Small-x Resummation and Factorisation Schemes

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- ## Aim of the work
  - Small-x resummed splitting functions in MS scheme at subleading $\log x$ level within RGI approach
  - Relation between MS-schemes (used in fixed order) and Q_0-scheme (natural in $\boldsymbol{k}$-factorisation for small-x resummation)
- ## Outline
  - Recall $\boldsymbol{k}$-factorisation and BFKL equation
  - Generalisation to arbitrary space-time dimension $D = 4 + 2\varepsilon$
  - Solution of the equation
  - MS vs. Q_0 scheme change
  - Generalisation to subleading level
  - Numerical results
  - Quark-gluon mixing
  - Numerical results
  - Conclusions

Recalling k-factorisation and Q_0-scheme

$\boldsymbol{k}$-factorisation

Structure functions in moment space $\quad N-1 \equiv \omega \sim \alpha_{\mathrm{s}} \ll 1$

$$F_{i,\omega}(Q^2) = \int \mathrm{d}^2\boldsymbol{k}\; h_{i,\omega}(Q^2,\boldsymbol{k})\mathcal{F}_\omega(\boldsymbol{k})$$

Unintegrated gluon density determined by BFKL equation

$$\mathcal{F}_\omega(\boldsymbol{k}) = \mathcal{F}_\omega^{(0)}(\boldsymbol{k}) + \frac{1}{\omega}\int \mathrm{d}\boldsymbol{k}'^2\; K(\boldsymbol{k},\boldsymbol{k}')\mathcal{F}_\omega(\boldsymbol{k}')$$

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Q_0-scheme: off-shell initial condition

- $\mathcal{F}_\omega^{(0)}(\boldsymbol{k}) = \delta^2(\boldsymbol{k} - Q_0)$
- $g_\omega(Q^2) = \int \mathrm{d}^2\boldsymbol{k}\; \Theta(Q^2 - \boldsymbol{k}^2)\mathcal{F}_\omega(\boldsymbol{k})$
- $q_\omega(Q^2) = \int \mathrm{d}^2\boldsymbol{k}\; H_\omega(Q^2, \boldsymbol{k})\mathcal{F}_\omega(\boldsymbol{k})$ $\qquad$ e.g., $H_\omega = h_{2,\omega} \Rightarrow$ DIS-subscheme

Remark: Q_0 anomalous dimensions are finite in the on-shell limit $Q_0 \to 0$

MS $\leftrightarrow Q_0$ relation (gluon)

Solve BFKL equation in $D = 4 + 2\varepsilon$ with γ-representation

$$\mathcal{F}_\omega(\boldsymbol{k}) = \int \frac{\mathrm{d}\gamma}{\sqrt{2\pi\varepsilon}} \quad \left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^\varepsilon \exp\left\{\frac{1}{\varepsilon}\int_0^\gamma \log\frac{\alpha_\mathrm{s}}{\omega}\chi(\gamma',\varepsilon)\mathrm{d}\gamma' - \frac{1}{2}\log\chi(\gamma,\varepsilon) + \mathcal{O}(\varepsilon)\right\}$$

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$$\stackrel{\text{s.p.}}{=} \frac{\exp\{\int_0^{\bar\gamma}\chi_1/\chi_0\}}{\bar\gamma\sqrt{-\chi_0'(\bar\gamma)}} \exp\left\{\frac{1}{\varepsilon}\int_0^{\frac{\alpha_s}{\omega}\left(\frac{Q^2}{\mu^2}\right)^\varepsilon} \frac{\mathrm{d}a}{a}\,\bar\gamma(a)\right\}$$

$$\chi(\gamma,\varepsilon) = \chi_0(\gamma) + \varepsilon\chi_1(\gamma)\cdots$$

$$1 = \frac{\alpha_s}{\omega}\left(\frac{Q^2}{\mu^2}\right)^\varepsilon \chi_0(\bar\gamma)$$

$$\equiv \quad R\Big(\bar\gamma\big(\frac{\alpha_s(Q^2)}{\omega}\big)\Big)\; g_\omega^{(\mathrm{MS})}(Q^2)$$

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Anomalous dimensions

$$\gamma^{(\overline{\mathrm{MS}})} = \gamma^{(Q_0)} + \beta_0\alpha_\mathrm{s}\frac{\partial\log R}{\partial\log\alpha_\mathrm{s}} + \text{NLL}x$$

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Pomeron singularity in $\bar\gamma$ for $\omega < 4\log(2)\alpha_s$; NLx hierarchy unstable

Scheme change resummation

Leading Pomeron singularities

$\Rightarrow$ small-x resummation required for the scheme change too

This can be achieved by implementing the scheme-change ...

$$g_\omega^{(\mathrm{MS})}(Q^2) = g_\omega^{(Q_0)}(Q^2) \Big/ R\Big(\bar{\gamma}\big(\alpha_\mathrm{s}(Q^2), \omega\big)\Big)$$

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Consistency at LLx $\quad \tilde\rho_0(\gamma) = 1/\gamma R(\gamma)$

$$\rho(Q^2/\boldsymbol{k}^2) = \int \frac{\mathrm{d}\gamma}{2\pi\mathrm{i}}\left(\frac{Q^2}{\boldsymbol{k}^2}\right)^\gamma \frac{1}{\gamma\,R(\gamma)}$$

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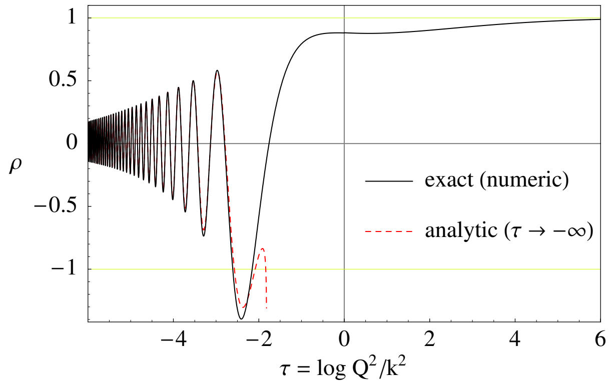

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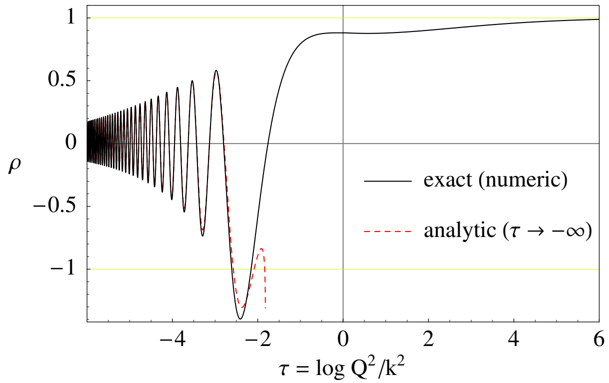
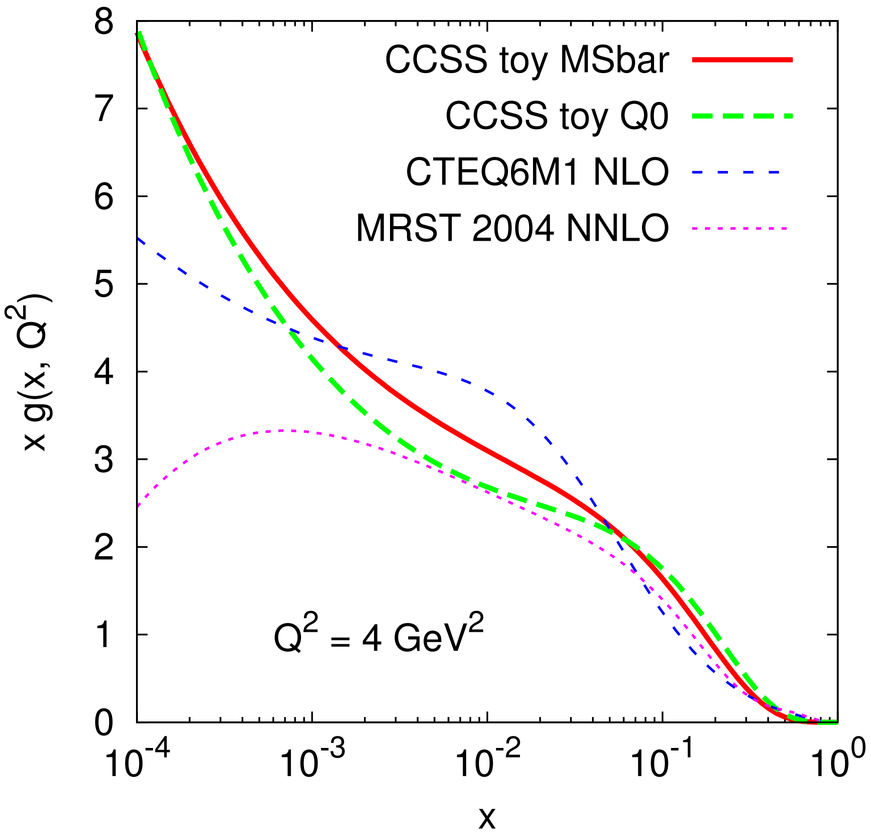


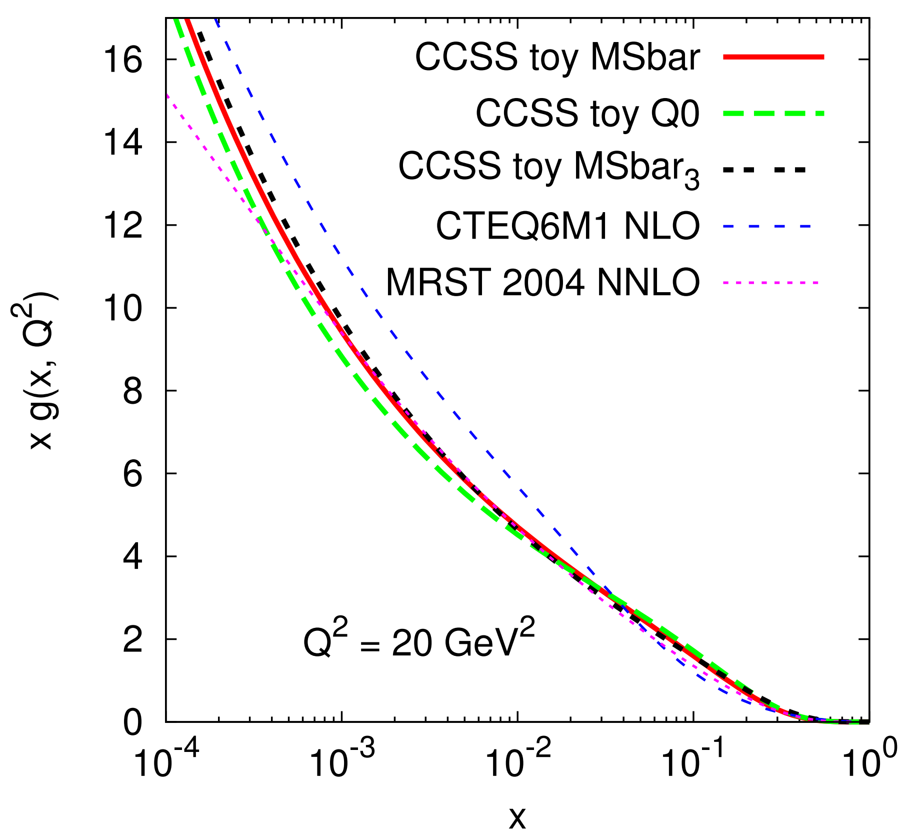
If $\mathcal{F}_\omega(\boldsymbol{k})$ is computed in the RGI approach,

the effective anomalous dimension γ_{eff} is much smoother than $\bar\gamma_{\mathrm{LL}x}$

Resummed gluon density

Valence-like initial condition $\mathcal{F}(x, \boldsymbol{k}) = A\, x^{0.5}(1-x)^5\, \delta(\boldsymbol{k} - \boldsymbol{k}_0)$, $(\boldsymbol{k}_0^2 = 0.55\ \mathrm{GeV}^2)$
not fine-tuned to get good agreement at large x. $(A \Rightarrow$ momentum sum rule = 1/2)

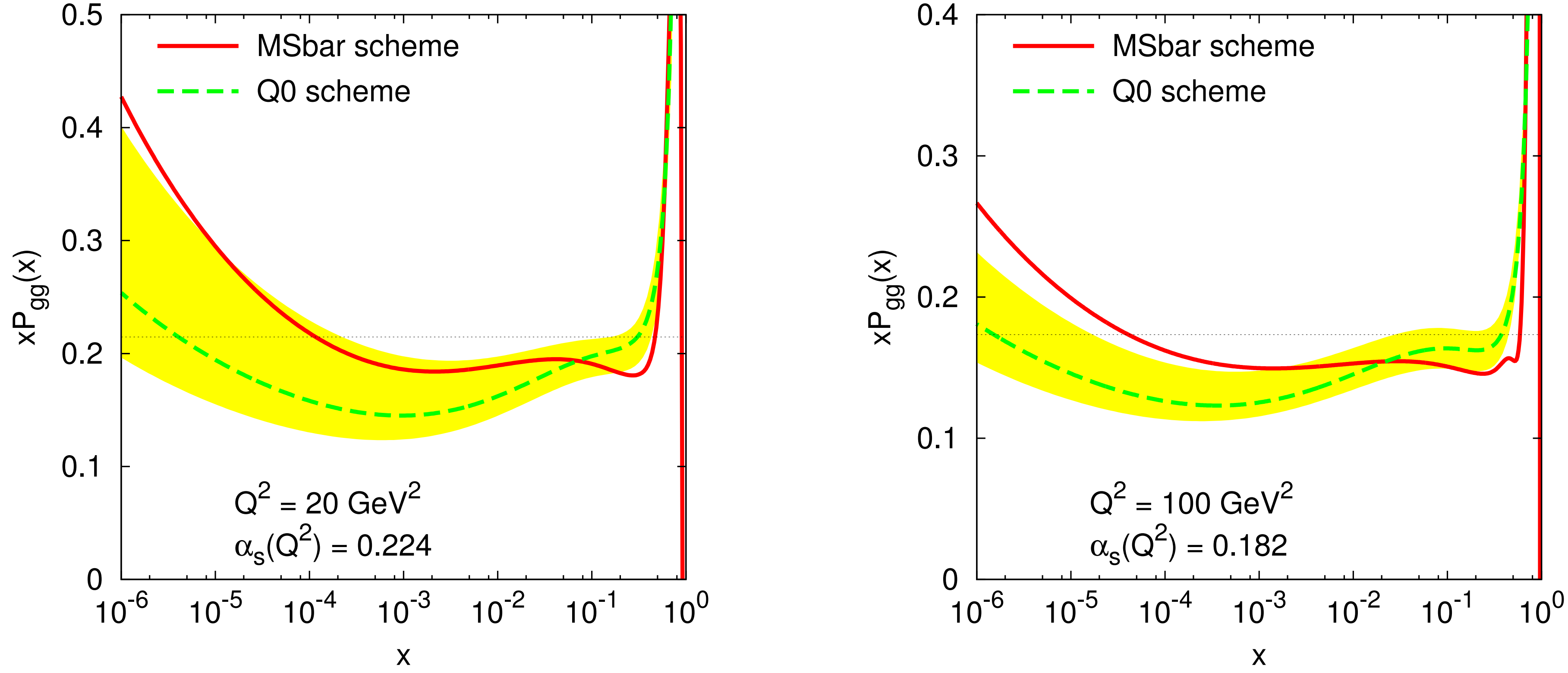




- Difference between $\overline{\mathrm{MS}}$ and Q_0 is modest compared to that between CTEQ and MRST
- No tendency for the $\overline{\mathrm{MS}}$ gluon to go negative (despite large oscillations of ρ)

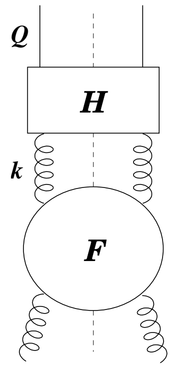
Resummed gluon splitting function

Reliable only for $x \lesssim 10^{-1}$ (ω-dependence of scheme change is important at finite x)



- Splitting function more sensitive to the scheme change than the density itself
- At low Q^2 scheme difference (NLx) within renormalisation scale uncertainty (NNLx); the latter decreases more rapidly as Q^2 is increased
- At large Q^2 the effect of factorisation scheme change is not negligible

Quark-Gluon mixing (approx)



In a generic p-scheme $$q_\omega^{(p)}(Q^2,\varepsilon) = \alpha_s(Q^2)\int \mathrm{d}^{2+2\varepsilon}\boldsymbol{k}\; H^{(p)}\Big(\frac{Q^2}{\boldsymbol{k}^2},\varepsilon\Big)\mathcal{F}_\omega(\boldsymbol{k},\varepsilon)$$

The residue of the characteristic function $\mathcal{H}(\gamma,\varepsilon)$ at the collinear pole $\gamma=-\varepsilon$ is universal

$$\mathcal{H}(\varepsilon) = \left[\frac{T_R}{2\pi}\,\frac{2}{3}\,\frac{1+\varepsilon}{(1+2\varepsilon)(1+\frac{2}{3}\varepsilon)}\right]\left[\frac{\mathrm{e}^{\varepsilon\psi(1)}\Gamma^2(1+\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(1+2\varepsilon)}\right] \equiv \mathcal{H}^{\mathrm{rat}}(\varepsilon)\mathcal{H}^{\mathrm{tran}}(\varepsilon)\;.$$

Define the transcendental coefficients T_n: $\quad \sum_{n=0}^\infty(-\varepsilon)^n T_n\Big(\frac{\alpha_s}{\omega}\Big) \equiv \mathcal{H}^{\mathrm{tran}}(\varepsilon) R\Big(\frac{\alpha_s}{\omega},\varepsilon\Big)$

$$\gamma_{qg}^{(\overline{\mathrm{MS}})}(\alpha_s,\omega) = \alpha_s\mathcal{H}^{\mathrm{rat}}\bigg(-\gamma_0\Big(\frac{\alpha_s}{\omega}\Big)\frac{1}{1+\partial_{\log\alpha_s}}\bigg)\sum_{n=0}^\infty\bigg(\gamma_0\Big(\frac{\alpha_s}{\omega}\Big)\frac{1}{1+\partial_{\log\alpha_s}}\bigg)^n T_n\Big(\frac{\alpha_s}{\omega}\Big)\;,$$

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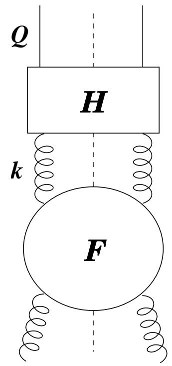

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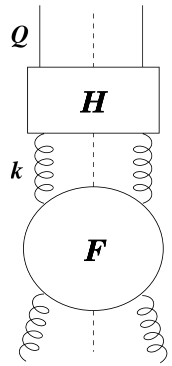
Rational approximation: Neglect transcendental coefficients (they start at relative $\mathcal{O}\left(\gamma^3\right)$)

$$\gamma_{qg}^{(\overline{\mathrm{MS}})}\Big|_{\mathrm{rat}} = \frac{\alpha_s T_R}{4\pi}\left(e^{2\frac{\bar{\alpha}_s}{\omega}} + \frac{1}{3}e^{\frac{2}{3}\frac{\bar{\alpha}_s}{\omega}}\right)$$

Quark-Gluon mixing (approx)



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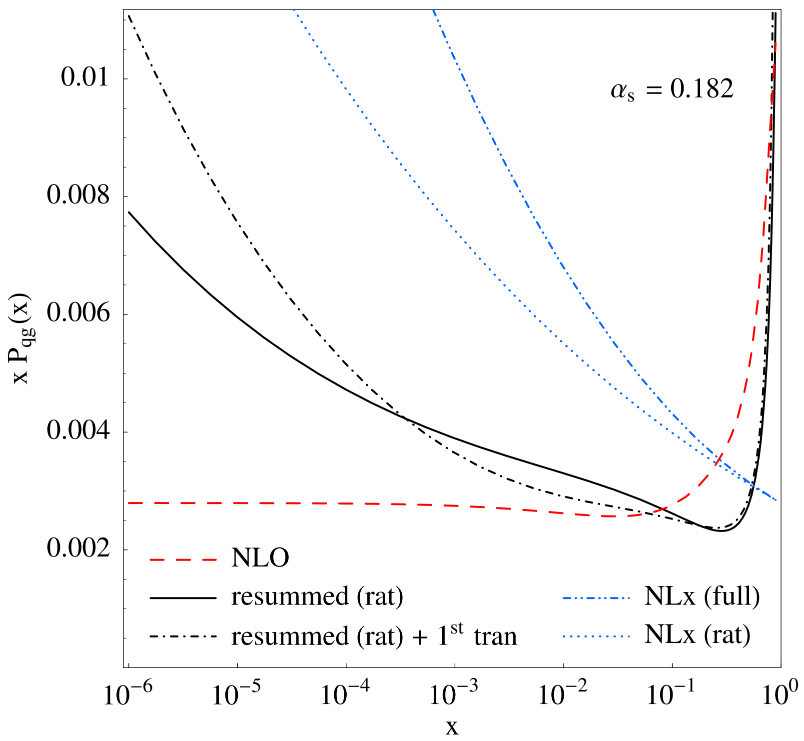
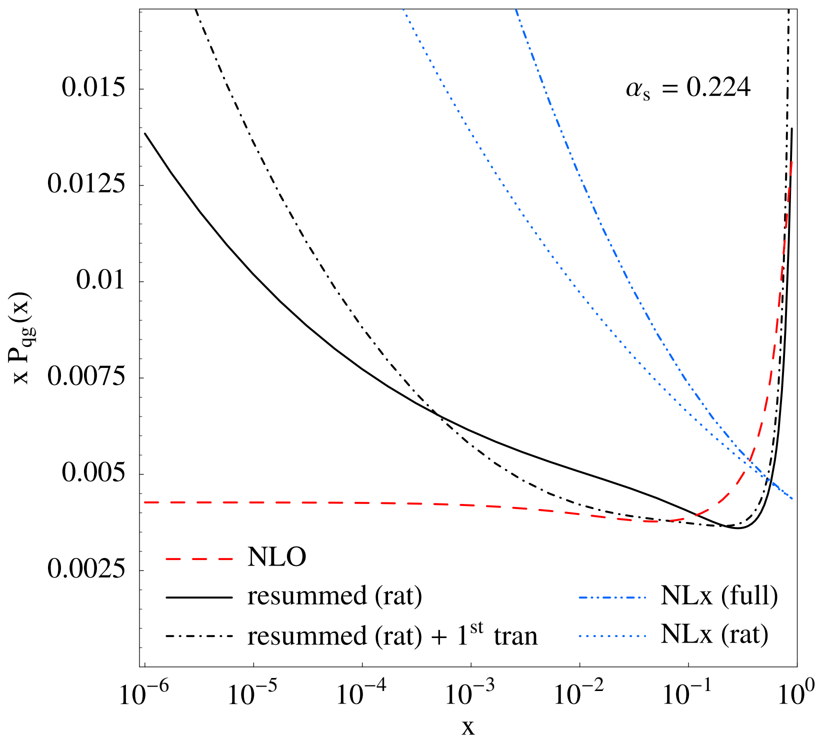
Interpretation in $\boldsymbol{k}$-factorised form: $\quad \frac{dq_{\mathrm{rat}}^{(\overline{\mathrm{MS}})}}{d\log Q^2} = \alpha_s(Q^2)\int d^2\boldsymbol{k}\; H_{\mathrm{rat}}^{(\overline{\mathrm{MS}})}\Big(\frac{Q^2}{\boldsymbol{k}^2}\Big)\mathcal{F}_\omega(\boldsymbol{k})$

with characteristic function $\quad \mathcal{H}_{\mathrm{rat}}^{(\overline{\mathrm{MS}})}(\gamma) = \frac{\alpha_s T_R}{4\pi}\,\frac{1}{\gamma}\left(e^{2\gamma}+\frac{1}{3}e^{\frac{2}{3}\gamma}\right)$

Quark splitting function (approx)

Proper comparison only for $x \lesssim 10^{-1}$ (finite-x PT terms not included)





- resummation effects are sizeable even around $x \sim 10^{-3}$ and somewhat larger than the gluonic ones
- resummation effects much smaller than pure NLx ones (confirm that resummed gluon anomalous dimension is pretty small)
- transcendental corrections are small (smaller than in pure NLx case)

Conclusions

- We have proposed a $\boldsymbol{k}$-factorised form of the $Q_0 \to \overline{\mathrm{MS}}$ scheme-change which allows a convergent leading log$(\boldsymbol{x})$ hierarchy, because of the smoothness of the resummed anomalous dimension
- The gluon density is rather insensitive to the scheme-change, while its splitting function is somewhat sensitive.
- Resummation effects for the $\overline{\mathrm{MS}}$ quark are more important, but much smaller than those at NLx level.

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The scheme-change can be calculated in a reliable way in a fully (matrix) resummed approach as well