Measurement of $F_L(x, Q^2)$ at HERA

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It would be vital to have a real accurate measurement of $F_L(x,Q^2)$ at HERA since it would then give an independent test of the gluon distribution at low x to go along with that determined from $dF_2(x,Q^2)/d\ln Q^2$. At present the fits to $F_2(x,Q^2)$ at low x are reasonably good (perhaps not perfect) but the gluon is free to vary to make them as good as possible. We need a cross-check.

Currently have consistency checks on the relationship between $F_2(x,Q^2)$ and $F_L(x,Q^2)$ at high y where both contribute to the total cross-section.

Extrapolate in y using either NLO perturbative QCD or using $(d\sigma/d\ln y)_{Q^2}$ whilst making assumption about $(dF_2(x,Q^2)/d\ln y)_{Q^2}$.

Good consistency test of a given theory (NLO QCD), and could show up major flaws. However, relies on small difference between two large quantities, so accuracy limited.

For extraction of $F_2(x,Q^2)$ has model-dependent uncertainties difficult to quantify fully.

Real measurement would be much more direct test of success of different theories in QCD.

$F_L(x,Q^2)$ is a much better discriminator of the gluon distribution, and/or of different theories, for given $F_2(x,Q^2)$ than charm contribution.

$F_2^c(x,Q^2)$ is constrained to evolve in exactly the same way as $F_2^{tot}(x,Q^2)$ (with appropriate charge weighting) for $W^2 \gg m_c^2$.

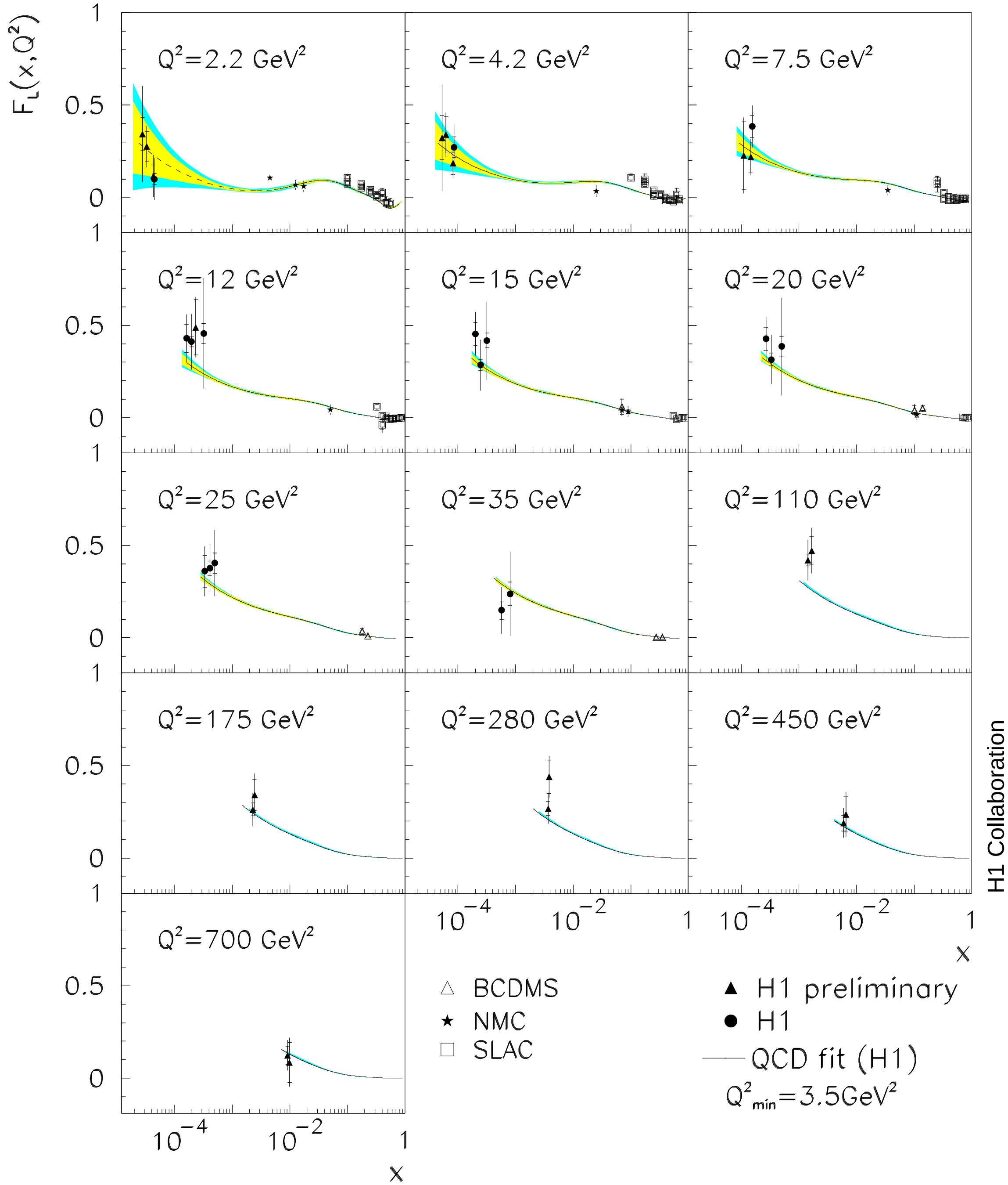
Empirically the evolution of $F_2^c(x,Q^2)$ is suppressed by a factor of

$$v = 1 - \frac{4m_c^2 z}{Q^2(1-z)}$$

i.e. the velocity of the heavy quark in the centre-of-mass frame and by the limit of integration in the convolution being $\xi = x(1+4m_c^2/Q^2)$ rather than x. Same amount of suppression seems to exist from order-to-order and when including resummations. Does not distinguish well between different approaches - unlike $F_L(x,Q^2)$.

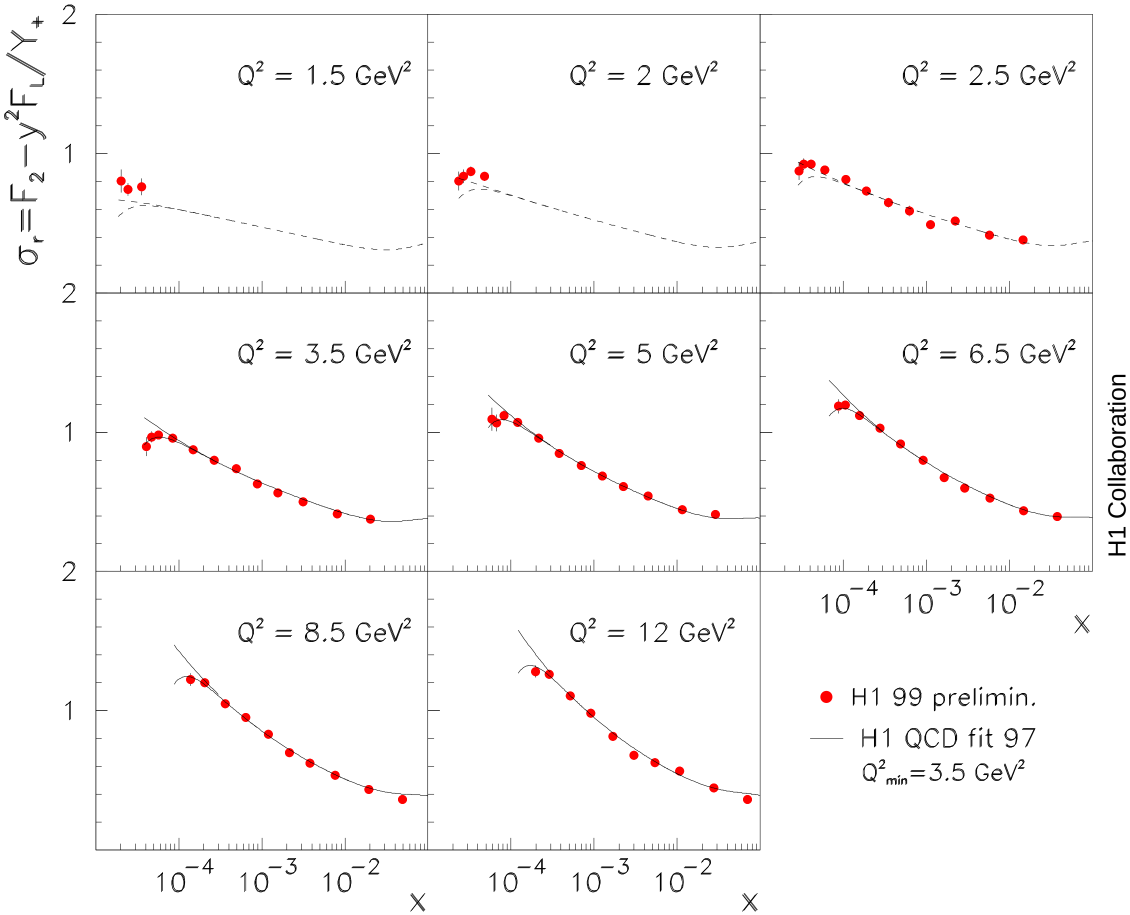
The consistency check of $F_L(x, Q^2)$ extracted versus $F_L(x, Q^2)$ predicted. Potentially large, strongly correlated, model-dependent errors.

Do not want to see plots like this.



Far more revealing to see plots like this.

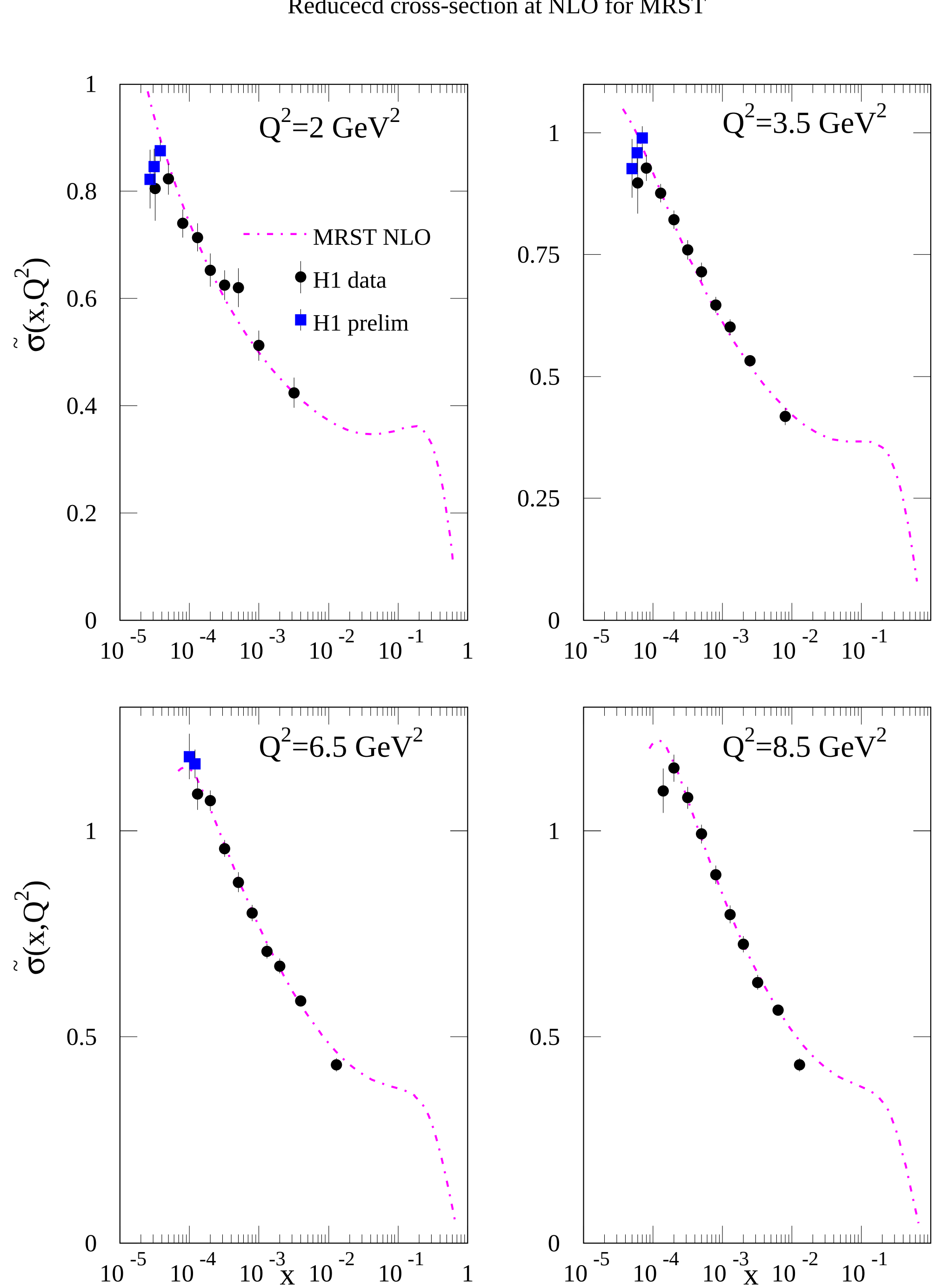
The NLO consistency check of $F_L(x,Q^2)$ for the H1 fit.



Turn over in $\tilde{\sigma}(x,Q^2) = F_2(x,Q^2) - y^2/(1+(1-y)^2)F_L(x,Q^2)$ clearly matched by $F_L(x,Q^2)$ contribution.

However, the consistency check for the fit of $\tilde{\sigma}(x, Q^2)$ for MRST partons at NLO fails at the lower Q^2 values.

Consistency check works well for H1 NLO fit, and some others (though not as well) but not for the MRST NLO fit.



Note standard perturbation theory (and errors determined using renormalization and/or factorization scale variation) is not necessarily reliable in general because of increasing logs at higher orders, e.g. at small x

$$P^1_{qg} \sim \alpha_S(\mu^2) \qquad\qquad P^2_{qg} \sim \frac{\alpha_s^2(\mu^2)}{x}$$
$$P^n_{qg} \sim \frac{\alpha_s^n(\mu^2)\ln^{n-1}}{x}$$

and similarly

$$C^1_{Lg} \sim \alpha_S(\mu^2) \qquad\qquad C^2_{Lg} \sim \frac{\alpha_s(\mu^2)}{x}$$
$$C^n_{Lg} \sim \frac{\alpha_s^n(\mu^2)\ln^{n-1}}{x}$$

and hence enhancements at small x are possible.

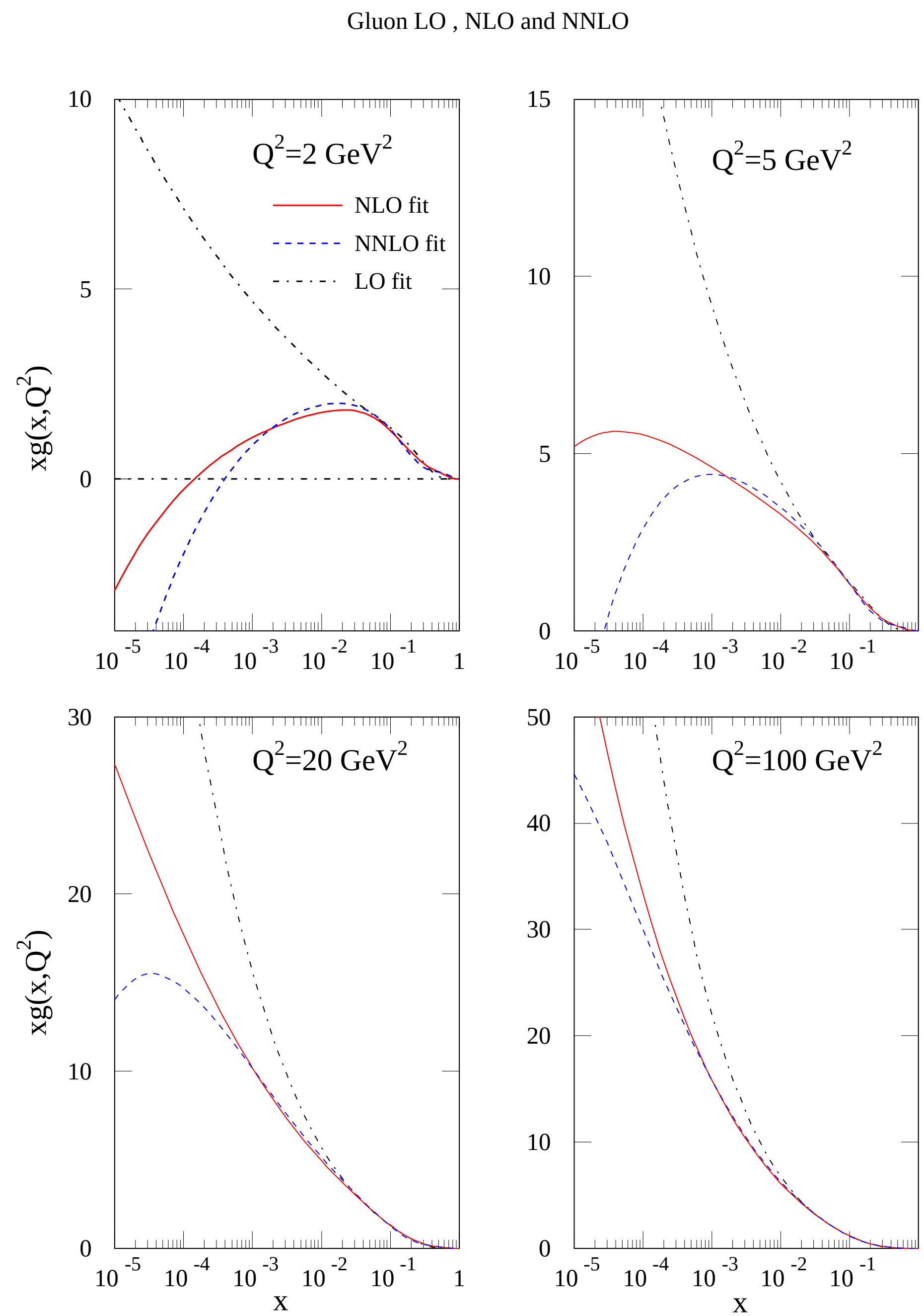
To some extent can see what happens at NNLO.

Splitting functions calculated at NNLO and recently coefficient functions for $F_L(x,Q^2)$ finished too Moch, Vermaseren, Vogt.

The gluon extracted from the global fit at LO, NLO and NNLO.

Additional and positive small-x contributions in P_{qg} at each order lead to smaller small-x gluon at each order.

Note - this conclusion relied on correct application of flavour thresholds in a General Variable Flavour Number Scheme at NLO not present in earlier approximate NNLO MRST fits. Correct treatment of flavour particularly important at NNLO because discontinuities in unphysical quantities appear at this order.

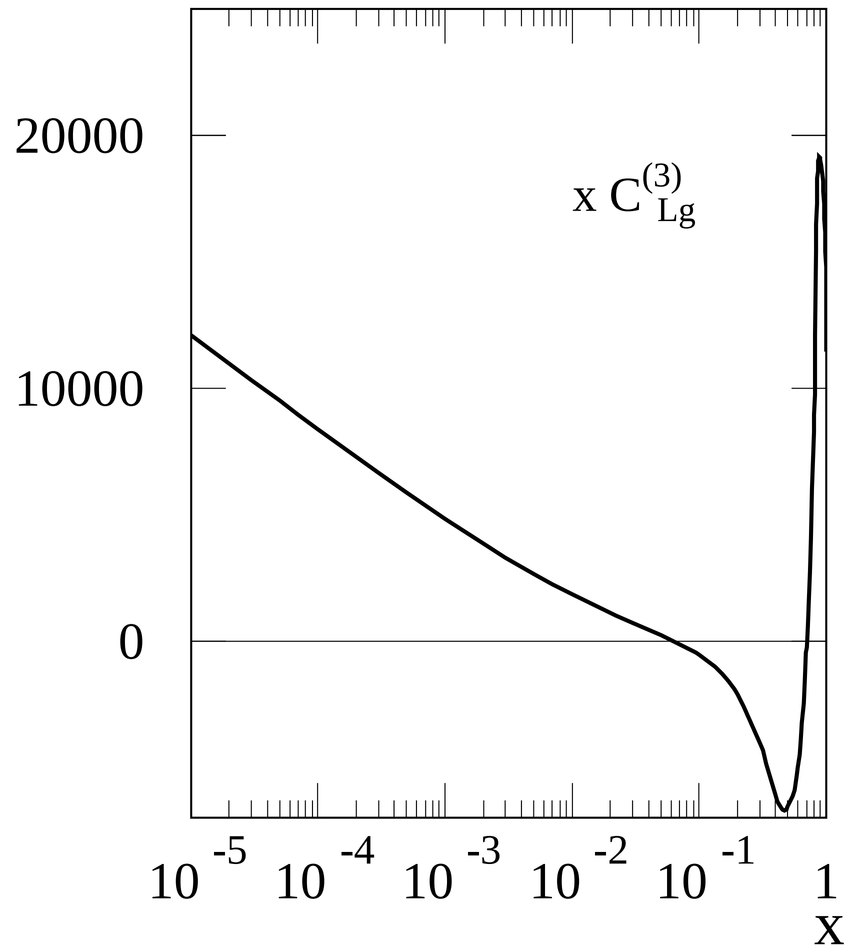


The NNLO $\mathcal{O}(\alpha_s^3)$ longitudinal coefficient function $C^3_{Lg}(x)$ given by

$$C^3_{Lg}(x) = n_f \left(\frac{\alpha_S}{4\pi}\right)^3 \left(\frac{409.5 \ln(1/x)}{x} - \frac{2044.7}{x} - \cdots\right.$$

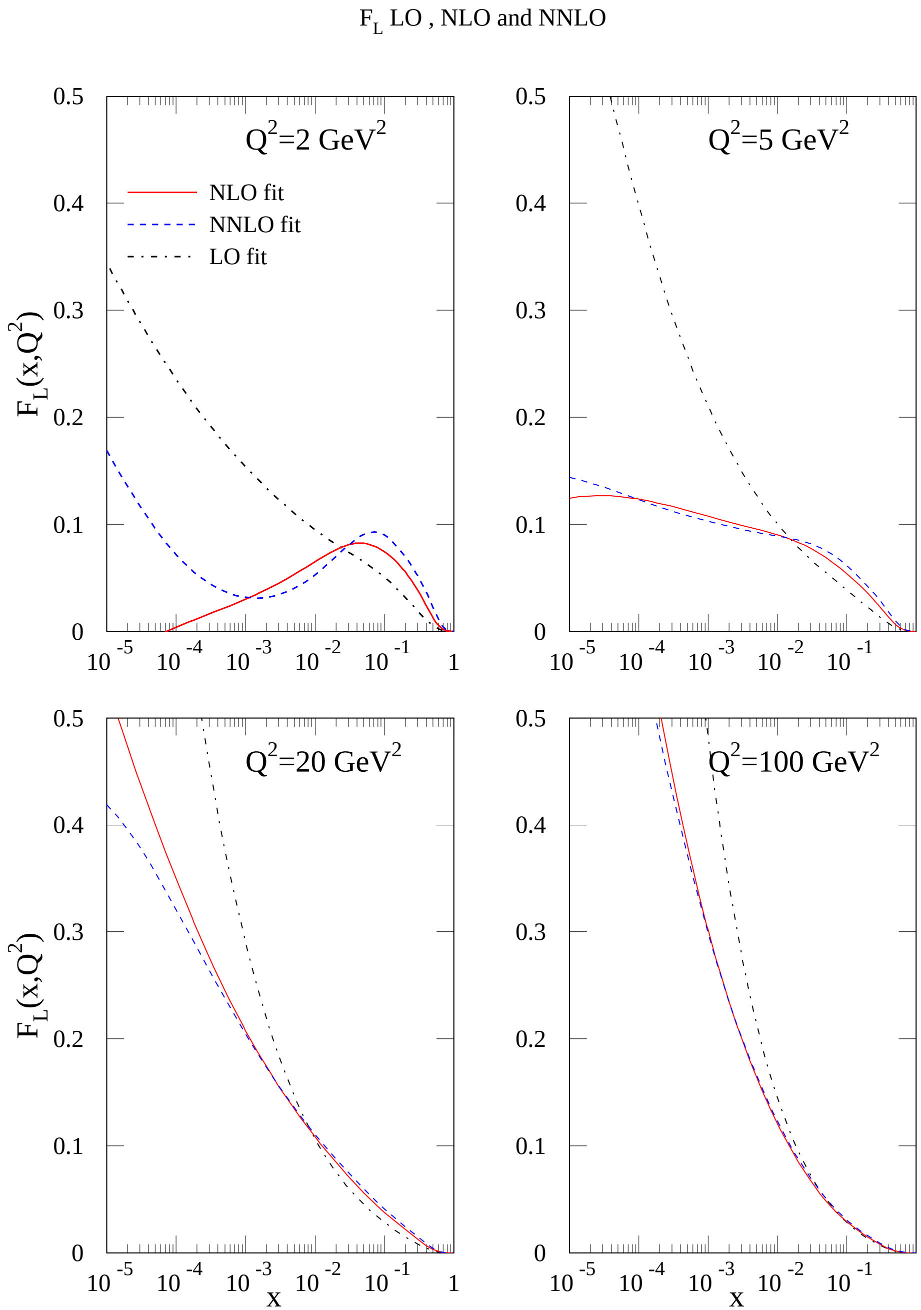
Clearly a significant positive contribution at small x.

Counters decrease in small-x gluon.



$F_L(x, Q^2)$ predicted from the global fit at LO, NLO and NNLO.

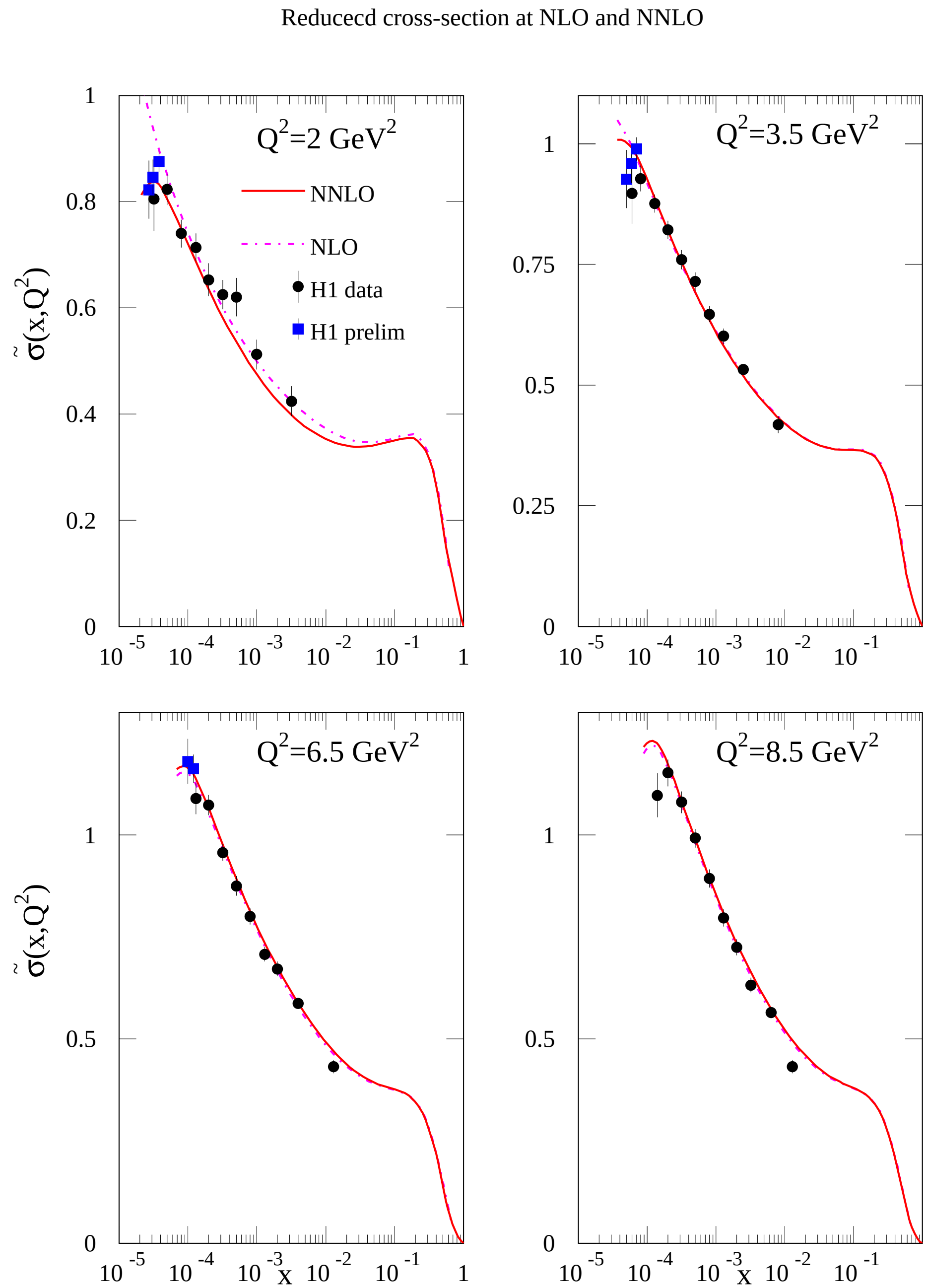
NNLO coefficient function more than compensates decrease in NNLO gluon.



Without considering high-y data, NNLO fit not much better than NLO fit.

NNLO contribution to $F_L(x,Q^2)$ solves previous high-y problem with $\tilde{\sigma}(x,Q^2)$. (Would fail with gluons from NLO fits which currently work at high-y - $F_L(x,Q^2)$ would be too big and turnover too great).

But this data not very precise. Effective error on $F_L(x,Q^2) \sim 30-40\%$.



If the NNLO correction is itself rather large, might not other corrections on top of this - higher orders still, or higher twist, also be quite large?

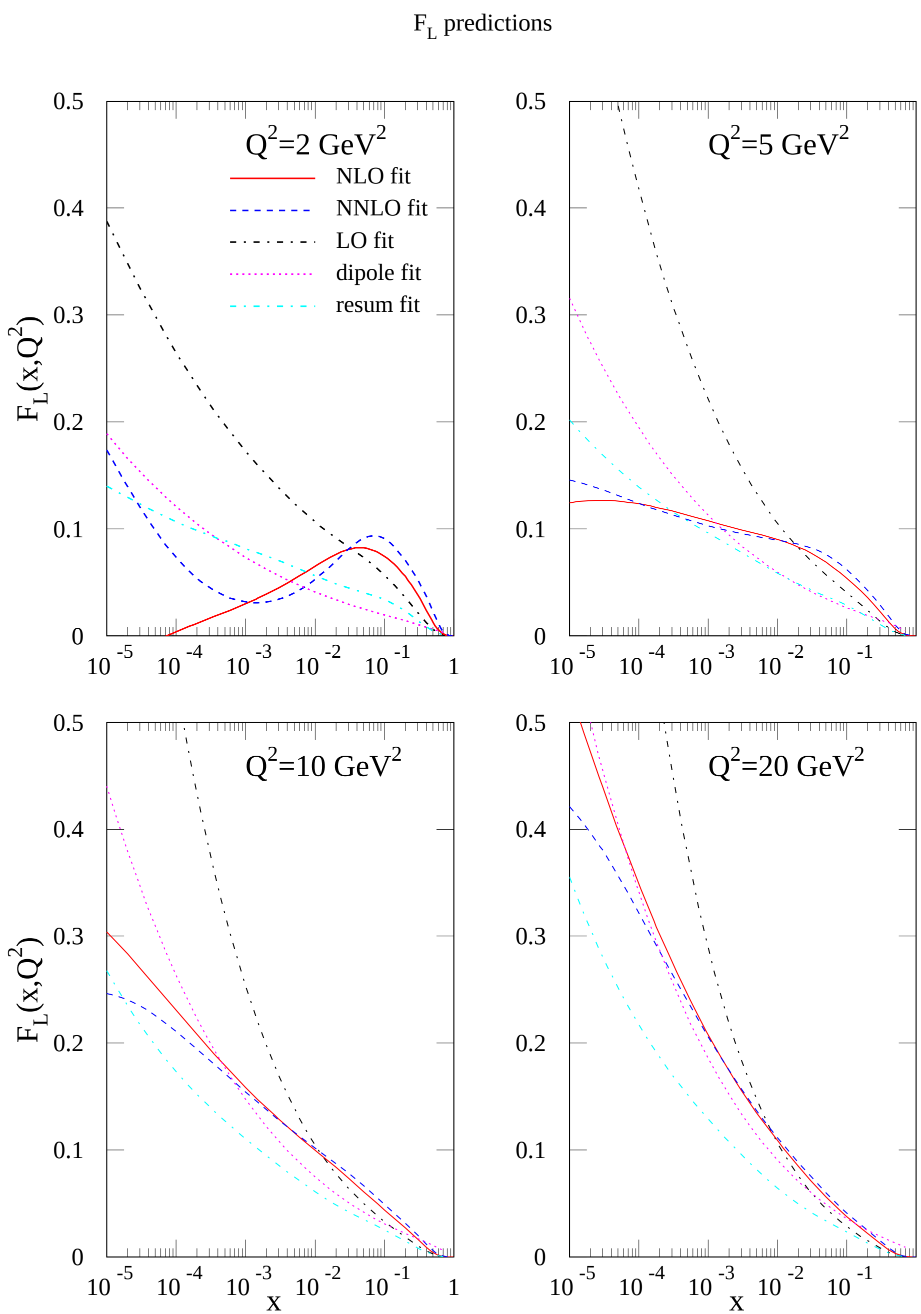
A fit which performs a double resummation of leading $\ln(1/x)$ and β_0 terms leads to a better fit to small-x data than a conventional perturbative fit. Resummation also seems to stabilize $F_L(x, Q^2)$ and small x and Q^2 (C White, RT). Prediction a bit approximate but correct trend.

Dipole motivated fit contains higher terms in $\ln(1/x)$ and higher twists. Guarantees sensible behaviour for $F_L(x, Q^2)$ at low Q^2 from form of wavefunction.

Can also examine the possibility of explicit higher twist (later).

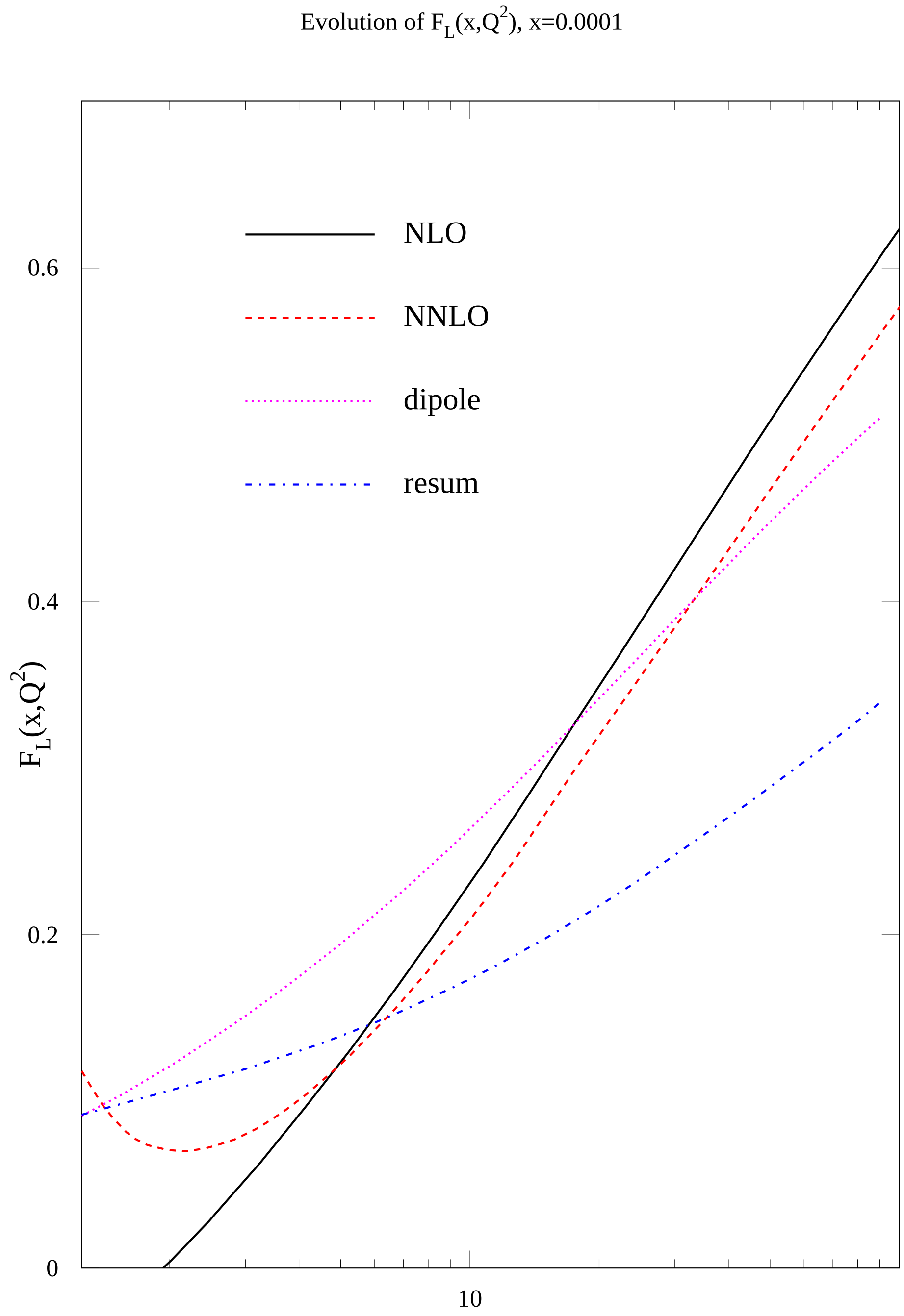
$F_L(x,Q^2)$ predicted from the global fit at LO, NLO and NNLO, from a fit which performs a resummation of small-x terms, and from a dipole model type fit.

Implies a measurement of $F_L(x,Q^2)$ over as wide a range of x and Q^2 as possible would be very useful.

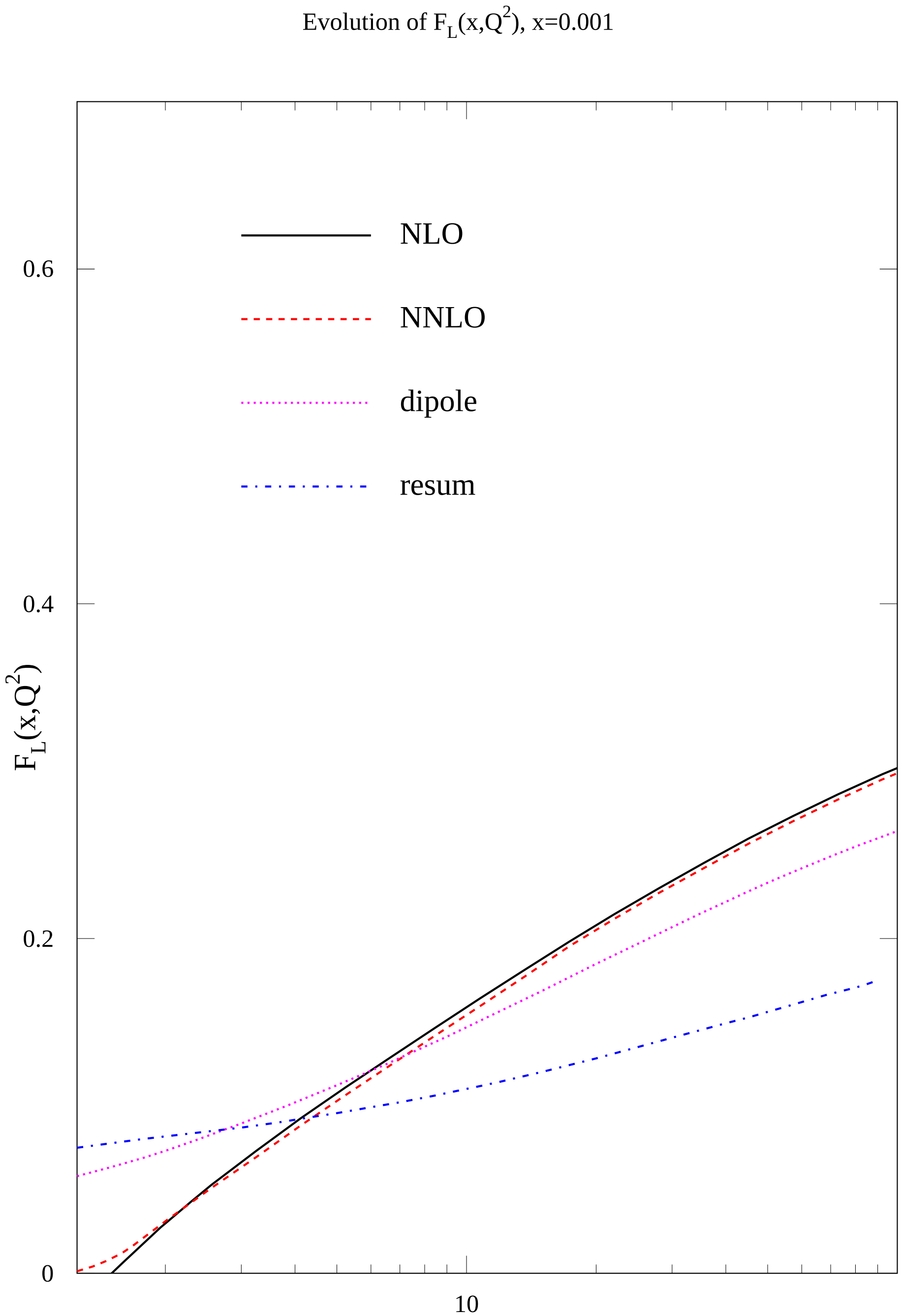


Evolution of various predictions for $F_L(x,Q^2)$ at $x = 0.0001$.

NNLO turning up at $Q^2 \sim 1\mathrm{GeV}^2$.

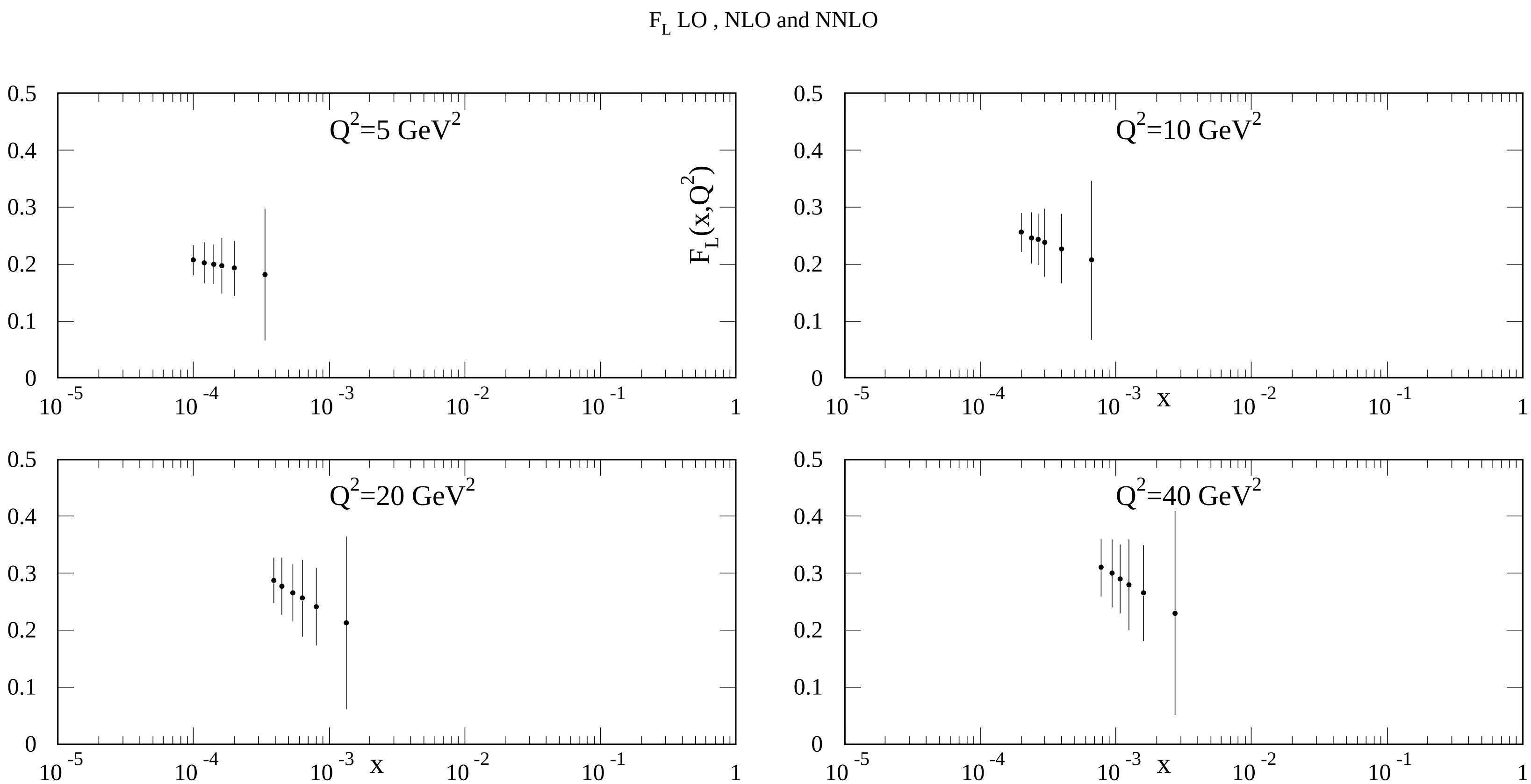


Evolution of various predictions for $F_L(x, Q^2)$ at $x = 0.001$.



HERA propose running at lower beam energy before finishing in order to make a direct measurement of $F_L(x, Q^2)$.

Measure data from $Q^2 = 5 - 40\mathrm{GeV}^2$ and $x = 0.0001 - 0.003$ with typical error of at best $12 - 15\%$. (H1 simulation, Klein).



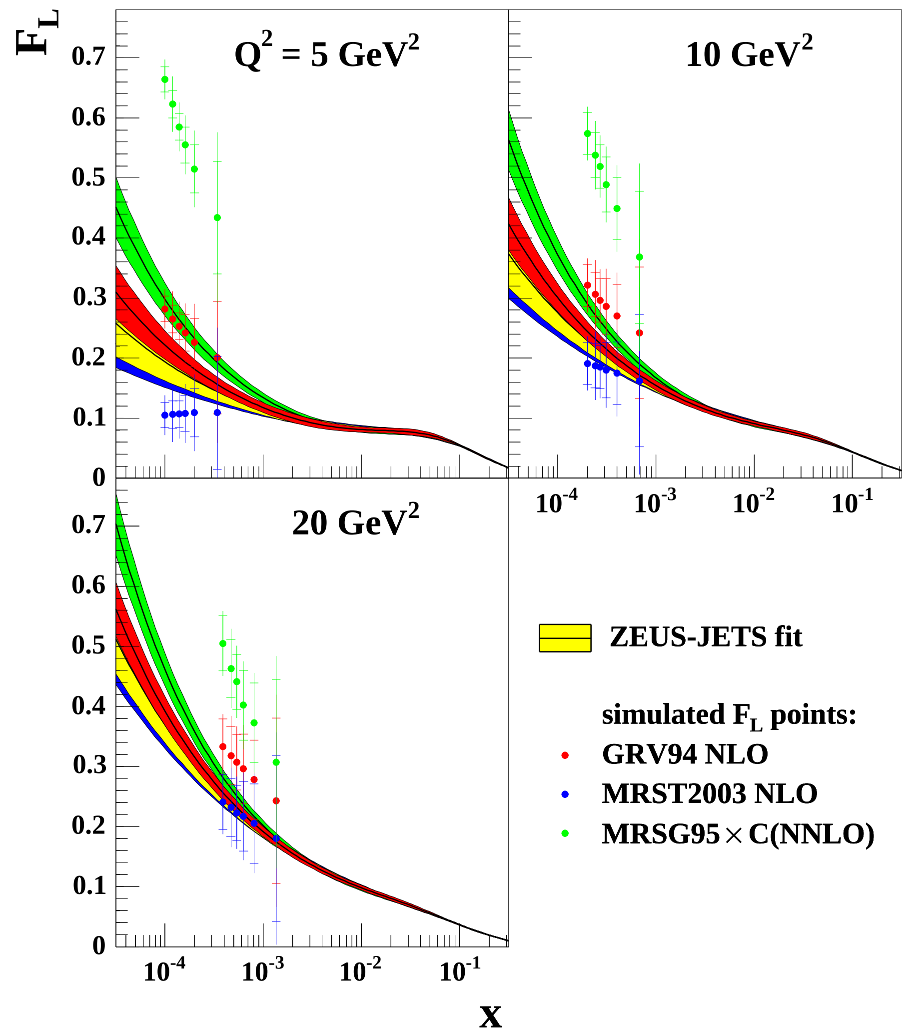
How important would this be in distinguishing between different theoretical approaches to structure functions?

Study of this by ZEUS (Gwenlan), with *extreme* theoretical predictions (RT).

Some discriminating power obvious. However, extremes based on unrealistic real models (out of data partons, partons from one order with coefficient functions from another).

Also all data points assumed to line up, i.e. χ^2 for correct theory would be 0.

Need more sophisticated approach.



Dipole fit produces rather different shape and size prediction for $F_L(x, Q^2)$ from that at NLO and NNLO.

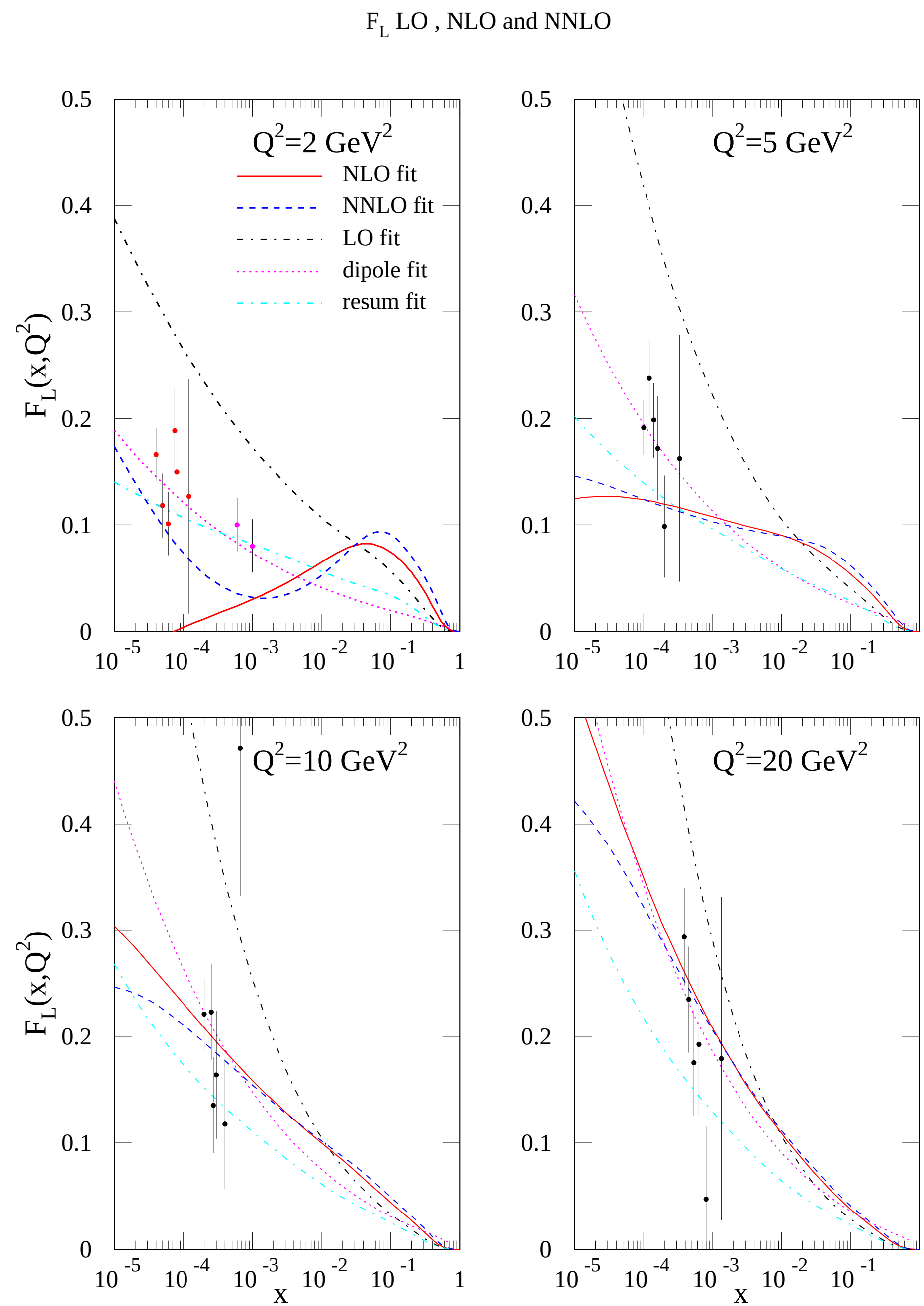
Generate a set of data based around central dipole prediction but with random scatter. $\chi^2 = 20/18$ for dipole prediction. Comparison to other predictions shown opposite.

Also show points at $Q^2 = 2\mathrm{GeV}^2$ which might have been measured at HERA III red and might be at eRHIC pink.

Any points at $40\mathrm{GeV}^2$ not as useful. Errors bigger, curves converging.

Clearly some reasonable differentiating power.

But these are central predictions.

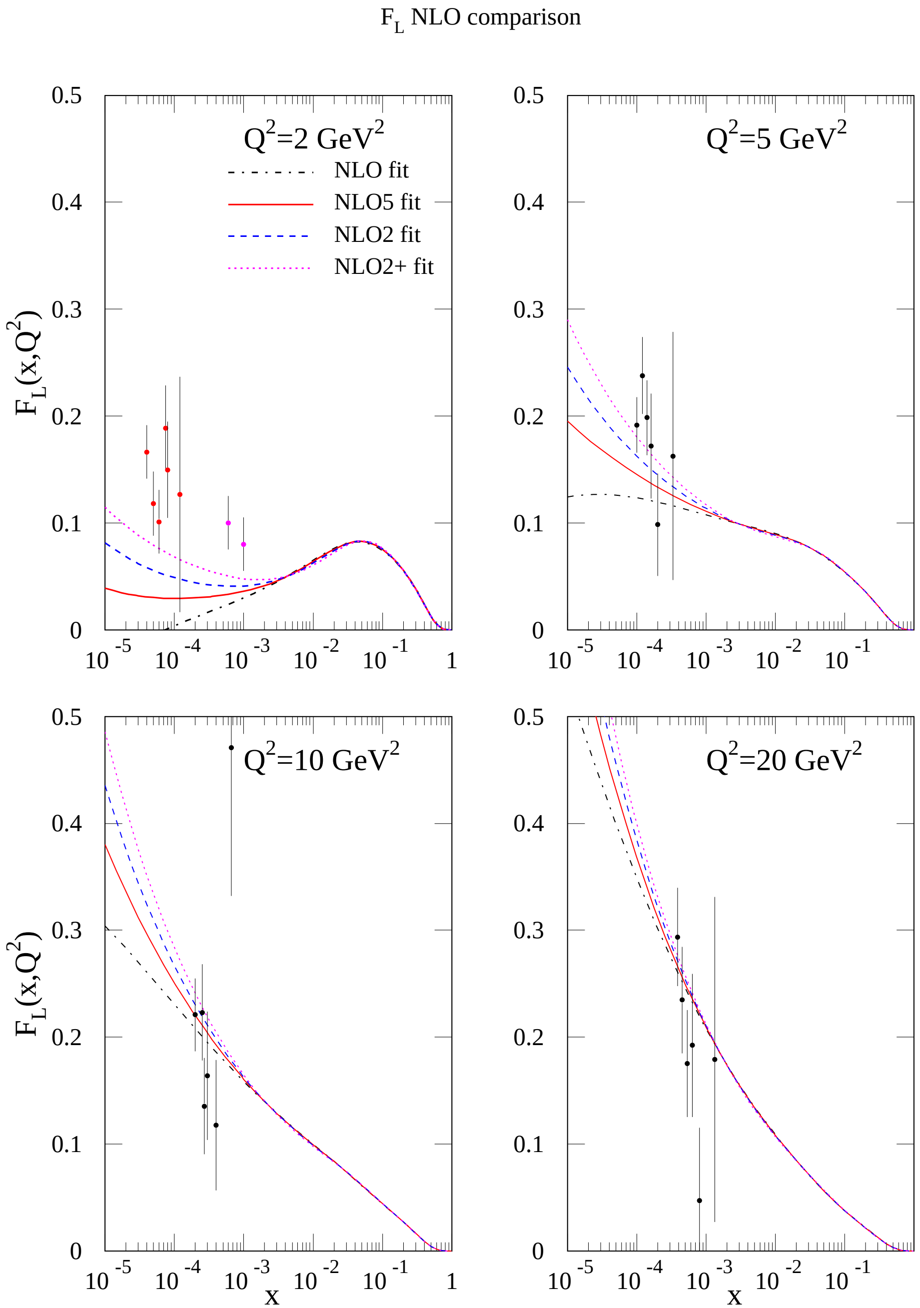


Comparison at NLO as weight of $F_L(x,Q^2)$ *data* is increased in the fit.

Best fit results in $\chi^2 = 27/18$ for $F_L(x,Q^2)$ *data* but is becoming unacceptable global fit.

Next best fit acceptable for global fit - $\chi^2 = 29/18$ for $F_L(x,Q^2)$ *data*.

NLO fit to $F_L(x,Q^2)$ *data* never that good because shape in Q^2 wrong.

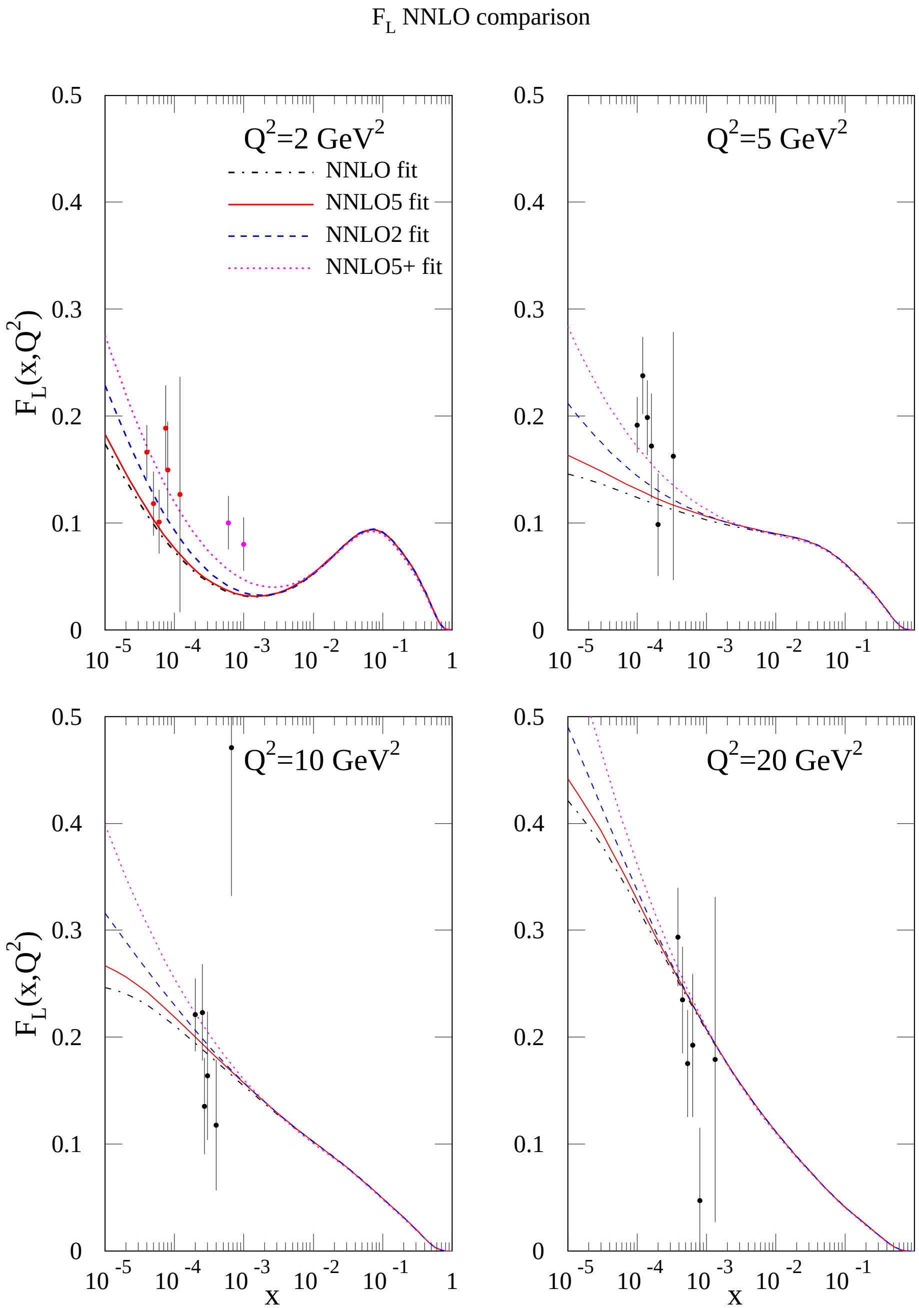


Comparison at NNLO as weight of $F_L(x,Q^2)$ *data* is increased in the fit.

Best fit results in $\chi^2 = 26/18$ for $F_L(x,Q^2)$ *data* but is becoming unacceptable global fit.

Next best fit acceptable for global fit - $\chi^2 = 31/18$ for $F_L(x,Q^2)$ *data*.

NNLO fit to $F_L(x,Q^2)$ *data* also never that good because shape in Q^2 wrong.



Can also look at explicit higher twist possibilities. Different picture than in $F_2(x,Q^2)$. There renormalon calculation of higher twist dies away at small x (from satisfying Adler sum rule).

Completely different picture for $F_L(x,Q^2)$. At small x contribution proportional to quark distributions, i.e. $F_L^{HT}(x,Q^2) \propto F_2(x,Q^2)$.

Explicit renormalon calculation (Stein *et al*) gives

$$F_L^{HT}(x,Q^2) = \frac{A}{Q^2}(\delta(1-x) - 2x^3) \otimes \sum_f Q_f^2 q_f(x,Q^2).$$

where estimate for A is

$$A = \frac{4C_f \exp(5/3)}{\beta_0}\Lambda_{QCD}^2 \approx 0.4 - 0.8\,\mathrm{GeV}^2.$$

This effect is nothing to do with the gluon distribution, and is not part of the higher twist contribution in the dipole approach.

Higher twist does mix with higher orders though. Best to add it to NLO or NNLO prediction?

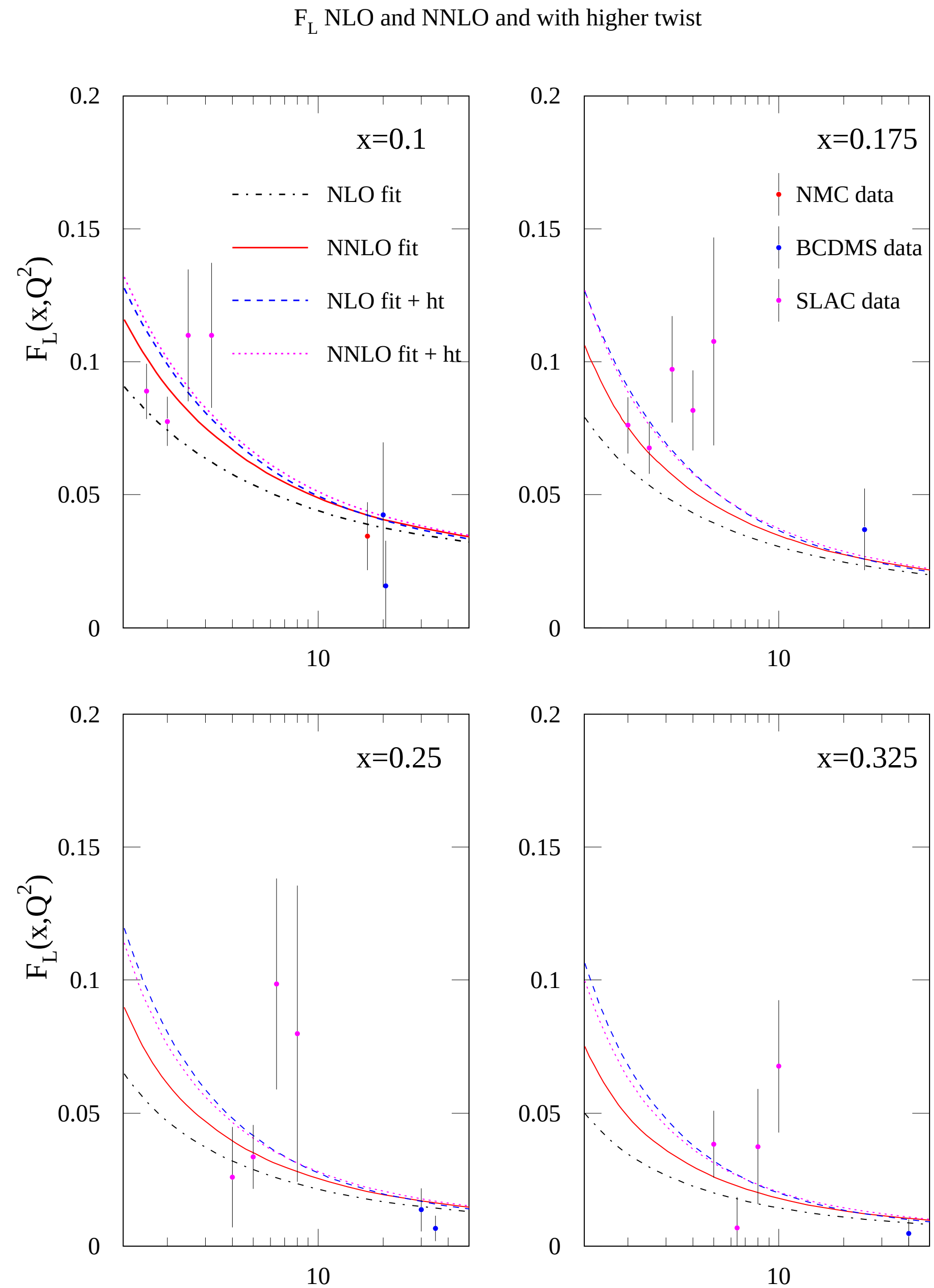
Try global fit to both high-y $\tilde{\sigma}(x, Q^2)$ HERA data and to higher-x fixed target data from NMC, BCDMS and SLAC.

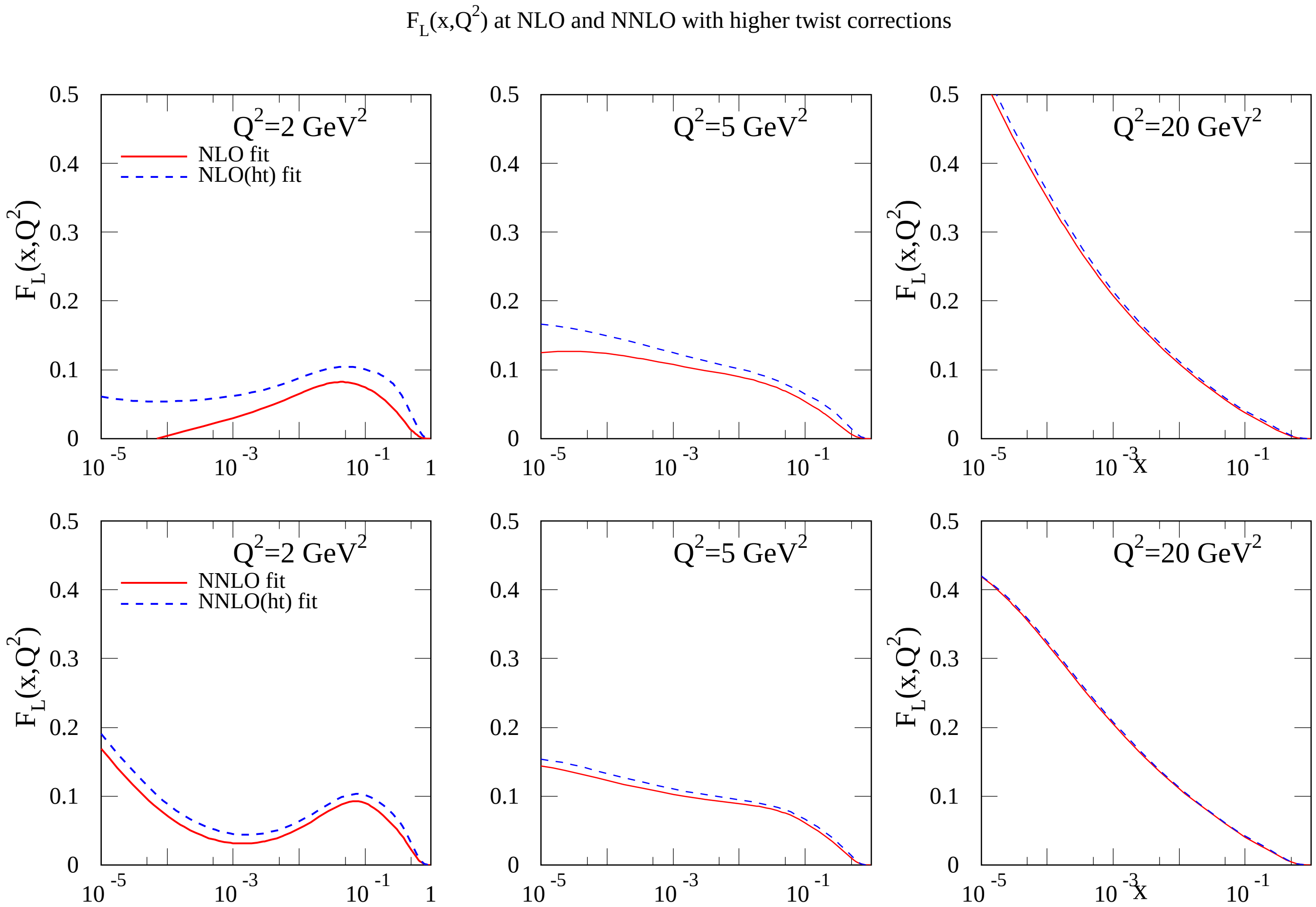
Standard NLO and NNLO both undershoot this data, they choose $A = 0.36 \pm 0.08$ or $A = 0.16 \pm 0.08$.

Improvement in $\chi^2 \sim 8(3)$ for direct data for 36 points.

Direct data keeps value of A low. Fitting only HERA data $\rightarrow A = 0.58(0.25)$.

Values quite consistent with estimates. However, analysis of divergence of perturbation series implies renormalon correction should be added to NNLO.





Corrections to leading twist NLO and NNLO when best fit values of A are used.

Can try analysing pseudodata generated using higher twist correction rather than dipole/resummation. Slightly more difficult to disentangle than dipole corrections.

However, can be both types of correction. All raise $F_L(x, Q^2)$ at low x and Q^2.

Conclusions

Measurement of $F_L(x, Q^2)$ seems to be best way to really determine the gluon distribution at low x particularly at low Q^2 (much better than charm structure function), and to determine whether fixed order calculations are sufficient or whether resummations, or other theoretical extensions, may be needed.

Currently can perform global fits to all up-to-date data over wide range of parameter space. Fit fairly good - some slight worries.

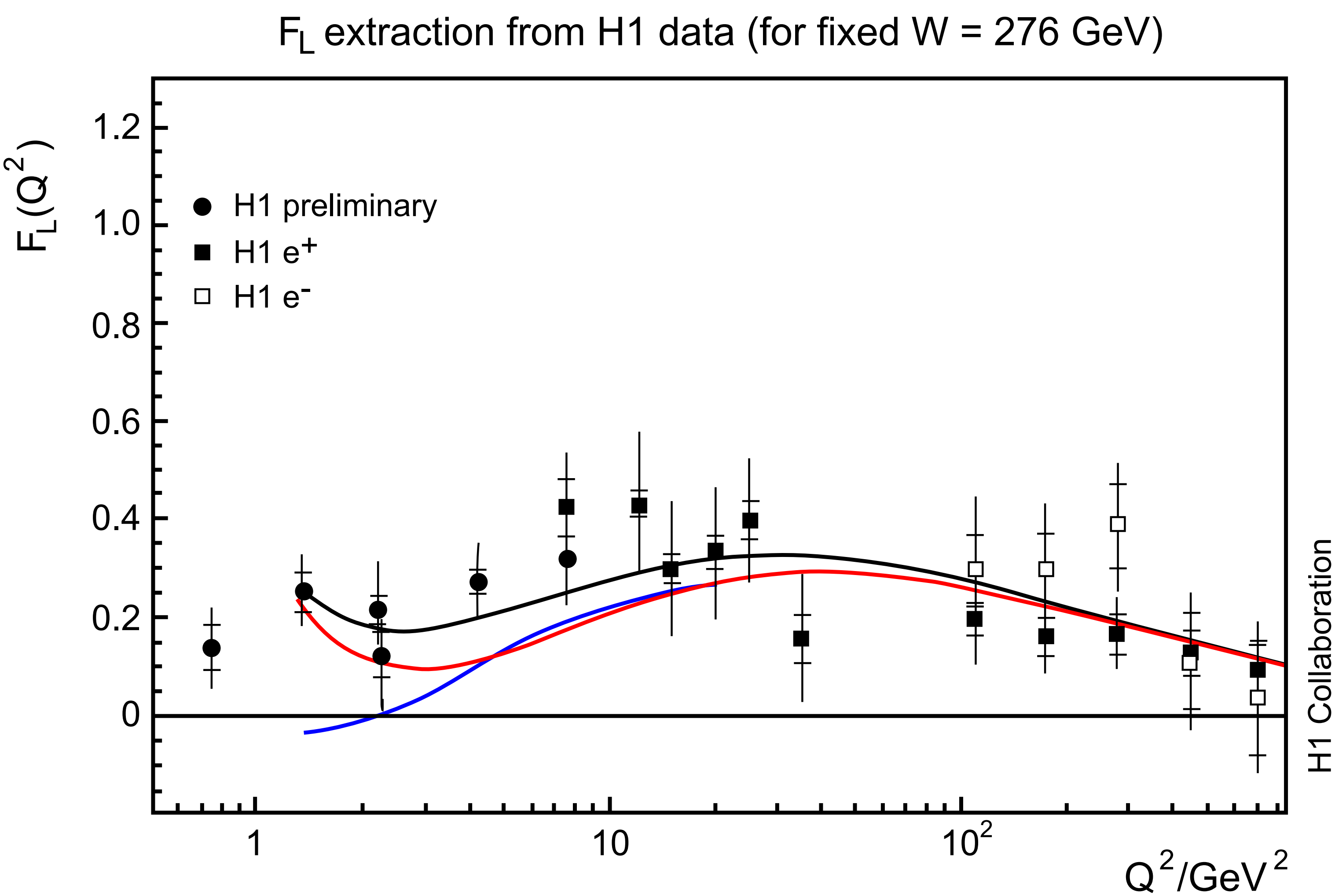
Could require higher orders, higher twist, and/or some type of resummation which have potentially large impact on the predicted $F_L(x, Q^2)$ and other quantities.

Vital measurement for our understanding of precisely how best to use perturbative QCD to describe the structure of the proton and also for making really reliable predictions and comparisons at the LHC. Lowest Q^2 possible would be useful.

Proposed measurement at HERA would have a reasonable ability to distinguish between different theoretical approaches, due to both the inability to fit $F_L(x, Q^2)$ due the shape and the deterioration in global fits needed in order to match the general features of $F_L(x, Q^2)$ data. Measurements at eRHIC at $Q^2 \sim 2\mathrm{GeV}^2$ and $x \sim 0.001$ also of some use.

Predictions for $F_L(x, Q^2)$ compared to $data$ extracted by H1.

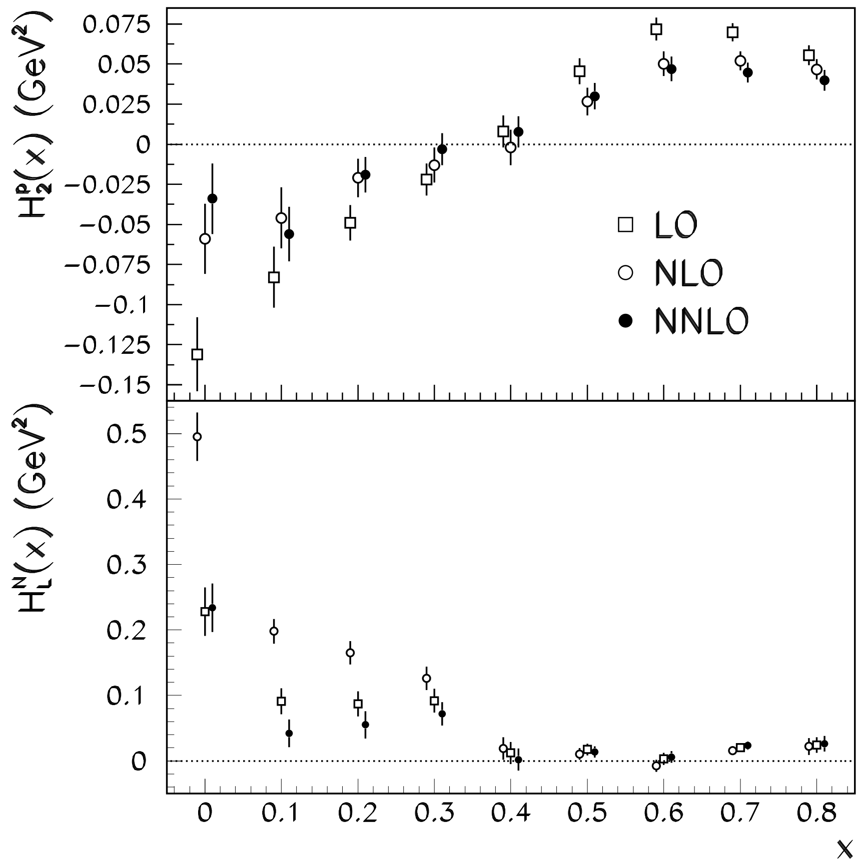
Black – H1 NLO fit. blue – MRST NLO fit. red – MRST NNLO fit.



Additionally, Alekhin performed fits to DIS data, using reduced cross-section for HERA data, and allows higher-twist corrections to be determined phenomenologically.

Finds unambiguous positive correction for $F_L(x,Q^2)$, i.e. consistency check fails for perturbative fit (though errors smaller than I believe).

My view, although higher twist may be important for $F_L(x,Q^2)$ at low x, there are other important effects. Have to consider higher orders in perturbation theory as well as possible higher twist.

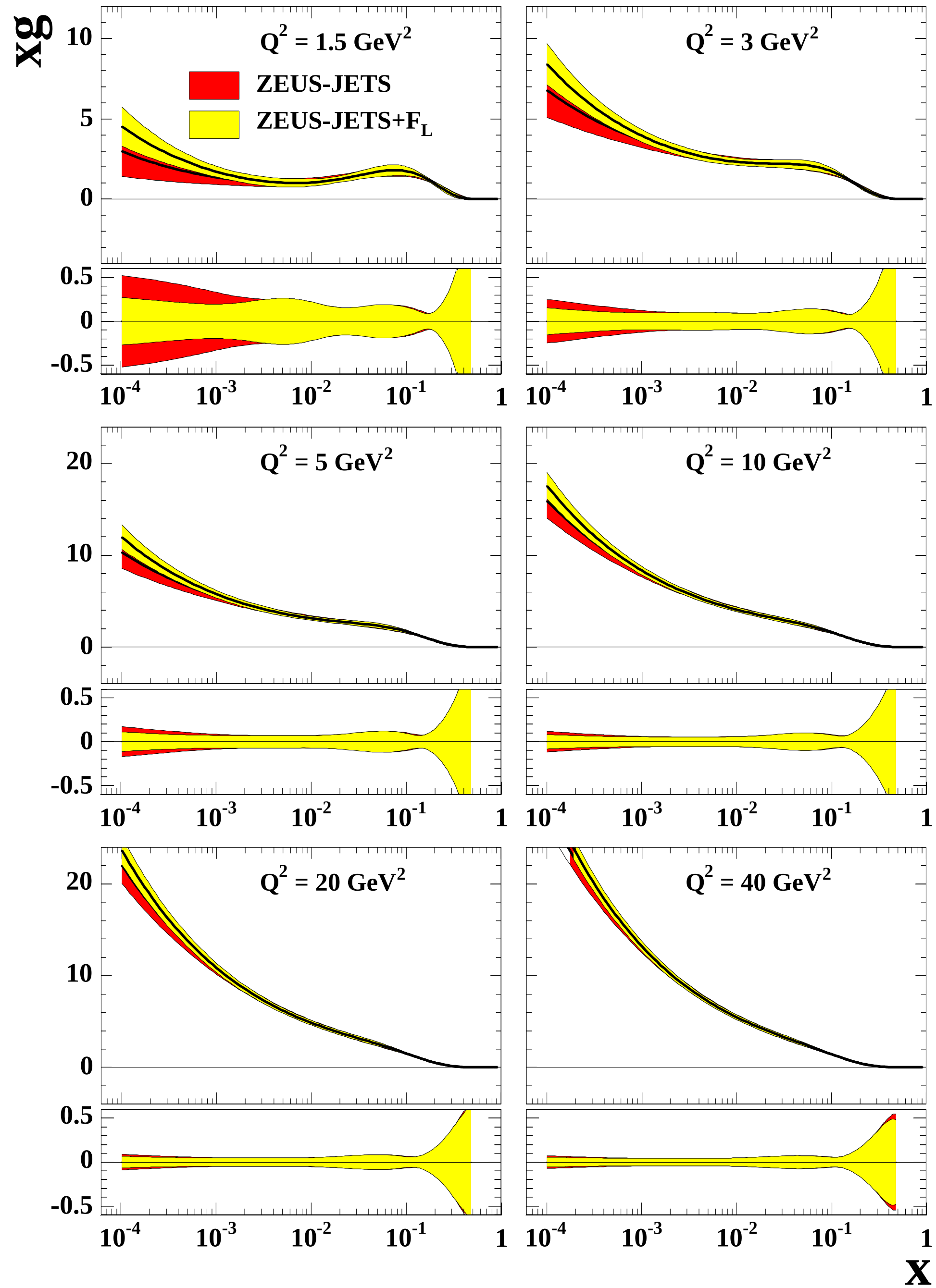


Study by ZEUS (Gwenlan) on the impact of such data on the accuracy with which $g(x, Q^2)$ is determined if $F_L(x, Q^2)$ is roughly as expected from an NLO fit.

Not an enormous improvement in gluon uncertainty.

However, not the most interesting question. Rather, want to see if potential measurement could tell apart different theoretical treatments.

Could it say if we need go beyond standard perturbation theory approach?



Applied to NLO prediction the renormalon correction, with *extreme* $A = 1.2$ is a very significant effect.

Generate a new set of data based around central higher twist prediction. Most similar to dipole prediction but data give $\chi^2 = 25/18$ for dipole prediction curve. Comparison to other predictions shown opposite.

Clearly able to rule out central NLO and NNLO. Repeat study done for *dipole data*.