Pomeron Loops, Black Spots & Diffusive Scaling in QCD at High Energy

Edmond Iancu
SPhT Saclay & CNRS

Pomeron Loops, Black Spots
& Diffusive Scaling in QCD at High Energy

Edmond Iancu
SPhT Saclay & CNRS

Based on:

E.I., A. Mueller and S. Munier (hep–ph/041018)

E.I., D. Triantafyllopoulos (hep-ph/0411405 & 0501193)

Y. Hatta, E.I., C. Marquet, G. Soyez, D. Triantafyllopoulos (hep-ph/0601150)

Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

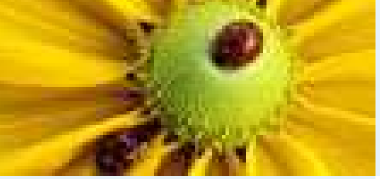
High–energy evolution in QCD

Pomeron loops

- Introduction :

The physical picture of the hadron in DIS at high energy

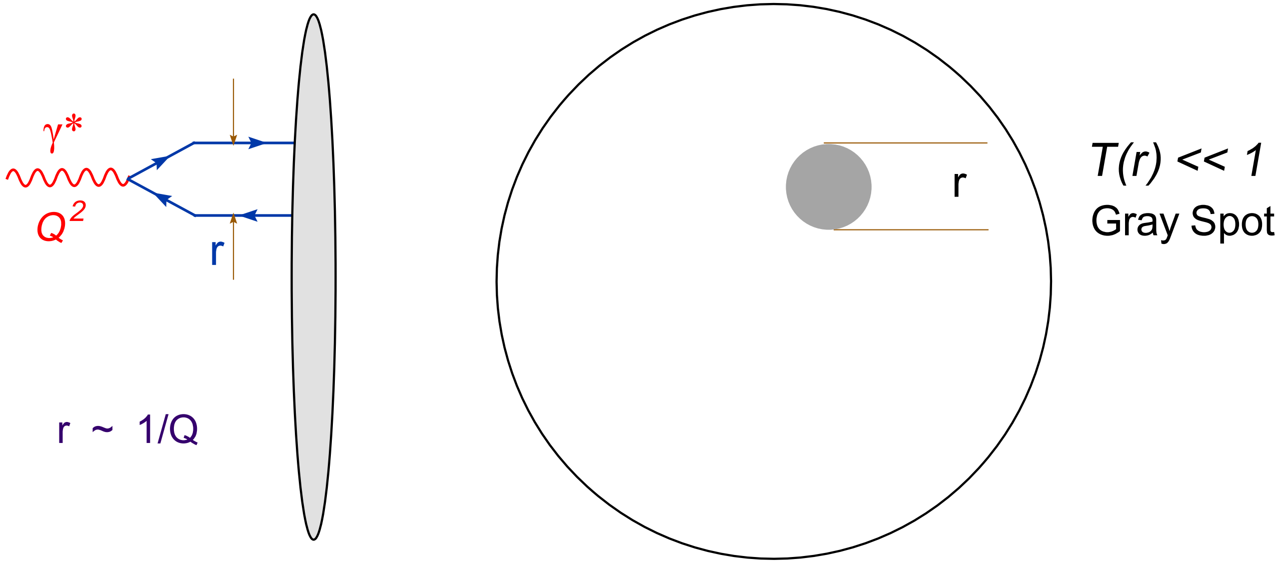
(in perturbative QCD)



Preliminaries

- How does a hadron look like when probed in (inclusive or diffractive) DIS at very high energy ?
- 'Large nucleus at nearly central impact parameters'

  $\Longrightarrow$ Quasi–homogeneous (at low energy)



- The dipole probes an area $\Sigma \sim r^2$ with a (nearly b–independent) scattering amplitude $T(r, Y)$

Preliminaries

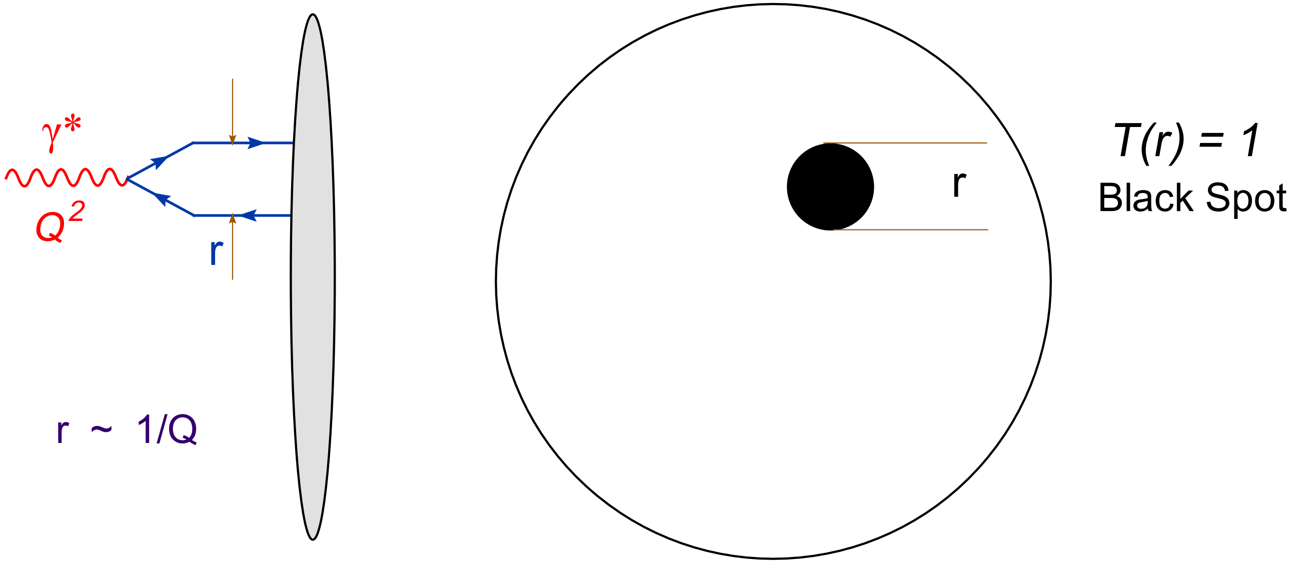
Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

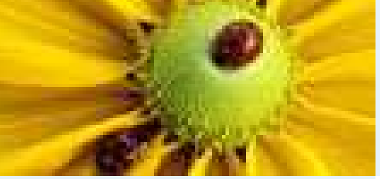
Pomeron loops

- How does a hadron look like when probed in (inclusive or diffractive) DIS at very high energy ?
- ‘Large nucleus at nearly central impact parameters’

  $\Longrightarrow$ Quasi–homogeneous (at low energy)



- The dipole probes an area $\Sigma \sim r^2$ with a (nearly b–independent) scattering amplitude $T(r, Y)$



Preliminaries

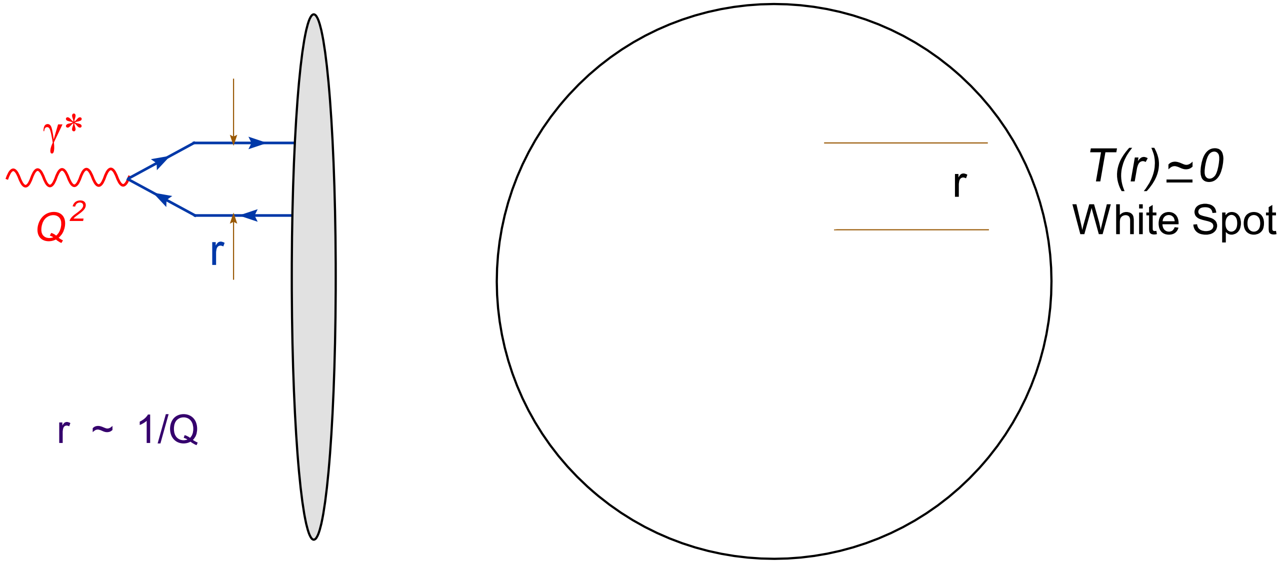
Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

Pomeron loops

- How does a hadron look like when probed in (inclusive or diffractive) DIS at very high energy ?
- ‘Large nucleus at nearly central impact parameters’

  $\Longrightarrow$ Quasi–homogeneous (at low energy)



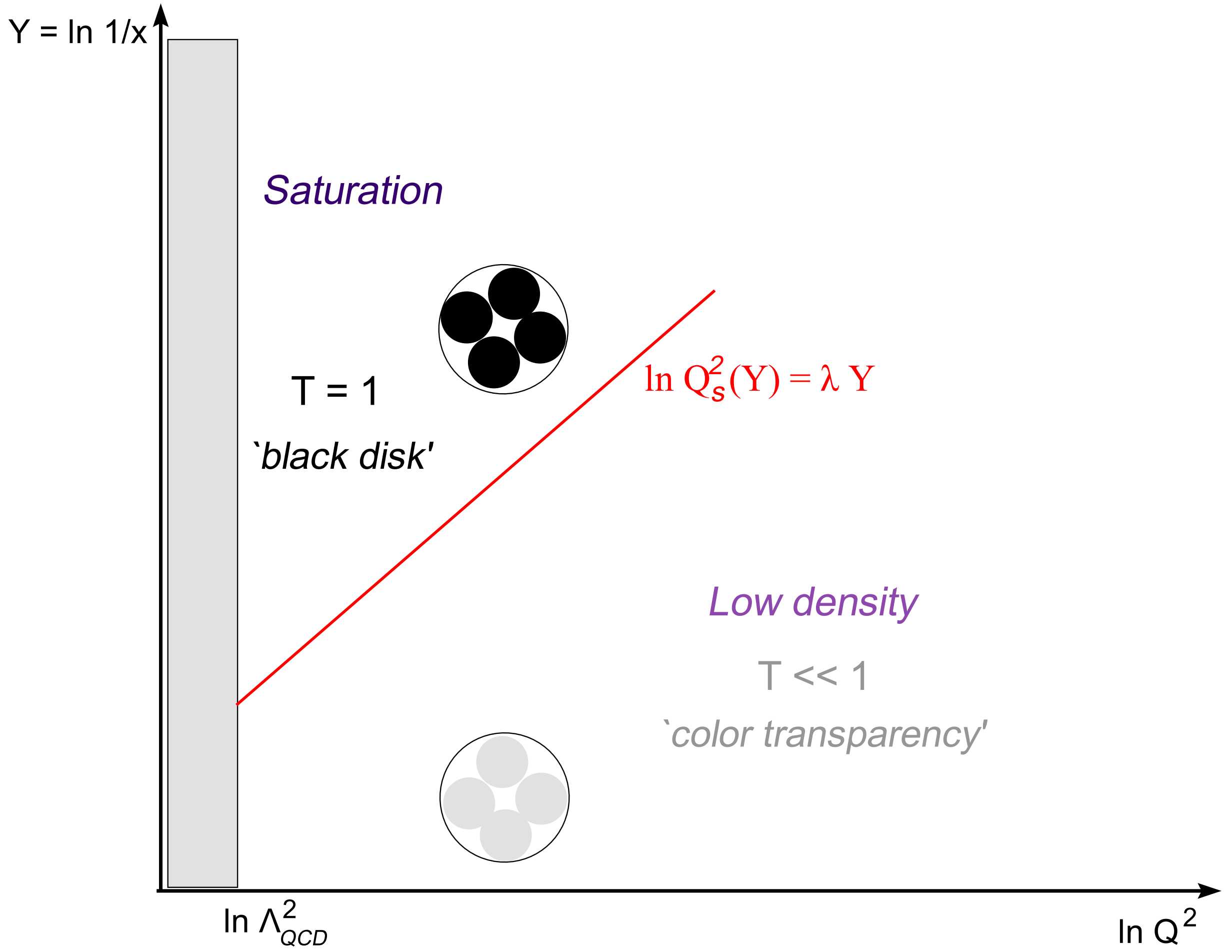
- For given r and b, all these situations can occur, depending upon the value of the rapidity $Y = \ln(1/x)$

Saturation line

Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

Pomeron loops



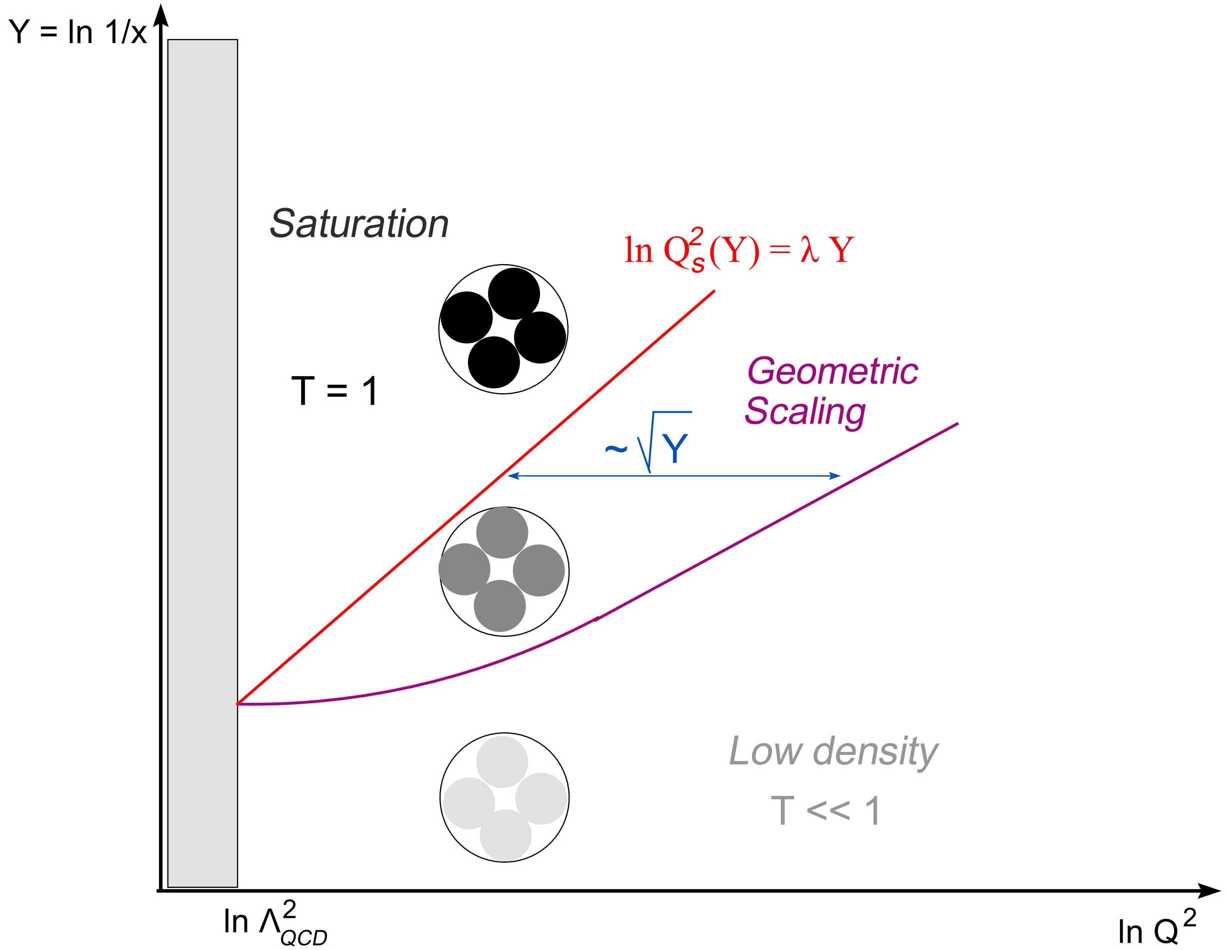
- Low energy: Color transparency (or leading–twist) : $T \propto r^2$
- High energy: Saturation (or black disk limit) : $T = 1$
- The only non–trivial information: The 'saturation line' (λ)

Geometric scaling

Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

Pomeron loops



- New physical regime at relatively large $Q^2 \gg Q_s^2(Y)$:

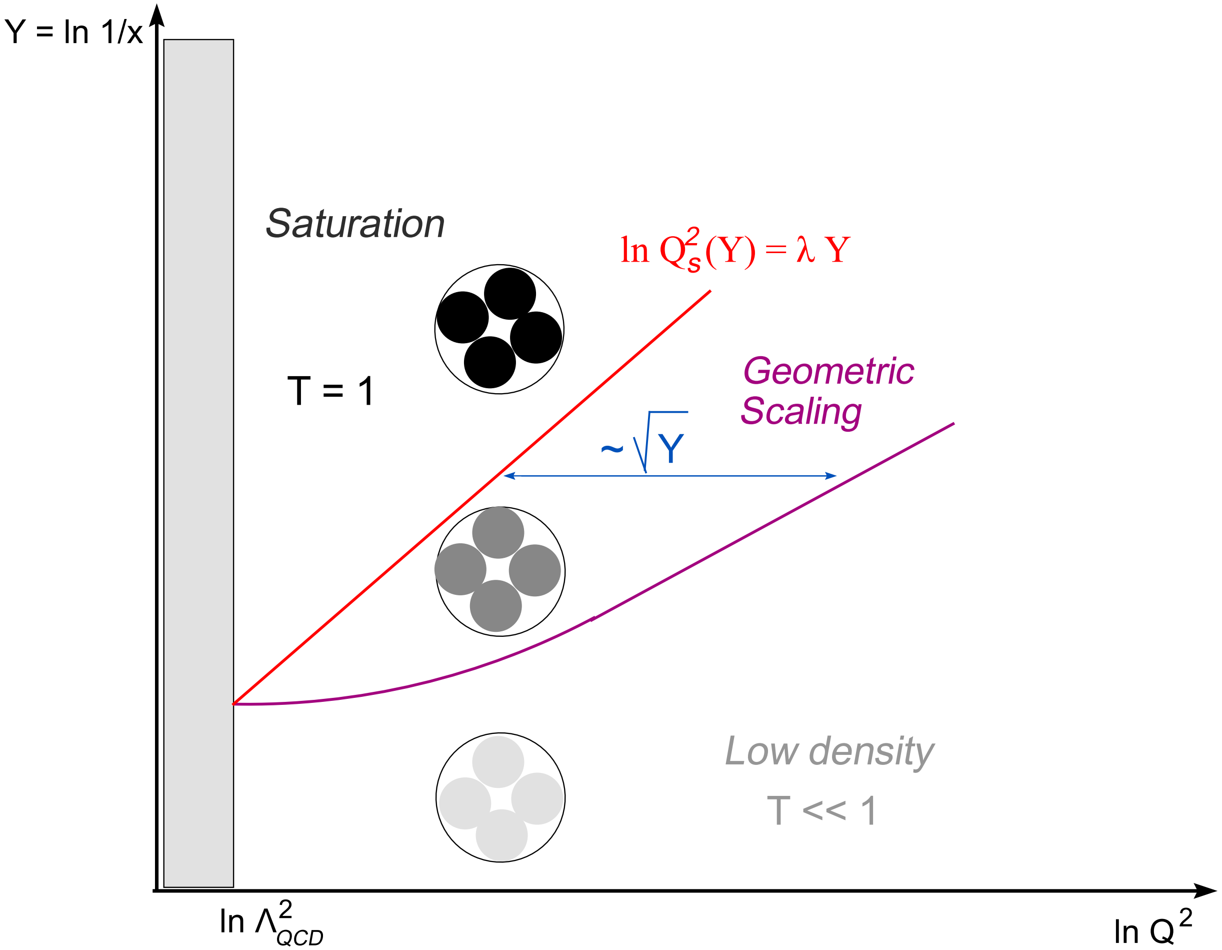
$$T(r, Y) \approx \left(r^2 Q_s^2(Y)\right)^\gamma \ll 1, \qquad \gamma \approx 0.63$$

Geometric scaling

Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

Pomeron loops



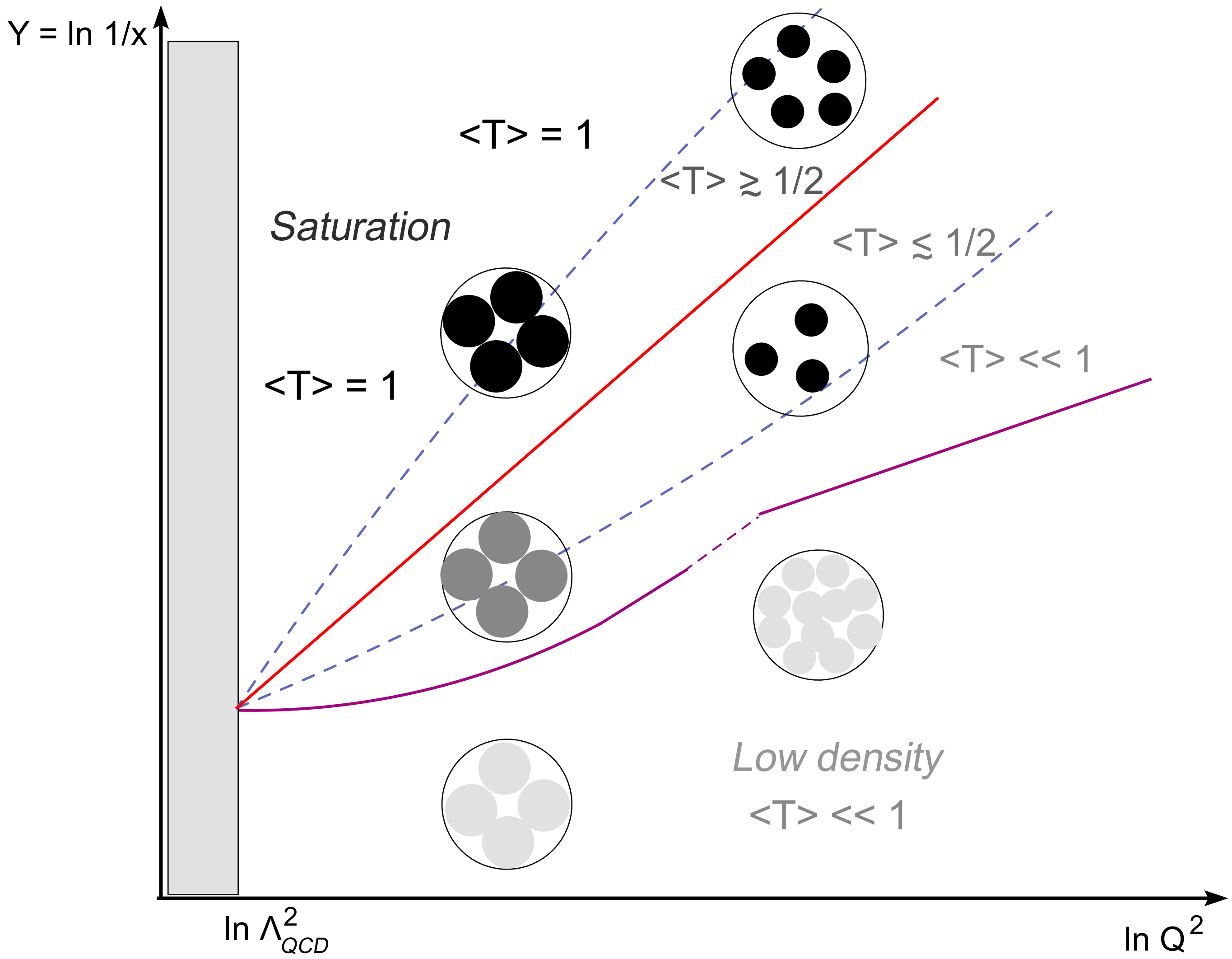
- Geometric scaling window: $\ln[Q^2/Q_s^2(Y)] \propto \sqrt{Y}$
- The influence of saturation extends well outside the saturation regime !

Black spots

Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

Pomeron loops



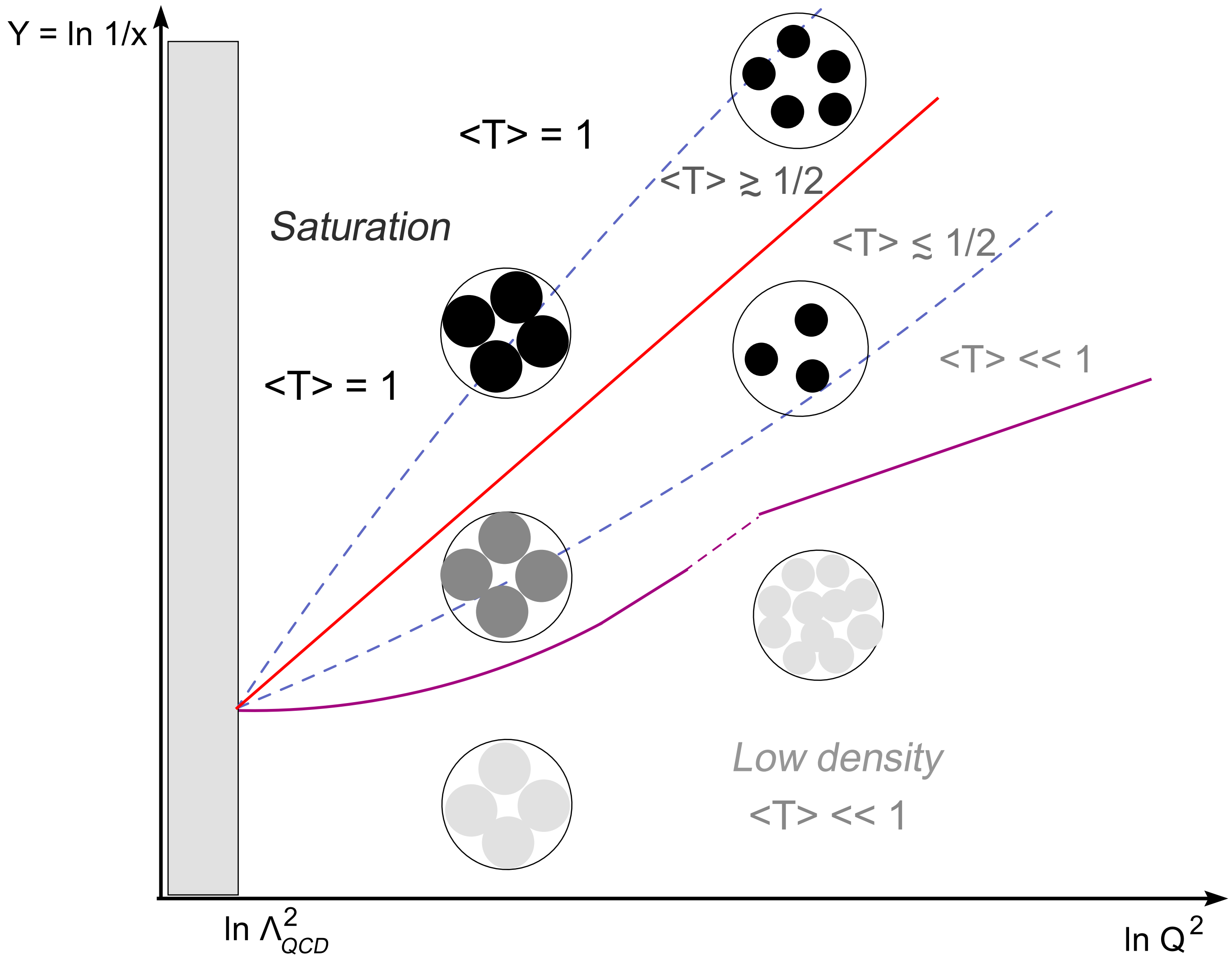
- New physical regime opens at higher energy & larger Q^2

Black spots

Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

Pomeron loops



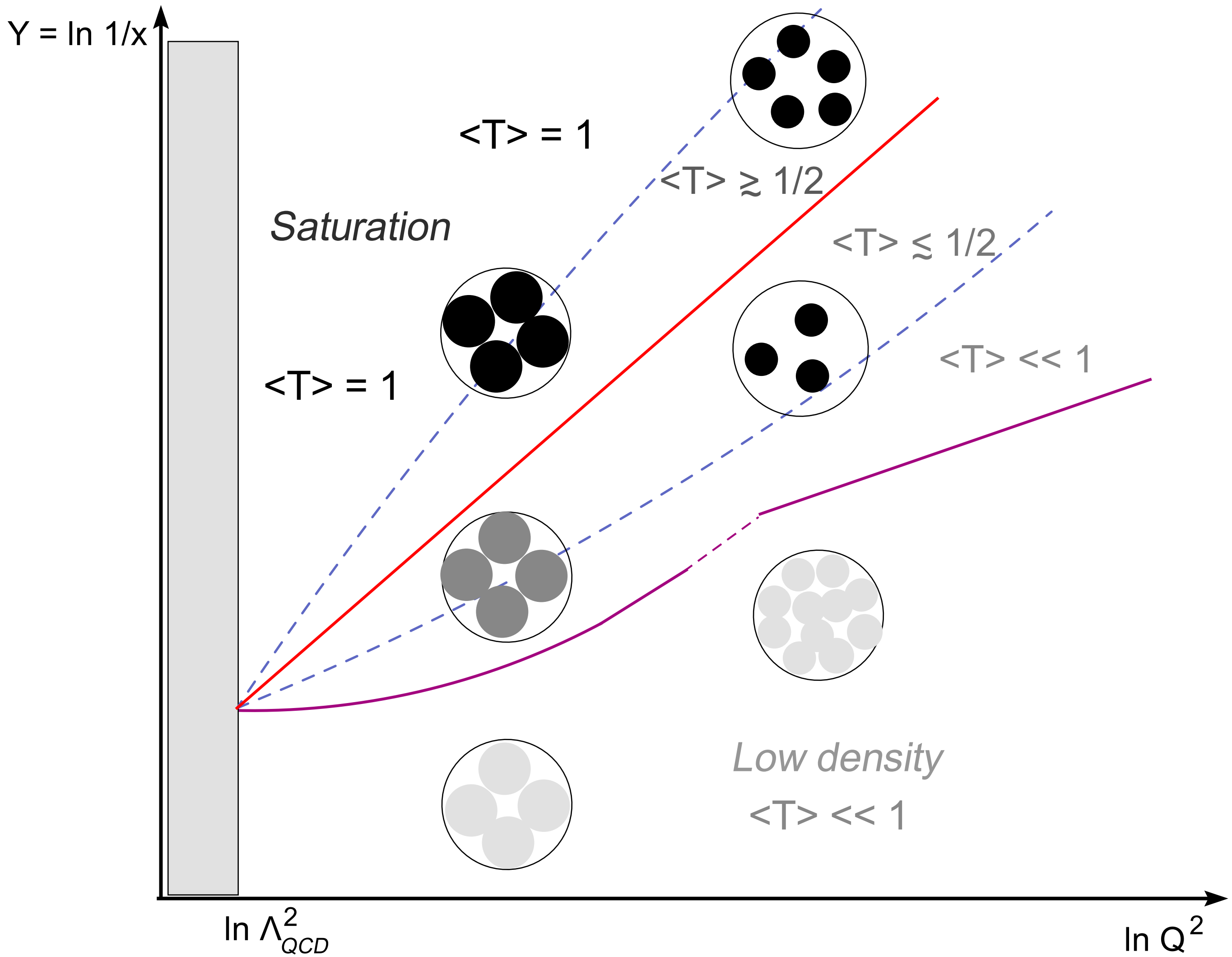
- Strong fluctuations (inhomogeneity) in the vicinity of the (average) saturation line: “Black spots”

Black spots

Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

Pomeron loops



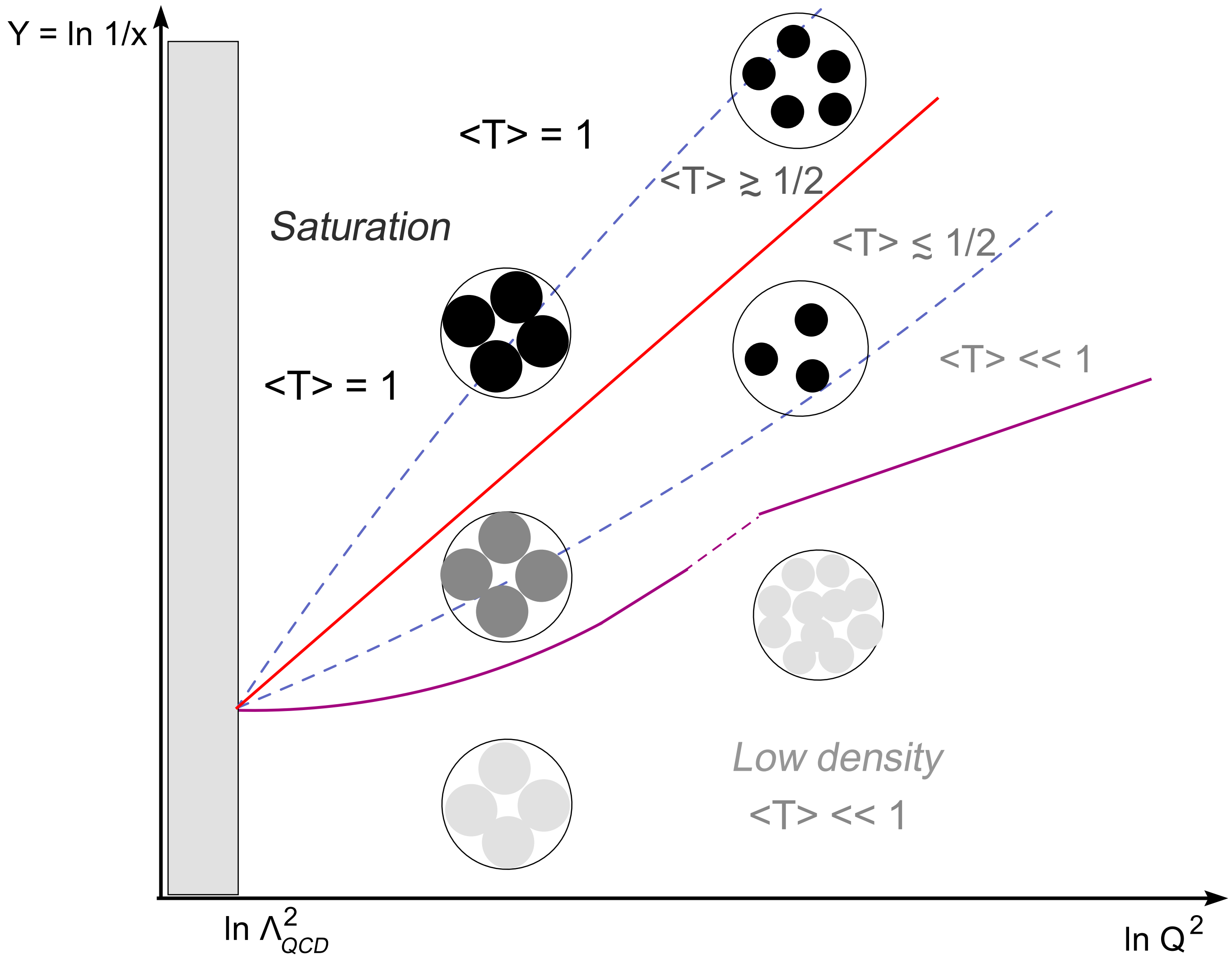
- $T \approx 1/2$ within a large window $\propto \sqrt{Y}$ around the (average) saturation line

Black spots

Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

Pomeron loops



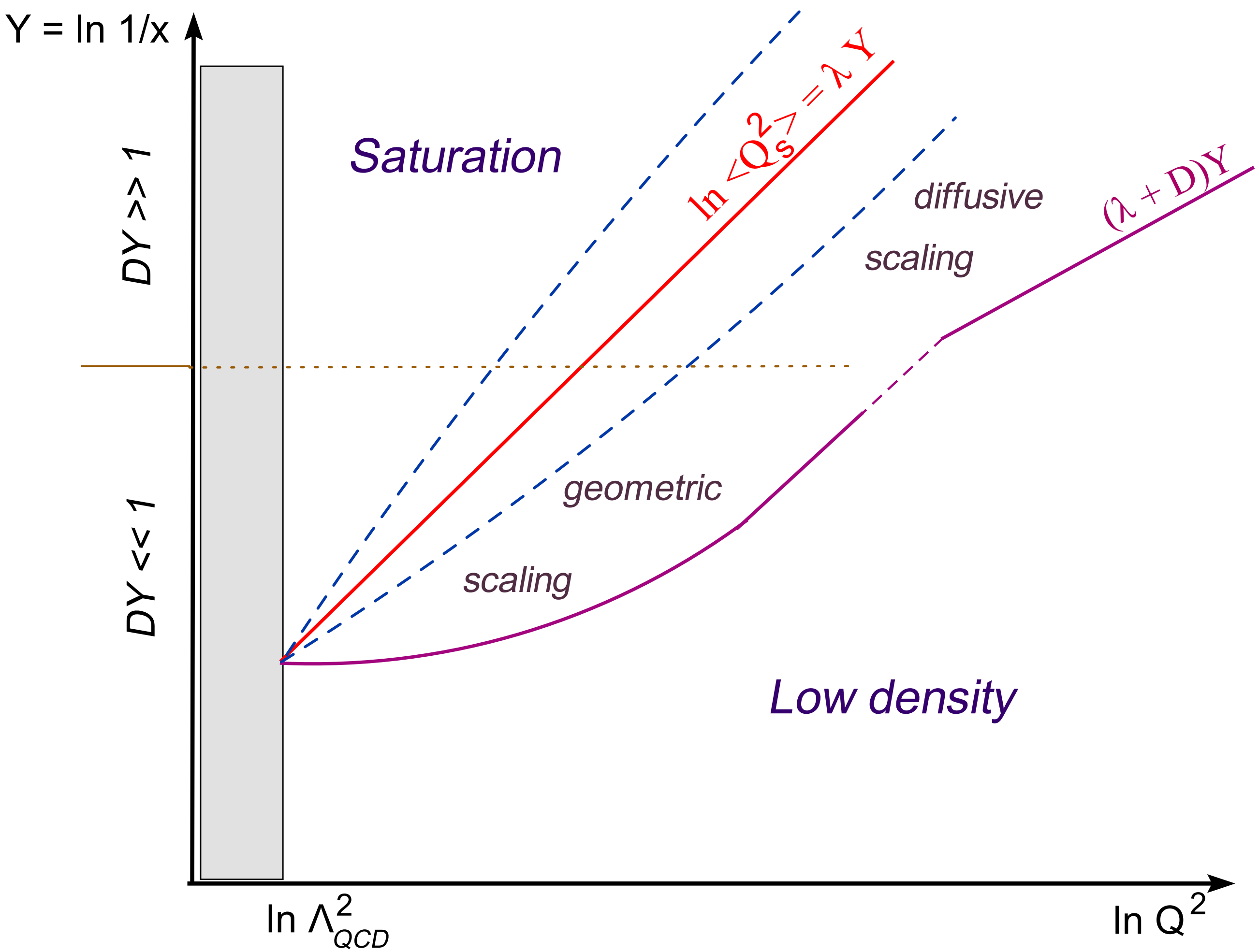
- Diffusive scaling within an even larger window $\propto Y$

Diffusive scaling

Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

Pomeron loops



$$\langle T(r,Y)\rangle \simeq \frac{\exp\left(-Z^2\right)}{Z} \ll 1/2, \qquad Z \equiv \frac{\ln\left[r^2\langle Q_s^2(Y)\rangle\right]}{\sqrt{D\,Y}}$$

Introduction
- Preliminaries
- Saturation line
- Geometric scaling
- Black spots
- Diffusive scaling

High–energy evolution in QCD

Pomeron loops

- Where is all that coming from ?!?

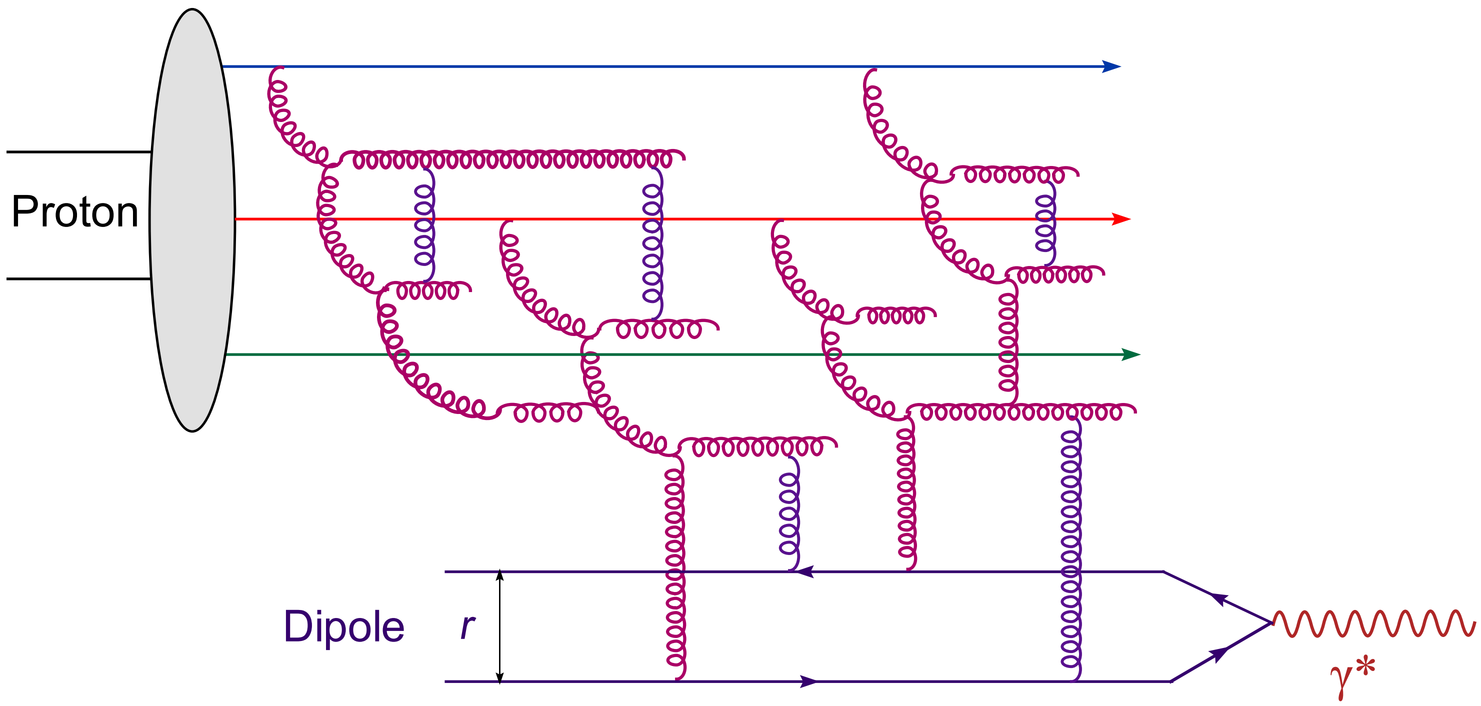
Non–linear gluon evolution in QCD at high energy

DIS in the high energy limit

Introduction

High–energy evolution in QCD
- DIS
- One–step

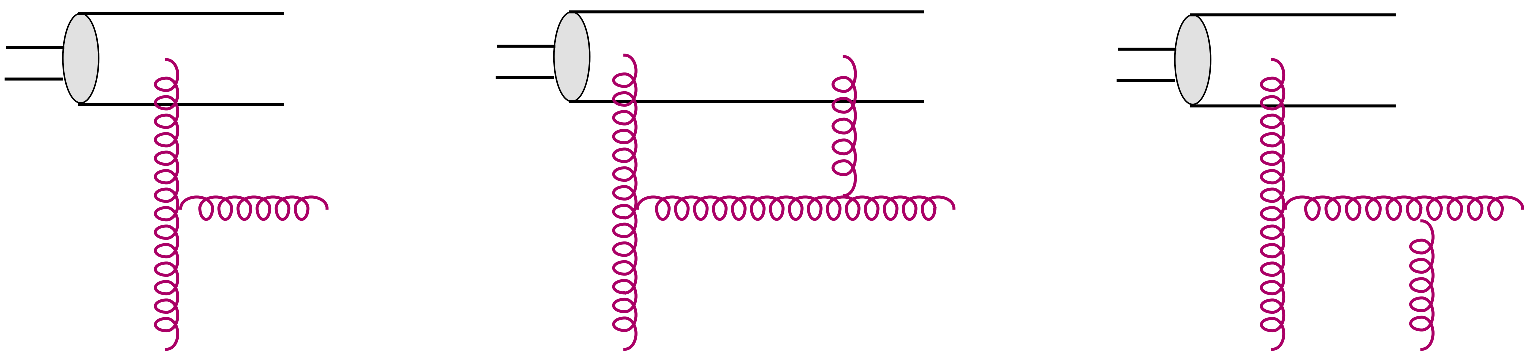
Pomeron loops



- At high energy, the dipole ‘sees’ a complex gluon configuration generated via QCD evolution
  - BFKL ladders: rapid rise of gluon density (and T)
  - Recombination: gluon saturation/unitarization of T
  - Splitting: gluon–number fluctuations/hot spots

One–step evolution in more detail

Introduction
High–energy evolution in QCD
DIS
One–step
Pomeron loops



BFKL recombination splitting

- Evolution equations for $n(Y, k_\perp)$:

  Gluon occupation number or "unintegrated distribution"

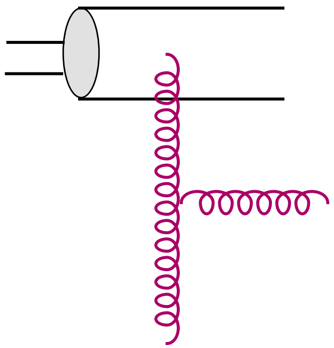
- Ignore $k_\perp$ for a while.

One–step evolution in more detail

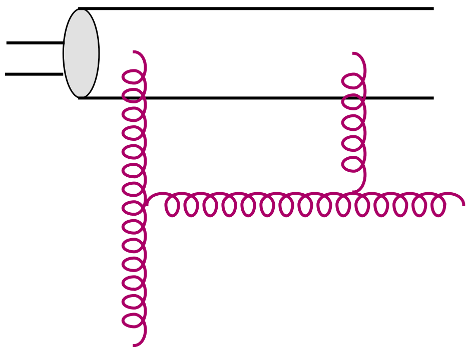
Introduction

High–energy evolution in QCD
- DIS
- One–step

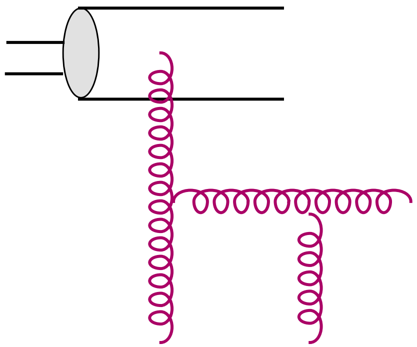
Pomeron loops



BFKL



recombination

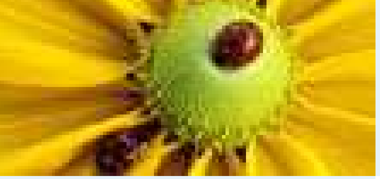


splitting

- BFKL evolution (alone) :

$$\frac{\partial n}{\partial Y} \simeq \alpha_s n \qquad \Longrightarrow \qquad n(Y) \propto \mathrm{e}^{\omega \alpha_s Y}$$

▷ Linear evolution, unstable

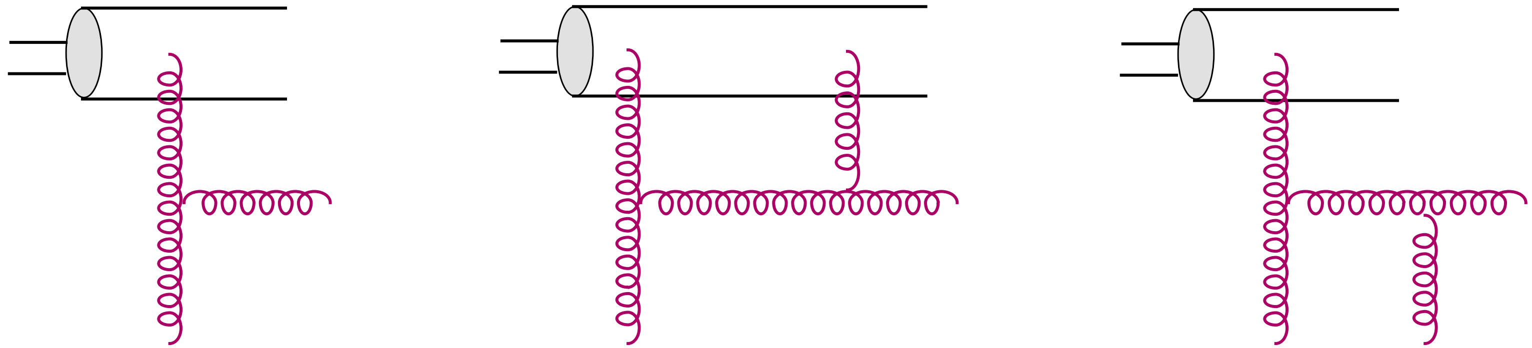


One–step evolution in more detail

Introduction

High–energy evolution in QCD
- DIS
- One–step

Pomeron loops



BFKL recombination splitting

- **BFKL + Gluon recombination :**

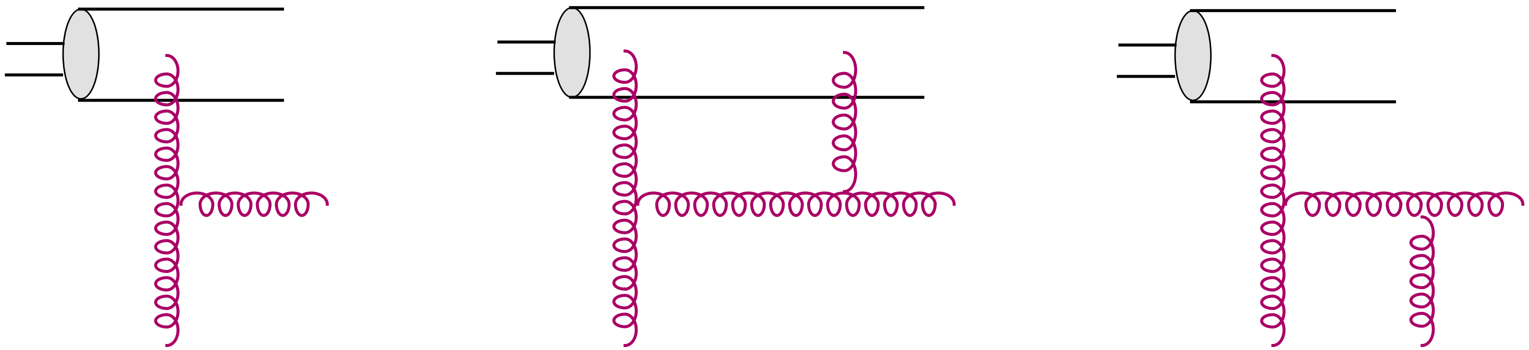
$$\frac{\partial n}{\partial Y} \simeq \alpha_s n - \alpha_s^2 n^2 = 0 \quad \text{when} \quad n \sim \frac{1}{\alpha_s} \gg 1$$

▷ Non–linear evolution, stable fixed point at high energy

▷ High gluon occupancy at saturation

One–step evolution in more detail

Introduction
High–energy evolution in QCD
DIS
One–step
Pomeron loops



BFKL recombination splitting

- However, the actual equations involve the average densities!

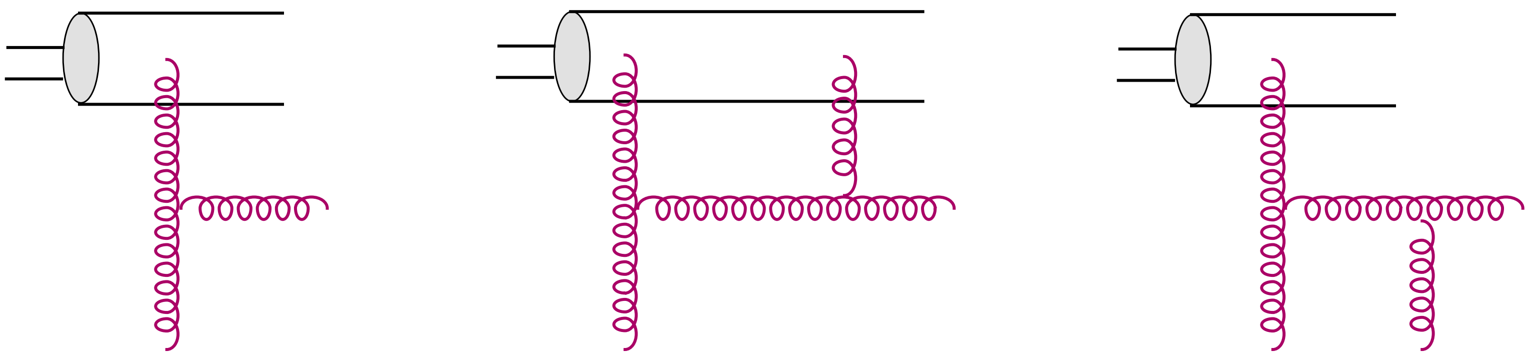
$$\frac{\partial \langle n \rangle}{\partial Y} \simeq \alpha_s \langle n \rangle - \alpha_s^2 \langle nn \rangle$$

- $\langle nn \rangle$: gluon pair density $\ni$ 2–body correlations
- The first equation in an infinite hierarchy !
- Mean field approximation: $\langle nn \rangle \approx \langle n \rangle \langle n \rangle$

... appropriate only for sufficiently high density: $\langle n \rangle \gtrsim 1$

One–step evolution in more detail

Introduction
High–energy evolution in QCD
DIS
One–step
Pomeron loops

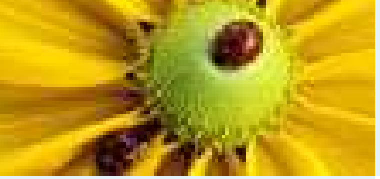


BFKL recombination splitting

- BFKL + Gluon recombination + Gluon splitting :
  - ▷ The 2–body correlations are originally generated through gluon splitting in the dilute regime (low energy)
- The second equation in the hierarchy :

$$\frac{\partial \langle nn \rangle}{\partial Y} \simeq 2\alpha_s \langle nn \rangle - \alpha_s^2 \langle nnn \rangle + \alpha_s^2 \langle n \rangle \ \ldots$$

- Fluctuations dominate the production of $\langle nn \rangle$ when $\langle n \rangle \lesssim \alpha_s$

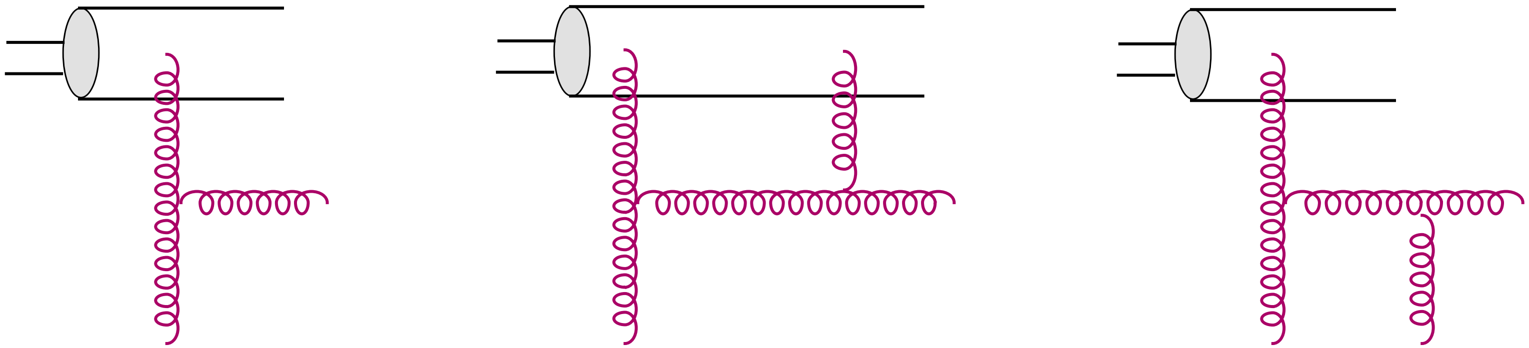


One–step evolution in more detail

Introduction

High–energy evolution in QCD
- DIS
- One–step

Pomeron loops



BFKL recombination splitting

- BFKL + Gluon recombination + Gluon splitting :

$$\frac{\partial\langle n\rangle}{\partial Y} \simeq \alpha_s\langle n\rangle - \alpha_s^2\langle nn\rangle$$

$$\frac{\partial\langle nn\rangle}{\partial Y} \simeq 2\alpha_s\langle nn\rangle - \alpha_s^2\langle nnn\rangle + \alpha_s^2\langle n\rangle \ \ldots$$

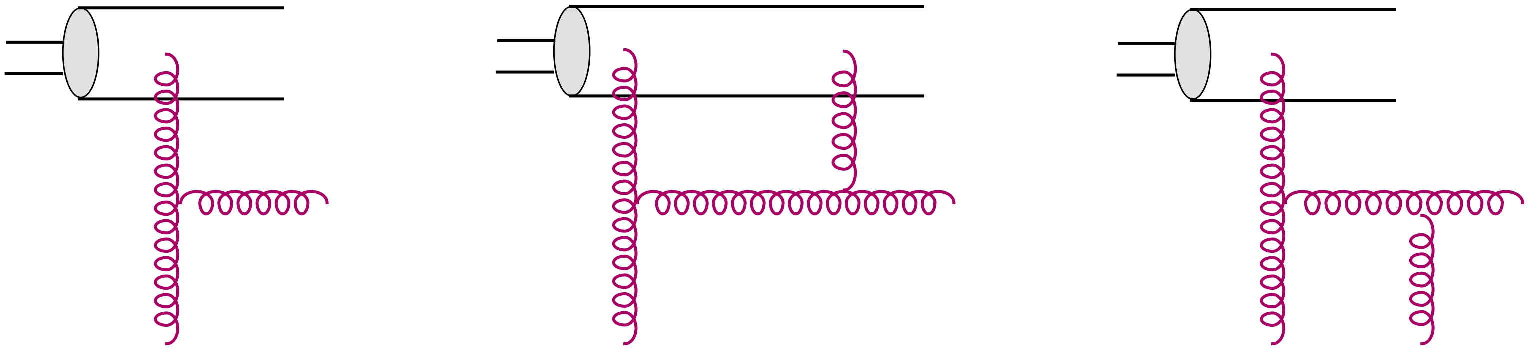
  - Early stages: Correlations are generated via fluctuations
  - Intermediate stages: ... then amplified by BFKL evolution
  - High density: ... and eventually lead to saturation !

One–step evolution in more detail

Introduction

High–energy evolution in QCD
- DIS
- One–step

Pomeron loops

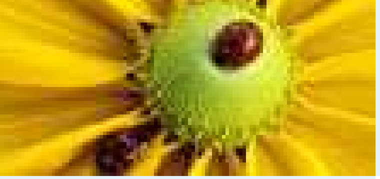


BFKL recombination splitting

- How to actually solve such an infinite hierarchy ?
- Replace it by an equivalent Langevin equation !
  (cf. the yesterday talk by G. Soyez)

$$\frac{\partial n}{\partial Y} \simeq \alpha_s n - \alpha_s^2 n^2 + \alpha_s \sqrt{\alpha_s n}\, \nu \,, \qquad \langle \nu(Y_1)\nu(Y_2)\rangle = \frac{1}{\alpha_s}\delta(Y_1 - Y_2)$$

- $n =$ event–by–event occupation number; $\nu =$ noise
- The high energy evolution is QCD is non–linear & stochastic!



The Pomeron loop equations

(E.I. and D. Triantafyllopoulos, 04)

Introduction

High–energy evolution in QCD

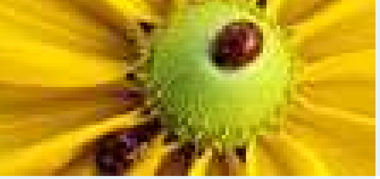
Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling

- Replace n by $T \sim \alpha_s n$ (dipole scattering amplitude).
- Restore $k_\perp$–dependence (here, the dipole size r)

  $T(r, Y) \equiv T(\rho, Y)$ with $\rho \equiv \ln(1/r^2) \sim \ln Q^2$
- 'Stochastic FKPP equation' (simplified version of QCD)

$$\partial_Y T(\rho, Y) = \underbrace{\partial_\rho^2 T + T}_{\text{'BFKL'}} \underbrace{- \; T^2(\rho, Y)}_{\text{unitarization}} + \underbrace{\alpha_s^2 \sqrt{T(1-T)}\,\nu(\rho, Y)}_{\text{fluctuation}}$$

- Unitarization ('black disk limit' $T = 1$) for sufficiently high energy, or sufficiently large dipole sizes
- Noise term $\Longrightarrow$ Fluctuations in the unitarization scale (Q_s)



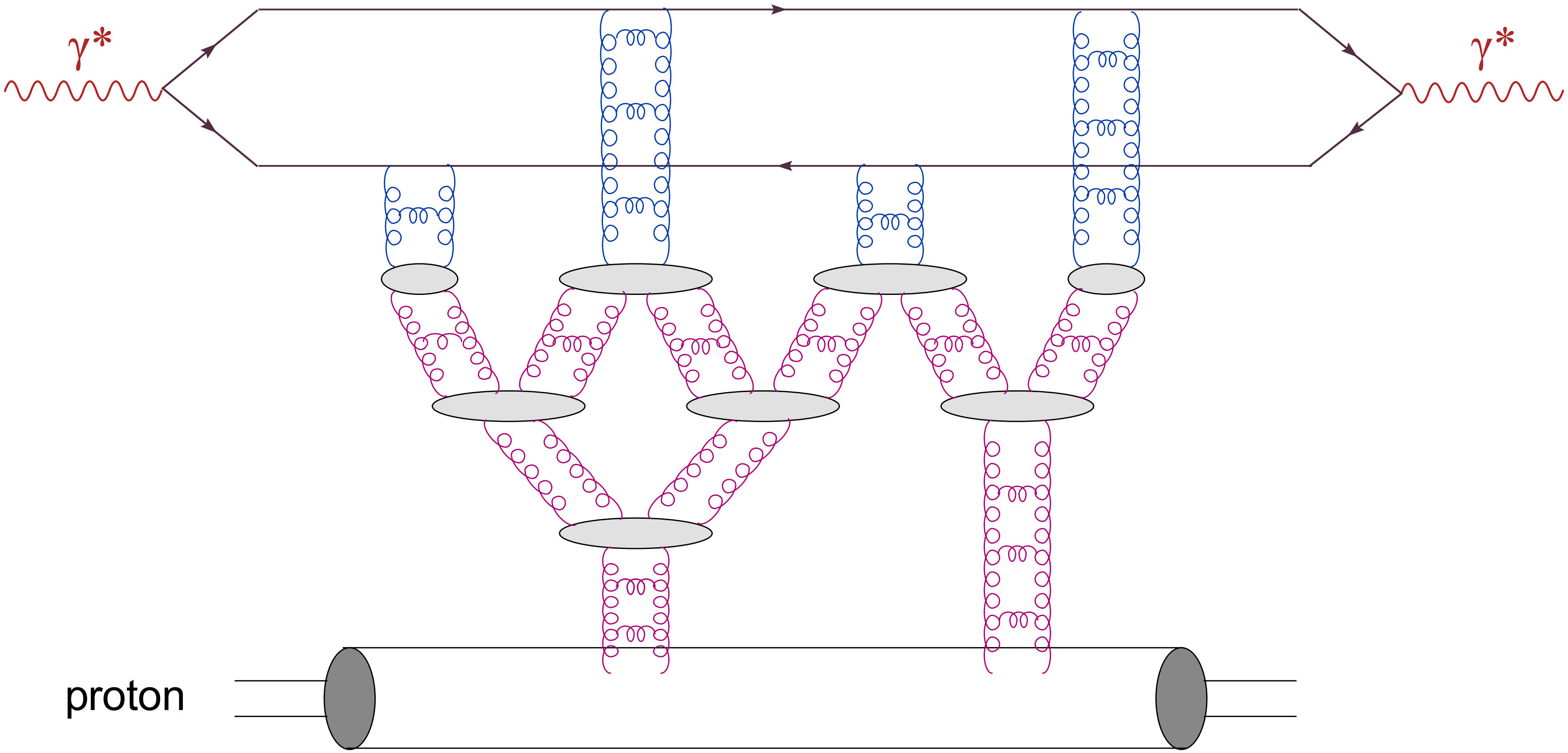
A cartoon of DIS with Pomeron loops

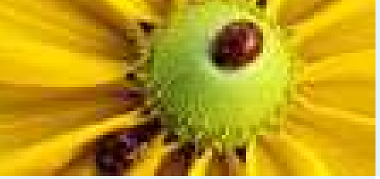
Introduction

High–energy evolution in QCD

Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling

▷ Effective theory for BFKL Pomerons: splitting, merging, loops





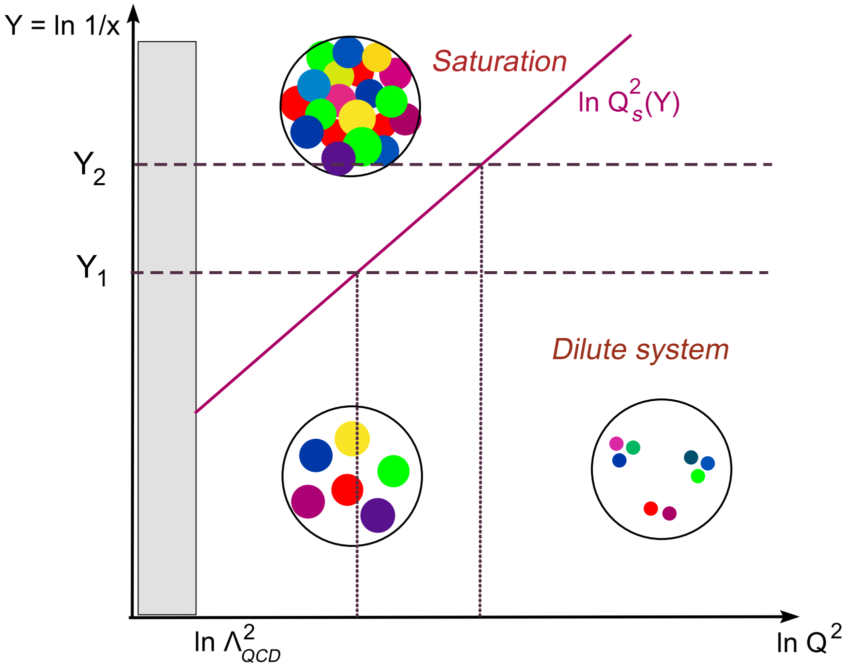
The event–by–event picture

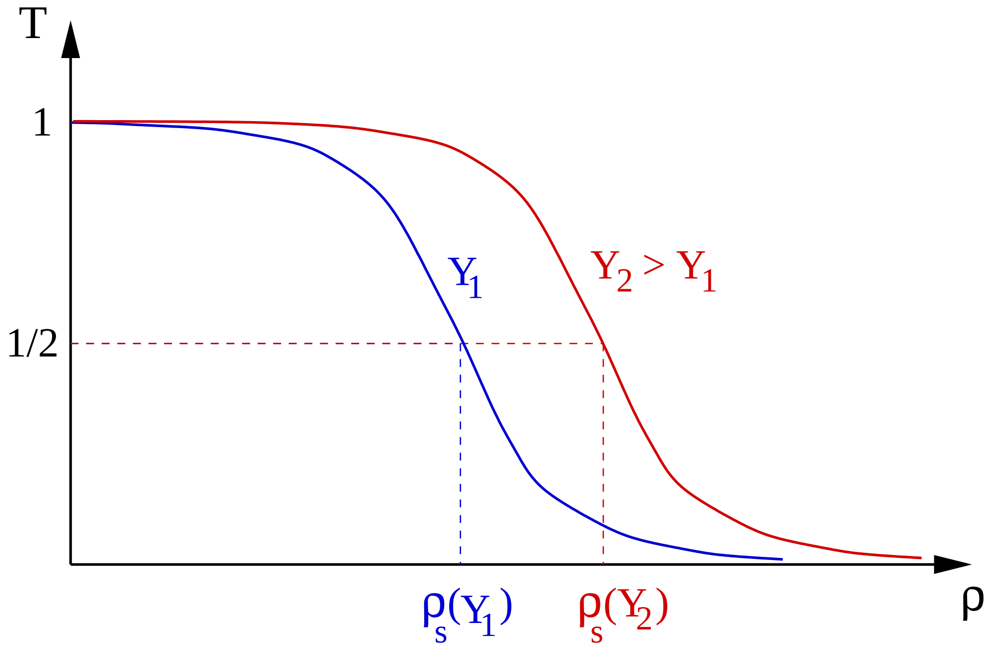
Introduction

High–energy evolution in QCD

Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling

- Two fixed points: $T = 0$ (unstable) and $T = 1$ (stable)





- A front propagating towards larger values of ρ
- The position of the front $\Longrightarrow$ Saturation momentum

$$\rho_s(Y) \equiv \ln Q_s^2(Y) = \lambda Y$$

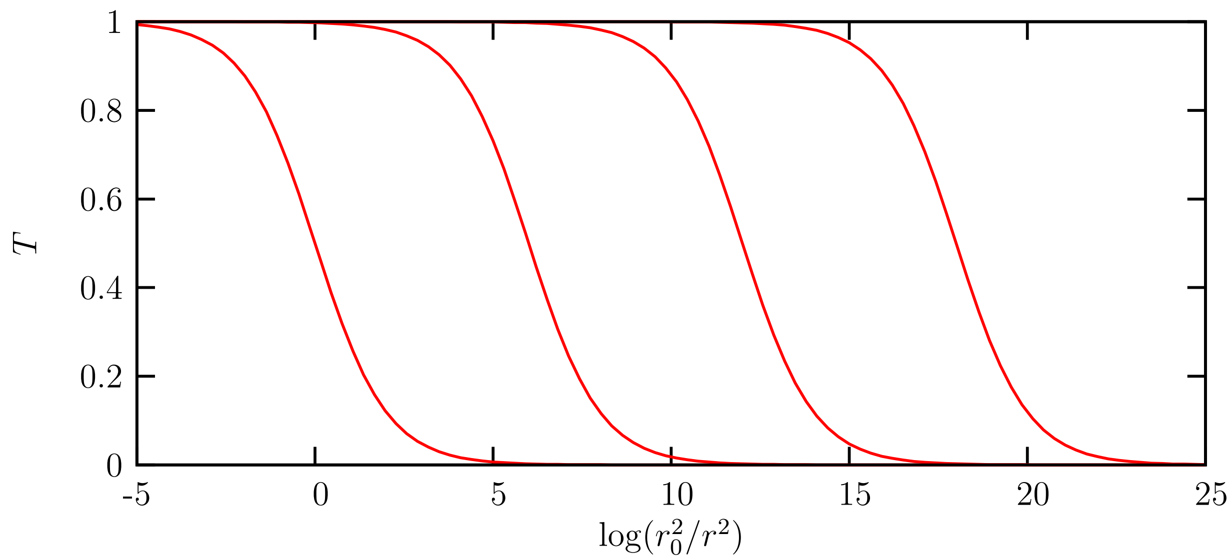
Geometric scaling

Introduction

High–energy evolution in QCD

Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling

- The shape of the front is not altered by the evolution



$$T(\rho, Y) \simeq T(\rho - \rho_s(Y)) \equiv T\big(r^2 Q_s^2(Y)\big)$$

- ‘Traveling wave’ or ‘Geometric scaling’

  E.I., Itakura, McLerran, 02 ; Mueller, Triantafyllopoulos, 02

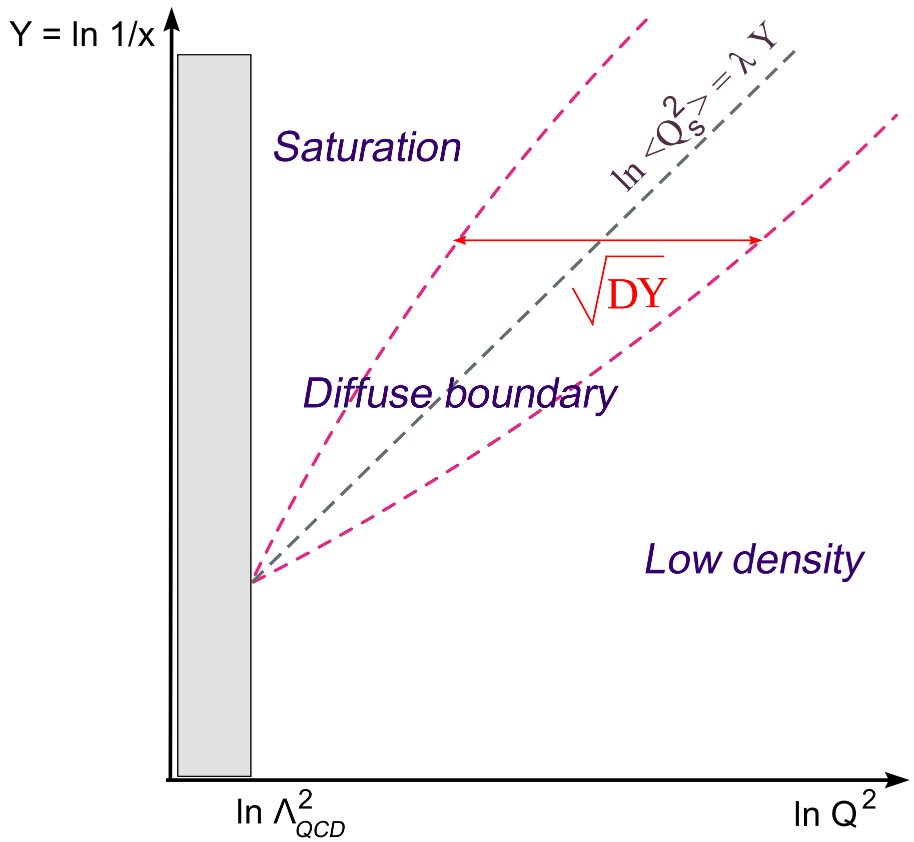
  Munier, Peschanski, 03 : relation to statistical physics (F–KPP)

Effects of fluctuations

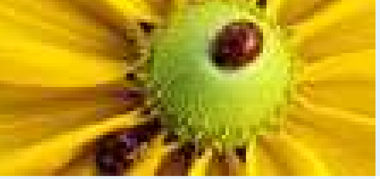
Introduction
High–energy evolution in QCD
Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling

- The saturation momentum $\rho_s(Y) \equiv \ln Q_s^2(Y)$ is random :

$$\langle \rho_s(Y) \rangle = \lambda Y, \qquad \sigma^2(Y) \equiv \langle \rho_s^2 \rangle - \langle \rho_s \rangle^2 = DY$$



- A diffuse saturation boundary, with a width increasing with Y



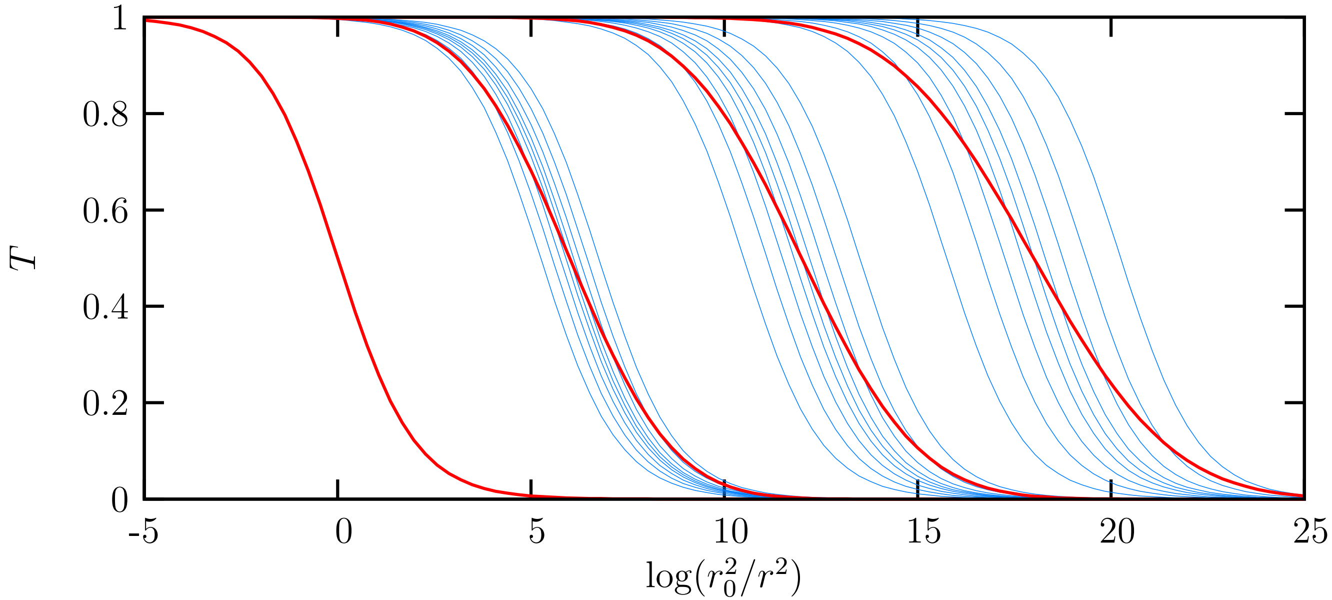
Front diffusion through fluctuations

Introduction

High–energy evolution in QCD

Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling

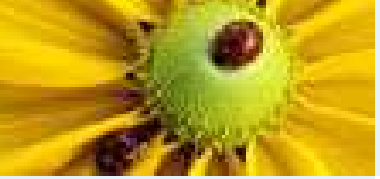
- A stochastic evolution generates un ensemble of fronts.
- One front = One event = Geometric scaling



- The average amplitude $\langle T(\rho, Y)\rangle$ gets flatter and flatter with increasing energy

  $\Longrightarrow$ geometric scaling is eventually washed out !

- Violations remain small, though, so long as $DY \ll 1$



Black spots

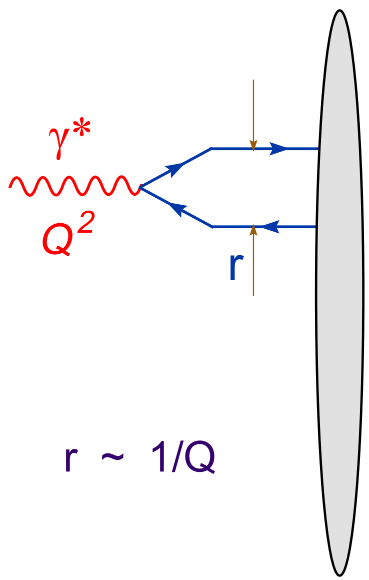
Introduction

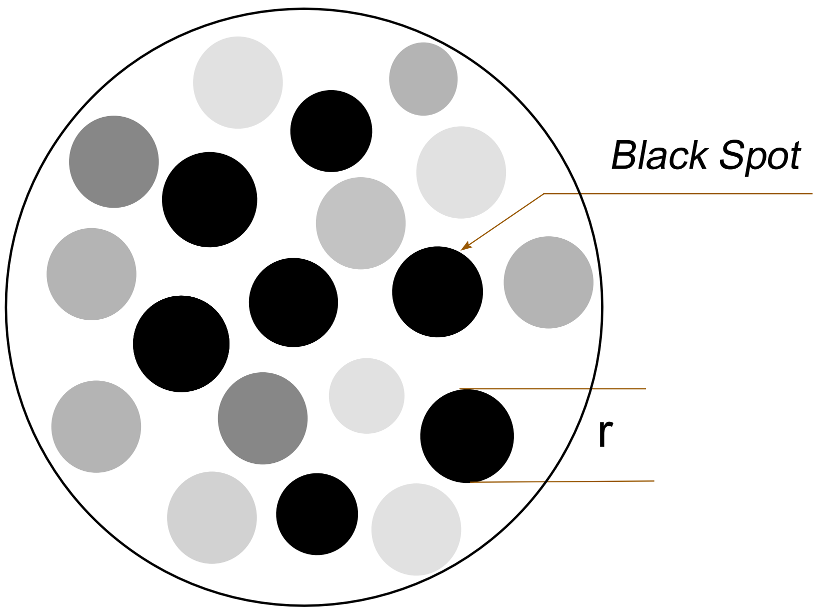
High–energy evolution in QCD

Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling

- High energy $DY \gg 1$: $\langle T(\rho, Y)\rangle$ is dominated by 'black spots' up to very high values of ρ ($\equiv$ large Q^2)

$$\rho - \langle \rho_s \rangle \equiv \ln \frac{Q^2}{\langle Q_s^2(Y)\rangle} \ll DY$$





- 'Black spots' : rare fluctuations having $Q_s^2 \gtrsim Q^2 \gg \langle Q_s^2(Y)\rangle$

Diffusive scaling

Introduction
High–energy evolution in QCD
Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling

(Y. Hatta, E.I., C. Marquet, G. Soyez, D. Triantafyllopoulos, hep-ph/0601150)

- A Gaussian average over the fronts (the values of ρ_s) :

$$\langle T(\rho, Y)\rangle = \int \mathrm{d}\rho_s \underbrace{\frac{1}{\sqrt{\pi}\sigma} \exp\left[-\frac{(\rho_s - \langle\rho_s\rangle)^2}{\sigma^2}\right]}_{\text{probability}} \underbrace{\Theta(\rho - \rho_s)}_{\text{black spots}}$$

$$\langle T(r, Y)\rangle \simeq \frac{1}{2}\,\mathrm{Erfc}(Z), \qquad Z \equiv \frac{\ln\left[r^2\langle Q_s^2(Y)\rangle\right]}{\sqrt{DY}}$$

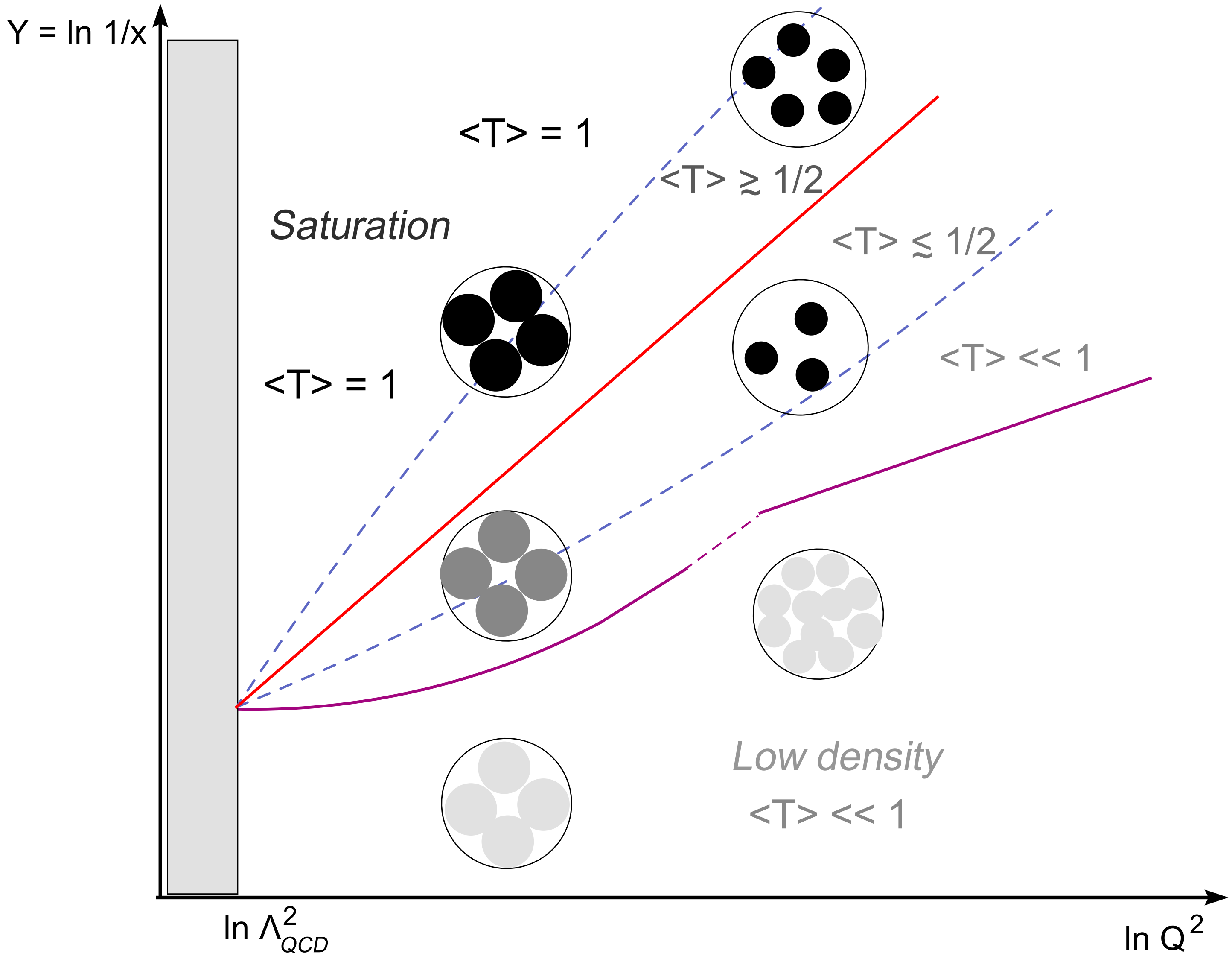
$$\langle T(r, Y)\rangle = \begin{cases} 1\,, & \text{for } Z \ll -1 \\ 1/2\,, & \text{for } -1 \ll Z \ll 1 \\ \exp\left(-Z^2\right)/Z\,, & \text{for } Z \gg 1 \end{cases}$$

Black spots

Introduction

High–energy evolution in QCD

Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling



Introduction

High–energy evolution in QCD

Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling

- See the following talk (by C. Marquet)

  for applications to diffraction !

Front diffusion in more detail

Introduction
High–energy evolution in QCD
Pomeron loops
- Langevin equation
- Ploops in DIS
- Geometric scaling
- Fluctuations
- Front diffusion
- Black spots
- Diffusive scaling