

Cross sections at NNLO

Vittorio Del Duca
INFN Torino

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NLO features

- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: μ_R, μ_F
 - predictive normalisation of observables
 - first step toward precision measurements
 - accurate estimate of signal and background for Higgs and new physics
- Matching with parton-shower MC's: MC@NLO

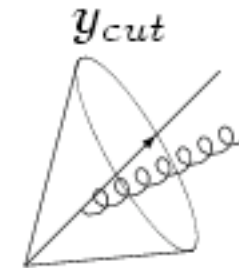
Jet structure

the **jet** non-trivial structure shows up first to **NLO**

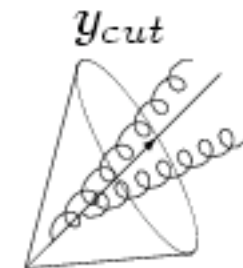
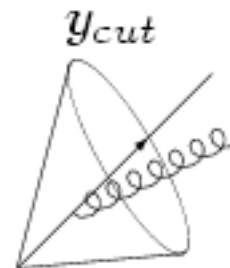
leading order



NLO



NNLO



Summary of $\alpha_S(M_Z)$

S. Bethke hep-ex/0407021

world average of $\alpha_S(M_Z)$

using $\overline{\text{MS}}$ and NNLO results only

$$\alpha_S(M_Z) = 0.1182 \pm 0.0027$$

(cf. 2002 $\alpha_S(M_Z) = 0.1183 \pm 0.0027$)

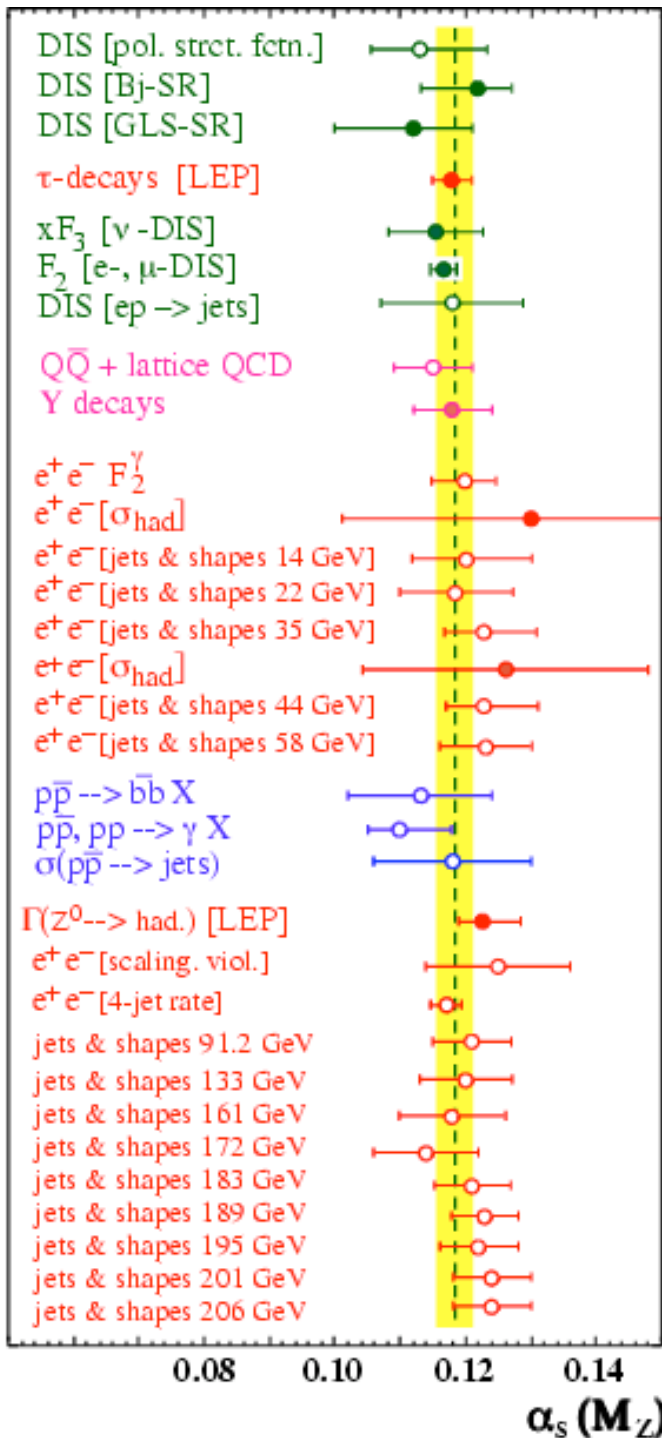
outcome almost identical

because new entries wrt 2002

- LEP jet shape observables and

4-jet rate, and HERA jet rates

and shape variables - are NLO)



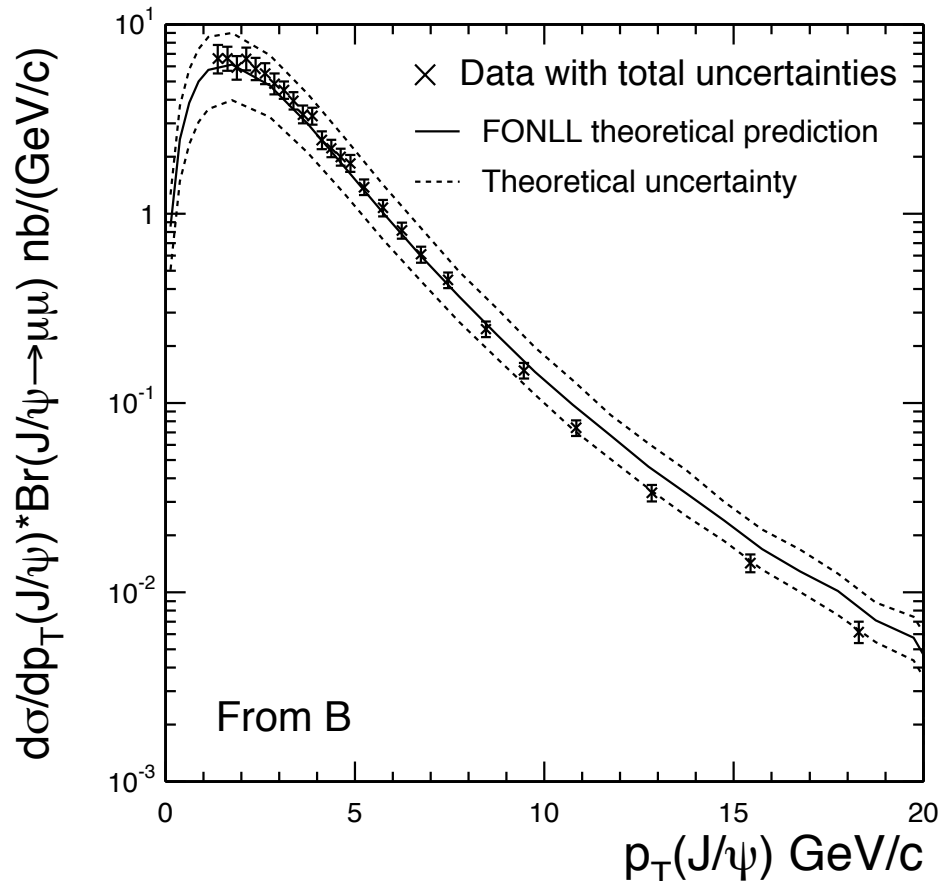
filled symbols are NNLO results

Is **NLO** enough to describe data ?

b cross section in $p\bar{p}$ collisions at 1.96 TeV

$$d\sigma(p\bar{p} \rightarrow H_b X, H_b \rightarrow J/\psi X)/dp_T(J/\psi)$$

CDF hep-ex/0412071



total x-sect is

$$19.4 \pm 0.3(stat)_{-1.9}^{+2.1}(syst) \text{ nb}$$

FONLL = NLO + NLL

Cacciari, Frixione, Mangano,
Nason, Ridolfi 2003

good agreement
with data (with use
of updated FF's by
Cacciari & Nason)




Is NLO enough to describe data ?

Drell-Yan W acceptances at LHC with leptonic decay of the W

$$\text{Cuts A} \longrightarrow |\eta^{(e)}| < 2.5, p_T^{(e)} > 20 \text{ GeV}, p_T^{(\nu)} > 20 \text{ GeV}$$

$$\text{Cuts B} \longrightarrow |\eta^{(e)}| < 2.5, p_T^{(e)} > 40 \text{ GeV}, p_T^{(\nu)} > 20 \text{ GeV}$$

	LO		LO+HW	NLO		MC@NLO
Cuts A	0.5249	$\xrightarrow{-7.7\%}$	0.4843	0.4771	$\xrightarrow{+1.5\%}$	0.4845
	$\downarrow 5.4\%$			$\downarrow 7.0\%$		$\downarrow 6.3\%$
Cuts A, no spin	0.5535			0.5104		0.5151
Cuts B	0.0585	$\xrightarrow{+208\%}$	0.1218	0.1292	$\xrightarrow{+2.9\%}$	0.1329
	$\downarrow 29\%$			$\downarrow 16\%$		$\downarrow 18\%$
Cuts B, no spin	0.0752			0.1504		0.1570

- 
 $|\text{MC@NLO} - \text{NLO}| = \mathcal{O}(2\%)$
S. Frixione M.L. Mangano 2004
- 
NNLO useless without spin correlations
- 
 Precisely evaluated Drell-Yan W, Z cross sections could be used as ``standard candles'' to measure the parton luminosity at LHC

Drell-Yan W acceptances at LHC with leptonic decay of the W

$p_{\perp}^{e,\min}$ (GeV)	A(NLO)	A(NNLO)
20	0.487,0.488,0.489	0.497,0.492,0.491
30	0.379,0.378,0.378	0.379,0.376,0.377
40	0.127,0.125,0.122	0.161,0.155,0.152
50	0.0312,0.0295,0.0277	0.0427,0.0397,0.0387

$$\mu = m_W/2, m_W, 2m_W$$

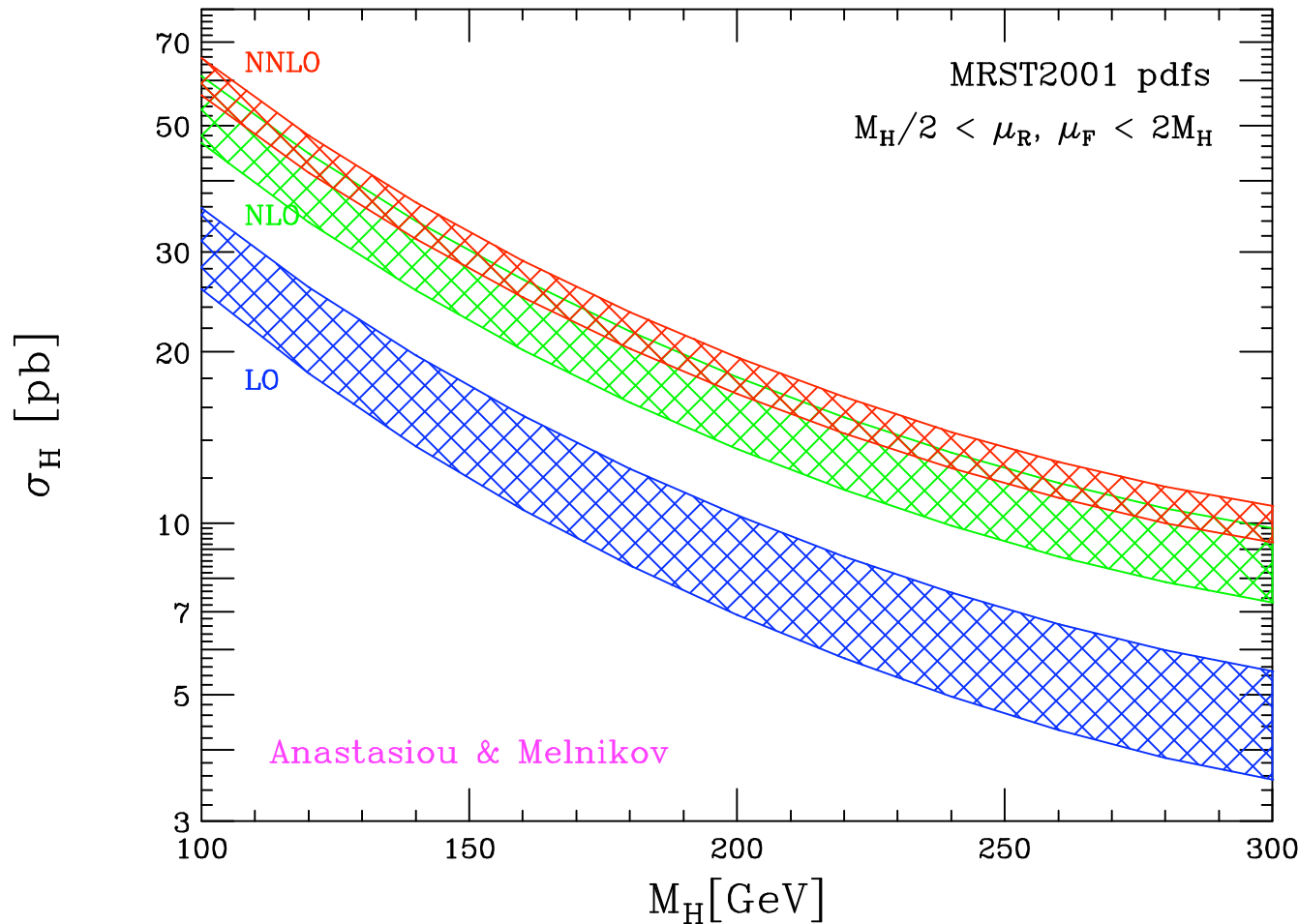
- At LO, $p_{e,\perp} \leq m_W/2$
- NNLO corrections are large for $p_{e,\perp} = 40, 50$ GeV
but are at the percent level for $p_{e,\perp} = 20, 30$ GeV

K. Melnikov, F. Petriello 2006

Is **NLO** enough to describe data ?

Total cross section for inclusive **Higgs** production at LHC

pp → H+X Cross section at LHC



contour bands are
lower

$$\mu_R = 2M_H \quad \mu_F = M_H/2$$

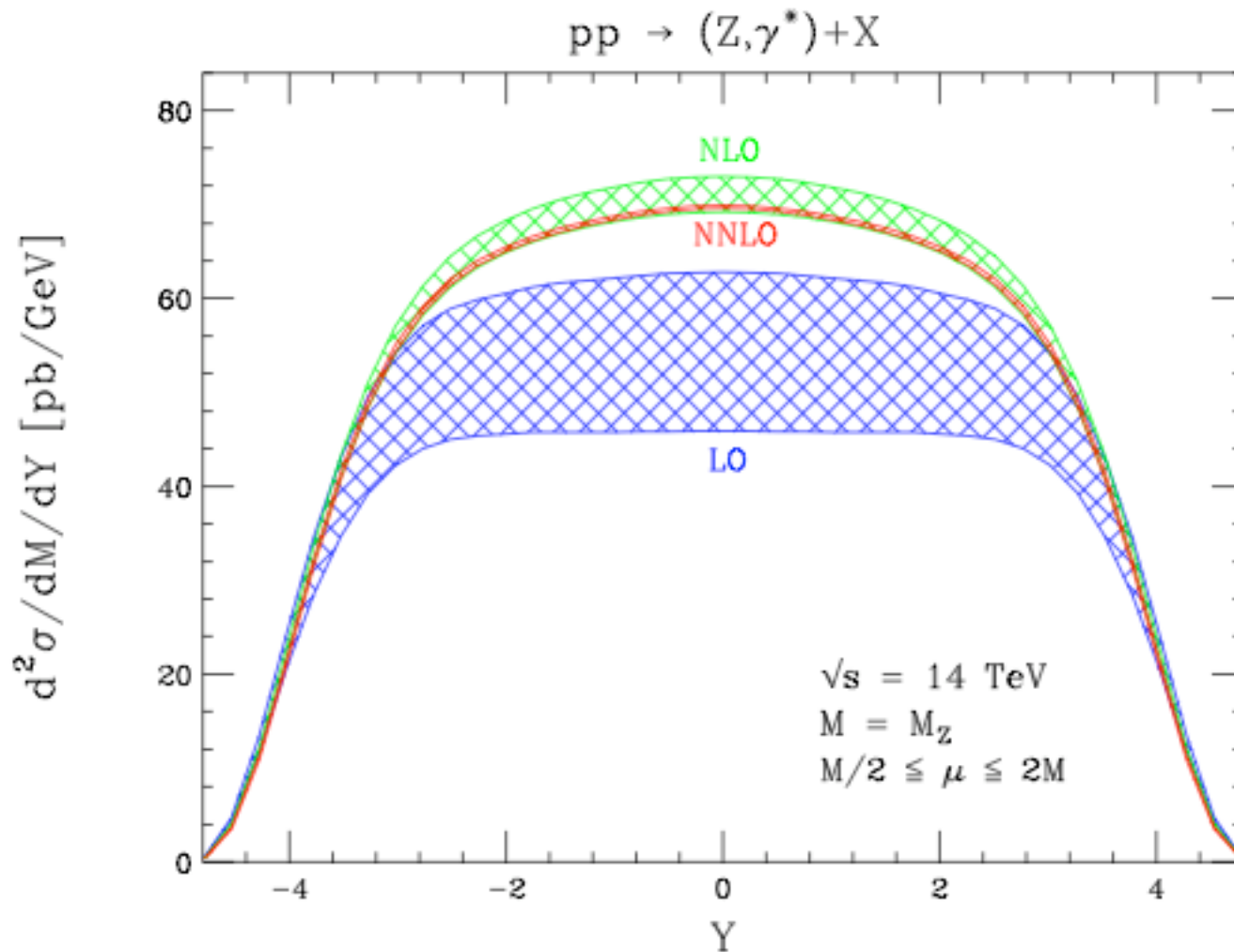
upper

$$\mu_R = M_H/2 \quad \mu_F = 2M_H$$

scale uncertainty
is about 10%

NNLO prediction stabilises the perturbative series

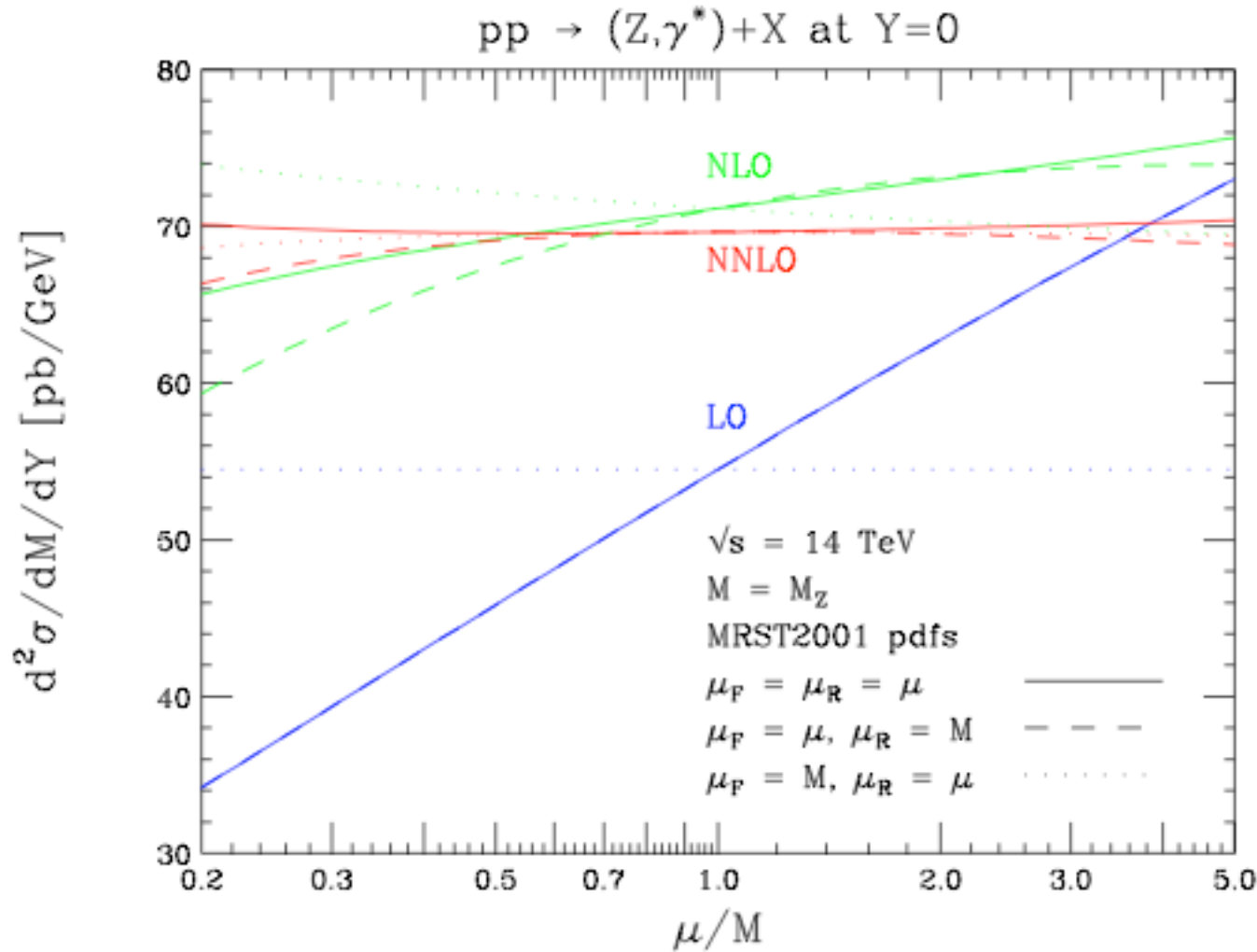
NNLO Drell-Yan Z production at LHC



Rapidity distribution for an on-shell Z boson

- 30% (15%) **NLO** increase wrt to LO at central Y 's (at large Y 's)
NNLO decreases **NLO** by 1 – 2%
- scale variation: $\approx 30\%$ at LO; $\approx 6\%$ at **NLO**; less than 1% at **NNLO**

Scale variations in Drell-Yan Z production



● solid: vary μ_R and μ_F together

● dashed: vary μ_F only

● dotted: vary μ_R only

NNLO corrections may be relevant if

- the main source of uncertainty in extracting info from data is due to NLO theory: α_s measurements
- NLO corrections are large:
Higgs production from gluon fusion in hadron collisions
- NLO uncertainty bands are too large to test theory vs. data: b production in hadron collisions
- NLO is effectively leading order:
energy distributions in jet cones

in short, NNLO is relevant where NLO fails to do its job

NNLO state of the art

● Drell-Yan W, Z production

● total cross section Hamberg, van Neerven, Matsuura 1990
Harlander, Kilgore 2002

● fully differential cross section Melnikov, Petriello 2006

● Higgs production

● total cross section Harlander, Kilgore; Anastasiou, Melnikov 2002
Ravindran, Smith, van Neerven 2003

● fully differential cross section
Anastasiou, Melnikov, Petriello 2004

● $e^+e^- \rightarrow 3$ jets

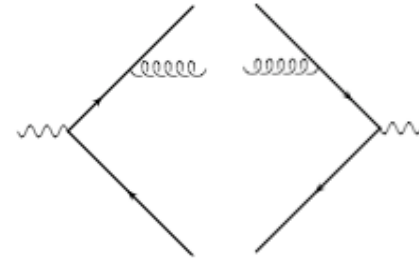
● the $1/N_c^2$ term the Gehrmanns, Glover 2004-5

NLO assembly kit

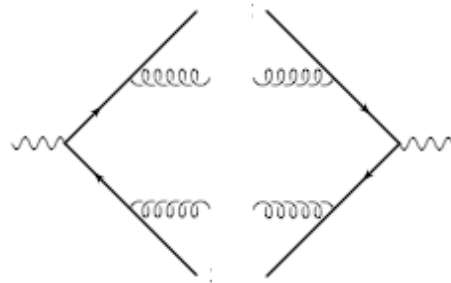
$e^+e^- \rightarrow 3 \text{ jets}$

leading order

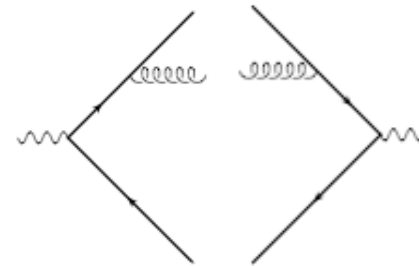
$$|\mathcal{M}_n^{\text{tree}}|^2$$



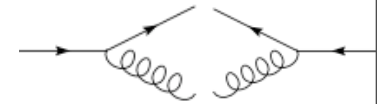
NLO real



IR
→



⊗



$$|\mathcal{M}_{n+1}^{\text{tree}}|^2$$

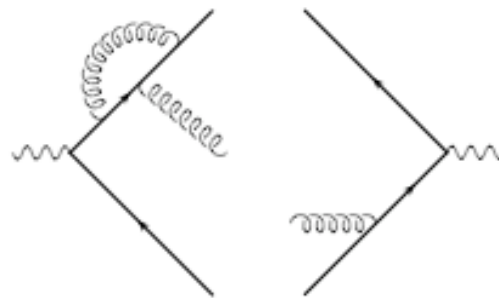
→

$$|\mathcal{M}_n^{\text{tree}}|^2$$

$$\times \int dPS |P_{\text{split}}|^2$$

$$= - \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} \right)$$

NLO virtual



$$d = 4 - 2\epsilon$$

$$\int d^d l \, 2(\mathcal{M}_n^{\text{loop}})^* \mathcal{M}_n^{\text{tree}} = \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} \right) |\mathcal{M}_n^{\text{tree}}|^2 + \text{fin.}$$

NLO production rates

Process-independent procedure devised in the 90's

- slicing Giele Glover & Kosower
- subtraction Frixione Kunszt & Signer; Nagy & Trocsanyi
 - dipole Catani & Seymour
 - antenna Kosower; Campbell Cullen & Glover

$$\sigma = \sigma^{\text{LO}} + \sigma^{\text{NLO}} = \int_m d\sigma_m^B J_m + \sigma^{\text{NLO}}$$

$$\sigma^{\text{NLO}} = \int_{m+1} d\sigma_{m+1}^R J_{m+1} + \int_m d\sigma_m^V J_m$$

the 2 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

$$\sigma^{\text{NLO}} = \int_{m+1} \left[d\sigma_{m+1}^R J_{m+1} - d\sigma_{m+1}^{\text{R,A}} J_m \right] + \int_m \left[d\sigma_m^V + \int_1 d\sigma_{m+1}^{\text{R,A}} \right] J_m$$

the 2 terms on the rhs are finite in d=4

Observable (jet) functions

J_m vanishes when one parton becomes soft or collinear to another one

$$J_m(p_1, \dots, p_m) \rightarrow 0, \quad \text{if } p_i \cdot p_j \rightarrow 0$$

➔ $d\sigma_m^{\text{B}}$ is integrable over 1-parton IR phase space

J_{m+1} vanishes when two partons become simultaneously soft and/or collinear

$$J_{m+1}(p_1, \dots, p_{m+1}) \rightarrow 0, \quad \text{if } p_i \cdot p_j \text{ and } p_k \cdot p_l \rightarrow 0 \quad (i \neq k)$$

R and V are integrable over 2-parton IR phase space

observables are IR safe

$$J_{n+1}(p_1, \dots, p_j = \lambda q, \dots, p_{n+1}) \rightarrow J_n(p_1, \dots, p_{n+1}) \quad \text{if } \lambda \rightarrow 0$$

$$J_{n+1}(p_1, \dots, p_i, \dots, p_j, \dots, p_{n+1}) \rightarrow J_n(p_1, \dots, p, \dots, p_{n+1}) \quad \text{if } p_i \rightarrow zp, p_j \rightarrow (1-z)p$$

for all $n \geq m$

NLO IR limits

collinear operator

$$C_{ir} |\mathcal{M}_{m+2}^{(0)}(p_i, p_r, \dots)|^2 \propto \frac{1}{S_{ir}} \langle \mathcal{M}_{m+1}(0)(p_{ir}, \dots) | \hat{P}_{f_i f_r}^{(0)} | \mathcal{M}_{m+1}(0)(p_{ir}, \dots) \rangle$$

soft operator

$$S_r |\mathcal{M}_{m+2}^{(0)}(p_r, \dots)|^2 \propto \frac{S_{ik}}{S_{ir} S_{rk}} \langle \mathcal{M}_{m+1}(0)(\dots) | T_i \cdot T_k | \mathcal{M}_{m+1}(0)(\dots) \rangle$$

counterterm

$$\sum_r \left(\sum_{i \neq r} \frac{1}{2} C_{ir} + S_r \right) |\mathcal{M}_{m+2}^{(0)}(p_i, p_r, \dots)|^2$$

performs double subtraction in overlapping regions

NLO overlapping divergences

$C_{ir}S_r$ can be used to cancel double subtraction

$$C_{ir} (S_r - C_{ir}S_r) |\mathcal{M}_{m+2}^{(0)}|^2 = 0$$

$$S_r (C_{ir} - C_{ir}S_r) |\mathcal{M}_{m+2}^{(0)}|^2 = 0$$

the **NLO** counterterm

$$A_1 |\mathcal{M}_{m+2}^{(0)}|^2 = \sum_r \left[\sum_{i \neq r} \frac{1}{2} C_{ir} + \left(S_r - \sum_{i \neq r} C_{ir} S_r \right) \right] |\mathcal{M}_{m+2}^{(0)}(p_i, p_r, \dots)|^2$$

• has the same singular behaviour as SME, and is free of double subtractions

$$C_{ir} (1 - A_1) |\mathcal{M}_{m+1}^{(0)}|^2 = 0 \quad S_r (1 - A_1) |\mathcal{M}_{m+1}^{(0)}|^2 = 0$$


• contains spurious singularities when parton $s \neq r$ becomes unresolved, but they are screened by J_m

NNLO cross sections

Analytic integration

Hamberg, van Neerven, Matsuura 1990
Anastasiou Dixon Melnikov Petriello 2003

 first method

 flexible enough to include a limited class of acceptance cuts by modelling cuts as “propagators”

Sector decomposition

Denner Roth 1996; Binoth Heinrich 2000
Anastasiou, Melnikov, Petriello 2004

 flexible enough to include any acceptance cuts

 cancellation of divergences is performed numerically

 can it handle many final-state partons ?

Subtraction

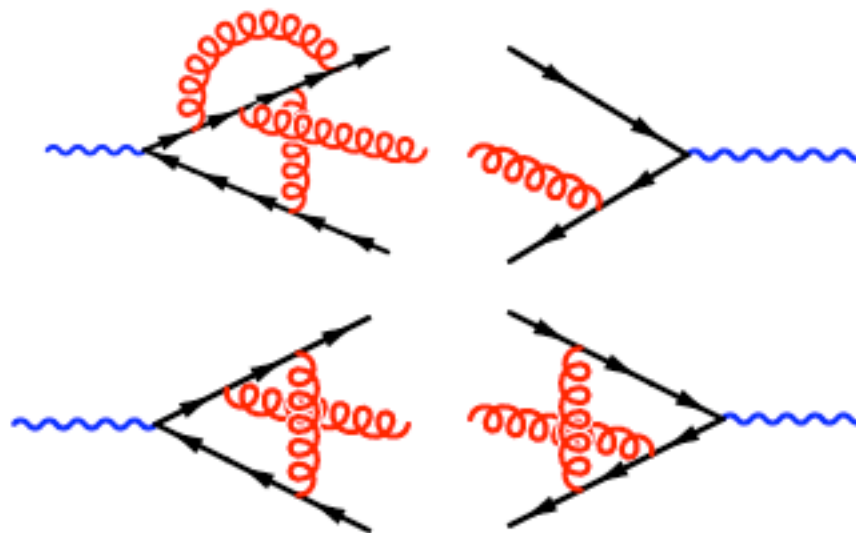
 process independent

 cancellation of divergences is semi-analytic
can it be fully automatised ?

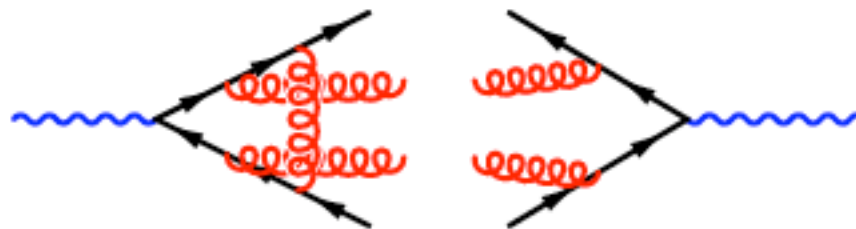
NNLO assembly kit

$e^+e^- \rightarrow 3 \text{ jets}$

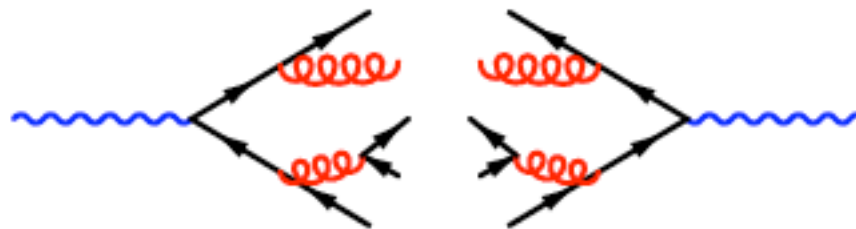
double virtual



real-virtual



double real



Two-loop matrix elements

● two-jet production $qq' \rightarrow qq', q\bar{q} \rightarrow q\bar{q}, q\bar{q} \rightarrow gg, gg \rightarrow gg$

C. Anastasiou N. Glover C. Oleari M. Tejada-Yeomans 2000-01

Z. Bern A. De Freitas L. Dixon 2002

● photon-pair production $q\bar{q} \rightarrow \gamma\gamma, gg \rightarrow \gamma\gamma$

C. Anastasiou N. Glover M. Tejada-Yeomans 2002

Z. Bern A. De Freitas L. Dixon 2002

● $e^+e^- \rightarrow 3$ jets $\gamma^* \rightarrow q\bar{q}g$

L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

● $V + 1$ jet production $q\bar{q} \rightarrow Vg$

T. Gehrmann E. Remiddi 2002

● Drell-Yan V production $q\bar{q} \rightarrow V$

R. Hamberg W. van Neerven T. Matsuura 1991

● Higgs production $gg \rightarrow H$ (in the $m_t \rightarrow \infty$ limit)

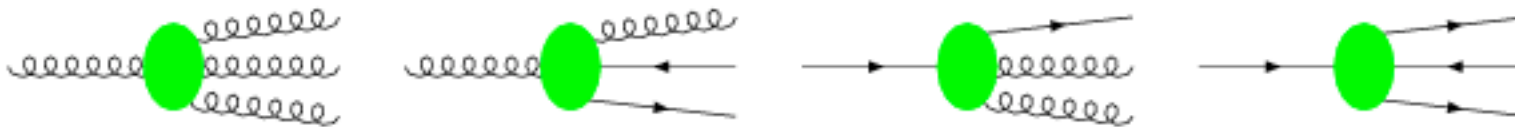
R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002

NNLO cross sections

universal IR structure \Rightarrow process-independent procedure

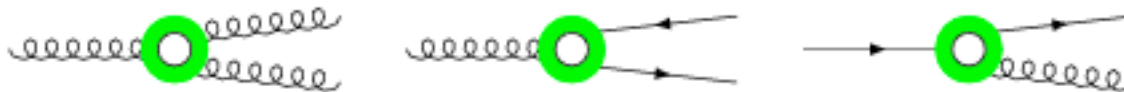
universal collinear and soft currents

3-parton tree splitting functions



J. Campbell N. Glover 1997; S. Catani M. Grazzini 1998; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

2-parton one-loop splitting functions



Z. Bern L. Dixon D. Dunbar D. Kosower 1994; Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; S. Catani M. Grazzini 1999; D. Kosower 2003

universal subtraction counterterms

several ideas and works in progress

D. Kosower; S. Weinzierl; the Gehrmanns & G. Heinrich 2003

S. Frixione M. Grazzini 2004; G. Somogyi Z. Trocsanyi VDD 2005

but completely figured out only for $e^+e^- \rightarrow 3$ jets

the Gehrmanns & N. Glover 2005

NNLO subtraction

$$\sigma^{\text{NNLO}} = \int_{m+2} d\sigma_{m+2}^{\text{RR}} J_{m+2} + \int_{m+1} d\sigma_{m+1}^{\text{RV}} J_{m+1} + \int_m d\sigma_m^{\text{VV}} J_m$$

the 3 terms on the rhs are divergent in $d=4$
use **universal IR** structure to subtract divergences

$$\sigma^{\text{NNLO}} = \int_{m+2} \left[d\sigma_{m+2}^{\text{RR}} J_{m+2} - d\sigma_{m+2}^{\text{RR},A_2} J_m \right]$$

takes care of doubly-unresolved regions,
but still divergent in singly-unresolved ones

$$+ \int_{m+1} \left[d\sigma_{m+1}^{\text{RV}} J_{m+1} - d\sigma_{m+1}^{\text{RV},A_1} J_m \right]$$

still contains $1/\epsilon$ poles in regions away from 1-parton IR regions

$$+ \int_m \left[d\sigma_m^{\text{VV}} + \int_2 d\sigma_{m+2}^{\text{RR},A_2} + \int_1 d\sigma_{m+1}^{\text{RV},A_1} \right] J_m$$

NNLO counterterm

- construct the 2-unresolved-parton counterterm using the IR currents

$$\begin{aligned}
 A_2 |\mathcal{M}_{m+2}^{(0)}|^2 &= \sum_r \sum_{s \neq r} \left\{ \sum_{i \neq r,s} \left[\frac{1}{6} C_{irs} + \sum_{j \neq i,r,s} \frac{1}{8} C_{ir;js} + \frac{1}{2} S_{rs} \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \left(\mathcal{C}S_{ir;s} - C_{irs} \mathcal{C}S_{ir;s} - \sum_{j \neq i,r,s} C_{ir;js} \mathcal{C}S_{ir;s} \right) \right] \right. \\
 &\quad \left. - \sum_{i \neq r,s} \left[\mathcal{C}S_{ir;s} S_{rs} + C_{irs} \left(\frac{1}{2} S_{rs} - \mathcal{C}S_{ir;s} S_{rs} \right) \right. \right. \\
 &\quad \left. \left. + \sum_{j \neq i,r,s} C_{ir;js} \left(\frac{1}{2} S_{rs} - \mathcal{C}S_{ir;s} S_{rs} \right) \right] \right\} |\mathcal{M}_{m+2}^{(0)}|^2
 \end{aligned}$$

G. Somogyi Z.Trocsanyi VDD 2005

performing double and triple subtractions in overlapping regions

$$C_{irs} (1 - A_2) |\mathcal{M}_{m+2}^{(0)}|^2 = 0$$

$$S_{rs} (1 - A_2) |\mathcal{M}_{m+2}^{(0)}|^2 = 0$$

$$C_{ir;js} (1 - A_2) |\mathcal{M}_{m+2}^{(0)}|^2 = 0$$

$$\mathcal{C}S_{ir;s} (1 - A_2) |\mathcal{M}_{m+2}^{(0)}|^2 = 0$$

needs a **NLO**-type subtraction
between the $m+2$ - and the $m+1$ -parton contributions

$$\sigma^{\text{NNLO}} = \sigma_{\{m+2\}}^{\text{NNLO}} + \sigma_{\{m+1\}}^{\text{NNLO}} + \sigma_{\{m\}}^{\text{NNLO}}$$

$$\sigma_{\{m+2\}}^{\text{NNLO}} = \int_{m+2} \left[d\sigma_{m+2}^{\text{RR}} J_{m+2} - d\sigma_{m+2}^{\text{RR},A_2} J_m \right.$$

must be finite in
the doubly-unresolved regions \rightarrow

$$\left. - d\sigma_{m+2}^{\text{RR},A_1} J_{m+1} + d\sigma_{m+2}^{\text{RR},A_{12}} J_m \right]_{d=4}$$

G. Somogyi Z.Trocsanyi VDD 2005

A_1 takes care of the singly-unresolved regions and A_{12} of the over-subtracting

$$\sigma_{\{m+1\}}^{\text{NNLO}} = \int_{m+1} \left[d\sigma_{m+1}^{\text{RV}} J_{m+1} - d\sigma_{m+1}^{\text{RV},A_1} J_m \right.$$

$$\left. + \int_1 \left(d\sigma_{m+2}^{\text{RR},A_1} J_{m+1} - d\sigma_{m+2}^{\text{RR},A_{12}} J_m \right) \right]_{d=4}$$

$$\sigma_{\{m\}}^{\text{NNLO}} = \int_m \left[d\sigma_m^{\text{VV}} + \int_2 d\sigma_{m+2}^{\text{RR},A_2} + \int_1 d\sigma_{m+1}^{\text{RV},A_1} \right]_{d=4} J_m$$

need to construct \mathbf{A}_{12} such that all overlapping regions in 1-parton and 2-parton IR phase space regions are counted only once

$$\mathbf{C}_{ir}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}_{m+2}^{(0)}|^2 = \mathbf{C}_{ir}|\mathcal{M}_{m+2}^{(0)}|^2$$

$$\mathbf{S}_r(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}_{m+2}^{(0)}|^2 = \mathbf{S}_r|\mathcal{M}_{m+2}^{(0)}|^2$$

$$\mathbf{C}_{irs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}_{m+2}^{(0)}|^2 = \mathbf{C}_{irs}|\mathcal{M}_{m+2}^{(0)}|^2$$

$$\mathbf{C}_{ir;js}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}_{m+2}^{(0)}|^2 = \mathbf{C}_{ir;js}|\mathcal{M}_{m+2}^{(0)}|^2$$

$$\mathbf{CS}_{ir;s}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}_{m+2}^{(0)}|^2 = \mathbf{CS}_{ir;s}|\mathcal{M}_{m+2}^{(0)}|^2$$

$$\mathbf{S}_{rs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}_{m+2}^{(0)}|^2 = \mathbf{S}_{rs}|\mathcal{M}_{m+2}^{(0)}|^2$$

the definition of \mathbf{A}_{12} is rather simple

$$\mathbf{A}_{12}|\mathcal{M}_{m+2}^{(0)}|^2 \equiv \mathbf{A}_1\mathbf{A}_2|\mathcal{M}_{m+2}^{(0)}|^2$$

but showing that it has the right properties is non trivial, and requires considering iterated singly-unresolved limits and strongly-ordered doubly-unresolved limits

Conclusions

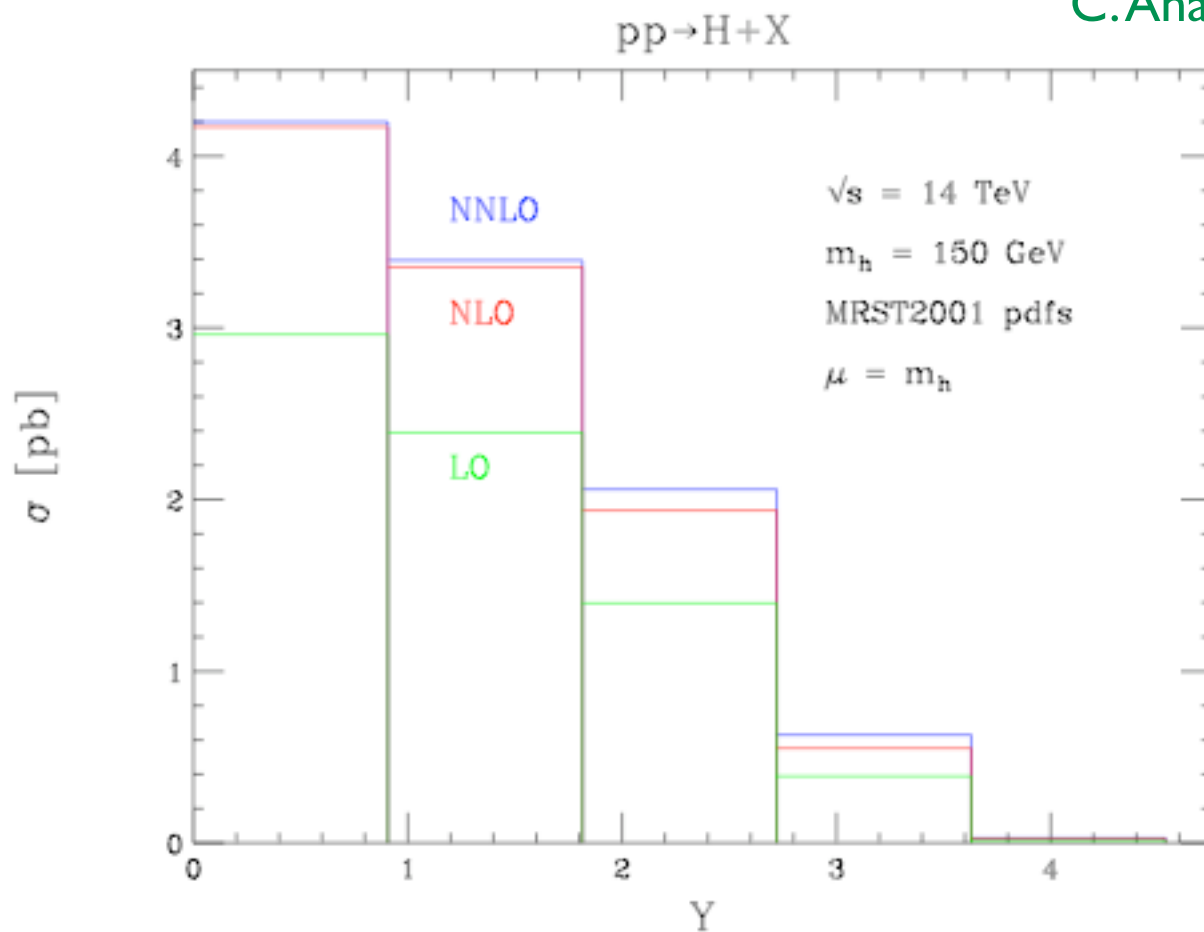
- need for **NNLO** corrections must be assessed process by process
- in the last few years, a lot of progress on the computation of **NNLO** cross sections
- sector decomposition is already up and running
- subtraction is making substantial progress

Higgs production at LHC

a fully differential cross section:

bin-integrated rapidity distribution, with a jet veto

C. Anastasiou K. Melnikov F. Petriello 2004



jet veto: require

$$R = 0.4$$

$$|\mathbf{p}_T^j| < p_T^{veto} = 40 \text{ GeV}$$

for 2 partons

$$R_{12}^2 = (\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2$$

if $R_{12} > R$

$$|\mathbf{p}_T^1|, |\mathbf{p}_T^2| < p_T^{veto}$$

if $R_{12} < R$

$$|\mathbf{p}_T^1 + \mathbf{p}_T^2| < p_T^{veto}$$

● $M_H = 150 \text{ GeV}$ (jet veto relevant in the $H \rightarrow W^+W^-$ decay channel)

● K factor is much smaller for the vetoed x-sect than for the inclusive one: average $|\mathbf{p}_T^j|$ increases from NLO to NNLO: less x-sect passes the veto