Gluon density from the impact parameter dependent BK equation

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1.Motivations

Reliable description of processes at region of very small values of x

$\downarrow$

- Constrains on parton distributions implied by HERA data.
- BFKL dynamics with subleading corrections.
- Non-linear screening effects built into the evolution equation.

2.Low x and nonlinear evolution equation

LO BFKL evolution in $Y = ln1/x$ predicts for unintegrated gluon density:

$$\frac{\partial f(Y,k)}{\partial Y} = \frac{\alpha_s N_c}{\pi} \int dk' K(k,k') f(Y,t') \qquad (1)$$

$$\downarrow$$

$$f(x,k) \simeq x^{-\omega_p}, \omega_P = \alpha_s 4ln2$$

Very strong rise with energy

Let us consider process:

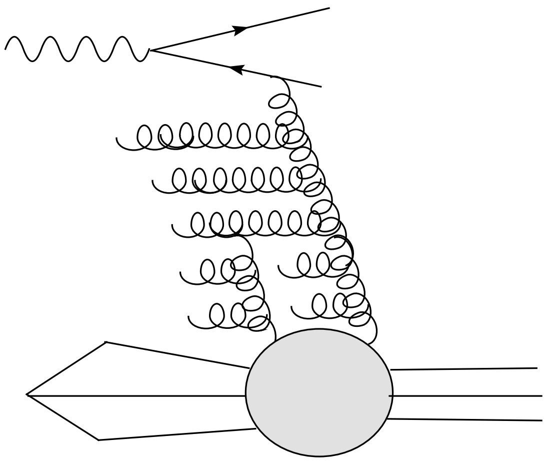


Figure 1: Left: Emitted gluons develop showers.

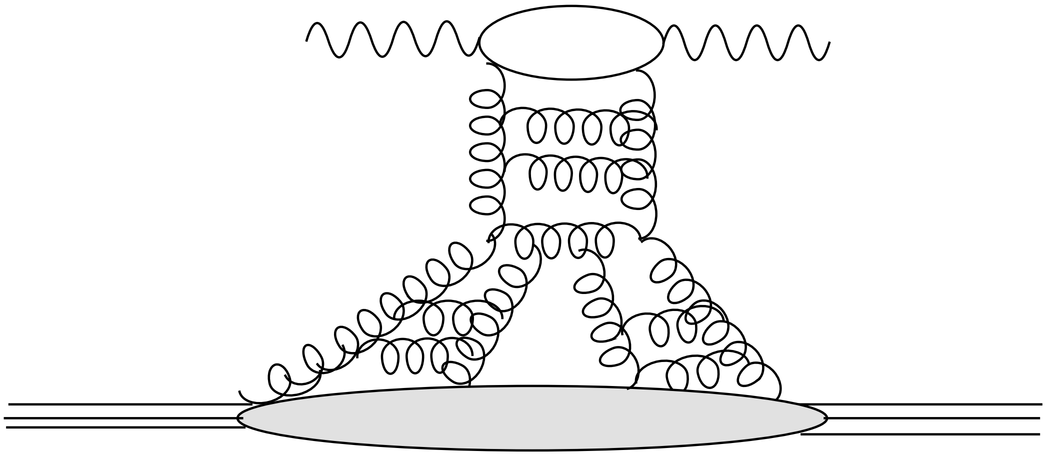


Figure 2: In the square amplitude gluons combine to ladders

Repeating this structure we get:

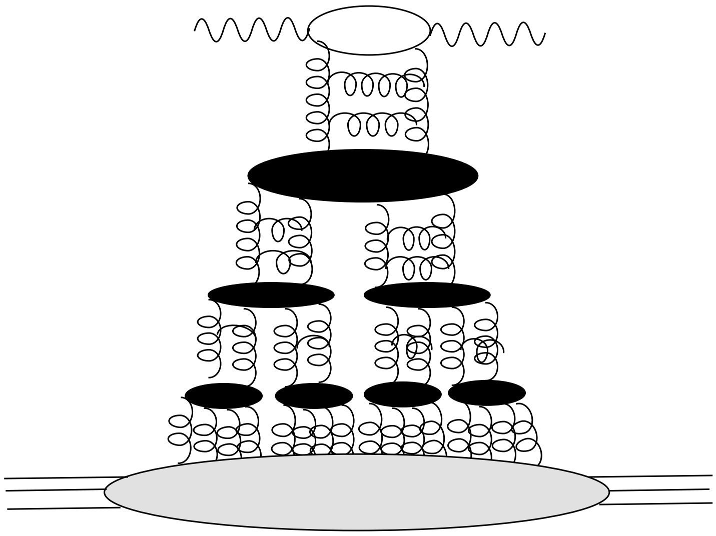


Figure 3: fan diagram.

$\uparrow$

BK equation

$$\frac{\partial f(x,k^2)}{\partial ln1/x} = K_{BFKL} \otimes f - K_2 \otimes f^2 \tag{2}$$

$$\frac{\partial f(x,k^2)}{\partial ln 1/x} = K_{BFKL} \otimes f - K_2 \otimes f^2 \qquad (3)$$

$$\downarrow$$

Balitsky Kovchegov equation

for unintegrated gluon distribution $f(x,k^2)$
(Kimber, Kwieciński, Martin)

$$\uparrow$$

linked to the dipole amplitude and the dipole density

- K_{BFKL} **is the LO BFKL kernel**
- K_2 **is the Triple Pomeron Vertex**
- **Sums up fan diagrams.**
- **The lower ladders of the fan diagram do not communicate in the multicolor limit.**

3. Extended BK-equation for gluon density with impact parameter dependence.

(Kutak, Motyka)

$$f_g(x,k^2,b) = \tilde{f}_g^{(0)}(x,k^2,b) + K_1 \otimes f_g - K_2 \otimes f_g^2 \qquad (4)$$

where

$$\tilde{f}_g^{(0)}(x,k^2,b) = S(b)\frac{\alpha_s(k^2)}{2\pi}\int_x^1 dz P_{gg}(z,b)\frac{x}{z}g(\frac{x}{z},k_0^2)$$

$$K_1 \otimes f_g = 2N_c\frac{\alpha_s(k^2)}{2\pi}k^2\int_x^1 \frac{dz}{z}\int_{k_0^2}\frac{dk'^2}{k'^2}$$

$$\left\{\frac{f_g\left(\frac{x}{z},k'^2,b\right)\Theta\left(\frac{k^2}{z}-k'^2\right)-f_g\left(\frac{x}{z},k^2,b\right)}{|k'^2-k^2|}+\frac{f_g\left(\frac{x}{z},k^2\right)}{[4k'^4+k^4]^{\frac{1}{2}}}\right\}$$

$$+\frac{\alpha_s(k^2)}{2\pi}\int_x^1\frac{dz}{z}\left[\left(zP_{gg}(z,b)-2N_c\right)\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}f_g\left(\frac{x}{z},k'^2,b\right)\right.$$

$$S(b) = \frac{1}{\pi R^2}e^{-b^2/R^2}$$

Where $R = 2.8 GeV^{-1}$ **and conventional unintegrated gluon density is obtained:**

$$f(x,k^2) = \int d^2b f_g(x,k^2,b) \qquad (5)$$

Extended because in the linear part we include:

- kinematic constraint on gluon emission $k'^2 < \frac{k^2}{z}$ (subleading BFKL effects)
- inclusion of DGLAP splitting function

appropriate input

$\downarrow$

b dependence

$\downarrow$

evolution local in b

4. Unintegrated gluon density

$f_g(x, k^2, b)$ **plotted as a function of** b **for** $k^2 = 1.4, ..5$ GeV^2 **and for** $x = 10^{-3}$ **and** $x = 10^{-8}$

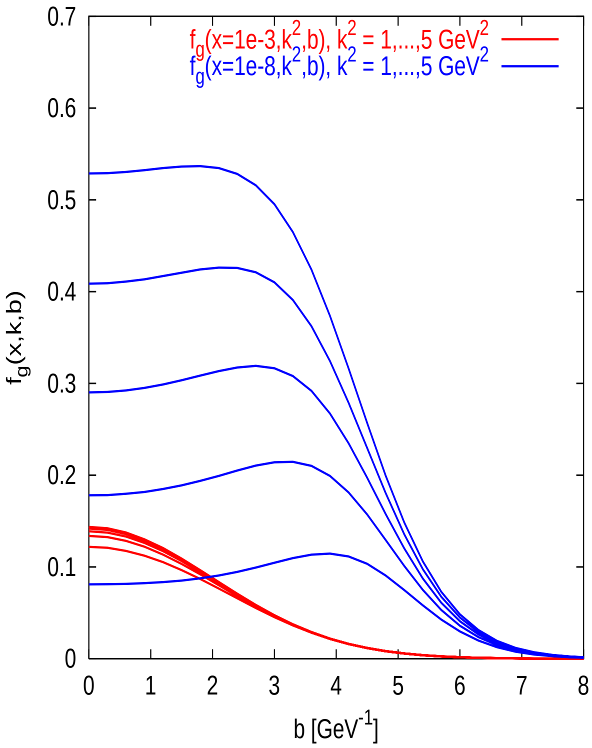


Figure 4:

- **Broadening and box-like shape of the proton at lower momenta and very low** x
- **At very low** x **gluon is a non-monotonical function of the distance from the center**

$f_g(x, k^2, b)$ **plotted as a function of x for** $k^2 = 1.4, ..5$ GeV^2 **and for b=0 and for** $b = 3GeV^{-1}$

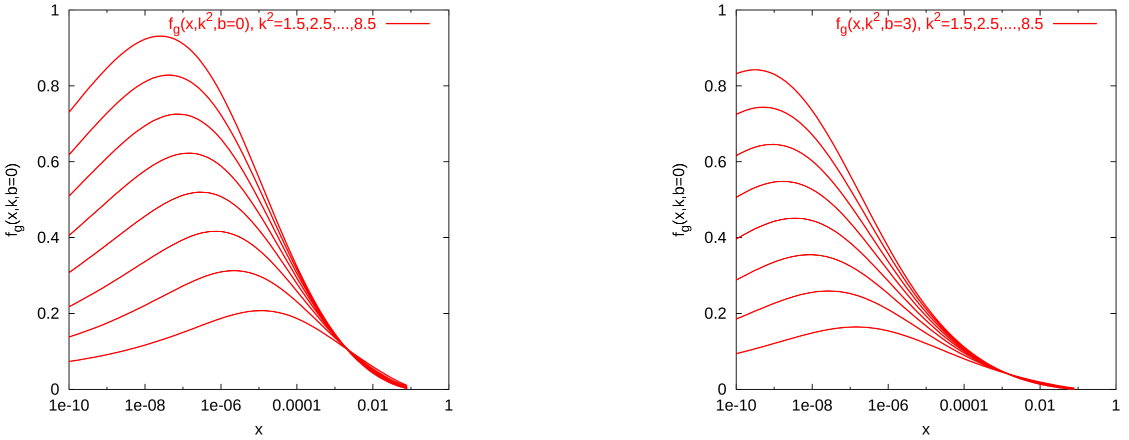


Figure 5:

- **The maxima signal that the saturation scale** $Q_s(x)$ **reached the momentum** k
- **Decreasing distribution for** $k^2 < Q_s^2(x)$
- **Saturation scale grows with decreasing x**
- **Saturation scale decreases with growing** b

The b dependent saturation scale can be defined as follows:

$$\frac{\partial f_g(x,k^2,b)}{\partial log(1/x)} = 0. \qquad (6)$$

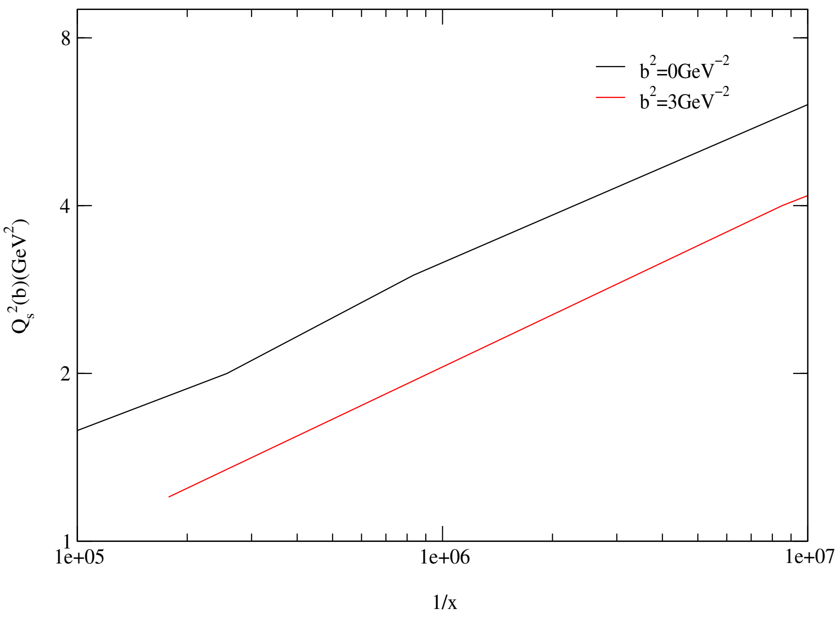


Figure 6:

$f_g(x,k^2,b)/k^2$ **plotted as a function of** k **for** $x = 10^{-3},..,10^{-8}$

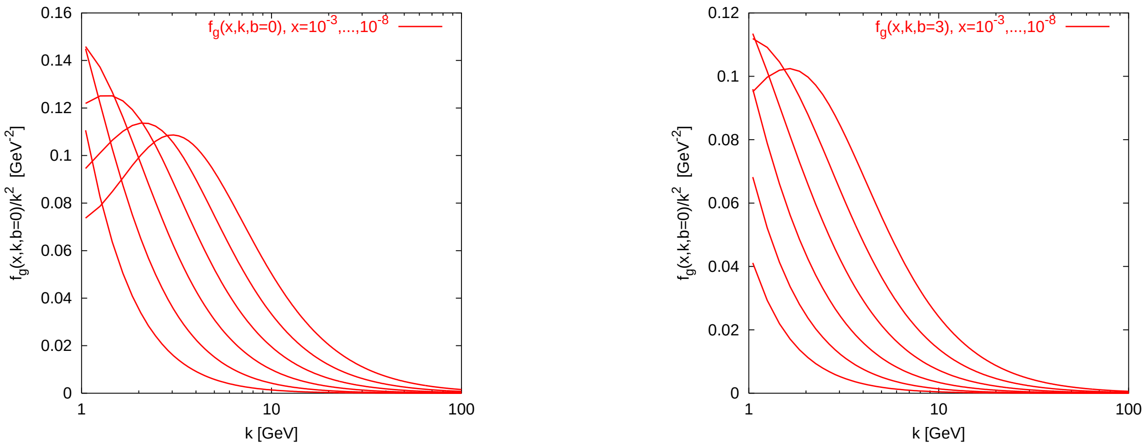


Figure 7:

- **Maxima set the saturation scale**
- $f_g(x,k^2,b)$ **decreases faster than** k^2 **as** $\rightarrow$ **0**
- **For** $x \simeq 10^{-7}$ **the saturation scale is deeply in the perturbative domain for** $b = 0$ **and** $Q_s \simeq$ **1 for** $b = 3GeV^{-1}$

Due to averaging over the impact parameter there is less saturation for $f(x,k^2)$:

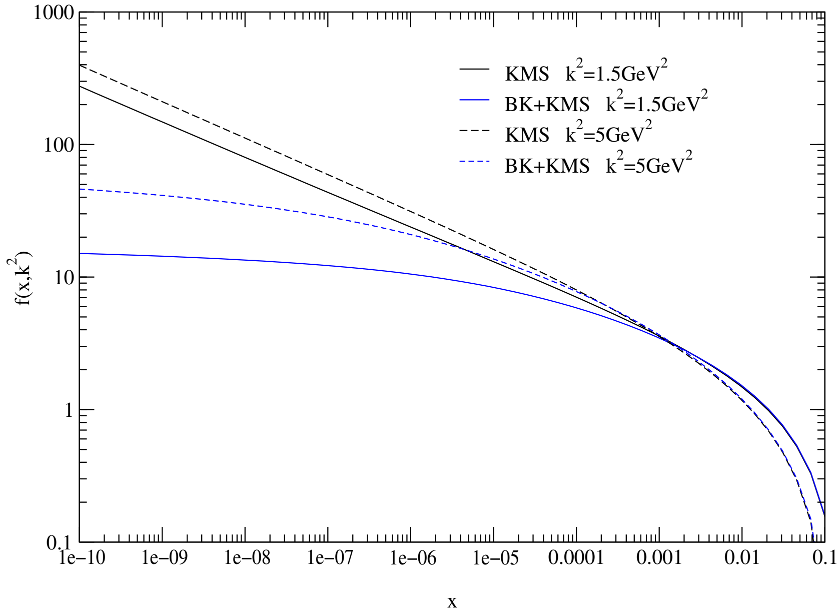
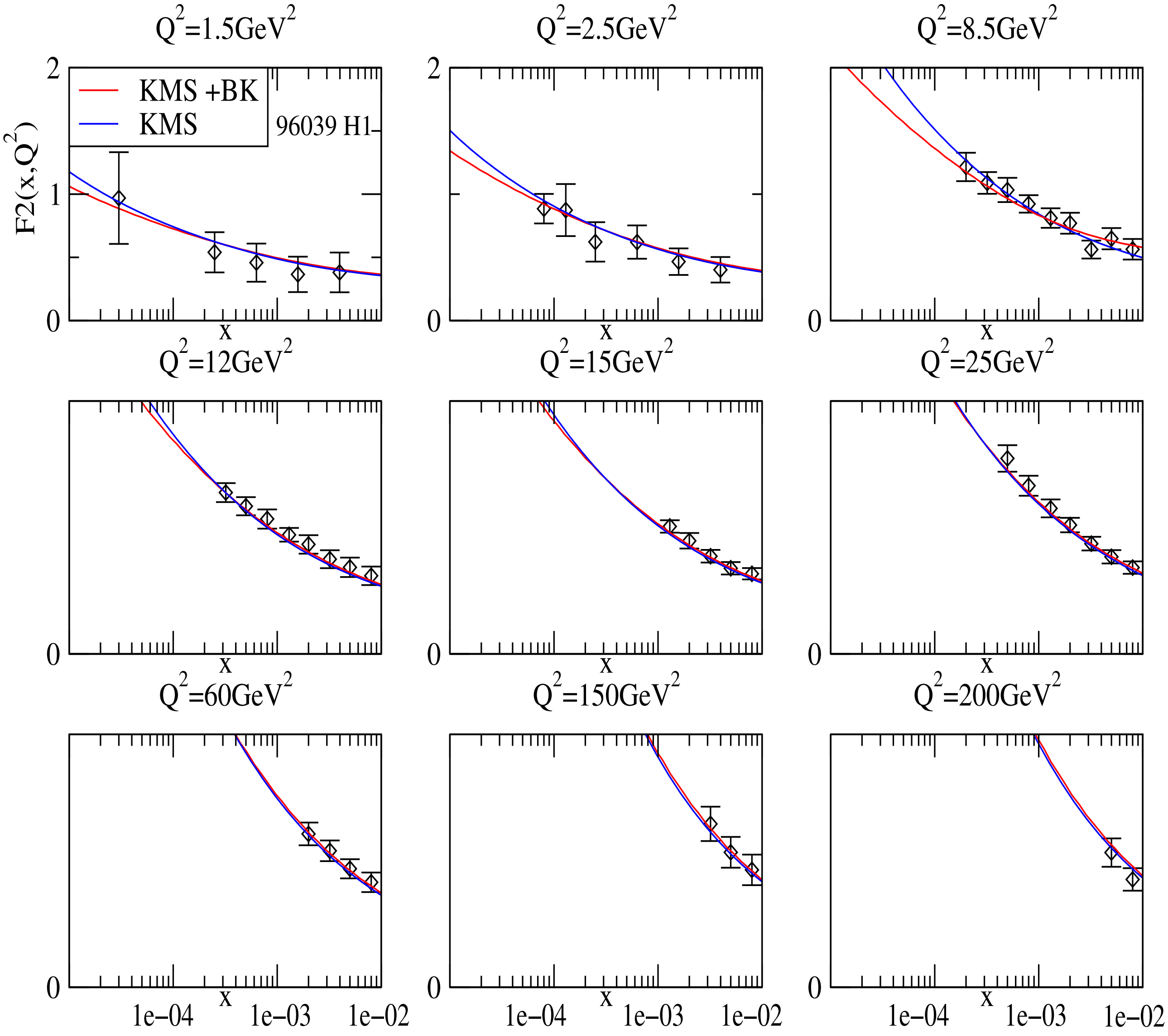


Figure 8:

To get F2 we average over b and using k_t factorisation

$$F_2 \simeq S \otimes f \tag{7}$$

we obtain...



Q^2=1.5GeV^2
Q^2=2.5GeV^2
Q^2=8.5GeV^2
KMS +BK
KMS
96039 H1
F2(x,Q^2)
Q^2=12GeV^2
Q^2=15GeV^2
Q^2=25GeV^2
Q^2=60GeV^2
Q^2=150GeV^2
Q^2=200GeV^2
x
1e−04
1e−03
1e−02

Figure 9:

5. Conclusions

- Position of the saturation line depends on the shape of the proton and is strongest for small distances and almost negligible for large.
- Due to averaging over impact parameter we hardly see saturation effects for F2.