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Fragmentation functions with heavy quark thresholds

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Outline

- Case for heavy quark thresholds
- Matching conditions to NLO
- Practical consequences

Why heavy quark thresholds in FF's

Fragmentation functions do change when a quark threshold is crossed, pretty much like parton distribution functions do: 4-flavour FF's evolve differently from 5-flavour ones (not to mention the existence of the fifth FF!)

The problem has however been pretty much ignored so far in fits to light hadron fragmentation functions:

- Data usually only span an energy range between ~30 and 90 GeV, void of heavy quark thresholds
- **All** flavours are parametrized by phenomenological forms, with charm and bottom being treated like light flavours
- When fitted initial conditions are given at a very low scale [O(1 GeV)] heavy quark contributions are simply "switched on" at the mass thresholds, i.e. mimicking the Collins-Tung space-like threshold condition, $F_h(x,m) = 0$

Why heavy quark thresholds in FF's

When considering heavy quark (perturbative) fragmentation functions it becomes more difficult to neglect heavy quark thresholds, if nothing else for consistency reasons

E.g. perturbative fragmentation functions for charmed hadrons at LEP:

m_c → m_b → 90 GeV

Start of evolution

4 flavours

5 flavours

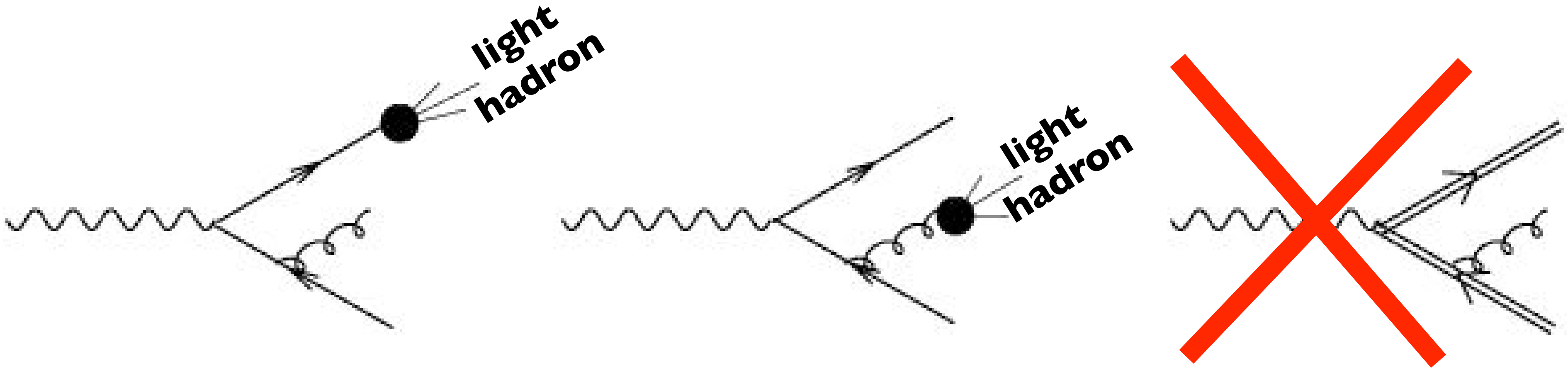
Initial condition available to NLO [Mele, Nason '91] (and now also NNLO [Melnikov, Mitov '04])

Threshold matching condition so far neglected in time-like case. Mimicking the space-like one only guarantees LO accuracy

Evolution available to NLL [Curci, Furmanski, Petronzio '80] (and VERY recently also NNLL [Mitov, Moch, Vogt, '06])

Below the threshold

Contributions:



Below the heavy flavour threshold the effect of the heavy flavour can be fully ignored using the decoupling scheme:

$$\frac{d\sigma}{dx} = \int_x^1 \frac{dy}{y} \sum_{i \in \mathbb{I}_{n_L}} D_i^{(n_L)}(x/y, \mu) \frac{d\hat{\sigma}_i^{(n_L)}(y, \mu)}{dy}$$

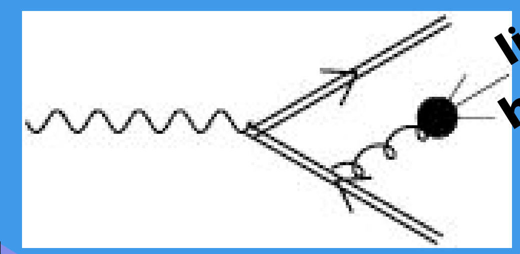
$$\mathbb{I}_{n_L} = \{q_1, \bar{q}_1, \ldots q_{n_L}, \bar{q}_{n_L}, g\}$$

n_L fragmentation functions of "kind" n_L

Light flavours and gluon only

Above the threshold

Above the heavy flavour threshold the production via fragmentation of a given light hadron can be considered either in the n_L or in the $n = n_L + 1$ schemes



n_L

$$\frac{d\sigma}{dx} = \int_x^1 \frac{dy}{y} \left\{ \sum_{i \in \mathbb{I}_{n_L}} D_i^{(n_L)}(x/y,\mu) \frac{d\hat{\sigma}_i^{(n_L)}(y,\mu)}{dy} + D_g^{(n_L)}(x/y,\mu) \frac{d\sigma_{h\bar{h}g}(y)}{dy} \right\}$$

MASSIVE QUARK h

$$\frac{d\sigma_{h\bar{h}g}(y)}{dy} = \sigma_{h\bar{h}} \frac{\alpha_S}{2\pi} C_F\, 2 \frac{1+(1-y)^2}{y} \left\{ \log\frac{Q^2}{m^2} + \log(1-y) - 1 \right\}$$

$n = n_L + 1$

$$\frac{d\sigma}{dx} = \int_x^1 \frac{dy}{y} \sum_{i \in \mathbb{I}_n} D_i^{(n)}(x/y,\mu) \frac{d\hat{\sigma}_i^{(n)}(y,\mu)}{dy}$$

$$\mathbb{I}_n = \mathbb{I}_{n_L} \cup \{h, \bar{h}\}$$

n fragmentation functions of "kind" n

'**MASSLESS**' QUARK h

Matching

The observable cross section must be the same in both schemes. Hence

$$\int_x^1 \frac{dy}{y} \sum_{i\in\mathbb{I}_{n_L},\, i\neq g} \left[D_i^{(n)}(x/y,\mu) - D_i^{(n_L)}(x/y,\mu)\right] \frac{d\hat{\sigma}_i(y,\mu)}{dy}$$
$$+ \int_x^1 \frac{dy}{y}\left[D_g(x/y,\mu)\left(\frac{d\hat{\sigma}_g^{(n)}(y,\mu)}{dy} - \frac{d\hat{\sigma}_g^{(n_L)}(y,\mu)}{dy}\right)\right]$$
$$+ \int_x^1 \frac{dy}{y}\left[\sum_{i\in\{h,\bar{h}\}} D_i^{(n)}(x/y,\mu)\frac{d\hat{\sigma}_i(y,\mu)}{dy} - D_g(x/y,\mu)\frac{d\sigma_{h\bar{h}g}(y)}{dy}\right] = 0$$

NB. n/n_L label omitted where overall difference is $O(\alpha_S^2)$

The MSBAR gluon coefficient function gives:

$$\frac{d\hat{\sigma}_g^{(n)}(y,\mu)}{dy} - \frac{d\hat{\sigma}_g^{(n_L)}(y,\mu)}{dy} = \frac{d\hat{\sigma}_{h\bar{h}g}(y,\mu)}{dy} = \sigma_{h\bar{h}}\frac{\alpha_S}{2\pi}C_F 2\frac{1+(1-y)^2}{y}\left\{2\log y + \log(1-y) + \log\frac{Q^2}{\mu^2}\right\}$$

MASSLESS MSBAR

different

For reference, compare to the **massive** one:

$$\frac{d\sigma_{h\bar{h}g}(y)}{dy} = \sigma_{h\bar{h}}\frac{\alpha_S}{2\pi}C_F\, 2\frac{1+(1-y)^2}{y}\left\{\log\frac{Q^2}{m^2} + \log(1-y) - 1\right\}$$

Matching

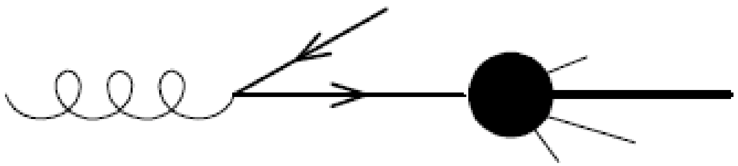
Eventually we find

from the mismatch between massive and massless gluon CF

$$D_h^{(n)}(x,\mu) = D_{\bar{h}}^{(n)}(x,\mu) = \int_x^1 \frac{dy}{y} D_g(x/y,\mu) \frac{\alpha_S}{2\pi} C_F \frac{1+(1-y)^2}{y} \left[\log\frac{\mu^2}{m^2} - 1 - 2\log y\right]$$

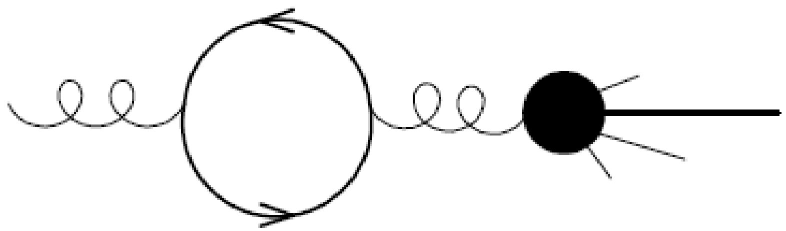
NOT vanishing at threshold!

The gluon fragmentation function



light quark: identical at $O(\alpha_S)$ in n and n_L schemes

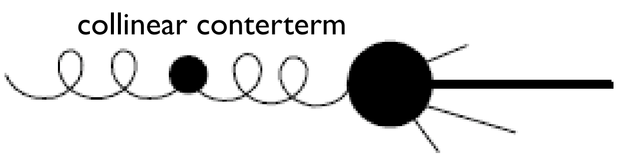
heavy quark: 0 in n_L scheme, $O(\alpha_S{}^2)$ in n scheme



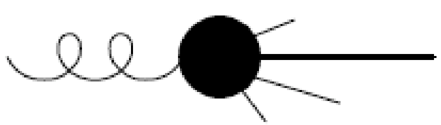
$$\Pi^{(n_L)} + C_r^{(n_L)} = 0 \,,$$ (subtraction at zero momentum, gluon on shell)

massless loop = 0

$$\Pi^{(n)} + C_r^{(n)} = C_r^{(n)} = \frac{1}{\epsilon}\frac{T_F \alpha_S}{3\pi} \times \mathrm{Born}$$



$$-\frac{1}{\epsilon}\frac{\alpha_S}{2\pi}\left[P_{gg}^{(n)}(z) - P_{gg}^{(n_L)}(z)\right] = -\frac{1}{\epsilon}\frac{T_F \alpha_S}{3\pi}$$



$$\alpha_S^{(n)}(\mu) D_g^{(n)}(z,\mu) = \alpha_S^{(n_L)}(\mu) D_g^{(n_L)}(z,\mu)$$

$$\alpha_S^{(n)} = \alpha_S^{(n_L)}\left(1 + \frac{T_F \alpha_S}{3\pi}\log\frac{\mu^2}{m^2}\right)$$

Finally:

$$D_g^{(n)}(z,\mu) = D_g^{(n_L)}(z,\mu)\left(1 - \frac{T_F \alpha_S}{3\pi}\log\frac{\mu^2}{m^2}\right)$$

All the threshold conditions

Summarizing:

$$D_h^{(n)}(x,\mu) = D_{\bar{h}}^{(n)}(x,\mu) = \int_x^1 \frac{dy}{y} D_g(x/y,\mu) \frac{\alpha_S}{2\pi} C_F \frac{1+(1-y)^2}{y} \left[\log\frac{\mu^2}{m^2} - 1 - 2\log y\right]$$

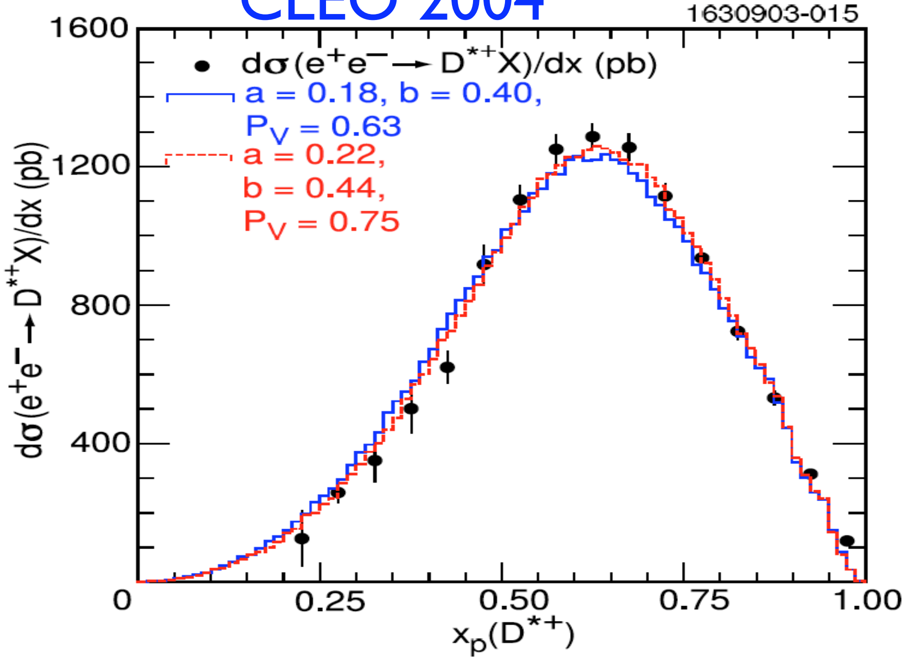
$$D_g^{(n)}(x,\mu) = D_g^{(n_L)}(x,\mu)\left(1 - \frac{T_F\alpha_S}{3\pi}\log\frac{\mu^2}{m^2}\right)$$

$$D_{i/\bar{i}}^{(n)}(x,\mu) = D_{i/\bar{i}}^{(n_L)}(x,\mu) \qquad \text{for } i = q_1,\ldots,q_{n_L}\,.$$

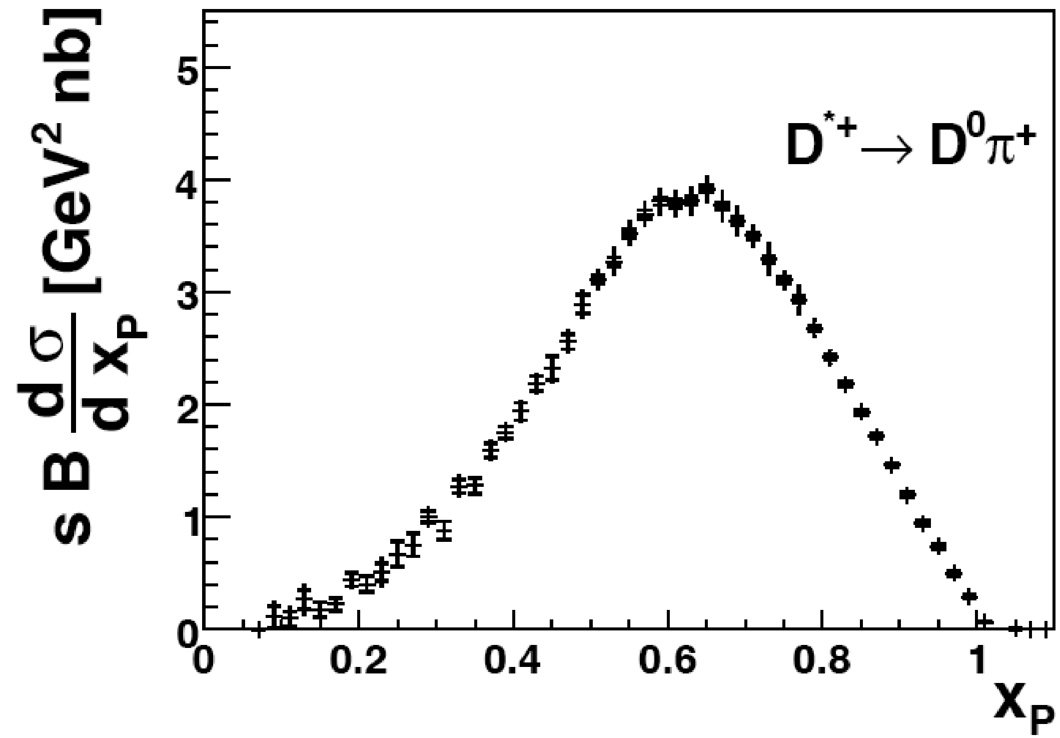
Time-like equivalent of Collins-Tung relations for parton distribution functions

Some data: D* from CLEO, BELLE and LEP

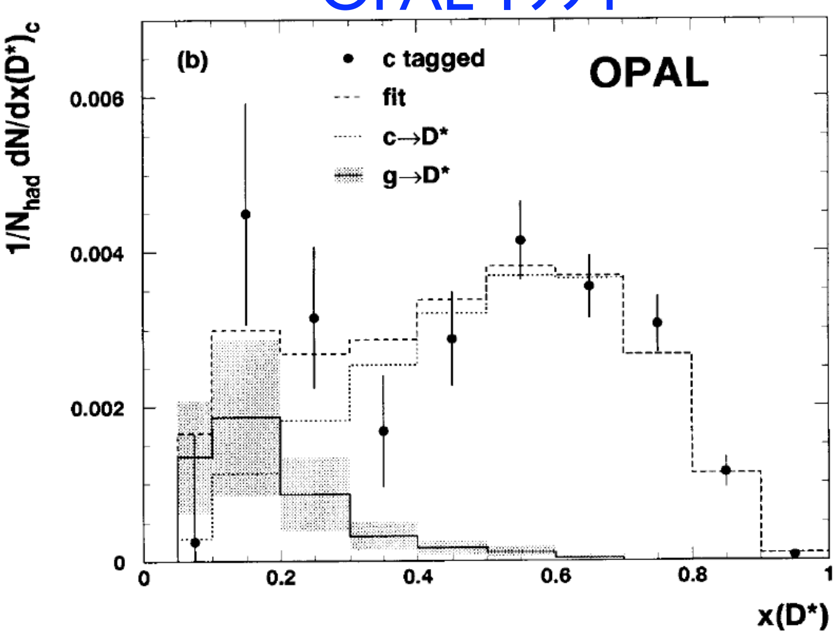
CLEO 2004



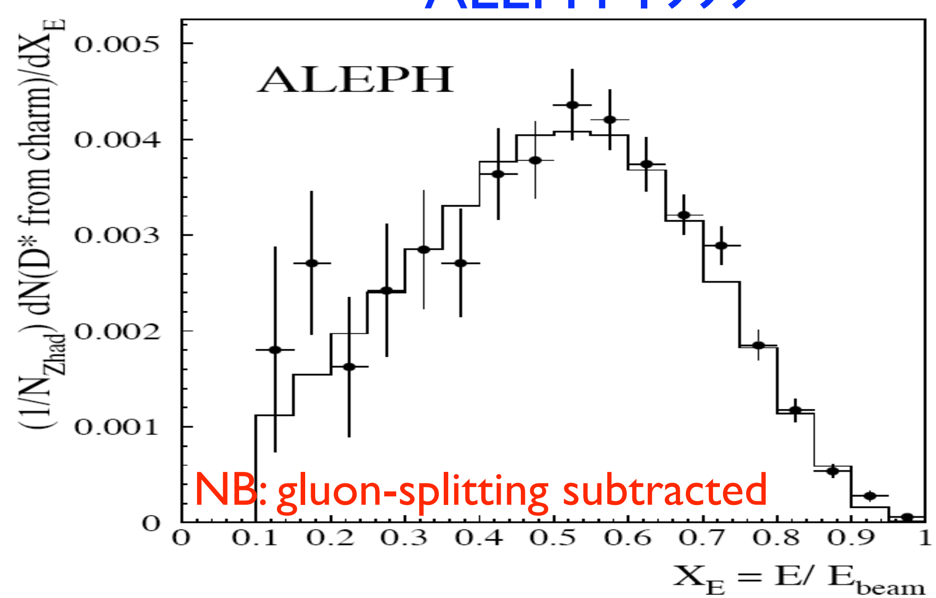
BELLE 2005



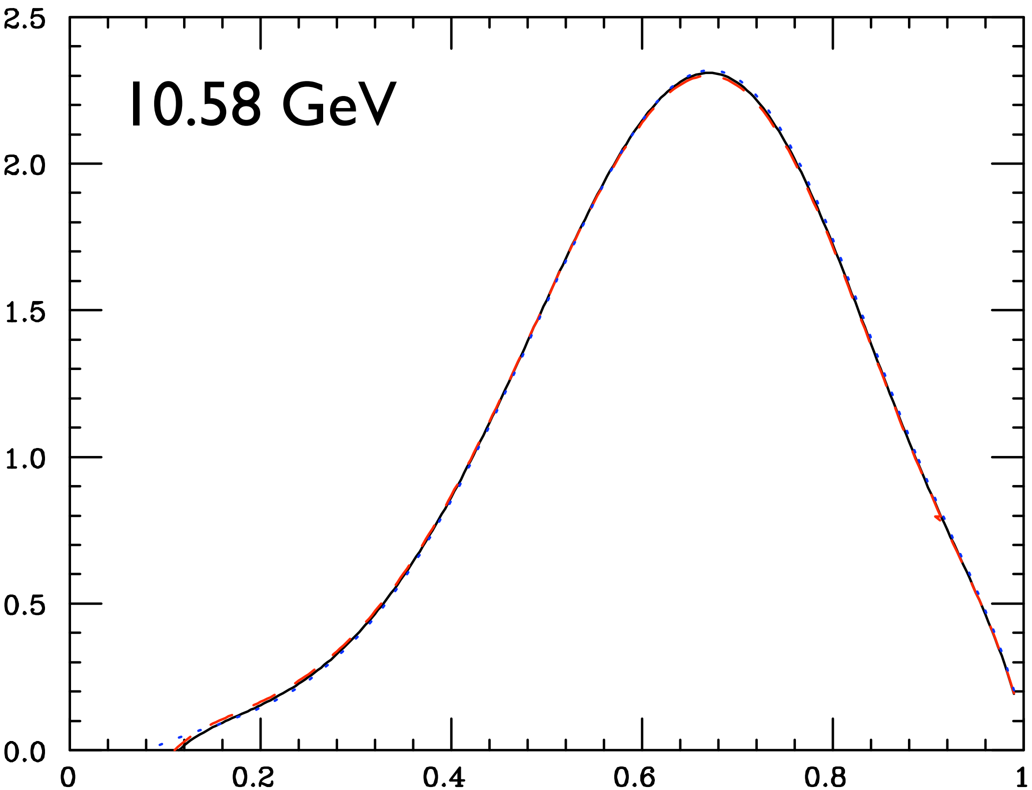
OPAL 1994



ALEPH 1999



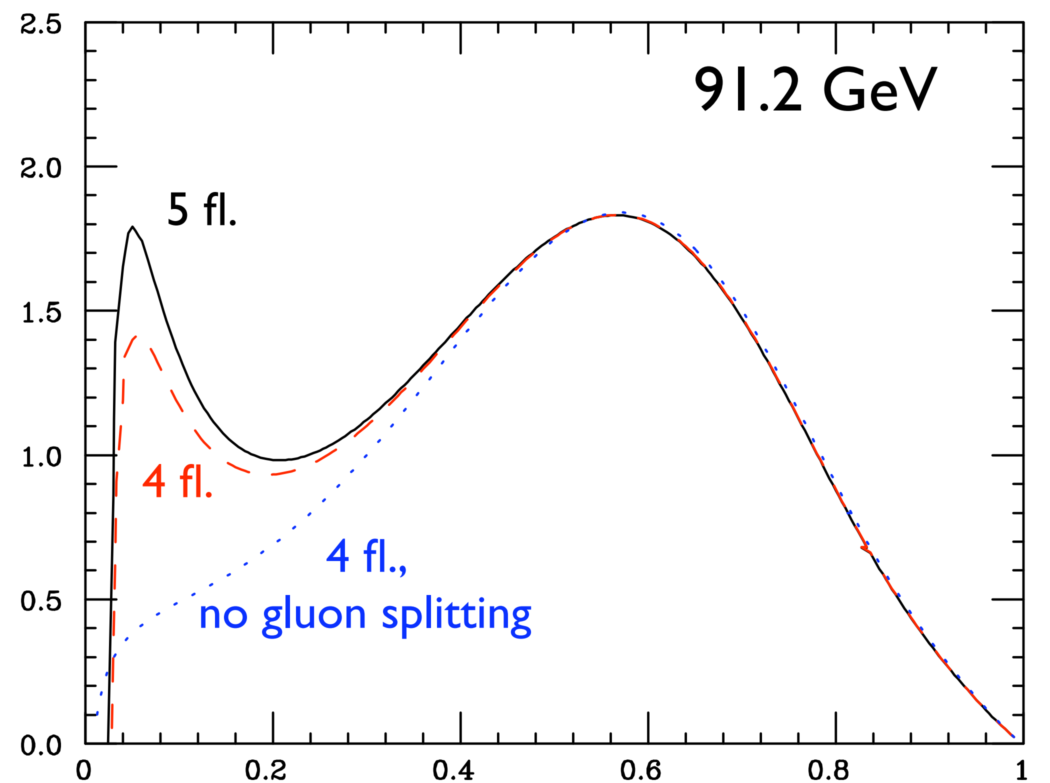
D* Fragmentation Functions at LEP and CLEO/BELLE



fit

[see Carlo Oleari's talk]

prediction



- Impact of bottom threshold numerically very limited
- Number of active flavours relevant in gluon splitting region and at high energy

Outlook: light hadron FF's fit

Analogously to the parton distribution functions case, evolving through a threshold allows to **dynamically generate** the heavy quark fragmentation functions

Recall:

$$D_h^{(n)}(x,\mu) = D_{\bar{h}}^{(n)}(x,\mu) = \int_x^1 \frac{dy}{y} D_g(x/y,\mu) \frac{\alpha_S}{2\pi} C_F \frac{1+(1-y)^2}{y} \left[\log\frac{\mu^2}{m^2} - 1 - 2\log y\right]$$

This will allow to perform fits to light hadron fragmentation data parametrizing only the **three light** quarks and the gluon FF's. The **charm** and **bottom** ones will be **radiatively generated**

Eventually, a comparison with the `five light flavours' sets available today will be possible

Kretzer, '00
Kniehl, Kramer, Poetter, '00
Bouhris, Fontannaz, Guillet, Werlen, '01
....

Work in progress

Conclusions

- NLO-accurate determination of heavy quark thresholds matching conditions for fragmentation functions
- Heavy quark FF's do **NOT** vanish at threshold, like PDF's instead do to NLO
- Numerical impact very limited when evolving charm FF through bottom threshold
- All ingredients now available for performing a NLO/NLL light flavour fragmentation analysis with dynamically generated heavy quark FF's. Work in progress