Deep Inelastic Scattering 2006

A Variable Flavour Number Scheme for Small x Physics

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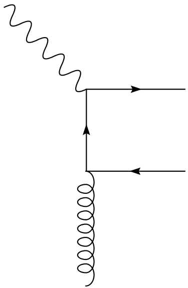
Variable Flavour Number Schemes



- In DIS, can produce final state heavy quarks by boson gluon fusion (Witten).
- Contribution to structure functions:

$$F_{i,H}^{FF} = C_{i,q}^{FF} \otimes \Sigma + C_{i,g}^{FF} \otimes g + C_{i,ns}^{FF} \otimes q_{ns}$$

- Can do this at all scales if we like.
- Number of light flavours constant - the fixed flavour (FF) scheme of Collins & Tung.

- At high Q^2 the FF coefficients contain terms $\sim \alpha_S^n \log(Q^2/M^2)^n$.
- Problem solved by defining a heavy quark distribution above a suitable matching scale e.g. $Q^2 = M^2$:

$$F_{i,H}^{VF} = C_{i,H}^{VF} \otimes q_{H+} + C_{i,q}^{VF,PS} \otimes \Sigma^{(n_f+1)} + C_{i,g}^{VF} \otimes g^{(n_f+1)}$$

- Heavy parton evolves according to DGLAP equations which resum problem logs.
- Number of flavours now varies - variable flavour (VF) number scheme. Coefficients tend to massless limits as $Q^2 \to \infty$.
- Equivalence of FF and VF descriptions:

$$\Rightarrow \begin{pmatrix} q_{H+} \\ g^{(n_f+1)} \\ \Sigma^{(n_f+1)} \end{pmatrix} = \begin{pmatrix} A_{Hq} & A_{Hg} \\ A_{gq} & A_{gg} \\ A_{qq} & A_{qg} \end{pmatrix} \begin{pmatrix} \Sigma^{(n_f)} \\ g^{(n_f)} \end{pmatrix}$$

  (Buza, Matiounine, Smith & van Neerven).
- Partons ambiguous - two choices to make...

Parton Choice 1: Collinear Factorisation Scheme

- Can transform partons from e.g. $\overline{\text{MS}}$ scheme:

$$\mathbf{f}'^{(n_f),(n_f+1)} = Z^{(n_f),(n_f+1)} \otimes \mathbf{f}^{(n_f),(n_f+1)},$$

- Example: The DIS scheme (Altarelli, Ellis & Martinelli). In N-space:

$$\begin{pmatrix} \Sigma^{\text{DIS}(n_f)} \\ g^{\text{DIS}(n_f)} \end{pmatrix} = \begin{pmatrix} C_{2,q}^{\overline{\text{MS}}} & 2n_f C_{2,g}^{\overline{\text{MS}}} \\ 1 - C_{2,q}^{\overline{\text{MS}}} & 1 - 2n_f C_{2,g}^{\overline{\text{MS}}} \end{pmatrix} \begin{pmatrix} \Sigma^{\overline{\text{MS}}(n_f)} \\ g^{\overline{\text{MS}}(n_f)} \end{pmatrix}.$$

- Can transform coefficient functions etc.
- In the DIS scheme:

$$F_2 = \sum_i e_i^2 [q_i + \bar{q}_i].$$

- Useful for massless BFKL calculations - impact factor for $\partial F_2 / \partial \ln Q^2$ gives P_{qg}.
- Hence use DIS scheme, although VF coefficient still ambiguous...

Parton Choice 2: Placement of M^2/Q^2 Terms

- There is an additional parton species in the variable flavour description, leading to an ambiguity in the VF coefficients. Using the $\{A_{ij}\}$:

$$F_{i,H} = (C^{VF}_{i,H} \otimes A_{Hq} + C^{VF,PS}_{i,q} \otimes A_{qq} + C_{i,g} \otimes A_{gq}) \otimes \Sigma^{(n_f)} + (C^{VF}_{i,H} \otimes A_{Hg} + C^{VF,PS}_{i,q} \otimes A_{qg} + C^{VF}_{i,g} \otimes A_{gg}) \otimes g^{(n_f)}.$$
$$= C^{FF}_{i,q} \otimes \Sigma^{(n_f)} + C^{FF}_{i,g} \otimes g^{(n_f)}$$

- Comparing with FF, one must have:

$$C^{VF}_{i,H} \otimes A_{Hg} + C^{VF,PS}_{i,q} \otimes A_{qg} + C^{VF}_{i,g} \otimes A_{gg} = C^{FF}_{i,g}.$$

- At $\mathcal{O}(\alpha_S)$, for example:

$$C^{VF(0)}_{2,H} \otimes A^{(1)}_{Hg} + C^{VF(1)}_{2,g} = C^{FF(1)}_{2,g}.$$

- Can shift terms of order M^2/Q^2 between the VF coefficients. Does not change the collinear factorisation scheme.

The DIS(χ) Scheme - F_2

- Moments of heavy flavour quantities are taken using the scaled variable $x' = x\left(1 + \frac{4M^2}{Q^2}\right)$.
- ACOT have suggested $C_{2,H}^{\overline{\mathrm{MS}}(0)VF} = \delta(1 - x')$ (the ACOT(χ) scheme).
- Use similar prescription in the DIS scheme - the DIS(χ) scheme.
- $C_{2,H}^{VF} = \delta(1 - x')$ with no higher order corrections. Specifies $C_{2,g}^{VF}$ and $C_{2,q}^{VF}$ completely.
- Can make a similar choice for F_L.
- Simplifies interpretation of impact factors at small x...

The DIS(χ) Scheme at Small x

- High energy factorisation in double Mellin space gives:

$$F_{2,H}(\gamma, N, Q^2/M^2) = h_2(\gamma, N, Q^2/M^2) f(N, \gamma, Q_0^2) g_B(Q_0^2, N)$$
$$= h_2(\gamma, N, Q^2/M^2) g(N, \gamma).$$

- Impact factors calculated by Catani, Ciafaloni & Hautmann.
- For $Q^2 < M^2$, identify $C^{FF}_{2,g} = h_2$.
- For $Q^2 > M^2$ cannot do this, as h_2 diverges.
- Instead use $F_{2,H} = A_{Hg} \otimes g + C^{VF}_{2,g} \otimes g \equiv C^{FF}_{2,g} \otimes g$ to get:

$$A_{Hg}(\gamma, N, Q^2/M^2) = C^{FF}_{2,g}(\gamma, N, Q^2/M^2) - C^{VF}_{2,g}(\gamma, N, Q^2/M^2).$$

- Must have $C^{VF}_{2,g} \to 0$ as $Q^2 \to \infty$, so:

$$A_{Hg}(\gamma, N, Q^2/M^2) = C^{FF}_{2,g}(\gamma, N, Q^2/M^2)\Big|_{\frac{Q^2}{M^2} \to \infty}.$$

The DIS(χ) Scheme at Small x

- Then obtain:

$$C_{2,g}^{VF}(\gamma, N, Q^2/M^2) = C_{2,g}^{FF}(\gamma, N, Q^2/M^2) - A_{Hg}(\gamma, N, Q^2/M^2)$$
$$= h_2(\gamma, N, Q^2/M^2) - h_2(\gamma, N, Q^2/M^2)|_{\frac{Q^2}{M^2}\to\infty}$$

- One can show consistency with the DIS(χ) scheme defined at fixed order.
- For $F_{L,H}$, things are somewhat simpler:

$$F_{L,H} = h_L(\gamma, N, Q^2/M^2)g(\gamma, N),$$

with h_L finite as $Q^2 \to \infty$.
- Identify $C_{L,g}^{FF} = C_{L,g}^{VF} = h_L$.
- Forms in x and Q^2 space found by solving BFKL equation.

LL BFKL Equation with Running Coupling

- The LO running coupling is:

$$\alpha_S(\mu_R^2) = \frac{1}{\beta_0 \ln \mu_R^2/\Lambda^2}$$

- Substitute $\alpha_S(k^2)$ into BFKL equation (Collins & Kwiecinski):

$$\frac{\partial f(\gamma, N)}{\partial \gamma} = \frac{\partial f_I(\gamma, Q_0^2)}{\partial \gamma} - \frac{1}{\bar{\beta}_0 N}\chi_0(\gamma, N),$$

  with $\bar{\beta}_0 = 3/(\pi\beta_0)$.
- Solve to give:

$$f(\gamma, N) = \exp[-X_0(\gamma)/(\bar{\beta}_0)] \int_\gamma^\infty \frac{df_I(\tilde{\gamma}, Q_0^2)}{d\tilde{\gamma}} \exp[X_0(\tilde{\gamma})/(\bar{\beta}_0 N)]d\tilde{\gamma}.$$

- Factorisation achieved by shifting the lower limit $\gamma \to 0$. Corrections are power-suppressed.

- For the integrated gluon distribution, one finds (Thorne):

$$g(Q^2, N) = \mathcal{G}_E(Q^2, N)\mathcal{G}_I(Q_0^2, N)g_B(Q_0^2, N)$$

with perturbative piece:

$$\mathcal{G}_E(Q^2, N) = \frac{1}{2\pi\imath}\int_{1/2-\imath\infty}^{1/2+\imath\infty} \frac{f^{\beta_0}}{\gamma} \exp[\gamma \ln(Q^2/\Lambda^2) - X_0(\gamma)/(\bar{\beta}_0 N)] d\gamma.$$

- Structure functions have an impact factor included:

$$\mathcal{F}_{E,i} = \frac{1}{2\pi\imath}\int \frac{h_i(\gamma, Q^2/M^2) f^{\beta_0}}{\gamma} \exp\left[\gamma \ln\left(\frac{Q^2}{\Lambda^2}\right) - \frac{X_0(\gamma)}{\bar{\beta}_0 N}\right] d\gamma.$$

- Coefficients and anomalous dimensions given by $\mathcal{C}_{i,g} = \mathcal{F}_{E,i}/\mathcal{G}_E$. Find numerically or (approximately) analytically by expanding $\chi_0(\gamma)$.

Resummed Coefficients

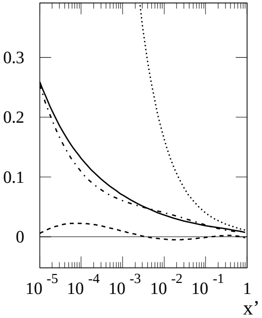


Figure: $x' C_{2,g}^{FF}$ for $Q^2 = m_c^2$, $n_f = 3$.

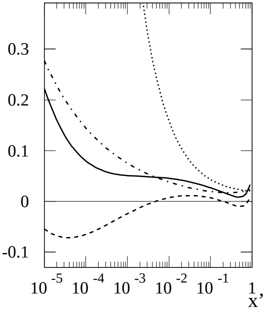


Figure: $x' C_{2,g}^{VF}$ for $Q^2 = m_c^2$, $n_f = 4$.

$C_{L,g}^{VF}$ and A_{Hg}

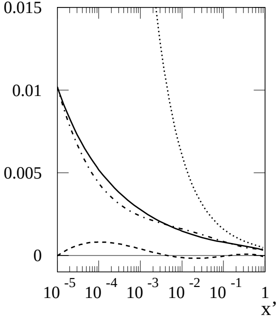


Figure: $x' C_{L,g}^{VF}$ for $Q^2 = m_c^2$, $n_f = 4$.

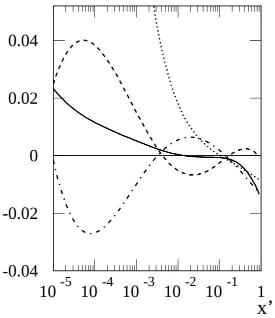
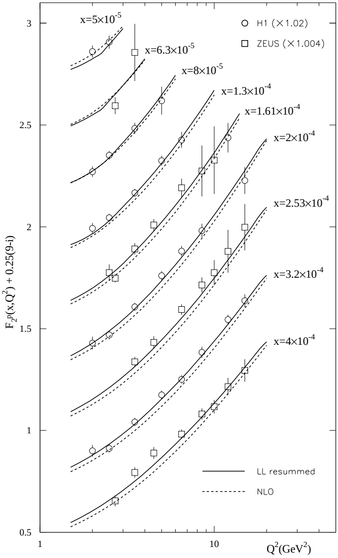
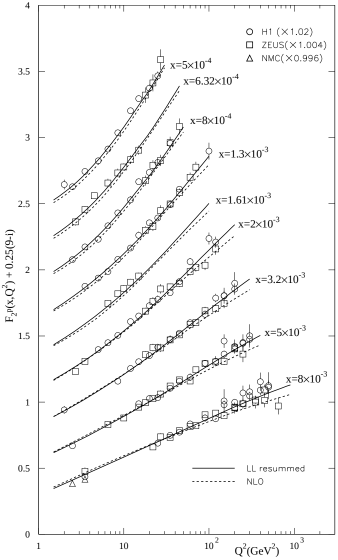


Figure: xA_{Hg} at $Q^2 = M^2$ for $n_f = 4$.

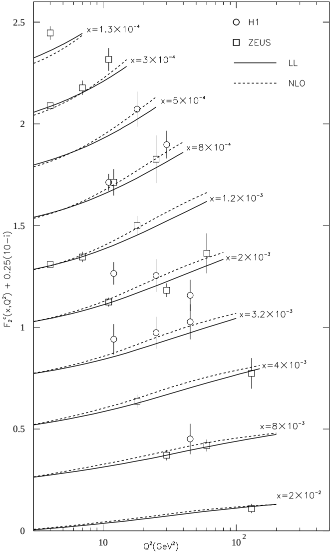
- Note positivity of A_{Hg} as $x \to 0$.
- In all quantities, running coupling moderates low x divergence.

Results - F_2


x=5×10^-5
x=6.3×10^-5
x=8×10^-5
x=1.3×10^-4
x=1.61×10^-4
x=2×10^-4
x=2.53×10^-4
x=3.2×10^-4
x=4×10^-4
H1 (×1.02)
ZEUS (×1.004)
LL resummed
NLO
F2p(x,Q2) + 0.25(9-i)
Q2(GeV2)

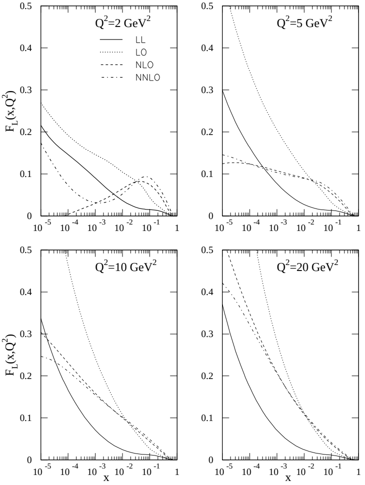

x=5×10^-4
x=6.32×10^-4
x=8×10^-4
x=1.3×10^-3
x=1.61×10^-3
x=2×10^-3
x=3.2×10^-3
x=5×10^-3
x=8×10^-3
H1 (×1.02)
ZEUS(×1.004)
NMC(×0.996)
LL resummed
NLO
F2p(x,Q2) + 0.25(9-i)
Q2(GeV2)

Results - F_2^c



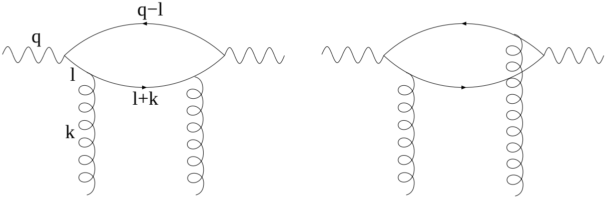
- Resummed fit performs better for small x data - note slope as Q^2 increases.
- However, problems at moderate and high x (not shown). Resummations not confined to low enough x.
- Need NLO as well as resummations...
- Can see this in F_L.

Predictions - F_L



- Can clearly see perturbative instability in the fixed order results.
- But resummed prediction much too small at high x (where DGLAP results should be reliable).
- Further evidence for needing both small x resummations and higher order terms...
- But resummations work very well at low x!

Towards NLL - Photon Impact Factor



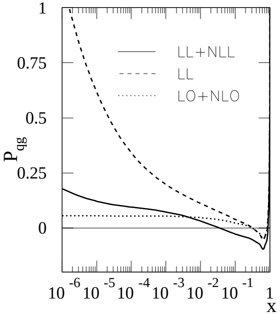
- Gluon momentum fraction x_g is not the same as Bjorken x.
- One can relate them by considering the kinematics to get:

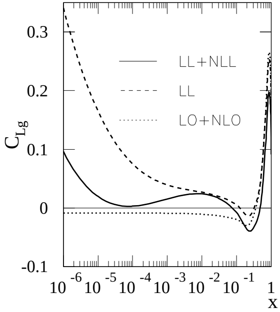
$$x_g = x \left[\frac{\hat{Q}^2 - \hat{k}_\perp^2 - {l'_\perp}^2}{\hat{Q}^2} \right] \quad ,$$

with $\hat{A} = \alpha(1-\alpha)A^2$, $l'_\perp = l_\perp + (1-\alpha)k_\perp$.

- Resulting impact factors calculated by Bialas, Navelet and Peschanski.
- They seem to approximate well the NLL impact factors.

NLL correction to P_{qg} and C_{Lg}





- NLL corrections soften the divergence in the resummed quantities.
- Just what is needed to alleviate the problems in the resummed fit at moderate x...

Conclusions

- Have defined a VF scheme for consistent implementation of small x heavy flavour effects alongside fixed order expansion.
- Impact factors interpreted in terms of $C_{2g}^{FF,VF}$, A_{Hg}.
- LL resummed fit (with running α_S) good at small x, but need for higher order corrections (e.g. F_L).
- Can perform an approximate NLL analysis by using impact factors with exact gluon kinematics.
- Preliminary results for massless P_{qg} and C_{Lg} are encouraging...