `Ariadne` is based on this kind of picture.[8] The latest version of `Pythia` is also based on a dipole picture.[2,9] The present authors have found that the Catani-Seymour dipole formalism[10] for generating the subtractions for perturbative NLO calculations is also quite useful as the basis for splittings in a parton shower.[11,12]

There is another way to do this. One can simply generate independent emissions from each parton and then impose a restriction on the angles of the emissions. This is the method of `Herwig`.[7] In `Herwig`, a wide angle soft gluon emission as depicted in the left-hand part of Figure 1 is generated first, before the splitting of the quark into a hard quark and a hard gluon. The algorithm enforces that the angles between daughter partons in a splitting decrease for splittings generated later in the algorithm evolution. The recognition of the importance of this ordering was important in the development of parton shower algorithms.[13]

## 4. Shower evolution in pictures

Shower evolution can be represented using an evolution equation of the form represented graphically as in Figure 2. The ovals represent the complete shower evolution operator $U(t_3, t_1)$ that, operating on a function representing the probability for the state to have a given partonic composition at Monte Carlo time $t_1$, produces a function representing the probability for the state to have a given partonic composition at Monte Carlo time $t_3$. The narrow rounded rectangles represent the a no splitting operator that inserts a Sudakov factor representing the probability that there was no splitting from time $t_1$ to time $t_3$. In the second term, there is a no splitting operator $N(t_2, t_1)$, followed by a splitting operator $\mathcal{H}(t_2)$ at time $t_2$, followed by complete evolution $U(t_3, t_2)$ for times after the splitting. There is an integration over the intermediate time $t_2$ at which the splitting occurs.

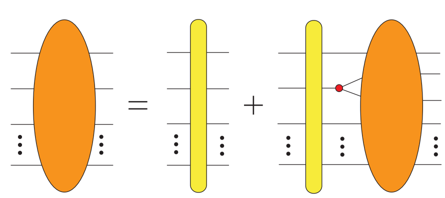

Figure 2. The evolution equation in pictures, as described in the text.

To generate a cross section with a shower Monte Carlo event generator, one can start with a hard squared matrix element for $2 \to 2$ scattering, then apply the shower operator to the two incoming and two outgoing partons. When the shower evolution equation is iterated, one obtains terms representing $n = 0, 1, 2, \ldots$ splittings with Sudakov factors for the intervals with no splittings, as depicted in Figure 3.

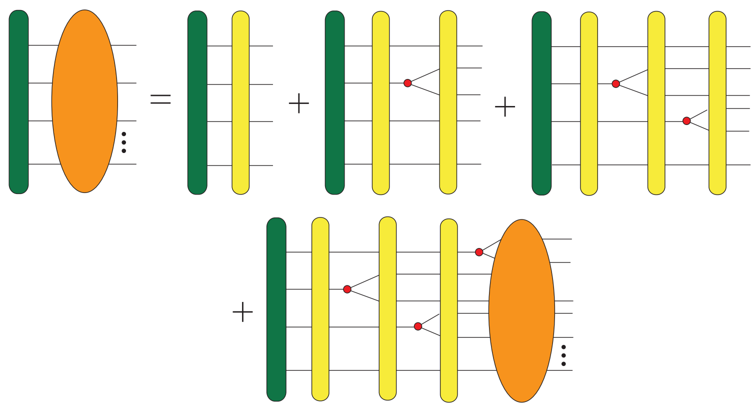

Figure 3. Calculation of a shower starting with a $2 \to 2$ hard cross section (dark rounded rectangle).

## 5. An improved shower

The standard shower depicted in Figure 3 has a deficiency. In a standard shower one has Sudakov factors and $1 \to 2$ parton splitting functions. These splittings are approximations based on the splitting angles being small or one of the daughter partons having small momentum. Thus the shower splitting probability with two splittings approximates the exact squared matrix element for $2 \to 4$ scattering. The approximation is good in parts of the final state phase space, but not in all of it. Thus one might want to replace the approximate squared matrix element with the exact squared matrix element. However, if we use the exact squared matrix element, we lack the Sudakov factors.

One can improve the approximation as illustrated in Figure 4. We reweight the exact squared matrix element by the ratio of the shower approximation with Sudakov factors to the shower approximation without

Sudakov factors. The idea is to insert the Sudakov factors into the exact squared matrix element. This is the essential idea in the paper of Catani, Krauss, Kuhn, and Webber.[14] They use the $k_T$ jet algorithm to define the ratio needed to calculate the Sudakov reweighting factor.

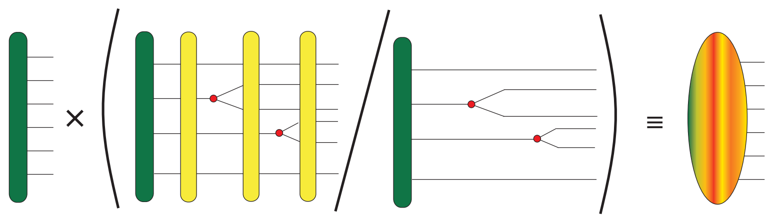

Figure 4. An improved version of the $2 \to 4$ cross section. We take the shower approximation, divide by the approximate collinear squared matrix element, and multiply by the exact tree level squared matrix element. The graphical symbol on the right hand side represents this Sudakov reweighted cross section.

There is a further step in implementing this idea. CKKW divide the shower evolution into two stages, $0 < t < t_{\rm ini}$ and $t_{\rm ini} < t < t_{\rm f}$, where $t_{\rm ini}$ is a parameter that represents a moderate $P_T$ scale and $t_{\rm f}$ represents the very small $P_T$ scale at which showers stop and hadronization is simulated.

With this division, the Sudakov reweighting can be performed for the part of the shower at scale harder than $t_{\rm ini}$, as depicted in Figure 5. The first term has no splittings at scale harder than $t_{\rm ini}$. In the second term there is one splitting, generated via the exact matrix element with a Sudakov correction as discussed above. In the next term there are two splittings. If we suppose that we do not have exact matrix elements for more than $2 \to 4$ partons, states at scale $t_{\rm ini}$ with more partons are generated with the ordinary parton shower. However, this contribution is suppressed by factors of $\alpha_s$. Evolution from $t_{\rm ini}$ to $t_{\rm f}$ is done via the ordinary shower algorithm.

Let $\sigma_m[F]$ be the contribution to the cross section for an infrared safe observable $F$ that comes from final states with $m$ jets at scale $t_{\rm ini}$. The CKKW calculation just described gets $\sigma_m[F]$ correct to leading perturbative order. The method can be extended. The present authors have shown (at least for the case of electron-positron annihilation) how to get $\sigma_m[F]$ for an infrared safe observable correct to next-to-leading order, $\alpha_s^{m+1}$.[11] The required NLO adjustments are a little complicated, so I do not discuss them here.

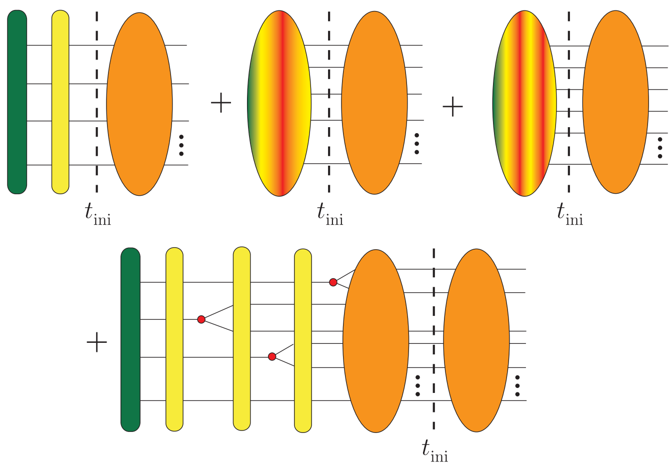


Figure 5. Shower with CKKW jet number matching. The calculation for $n$ jets at scale $t_{\rm ini}$ is based on the Sudakov reweighted tree level cross section for the production of $n$ partons. Evolution from $t_{\rm ini}$ to $t_{\rm f}$ is done via the ordinary shower algorithm.

## 6. An alternative shower improvement

There is an alternative way to organize the shower improvement so as to include exact tree level matrix elements.[12] One does not really need to split the evolution at a scale $t_{\rm ini}$. Suppose that one has the exact tree level matrix elements for $2 \to n$ partons for $n \le N$. Then the partonic cross section at a final very soft scale $t_{\rm f}$ before hadronization is the sum of the $2 \to 2$, $2 \to 3$, $\dots 2 \to N$ cross sections with Sudakov factors plus one more term, which is the most important term. In the last term, we have the Sudakov improved $2 \to N$ squared matrix element in which the softest splitting has scale $t$ and we integrate over $t$. This is convoluted with the simple shower approximation for splittings softer than $t$, down all the way to $t_{\rm f}$. This is depicted in Figure 6. The terms before the last one are included in the calculation but are not important because they contain the Sudakov suppression for only a small number of splittings to occur down to a very soft scale $t_{\rm f}$. In the term that really matters, we use the Sudakov improved $2 \to N$ squared matrix element with an arbitrary number of further splittings generated in the collinear/soft approximation, all of this with Sudakov suppression factors.

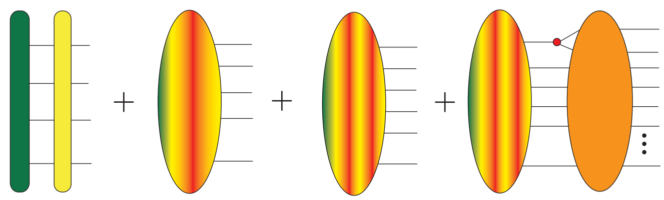

Figure 6. Alternative shower improvement that does not involve a scale $t_{\rm ini}$.

## Acknowledgments

This work was supported in part the United States Department of Energy and by the Swiss National Science Foundation (SNF) through grant no. 200020-109162 and by the Hungarian Scientific Research Fund grants OTKA K-60432.

## References

1. D. E. Soper, Phys. Rev. Lett. **81**, 2638 (1998) [arXiv:hep-ph/9804454].
2. T. Sjostrand, S. Mrenna and P. Skands, JHEP **0605**, 026 (2006) [arXiv:hep-ph/0603175].
3. M. Kramer and D. E. Soper, Phys. Rev. D **69**, 054019 (2004) [arXiv:hep-ph/0306222]; D. E. Soper, Phys. Rev. D **69**, 054020 (2004) [arXiv:hep-ph/0306268]; M. Kramer, S. Mrenna and D. E. Soper, Phys. Rev. D **73**, 014022 (2006) [arXiv:hep-ph/0509127].
4. S. Frixione, P. Nason and B. R. Webber, JHEP **0308**, 007 (2003) [arXiv:hep-ph/0305252]; S. Frixione and B. R. Webber, JHEP **0206**, 029 (2002) [arXiv:hep-ph/0204244].
5. For example, R. Odorico, Nucl. Phys. B **172**, 157 (1980).
6. T. Sjostrand, Phys. Lett. B **157**, 321 (1985); T. D. Gottschalk, Nucl. Phys. B **277**, 700 (1986).
7. G. Corcella *et al.*, JHEP **0101**, 010 (2001) [arXiv:hep-ph/0011363].
8. L. Lonnblad, Comput. Phys. Commun. **71**, 15 (1992).
9. T. Sjostrand and P. Z. Skands, Eur. Phys. J. C **39**, 129 (2005) [arXiv:hep-ph/0408302].
10. S. Catani and M. H. Seymour, Nucl. Phys. B **485**, 291 (1997) [Erratum-ibid. B **510**, 503 (1997)] [arXiv:hep-ph/9605323].
11. Z. Nagy and D. E. Soper, JHEP **0510**, 024 (2005) [arXiv:hep-ph/0503053];
12. Z. Nagy and D. E. Soper, arXiv:hep-ph/0601021.
13. G. Marchesini and B. R. Webber, Nucl. Phys. B **238**, 1 (1984); R. K. Ellis, G. Marchesini and B. R. Webber, Nucl. Phys. B **286**, 643 (1987) [Erratum-ibid. B **294**, 1180 (1987)].
14. S. Catani, F. Krauss, R. Kuhn and B. R. Webber, JHEP **0111**, 063 (2001) [arXiv:hep-ph/0109231].