Review on hard exclusive reactions

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1. Impact parameter
2. Evolution and higher orders
3. Summary

Impact parameter

- form states with definite light-cone momentum $p^+ = p^0 + p^3$ and transverse position (impact parameter):

$$|p^+, \boldsymbol{b}\rangle = \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{b}\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle$$

- can exactly localize in 2 dimensions
  no limitation by Compton wavelength
- can go to frame where particles moves fast
  $\rightsquigarrow$ parton interpretation
- same for internal lines in Feynman diagram

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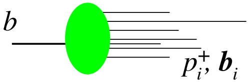
- composite systems: $\boldsymbol{b}$ is center of momentum of constituents



$$\boldsymbol{b} = \frac{\sum_i p_i^+ \boldsymbol{b}_i}{\sum_i p_i^+}$$

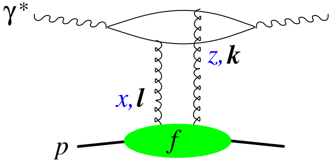
consequence of Lorentz invariance: transverse boosts

$$k^+ \to k^+ \qquad \boldsymbol{k} \to \boldsymbol{k} - k^+\boldsymbol{v}$$

nonrelativistic analog: Galilei invariance $\xrightarrow{\text{Noether}}$ center of mass

Dipole representation

- e.g. in forward Compton amplitude
- Fourier transform $\boldsymbol{k} \to \boldsymbol{r}$
  $=$ relative distance of q and $\bar{q}$
- high-energy limit:
  $\boldsymbol{r}$ conserved in interaction
  essential: approx $x \rightsquigarrow x_B$ in $f(x, \boldsymbol{l})$
  $x = \text{fct}(x_B, \boldsymbol{k}, \boldsymbol{l}, z)$



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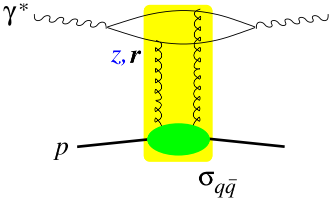

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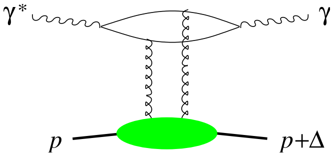

- nonforward kinematics (e.g. DVCS): extra variable $t \approx -\boldsymbol{\Delta}^2$
- Fourier transform $\boldsymbol{\Delta} \to \boldsymbol{b} =$ distance between γ^* and p
  (impact parameter)
- dipole scattering amplitude $N(x_B, \boldsymbol{r}, \boldsymbol{b})$
- $t = 0 \;\leftrightarrow\; \int d^2\boldsymbol{b}$:

$$\sigma_{q\bar{q}}(x_B, \boldsymbol{r}) = 2\int d^2\boldsymbol{b}\, N(x_B, \boldsymbol{r}, \boldsymbol{b})$$

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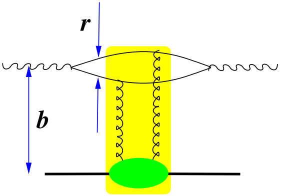

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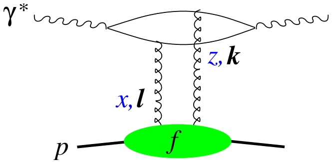
Collinear factorization (not limited to small x)

- large Q^2 limit
- symmetric dipoles $\longrightarrow$ small $\boldsymbol{r}$

  $\longrightarrow$ gluon distribution

  $xg(x, Q^2) = \int^{Q^2} d\boldsymbol{l}^2\, f(x, \boldsymbol{l}^2)/\boldsymbol{l}^2$



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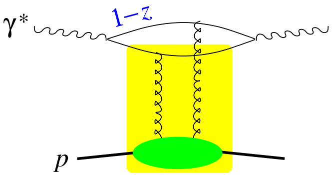
- asymmetric dipoles $(z \rightarrow 0)$
  $\rightarrow$ quark distribution



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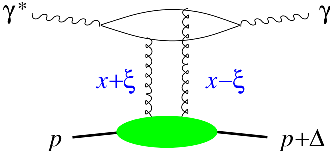


- nonforward kinematics (e.g. DVCS):
  generalized parton distributions $H^q(x,\xi,t)$, $H^g(x,\xi,t)$
- Fourier transform $\boldsymbol{\Delta} \rightarrow \boldsymbol{b} =$ distance between γ^* and p
  $=$ distance of struck parton from center of p
  $\rightsquigarrow$ impact parameter distributions
- for $\xi = 0$: probability to find parton with momentum fraction x and impact parameter $\boldsymbol{b}$

Scale evolution $\xi = 0$ for simplicity

- $q(x, b^2; \mu^2)$ fulfills usual DGLAP evolution equation
  evolution local in b (take $1/\mu \ll b$ to be safe)

Scale evolution $\xi = 0$ for simplicity

- $q(x, b^2; \mu^2)$ fulfills usual DGLAP evolution equation
  evolution local in b (take $1/\mu \ll b$ to be safe)
- average impact parameter

$$\langle b^2 \rangle_x = \frac{\int d^2b \; b^2 \, q(x, b^2)}{\int d^2b \; q(x, b^2)} = 4 \frac{\partial}{\partial t} \log H^q(x, \xi = 0, t) \Big|_{t=0}$$

evolves as (non-singlet quark for simplicity)

$$\mu^2 \frac{d}{d\mu^2} \langle b^2 \rangle_x = - \int_x^1 \frac{dz}{z} \, P_{qq}\Big(\frac{x}{z}\Big) \, \frac{q_{\text{NS}}(z)}{q_{\text{NS}}(x)} \Big[\langle b^2 \rangle_x - \langle b^2 \rangle_z \Big]$$

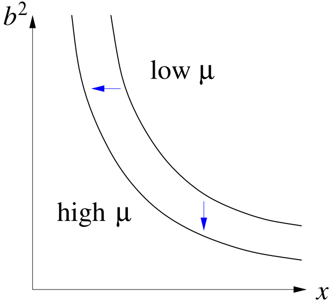
M.D. et al. '04

analogous for quark singlet and gluons

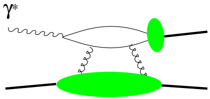
Scale Evolution

$$\mu^2 \frac{d}{d\mu^2} \langle b^2 \rangle_x = -\int_x^1 \frac{dz}{z} P_{qq}\Big(\frac{x}{z}\Big) \frac{q_{\rm NS}(z)}{q_{\rm NS}(x)} \Big[\langle b^2 \rangle_x - \langle b^2 \rangle_z \Big]$$

- if $\langle b^2 \rangle_x$ decreases with x $\Rightarrow$ $\langle b^2 \rangle_x$ decreases with μ^2



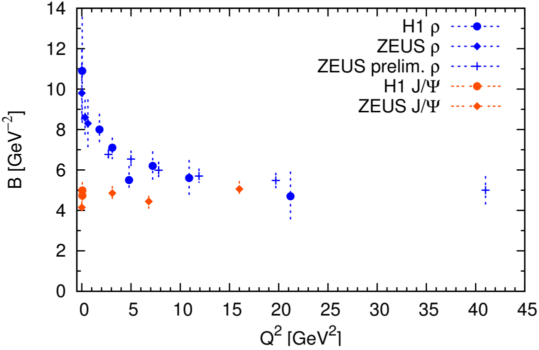
The t dependence: measurements



- ρ and ϕ: $\gamma^* \to q\bar{q}$ "pointlike" for large Q^2

  smaller Q^2: $q\bar{q}$ dipole size contributes to B

  $\leftrightarrow$ large power corrections to collin. approx.
- J/Ψ: $\gamma \to c\bar{c}$ "pointlike" even for $Q^2 = 0$



- parameterize

  $d\sigma/dt \propto e^{-B|t|}$

  for small x and t

  $\langle b^2 \rangle = 4B$

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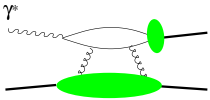

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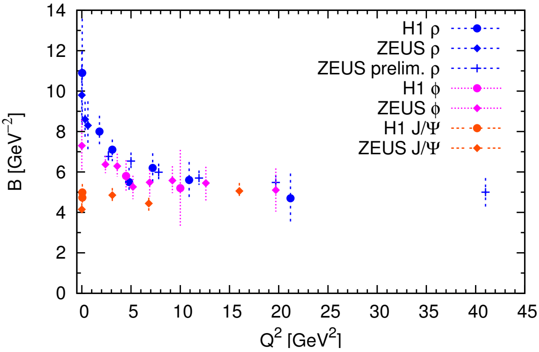

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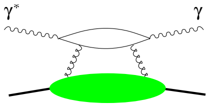
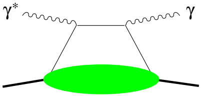
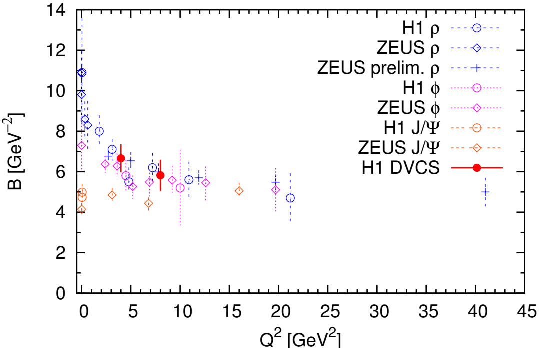




- first measurement for DVCS H1 '05

  dipole pict: $\gamma^*_T \to q\bar{q}$ dipole not small

  collin. fact: gluons and sea quarks



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Correlations between x and t

neglect interplay of x and ξ in following

Small x:

simple ansatz: GPD $\sim \left(\frac{1}{x}\right)^{\alpha+\alpha' t} = x^{-\alpha}\, e^{t\alpha' \log(1/x)}$

- exclusive J/Ψ production (gluons)
  - photoproduction (H1' 05)

    $\alpha = 1.224 \pm 0.010 \pm 0.012$

    $\alpha' = 0.164 \pm 0.028 \pm 0.030$ GeV^{-2}
  - similar in electroproduction
- values very different in soft processes $\gamma p \to \rho p$, $pp \to pp$, ...

  for α is well-known from inclusive $\gamma^* p \to X$ vs. $\gamma p \to X$

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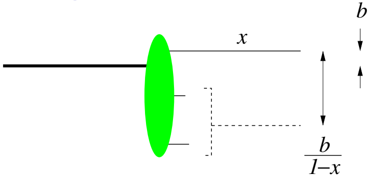
- in quark nonsinglet sector (no mixing with gluons)
  $\alpha \sim 0.4 \ldots 0.5$ in parton distrib's at low scale
  similar to soft processes (meson trajectories)
- α' in partonic regime? have indirect information $\rightsquigarrow$

Correlations between x and t

Large x:



- for $x \to 1$ get $b \to 0$
  nonrel. analog:
  center of mass of atom
- $\Leftrightarrow \quad t$ dependence becomes flat

- $d = b/(1-x)$
  $=$ distance of selected parton from spectator system
  gives lower bound on overall size of proton
- finite average size of configurations with $x \to 1$ implies

$$\langle b^2 \rangle_x \sim (1-x)^2$$

M. Burkardt, '02, '04

Information from electromagnetic form factors

- ff's constrain interplay of x and b dependence

  M.D. et al. '04, M. Guidal et al. '04

- e.m. current $\rightsquigarrow$ only $q - \bar{q}$

  $$H_v^q(x,t) = H^q(x,\xi=0,t) - H^{\bar{q}}(x,\xi=0,t)$$

  $$F_1^p(t) = \int_0^1 dx \left[\tfrac{2}{3} H_v^u(x,t) - \tfrac{1}{3} H_v^d(x,t) \right]$$
  $$F_1^n(t) = \int_0^1 dx \left[\tfrac{2}{3} H_v^d(x,t) - \tfrac{1}{3} H_v^u(x,t) \right]$$

- ansatz: $H_v^q(x,t) = q_v(x) \exp[\, t f_q(x) \,]$ $\qquad \langle b^2 \rangle_x^q = 4 f_q(x)$
- ansatz for $f_q(x)$ interpolates between

  $f_q(x) \to \alpha' \log(1/x)$ for $x \to 0$

  $f_q(x) \sim (1-x)^2$ for $x \to 1$

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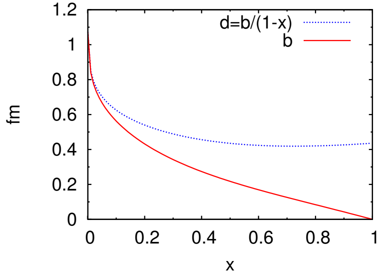
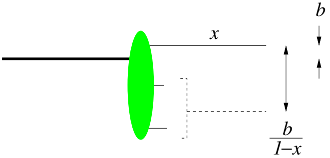
- ansatz: $H_v^q(x,t) = q_v(x) \exp[\, t f_q(x) \,]$ $\quad \langle b^2 \rangle_x^q = 4 f_q(x)$
- good description of data with $\alpha' = 0.9$ to 1 GeV^{-2}

  $\rightsquigarrow$ same α' as leading meson trajectories

Information from electromagnetic form factors

for $u - \bar{u}$





- clear drop with x of average distance $d = b/(1-x)$
  $\leftrightarrow$ strong correlation of x and t dependence

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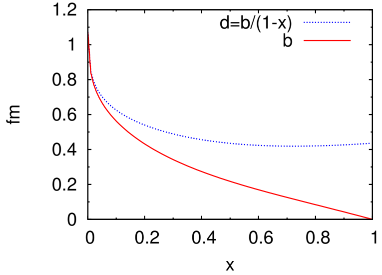
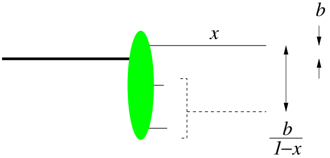
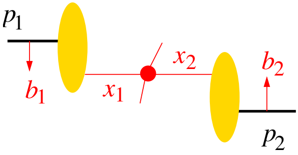




- clear drop with x of average distance $d = b/(1-x)$
  $\leftrightarrow$ strong correlation of x and t dependence
- consistent with lattice QCD results:
  t slopes of moments $\int dx\, x^n\, H^q(x, \xi, t)$ increase with n
  QCDSF Coll., LHPC Coll., '03 – '06

Consequences for hadron-hadron collisions



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  $\to$ no impact parameter dependence
  integrate over b_1 and b_2 independently

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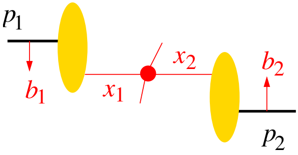
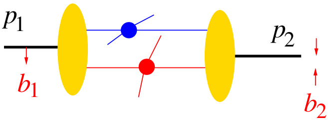




- hard inclusive process, e.g. $pp \to \text{jet jet} + X$
  $\to$ no impact parameter dependence
  integrate over b_1 and b_2 independently
- secondary soft or hard interactions
  do not affect inclusive cross section
  but change event structure
- larger mom. fractions x_1, x_2 in hard subprocess
  $\rightsquigarrow$ more central collision
  $\rightsquigarrow$ more secondary interactions
  $\rightsquigarrow$ enhanced saturation effects Frankfurt, Strikman, Weiss '03 – '05

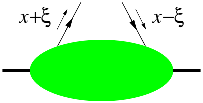
Evolution

- for non-singlet combinations (e.g. $q - \bar{q}$ or $u - d$)

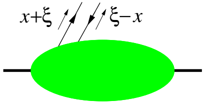
$$\mu^2 \frac{d}{d\mu^2} H^{NS}(x,\xi,t) = \int \frac{dx'}{|\xi|} V^{NS}\Big(\frac{x}{\xi},\frac{x'}{\xi}\Big) H^{NS}(x',\xi,t)$$

for singlet: matrix eq'n for mixing with gluon GPD

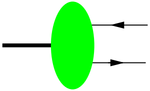
- evolution local in t (take $-t \ll \mu^2$ to be safe)



generalization of DGLAP evolution to $\xi \neq 0$



ERBL evolution as for meson distribution amplitudes



- conformal (Gegenbauer) moments

$$\mathcal{C}_n^{NS}(\xi,t;\mu^2) = \xi^n \int dx\, C_n^{3/2}\Big(\frac{x}{\xi}\Big) H^{NS}(x,\xi,t;\mu^2)$$

evolve multiplicatively at LO(α_s)

$$\mu^2 \frac{d}{d\mu^2} \mathcal{C}_n^{NS}(\xi,t;\mu^2) = -\gamma_n\, \mathcal{C}_n^{NS}(\xi,t;\mu^2)$$

- for $\xi \to 0$ recover Mellin moments

- conformal (Gegenbauer) moments

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- explicit solution of LO evolution   A. Manashov et al. '05
  - usual parton densities: Mellin transform

    $M^j(\mu) = \int dx\; x^{j-1}\, q(x,\mu)$ evolves multiplicatively

    $q(x,\mu) = -\frac{1}{2\pi i} \int_C dj\; x^{-j}\, M^j(\mu)$
  - generalization for $\xi \neq 0$ involves Legendre functions in moments and their inversion
  - suitable for fast numeric implementation

Asymptotics quark non-singlet for simplicity

- explicit solution $\rightsquigarrow$ can take limits analytically
- take large

$$Y = \log \frac{2}{\xi} \quad \text{and} \quad L = \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \sim \log \mu^2$$

with $Y \gg \log L$ and fixed x/ξ

- find

$$H_{NS} \sim \frac{\log^{1/4} L}{Y^{3/4}} \exp \sqrt{\frac{8C_F}{\beta_0} Y \log L}\; f\Big(\frac{x}{\xi}, 1 + \sqrt{\frac{2C_F}{\beta_0} \frac{\log L}{Y}}\Big)$$

$f =$ known function

$\rightsquigarrow$ double logarithmic scaling

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- take large

$$Y = \log\frac{2}{\xi} \quad \text{and} \quad L = \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \sim \log\mu^2$$

with $Y \gg \log L$ and fixed x/ξ

- but if $q(x;\mu_0^2) \sim x^{-\alpha}$ with $\alpha > \sqrt{\frac{2C_F}{\beta_0}\frac{\log L}{Y}}$ then

$$H_{NS} \sim \xi^{-\alpha}\, L^{\gamma(1+\alpha)}\, f\Big(\frac{x}{\xi}, 1+\alpha\Big)$$

$\gamma(j) =$ anomalous dimension, positive for $0 < \alpha < 1$

$\Leftrightarrow$ Shuvaev result — Shuvaev al, '99

$\rightsquigarrow$ skewness factor $\frac{H_{NS}(x=\xi,\xi)}{q_{NS}(2\xi)} = \frac{2^{2\alpha+1}}{\sqrt{\pi}}\,\frac{\Gamma(\alpha+\frac{3}{2})}{\Gamma(\alpha+2)}$

higher loop corrections to DVCS

- write $\gamma^* p \to \gamma p$ amplitude in terms of conformal moments
- use conformal symmetry and
  NNLO DGLAP kernels Moch, Vermaseren, Vogt '04
  $\rightsquigarrow$ DVCS amplitude at NNLO
  except for $O(\alpha_s^3)$ mixing of conf. moments under evolution
- results for quark non-singlet sector D. Müller, '05
  $\rightsquigarrow$ NNLO corrections well behaved:
  - $< 5\%$ of NLO for amplitude $\mathcal{A}$
  - $< 20\%$ of NLO for $\frac{\partial}{\partial Q^2} \mathcal{A}$

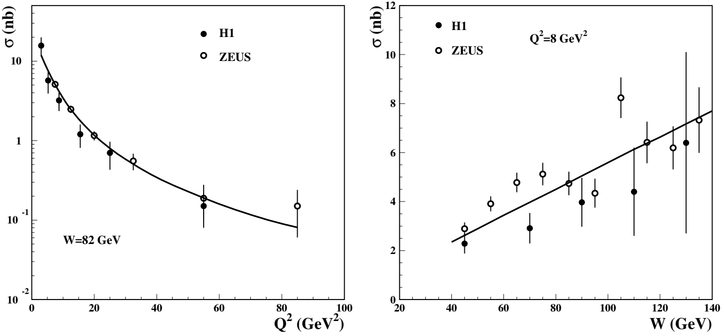
  for $10^{-2} < \xi < 0.6$

Modeling GPDs

- can make model of conformal moments
  - $\rightsquigarrow$ invert to obtain model of GPDs — Müller, Schäfer '05
  - $\oplus$ can implement Lorentz invariance (polynomiality) relations
  - $\oplus$ easy to perform evolution
  - $\ominus$ physical intuition?

Modeling GPDs

- new model for $g(x), q(x) \rightsquigarrow$ GPD
  V. Guzey and M. Polyakov '05, based on M. Polyakov and A. Shuvaev '02
- satisfies Lorentz invariance relations
- good description of DVCS data from HERA (LO calculation)



hep-ph/0507183

Many other things not covered in this talk:

- exclusive dijet production Braun, Ivanov '05
- progress in non diffractive aspects of generalized parton distributions
  - spin and orbital angular momentum M.D., Hägler '05; Burkardt '05; Burkardt, Schnell '05; Enberg et al. '06
  - lattice calculations Dalley '04; QCDSF; LHPC
  - dynamical models Kvinihkidze, Blankleider '04; Pasquini et al. '04; Tiburzi et al. '04; Noguera et al. '04, Scepetta, Vento '04; Mineo et al '05; Ossmann et al. '05; C.-R. Ji et al. '06
  - nuclear GPDs Scopetta '04; Liuti, Taneja '04, '05; Guzey, Siddikov '05
  - exotic meson production Anikin et al. '04, '05
  - large-angle processes Miller '04; M.D. et al. '04; Kroll, Schäfer '05
  - hadron-photon and baryon-meson transitions, $\gamma\gamma$ physics Pire, Szymanowski '04, '05; Tiburzi '05; Lansberg et al. '06
  - νp scattering Amore et al. '04; Psaker '04

Summary

impact parameter representation

- well-known in dipole picture
- also in collin. factorization $\rightsquigarrow$ impact parameter distributions
- DGLAP dynamics $\rightsquigarrow$ impact param. profile
  depends on resolution scale
- exp. information from exclusive meson production and DVCS
  for valence quarks: indirectly from form factors
- in general: impact parameter strongly correlated with x
- consequences for hadron-hadron collisions

Summary

better understanding of evolution for GPDs:

moment methods

- sheds new light on “skewness factor
- use to construct models of GPDs
- use to evaluate NNLO corrections to DVCS