HOW RESUMMATION STABILIZES

PERTURBATIVE EVOLUTION AT SMALL x

Stefano Forte
Università di Milano

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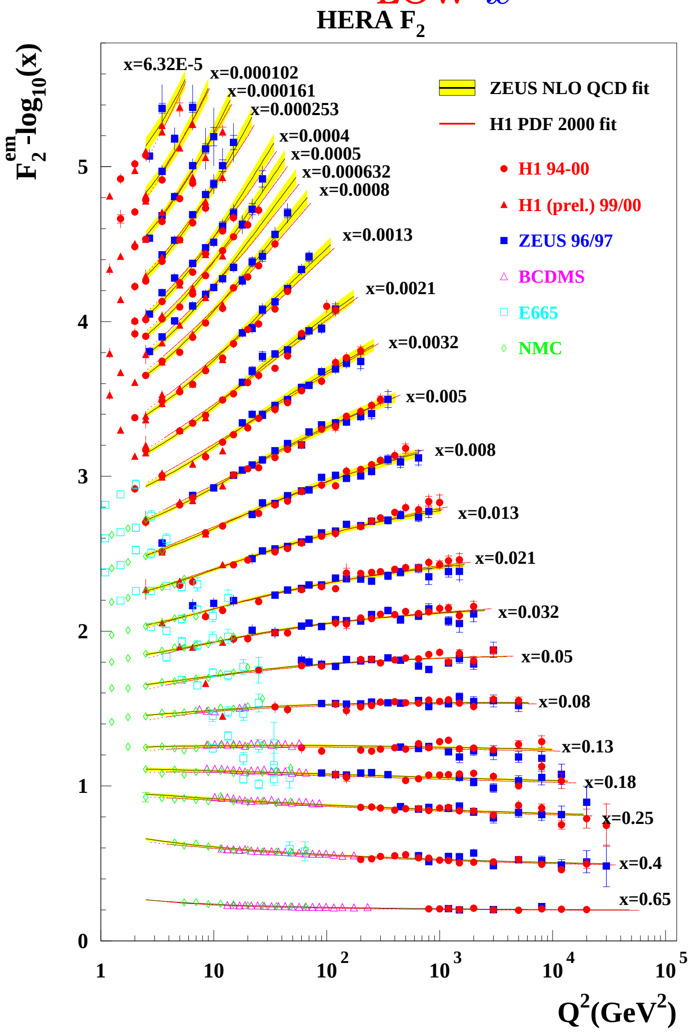
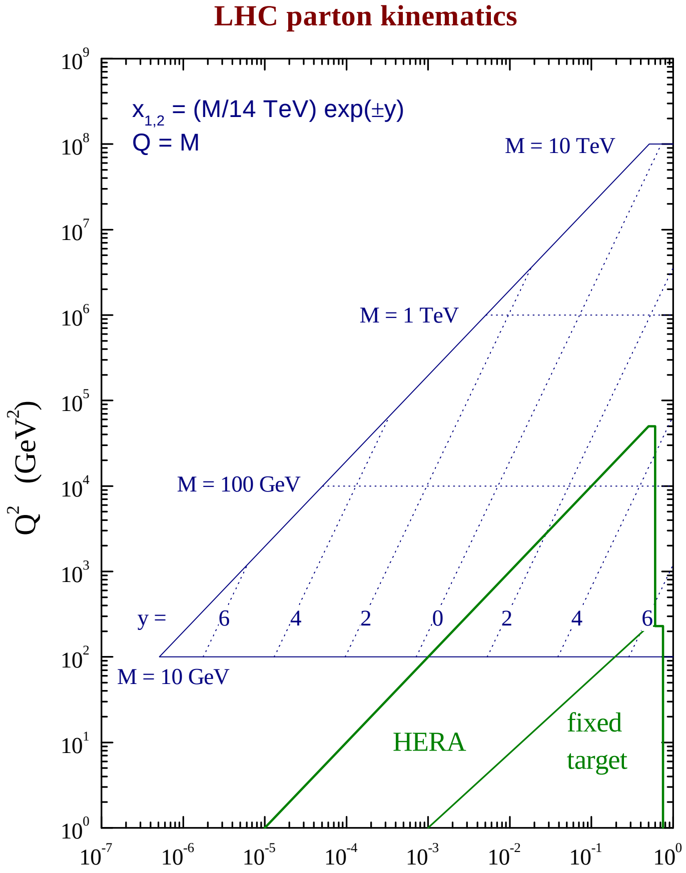
WHAT'S THE PROBLEM?
NLO QCD LOOKS OK, FROM HERA TO LHC

FIXED–ORDER PERTURBATIVE QCD DESCRIBES

PERFECTLY THE HERA DATA DOWN TO VERY

LOW x





DOES LHC PHENOMENOLOGY

REALLY REQUIRE

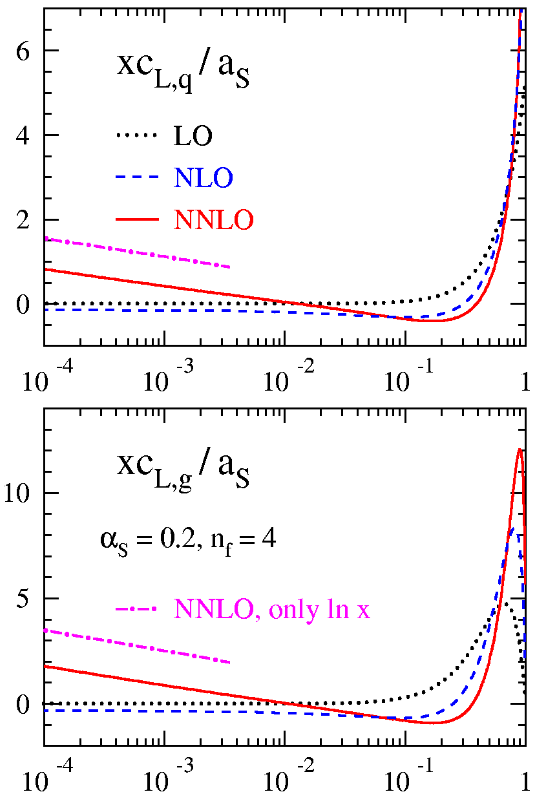
SMALL x RESUMMATION?

THE NNLO CORRECTIONS

DIS HAS BEEN COMPUTED TO THREE LOOPS BY MOCH, VERMASEREN AND VOGT (2005)

THEORY

- Perturbation theory unstable
- leading log approx no good

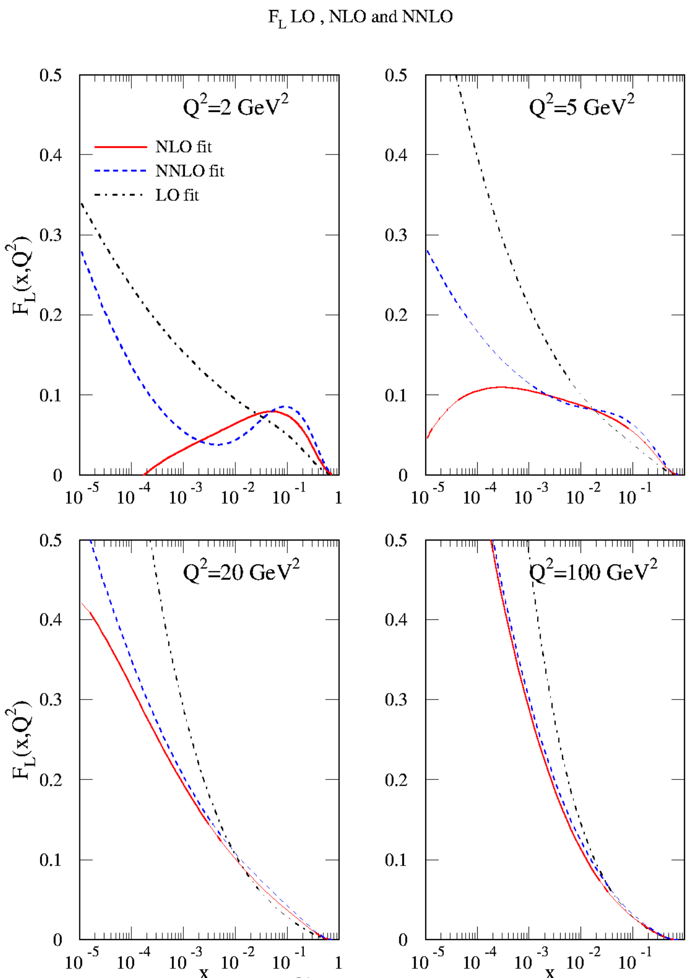


the coefficient function C_L

Moch, Vermaseren, Vogt 2005

PHENOMENOLOGY

F_L FIT



MRST 2004

AS ONE WOULD EXPECT!...

LARGE $\ln 1/x$ $\Rightarrow$ LARGE PERTURBATIVE CORRECTIONS

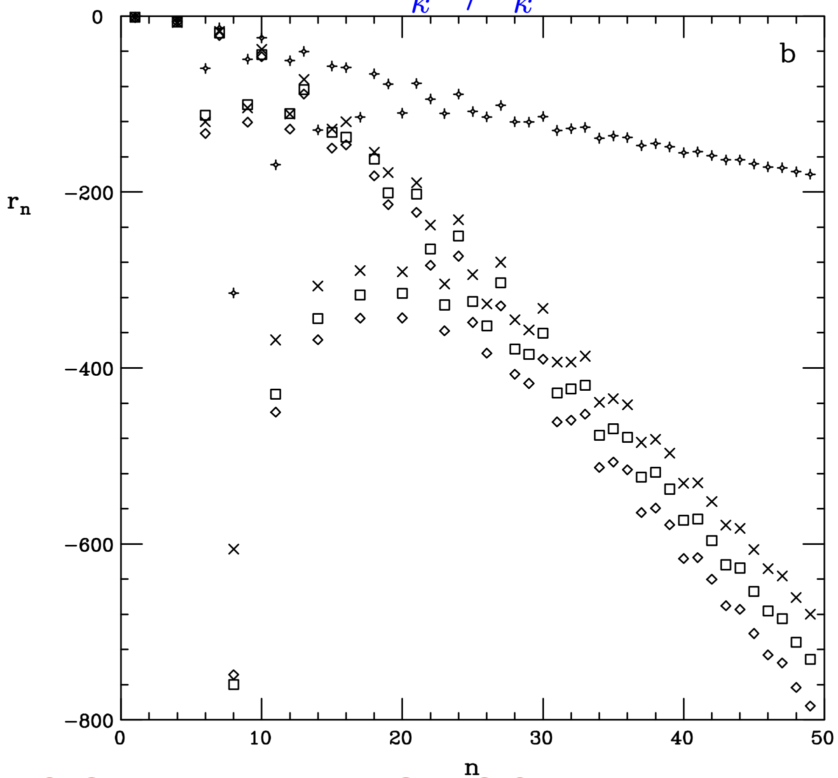
$\Rightarrow$ POORLY BEHAVED EXPANSION

$$\gamma_N \sim \sum_{k=1}^{\infty} c_k \left(\frac{\alpha_s}{N}\right)^k; \quad c_k = c_k^{(0)} + \alpha_s c_k^{(1)}$$

$c_k^{(0)} \downarrow$ BFKL $\qquad \alpha_s c_k^{(1)} \downarrow$ FADIN-LIPATOV

- LO SERIES CONVERGES SLOWLY: NEED $\sim 3.6\alpha_s \ln \frac{1}{x}$ TERMS
- NL SMALL x CORRECTION GROWS LINEARLY WITH PERT. ORDER: $\lim_{k\to\infty} c_k^{(1)}/c_k = -Ck$; $C \sim 6 \div 16$ according to factn. scheme
  @HERA, ($\alpha_s\times$NLO)/LO$\sim$ $-1.5 \div 15$!

$c_k^{(1)}/c_k^{(0)}$



WHERE ARE THESE CONTRIBUTIONS?

PROGRESS

CONSISTENT THEORY $\Rightarrow$ CONSISTENT PHENOMENOLOGY

- MOMENTUM CONSERVATION $\Leftrightarrow$ DUALITY ABF '99
  MATCHING OF ALTARELLI–PARISI & BFKL EVOLUTION ABF '99, CCSS '03
- RUNNING COUPLING $\Leftrightarrow$ FACTORIZATION ABF '01
  DETERMINATION OF ANOMALOUS DIMENSION FROM BFKL SOLUTION ABF '02, CCS '99
- EXCHANGE SYMMETRY Salam '98 $\Leftrightarrow$ PERTURBATIVE STABILITY
  RESUMMED PERTURBATIVE EXPANSION WELL–DEFINED ORDER BY ORDER ABF '05

G. ALTARELLI, R. BALL, S.F.

CIAFALONI, COLFERAI, SALAM, STAŚTO

CATANI, HAUTMANN (93), FADIN, LIPATOV (95-98) + CAMICI, CIAFALONI, DEL DUCA, SCHMIDT (98)

$\Rightarrow$ SUBLEADING RESUMMATION COEFFICIENTS

RELATED WORK BY THORNE, C. SCHMIDT, FORSHAW, ROSS, SABIO VERA, BARTELS, KANCHELI, K. ELLIS, HAUTMANN. . .

THE FIRST INGREDIENT: DUALITY (fixed coupling)

THE ALTARELLI-PARISI EQN IS AN INTEGRO–DIFFERENTIAL EQUATION $\Rightarrow$ IT CAN BE EQUIVALENTLY VIEWED AS Q^2–EVOLUTION EQUATION FOR x–MOMENTS (usual RG eqn.), OR x–EVOLUTION EQUATION FOR Q^2–MOMENTS(BFKL eqn.)

EVOLUTION IN $t = \ln Q^2$

$$\frac{d}{dt} G(N,t) = \gamma(N,\alpha_s)\; G(N,t)$$

MELLIN x–MOMENTS

$$G(N,t) = \int_0^\infty d\xi\, e^{-N\xi}\; G(\xi,t)$$

EVOLUTION IN $\xi = \ln 1/x$

$$\frac{d}{d\xi} G(\xi,M) = \chi(M,\alpha_s)\; G(\xi,M)$$

MELLIN Q^2–MOMENTS

$$G(\xi,M) = \int_{-\infty}^\infty dt\, e^{-Mt}\; G(\xi,t)$$

THE TWO EQUATIONS HAVE THE SAME SOLUTIONS

PROVIDED THE EVOLUTION KERNELS ARE RELATED BY

$$\chi(\gamma(N,\alpha_s),\alpha_s) = N$$

$$\gamma(\chi(M,\alpha_s),\alpha_s) = M$$

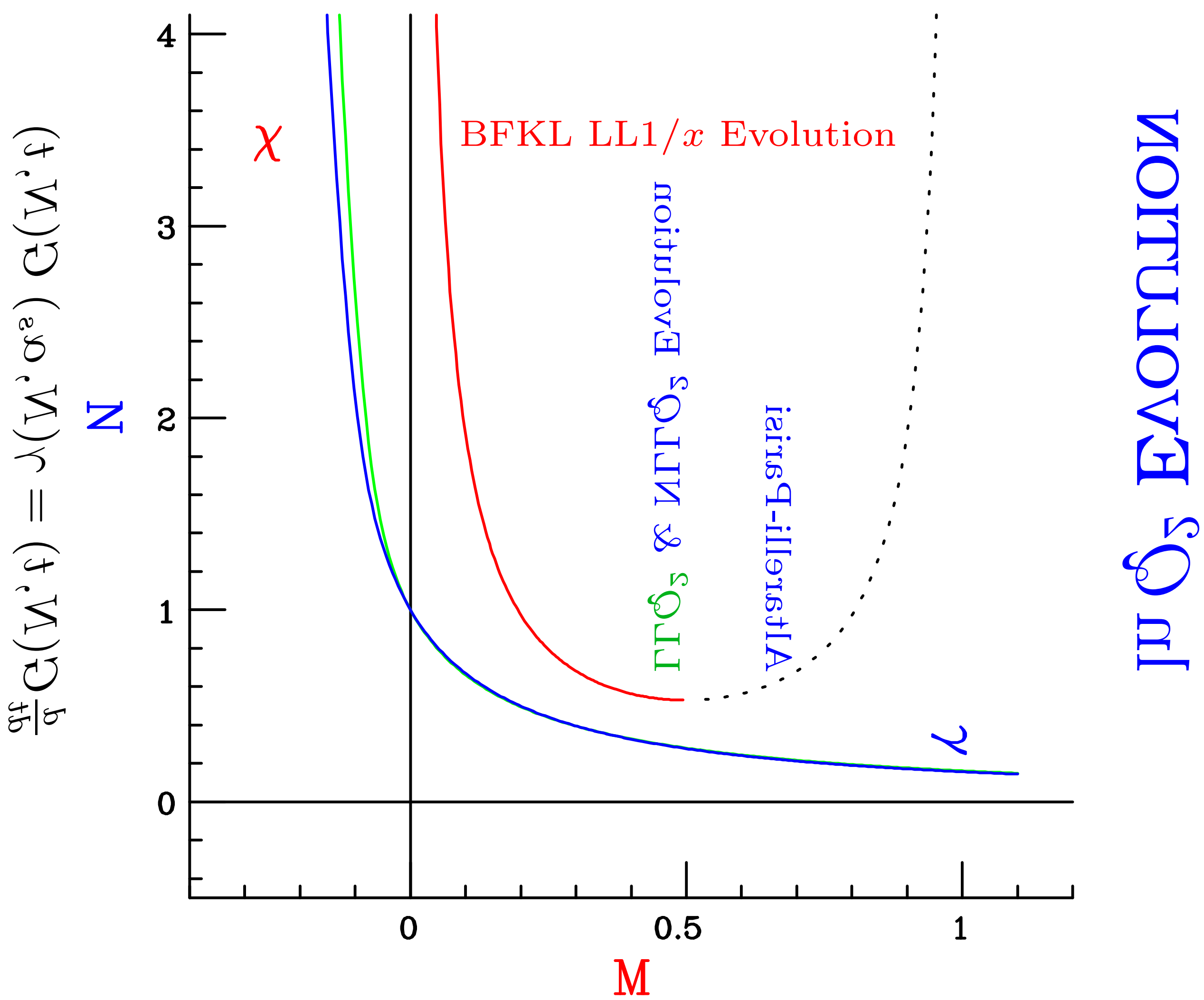
& BOUNDARY CONDITIONS RELATED BY

$$H_0[M] \to G_0(N) = H_0[\gamma(N,\alpha_s)]/\chi'(\gamma(N,\alpha_s))$$

. . . CAN SWITCH FROM LLQ^2 TO LL$1/x$

CHOOSING THE EVOLUTION KERNEL

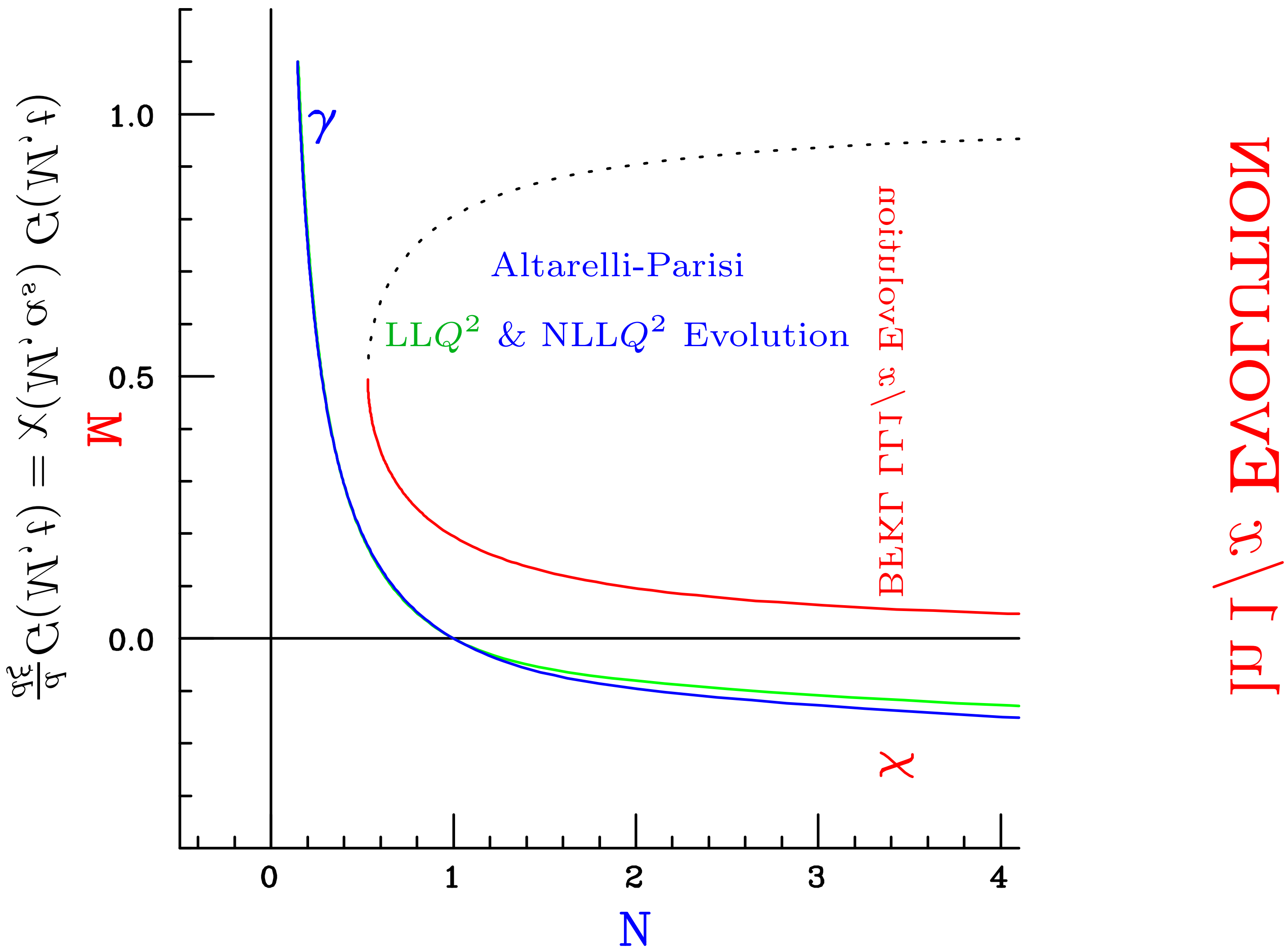
$\ln 1/x$ EVOLUTION



$$\frac{d}{d\xi} G(M,t) = \chi(M,\alpha_s)\ G(M,t)$$

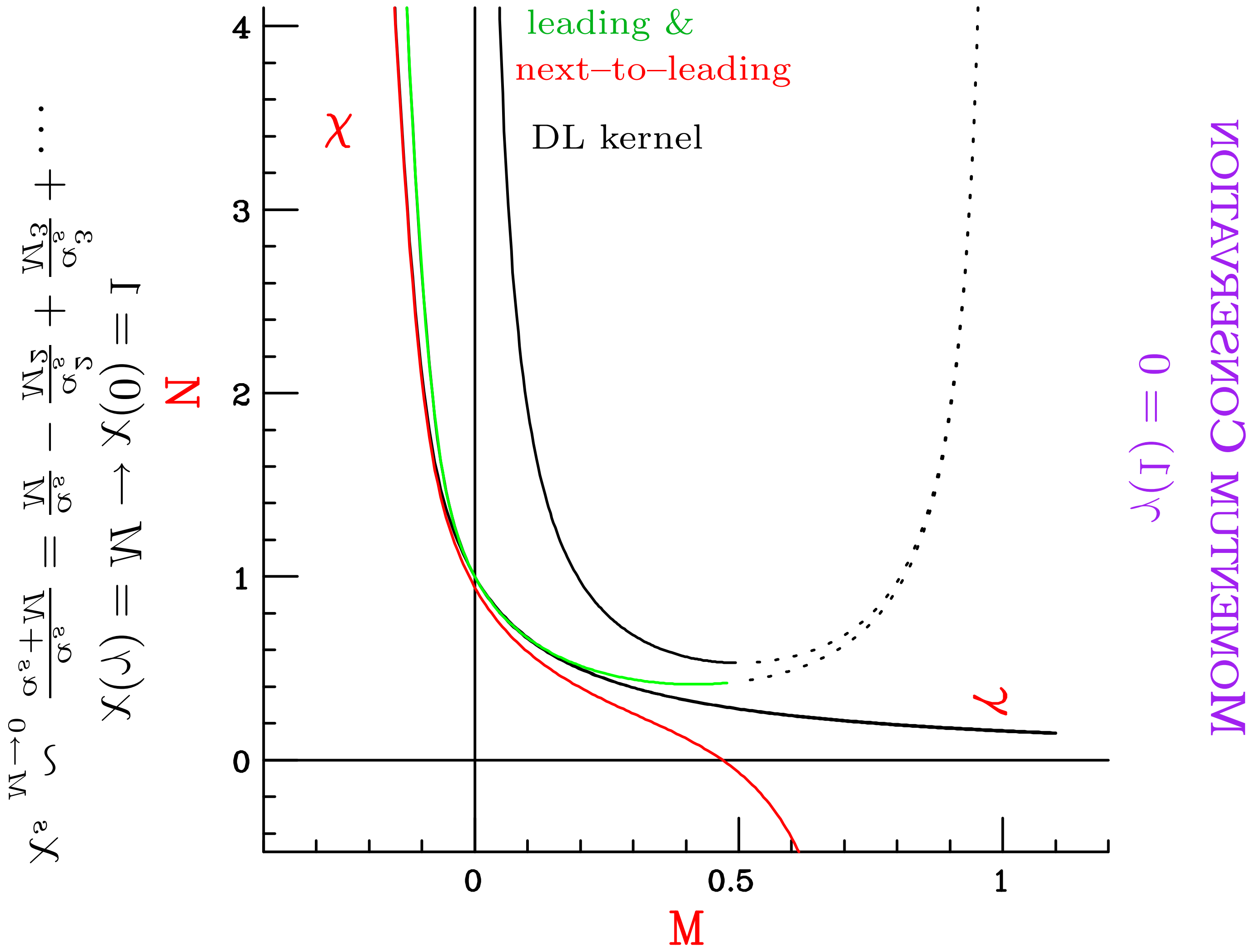
. . . IN EITHER EQUATION!

$\ln Q^2$ EVOLUTION



$$\frac{d}{dt}G(N,t) = \gamma(N,\alpha_s)\ G(N,t)$$

Double–Leading Evolution

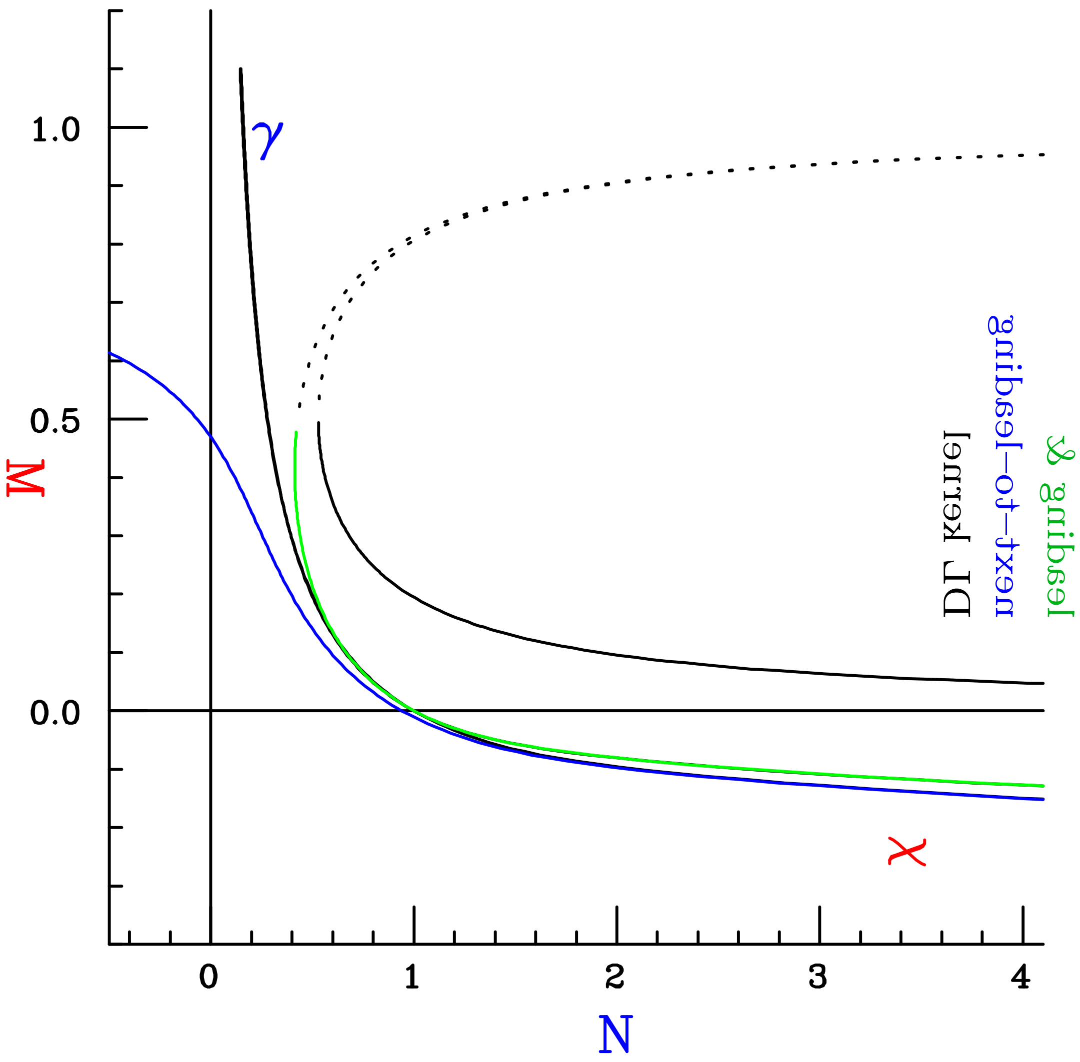


- The DL kernel has a well–behaved perturbative expansion
- DL is close to the LLQ^2 result for $N \gtrsim 0.3 \leftrightarrow M \lesssim 0.2$, close to $LL1/x$ for $M \sim 1/2$

Double–Leading Evolution

Momentum Conservation!

$$\gamma(1) = 0$$



$$\chi(\gamma) = N \rightarrow \chi(0) = 1$$

$$\chi_s(M) \underset{M \to 0}{\sim} \frac{\alpha}{\alpha + M} = \frac{\alpha}{M} - \frac{\alpha^2}{M^2} + \frac{\alpha^3}{M^3} + \ldots$$

THE SECOND INGREDIENT: RUNNING COUPLING

- THE RUNNING OF THE COUPLING $\alpha(t) = \alpha_\mu[1 - \beta_0\alpha_\mu t + \ldots]$ ($t \equiv \ln \frac{Q^2}{\mu^2}$) IS LEADING LOG Q^2, BUT NEXT–TO–LEADING LOG $\frac{1}{x}$
- UPON M-MELLIN TRANSFORMATION ($\ln x$ EVOLUTION) $\alpha_s(t)$ BECOMES AN OPERATOR:

$$\hat{\alpha}_s \equiv \frac{\alpha_{\mu^2}}{1 - \beta_0\alpha_{\mu^2}\frac{d}{dM}} + O(\alpha_{\mu^2})$$

$\Rightarrow$ EVOLUTION EQUATION

for $G(N, M)$ with b.c. $H_0(M)$

$$NG(N, M) = \chi(\hat{\alpha}_s, M)G(N, M) + H_0(M)$$

- NOTE: OPERATOR ORDERING $\Leftrightarrow$ ARGUMENT OF THE COUPLING

$$\int_{-\infty}^{\infty} \frac{dk^2}{k^2} \sum_{p=1}^{\infty} \left[\alpha_s^p(Q^2)K_L^{(p)}(Q^2/k^2) + \alpha_s^p(k^2)K_R^{(p)}(Q^2/k^2)\right] G(\xi, k^2) \Leftrightarrow$$

$$\Leftrightarrow \sum_{p=1}^{\infty} \left[\hat{\alpha}_s^p\chi_L^{(p)}(M) + \chi_R^{(p)}(M)\hat{\alpha}_s^p\right] G(\xi, M)$$

RUNNING COUPLING DUALITY

THE OPERATOR APPROACH: (Ball, S.F. 06)

DUAL KERNEL INVERSION

$$\chi(\hat\alpha_s, \gamma(\hat\alpha_s, N)) = N$$
$$\gamma(\hat\alpha_s, \chi(\hat\alpha_s, M)) = M$$

ACTING ON $G(N, M)$

DUALITY STILL HOLDS TO ALL ORDERS!:

$\Rightarrow$ CAN DETERMINE γ (χ) AS A FUNCTIONAL OF FIXED-COUPLING DUAL γ_s (χ_s):

at LO, using $\chi_0 = N\hat\alpha^{-1}$, $\gamma \neq \gamma_s$ because $[N\hat\alpha^{-1}, \chi_0(M)] \neq 0$, so

$$\gamma\left(N\hat\alpha^{-1}\right) = \gamma_s\left(N\hat\alpha^{-1}\right) - \frac{1}{2}N\beta_0\gamma_s''\left(N\hat\alpha^{-1}\right)/\gamma_s'\left(N\hat\alpha^{-1}\right) + \ldots$$

RESULTS UP TO NLO FOR χ, NNNLO FOR γ (including β_1 terms)

(Ball, Falgari, S.F., Marzani 05)

EXACT ASYMPTOTIC SOLUTION

ASYMPTOTIC BEHAVIOUR CONTROLLED BY

MINIMUM OF $\chi(M) \Leftrightarrow$ RIGHTMOST SING. OF $\gamma(N)$

QUADRATIC KERNEL $\chi_q(\hat\alpha_s, M) = [c(\hat\alpha_s) + \frac{1}{2}\kappa(\hat\alpha_s)(M - M_s)^2]$

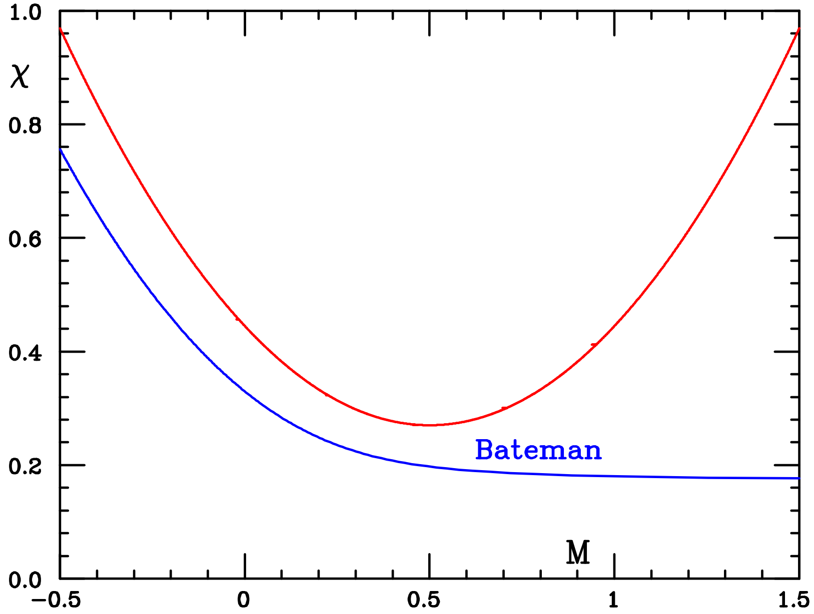
CAN SOLVE EXACTLY WITH LINEARIZED $c(\hat\alpha_s)$, $\kappa(\hat\alpha_s)$

IN TERMS OF BATEMAN FUNCTION $K_\nu(x)$:

- $G(N,t) \propto K_{2B(\alpha_s,N)}\left[\frac{1}{\beta_0\bar\alpha_s(t)A(\alpha_s,N)}\right]$

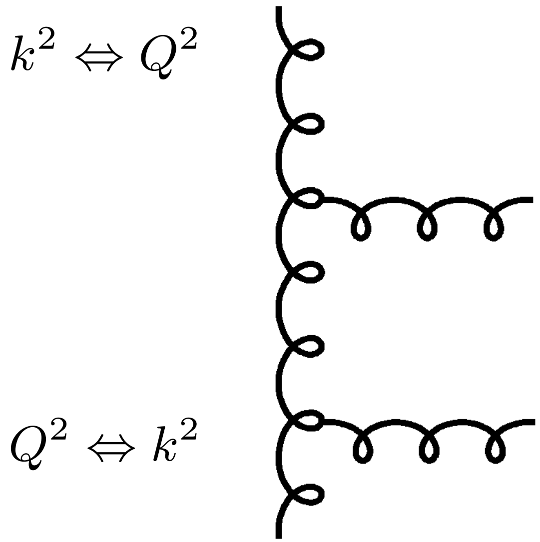
  A, B DEPEND ON α_s, N TRHOUGH c, κ
- ASYMPTOTIC LEADING LOG SMALL x SERIES RESUMMED
- BRANCH CUT IN γ REPLACED BY SIMPLE POLE

THE EFFECTIVE RESUMMED KERNEL



THE THIRD INGREDIENT: EXCHANGE SYMMETRY

DIAGRAMS FOR $\ln 1/x$ EVOLUTION KERNEL



$$\frac{d}{d\xi} G(\xi, M) = \chi(M, \alpha_s)\; G(\xi, M)$$

$$\chi(\xi, M) = \int_{-\infty}^{\infty} \frac{dQ^2}{Q^2} \left(\frac{Q^2}{k^2}\right)^{-M} \chi(\xi, \tfrac{Q^2}{k^2})$$

SYMMETRIC UPON INTERCHANGE

OF INITIAL AND FINAL PARTON VIRTUALITIES

$$Q^2 \leftrightarrow k^2 \Leftrightarrow M \leftrightarrow 1 - M$$

COLLINEAR RES. OF $\frac{1}{M}$ POLES $\leftrightarrow$ ANTICOLLINEAR RES. OF $\frac{1}{1-M}$ POLES

SYMMETRY BREAKING

- DIS KINEMATIC VARIABLES $s = \frac{Q^2}{x}$ (small x)
- RUNNING OF THE COUPLING $\alpha_s(Q^2)$

BOTH CAN BE DETERMINED EXACTLY

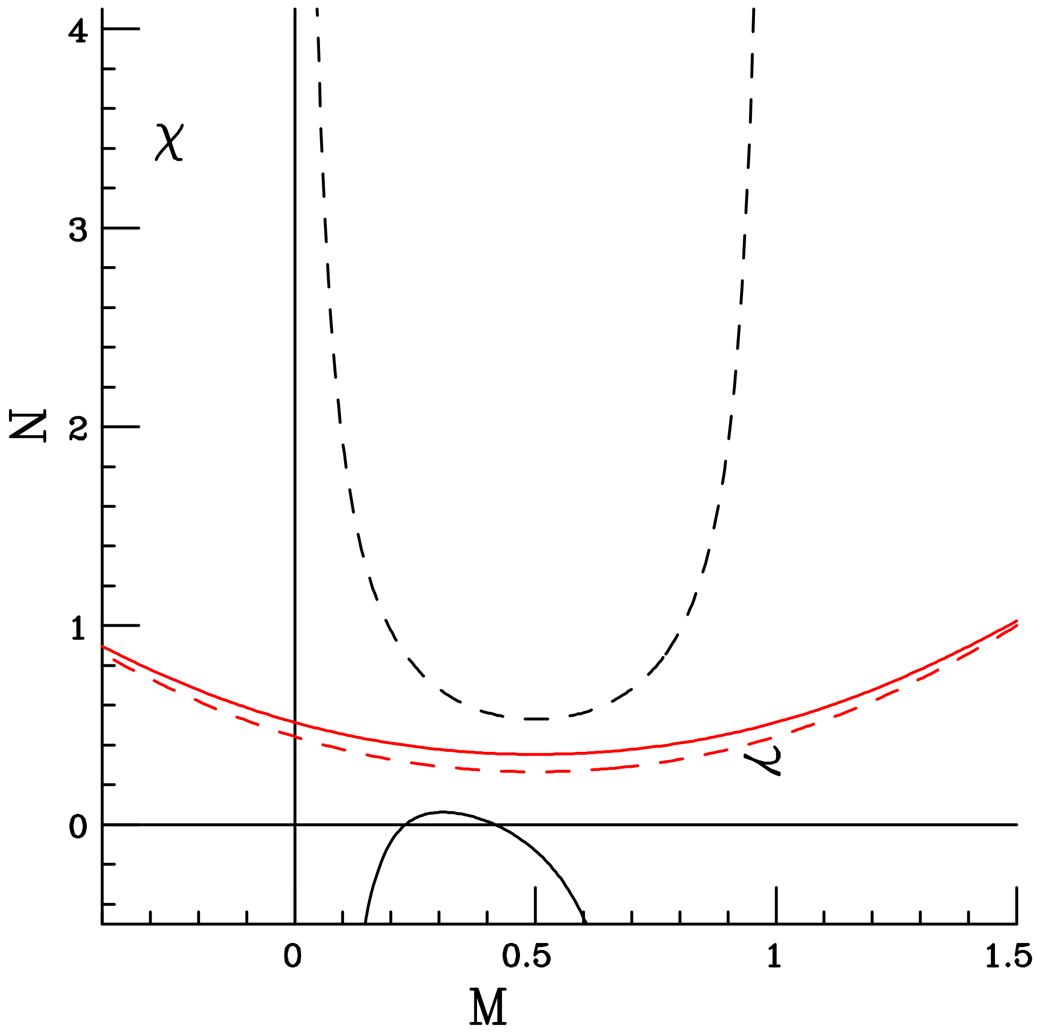
SYMMETRIZED EXPANSION

THE χ KERNEL

MOMENTUM CONSERVATION + SYMMETRY$\Rightarrow\chi$ ALWAYS HAS A MINIMUM

SYMMETRIC VARIABLES

- LO, NLO SYMMETRIC RESUMMED CLOSE TO EACH OTHER
- χ IS AN ENTIRE FCTN (QUADRATIC APPROX. IS EXCELLENT!)
- RESUMMED NLO HIGHER THAN LO



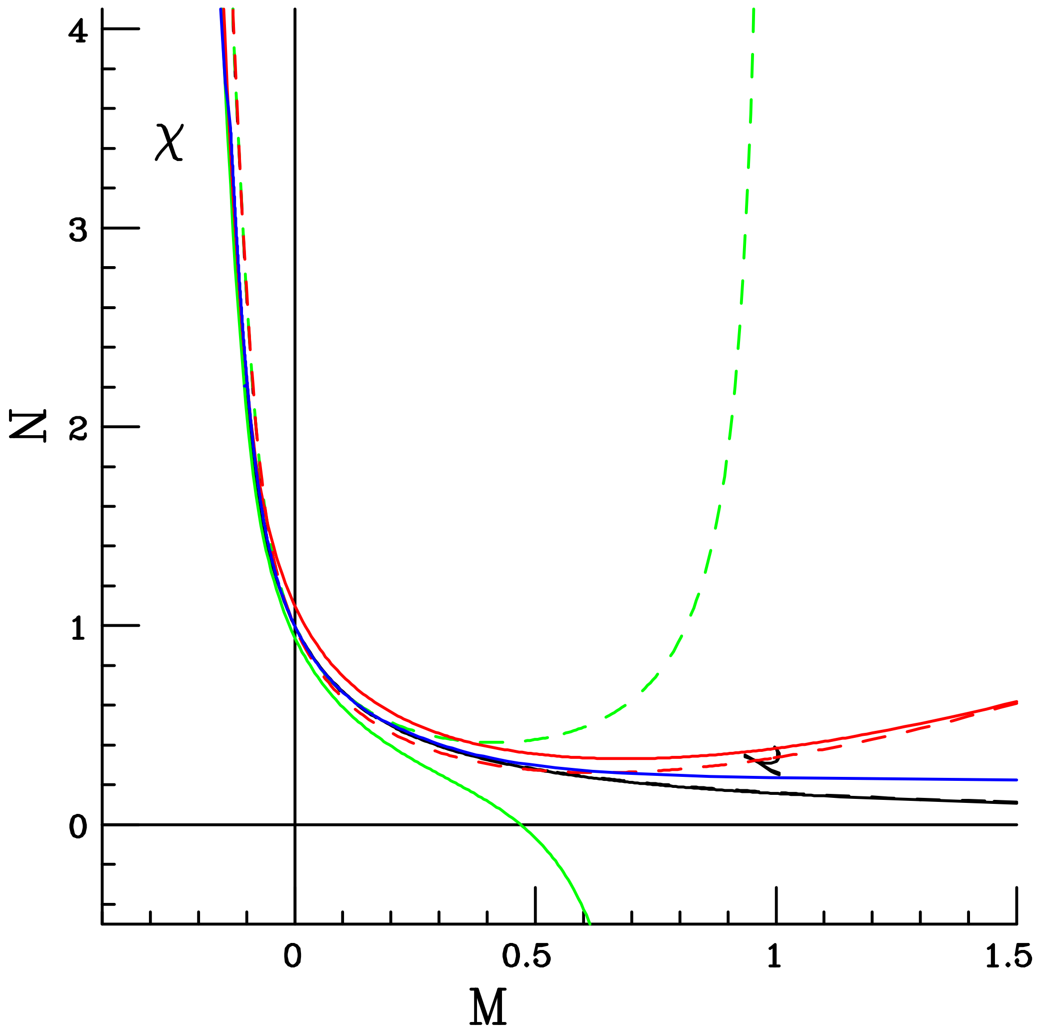
SYMMETRIZED EXPANSION

THE χ KERNEL

MOMENTUM CONSERVATION + SYMMETRY$\Rightarrow\chi$ ALWAYS HAS A MINIMUM

ASYMMETRIC VARIABLES

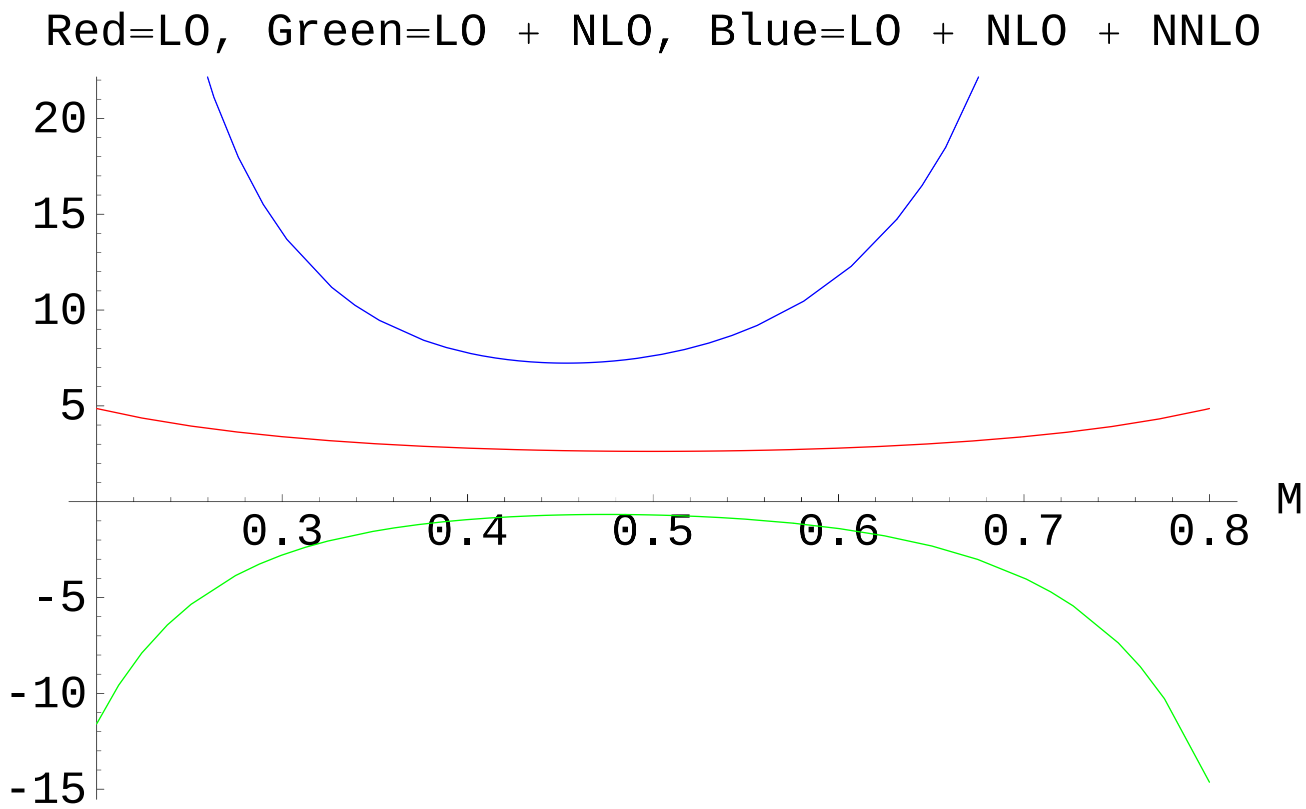
- LO, NLO SYM. CLOSE TO EACH OTHER
- LO, NLO SYM. CLOSE TO AP & RC RESUMMED
- CURVATURE & INTERCEPT SAME IN SYM. & ASYM. VARIABLES



EXPLOITING THE SYMMETRY

CAN DETERMINE THE APPROXIMATE NNLO BFKL KERNEL!

duality+symmetry+RC corrections (Ball, Falgari, S.F., Marzani, in preparation)



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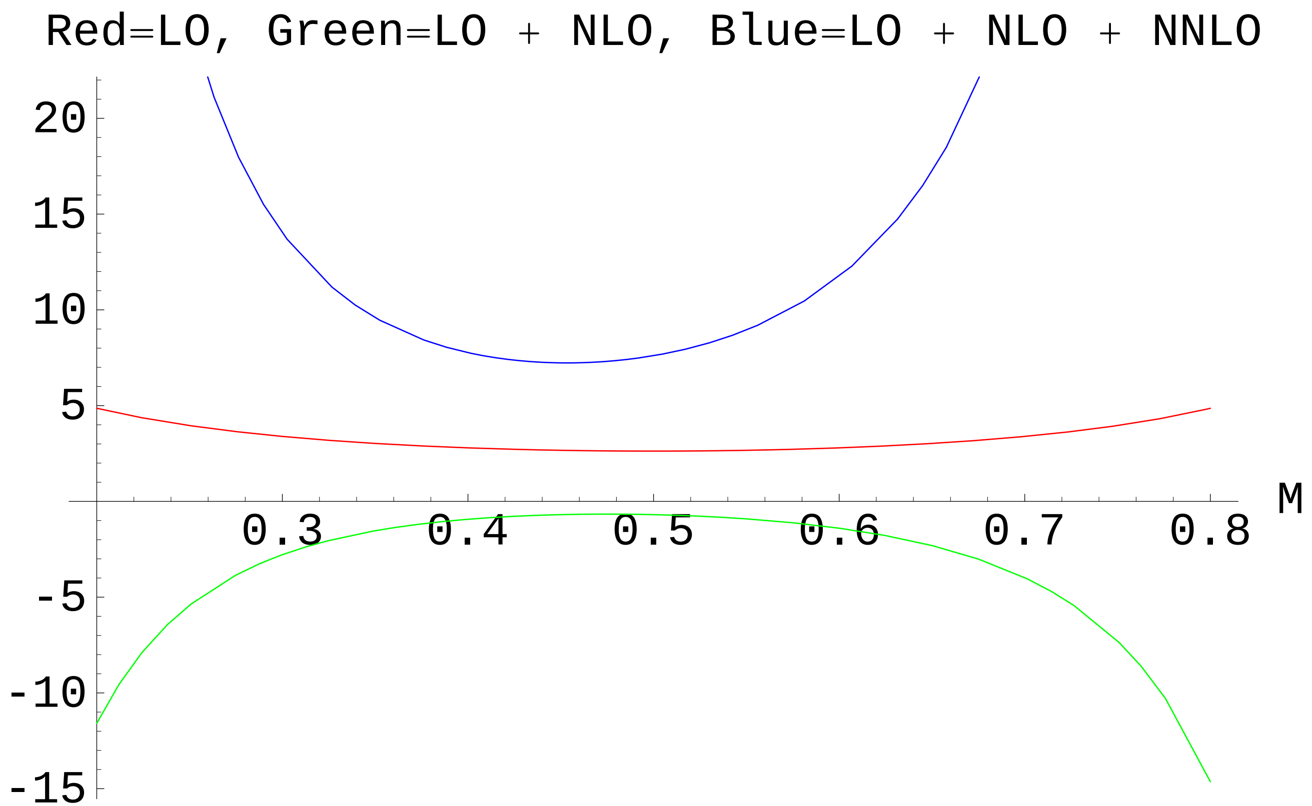


...THE BFKL EXPANSION IS NOT SO GOOD....

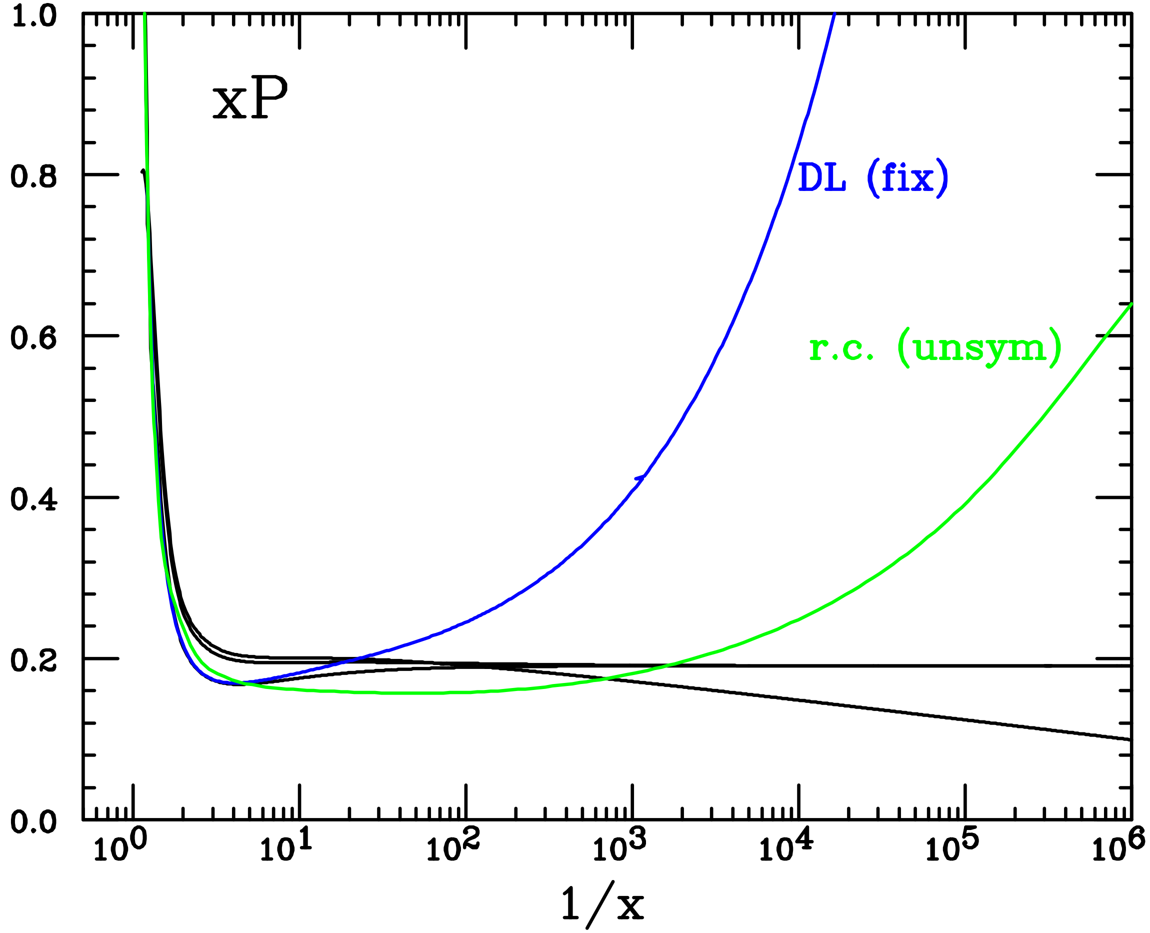
COMBINING THE INGREDIENTS...

THE RESUMMED EXPANSION OF THE SPLITTING FUNCTION

MOMENTUM CONS.+SYMMETRY+ R.G. RESUMMATION

⇒ SOFT SIMPLE POLE IN ANOMALOUS DIMENSION (REGGE BEHAVIOUR)

- BFKL RISE OF SPLITTING FUNCTION TAMED BY RUNNING COUPLING
- .
- .



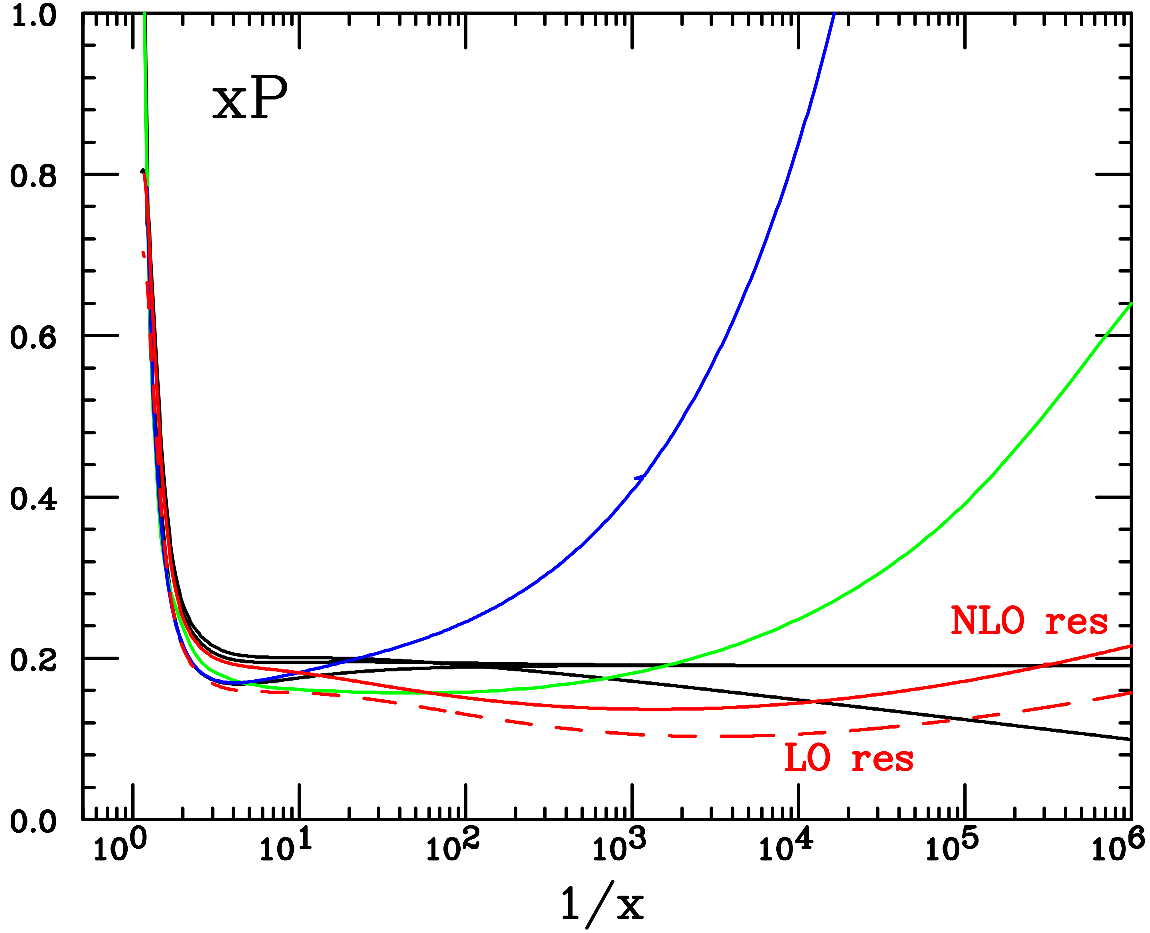
COMBINING THE INGREDIENTS...

THE RESUMMED EXPANSION OF THE SPLITTING FUNCTION

MOMENTUM CONS.+SYMMETRY+ R.G. RESUMMATION

$\Rightarrow$ SOFT SIMPLE POLE IN ANOMALOUS DIMENSION (REGGE BEHAVIOUR)

- BFKL RISE OF SPLITTING FUNCTION TAMED BY RUNNING COUPLING
- RESUMMED RESULT CLOSE TO NLO GLAP
- PERTURBATIVE RESUMMED EXPANSION STABLE

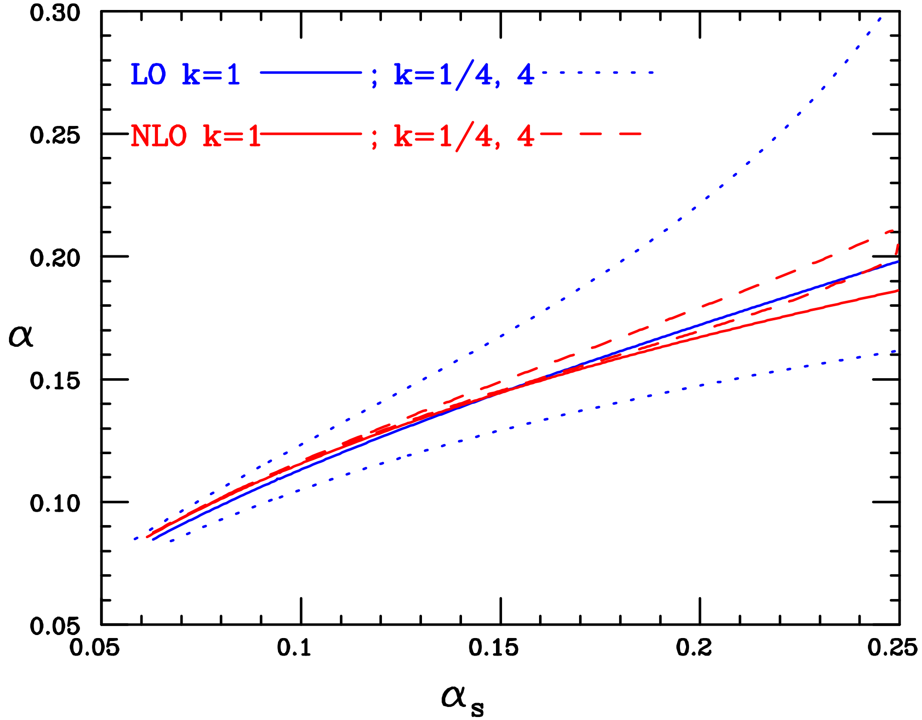


PHENOMENOLOGY: PERTURBATIVE STABILITY

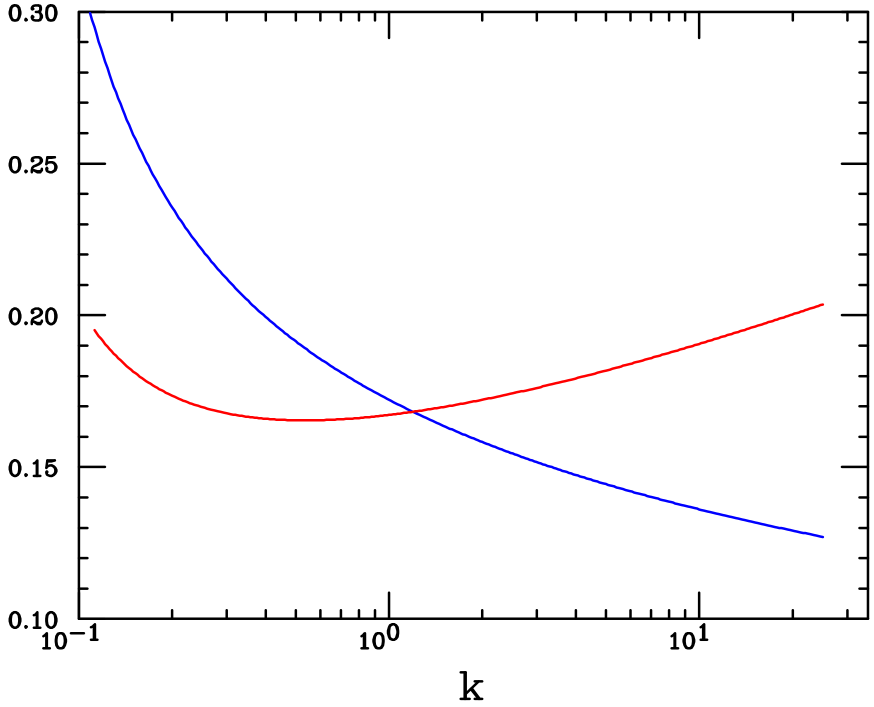
SIMPLE POLE IN ANOMALOUS DIMENSION $\Rightarrow$ ASYMPTOTIC SMALL-x POWER $G \sim x^{-\alpha}$

- LEADING AND NEXT-TO-LEADING POWER QUITE FLAT, CLOSE TO EACH OTHER
- DEP. ON RENORMALIZATION GETS WEAKER $\Rightarrow$ NLO STABILITY

SMALL-x POWER VS. α_s



SMALL-x POWER VS. REN. SCALE

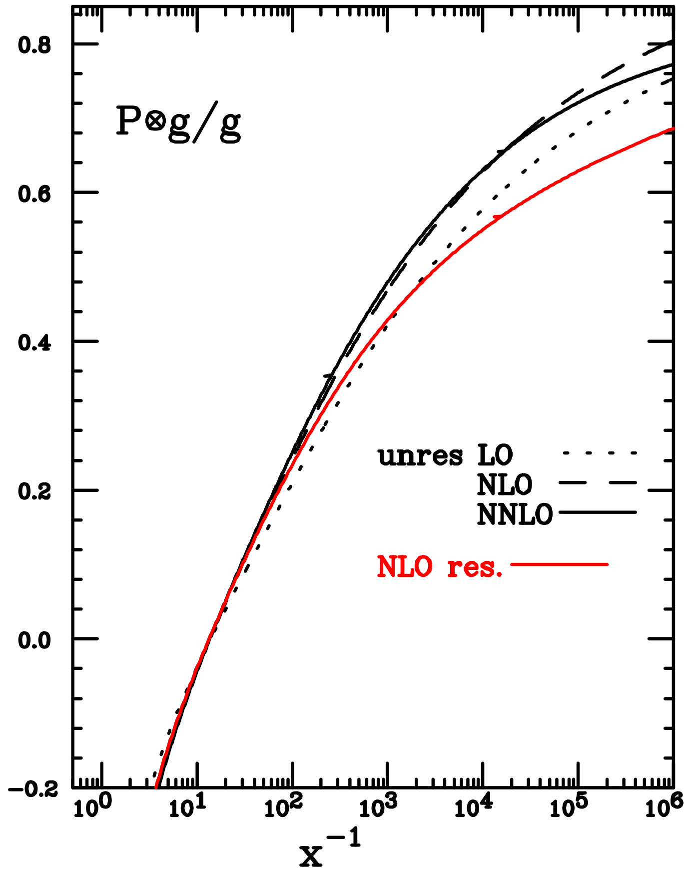


PHENOMENOLOGY: GLUON EVOLUTION

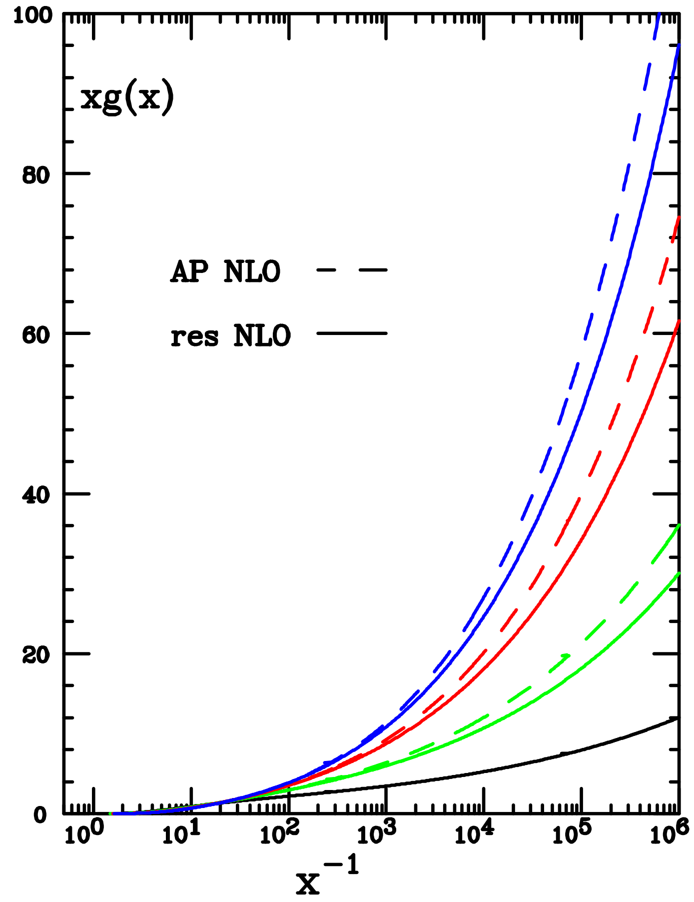
EVOLVE STARTING GLUON DISTRIBUTION: $xg(x) = x^{-0.18}(1-x)^5$ AT $Q^2 = (4\ \mathrm{GeV})^2$

TO $Q^2 =$ $(10\ \mathrm{GeV})^2$ $(100\ \mathrm{GeV})^2$ $(1000\ \mathrm{GeV})^2$

CONVOLUTION



EVOLUTION



- RESUMMATION SOFTENS THE SMALL-x BEHAVIOUR OF PDFS
- ASYMPTOTIC POWER RISE ONLY SETS IN VERY LATE

CONCLUSIONS

- Matching of Altarelli-Parisi and BFKL fully understood at running coupling level
- Generic small x behaviour: (soft) simple pole
- resummed perturbative expansion stable
- Resummation softens gluon down to pretty low x

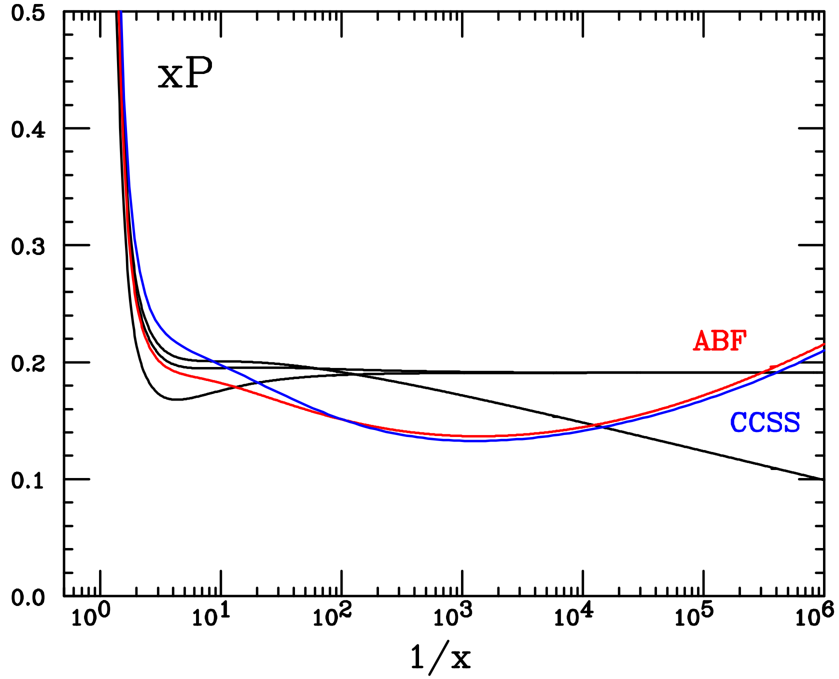
EXTRAS

ABF vs. CCSS

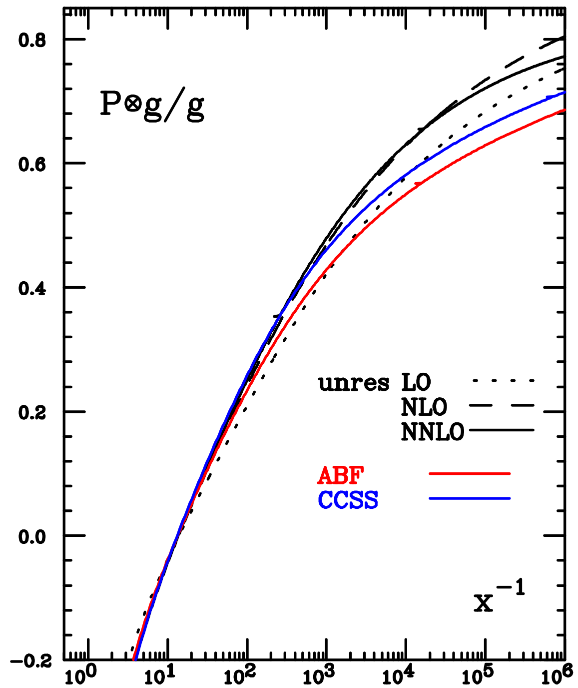
the CCSS approach:

- numerical but exact solution of x-space BFKL
- ABF & CCSS very close at fixed coupling level
- CCSS treats running BFKL exactly, but GLAP at fixed coupling

SPLITTING FUNCTION



GLUON CONVOLUTION

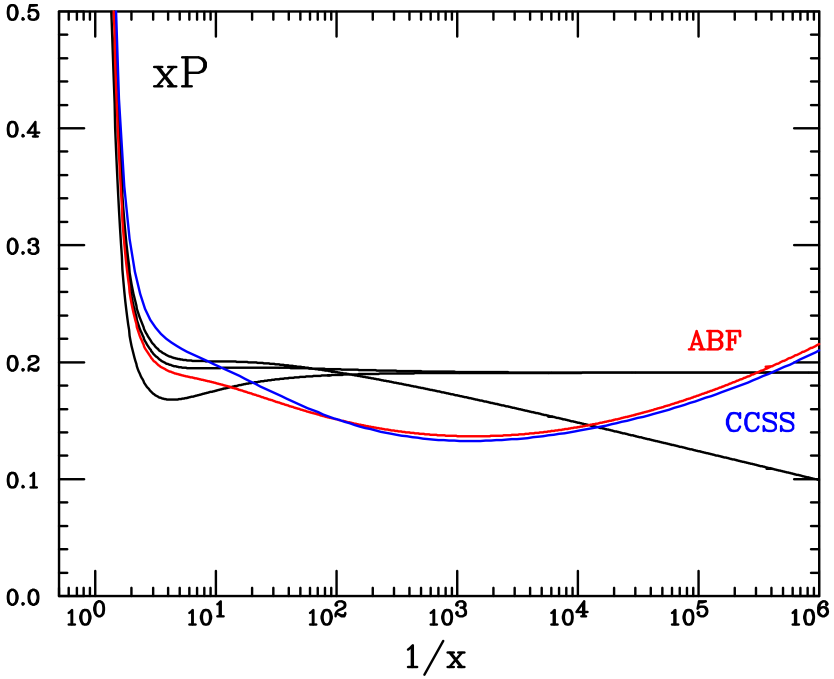


ABF VS. CCSS

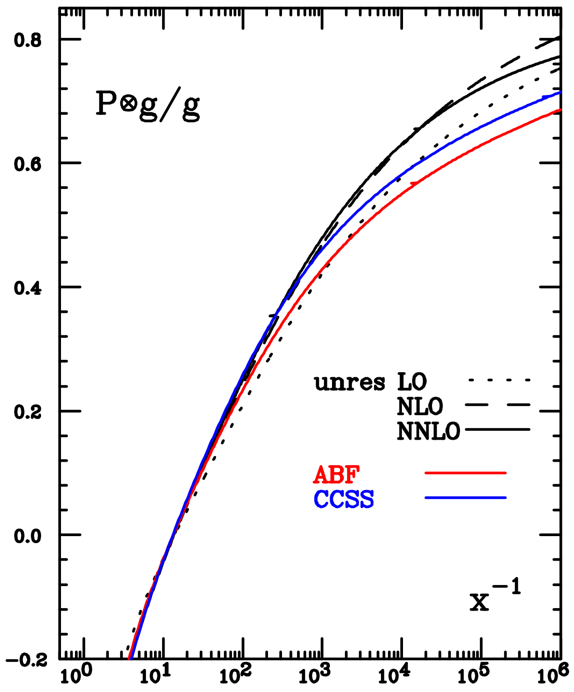
THE CCSS APPROACH:

- NUMERICAL BUT EXACT SOLUTION OF x-SPACE BFKL
- ABF & CCSS VERY CLOSE AT FIXED COUPLING LEVEL
- CCSS TREATS RUNNING BFKL EXACTLY, BUT GLAP AT FIXED COUPLING

SPLITTING FUNCTION



GLUON CONVOLUTION



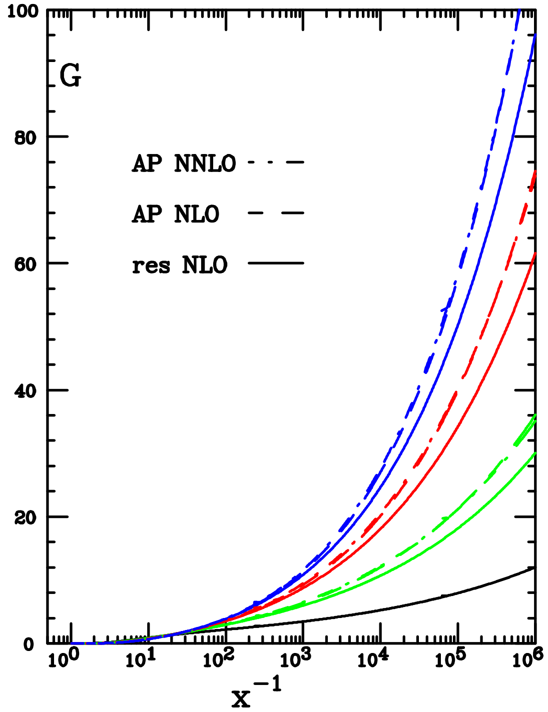
- SPLITTING FUNCTIONS CLOSER THAN TH. ERROR (!)
- DIFFERENCE IN GLUON DUE TO LARGE-x TERMS

NNLO corrections & K-factors

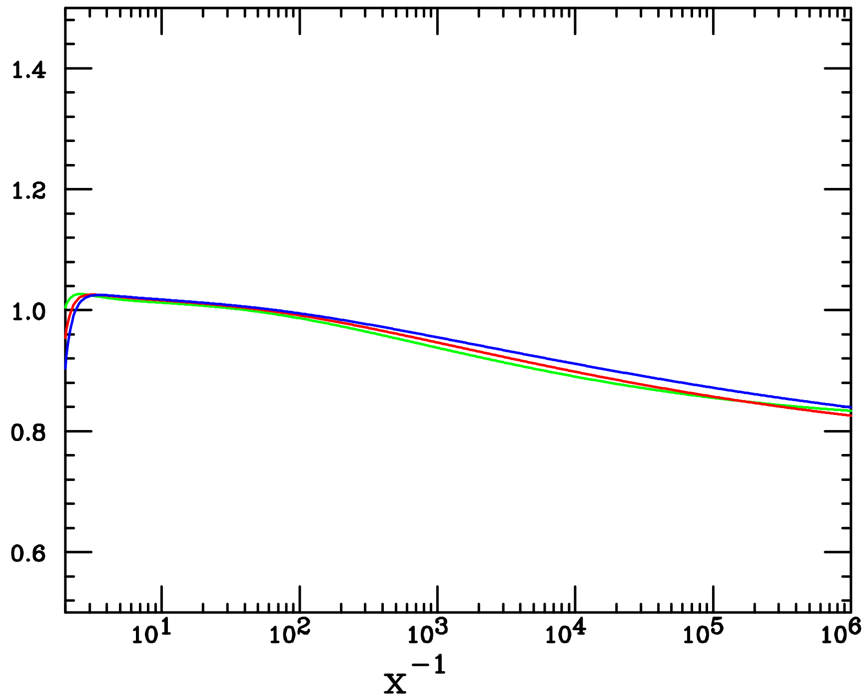
evolve starting gluon distribution: $xg(x) = x^{-0.18}(1-x)^5$ at $Q^2 = (4\ \mathrm{GeV})^2$

to $Q^2 = (10\ \mathrm{GeV})^2\ (100\ \mathrm{GeV})^2\ (1000\ \mathrm{GeV})^2$

gluon evolution



res/AP NLO K-factor

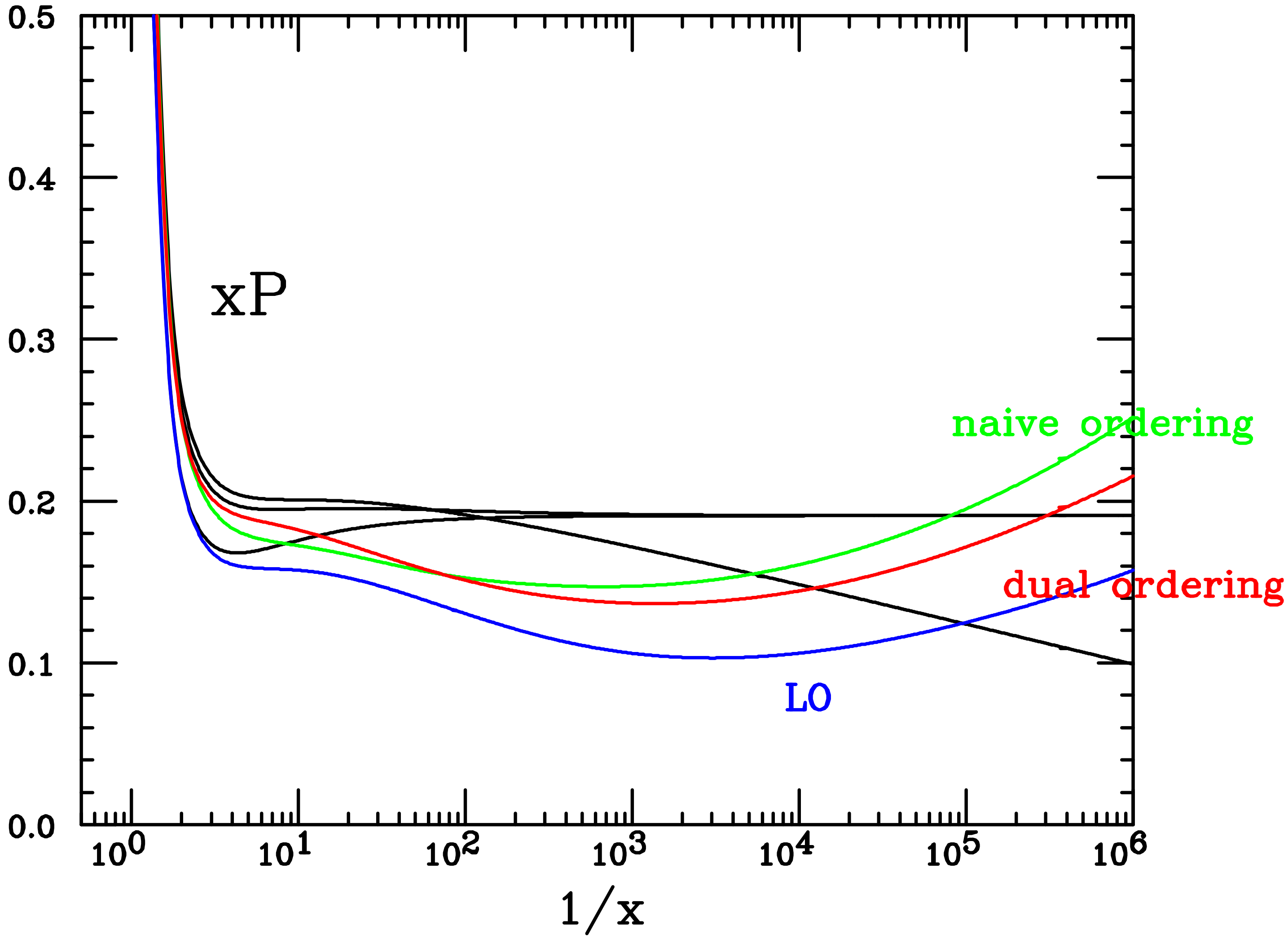


- NNLO corrections on evolution very small
- K-factor almost scale independent

coefficient function/scheme choice must be studied!

EFFECTS OF OPERATOR ORDERING

$$\bar{\chi}_{\sigma\, LO}(\hat{\alpha}_s, M, N) = \chi_s\left(\hat{\alpha}_s\left(M+\frac{N}{2}\right)^{-1}\right)+\chi_s\left(\left(1-M+\frac{N}{2}\right)^{-1}\hat{\alpha}_s\right)+\tilde{\chi}_0(\hat{\alpha}_s, M, N)$$



RESUMMATION 2000

- DOUBLE-LEADING PERTURBATIVE EXPANSION STABLE
- STRONG DEPENDENCE ON NONPERTURBATIVE ALL-ORDER INTERCEPT λ (value of $\chi(M)$ at min.)
- GOOD AGREEMENT WITH DATA IF λ FITTED (FINE-TUNED)