

Soft-gluon corrections in hard-scattering processes through NNNLO

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- **Soft-gluon corrections**
- **NLO and NNLO corrections**
- **NNLO soft-gluon corrections**
- **Applications to top quark production**

N. Kidonakis, hep-ph/0509079, Phys. Rev. D 73, 034001 (2006)

Soft-gluon corrections

Factorized cross section for $h_1 + h_2 \rightarrow F(p) + X$

$$\sigma = \sum_f \int \left[\prod_i dx_i \Phi_{f|h_i}(x_i, \mu_F^2) \right] \hat{\sigma}(s, t_i, \mu_F, \mu_R)$$

Hard-scattering cross section - perturbatively calculable

Near threshold for production of the system there is restricted phase space for real gluon emission

Incomplete cancellation of infrared divergences between real and virtual graphs \rightarrow large logarithms

Soft and collinear corrections \rightarrow plus distributions

For $f_1(p_1) + f_2(p_2) \rightarrow F(p) + X$ define $s = (p_1 + p_2)^2$, $t_1 = (p_1 - p)^2$, $t_2 = (p_2 - p)^2$, $s_4 = s + t_1 + t_2 - \sum m^2$ At threshold $s_4 \rightarrow 0$

$$\mathcal{D}_l(s_4) \equiv \left[\frac{\ln^l(s_4/M^2)}{s_4} \right]_+$$

with $l \leq 2n - 1$ for the n -th order corrections

Define moments of the cross section

$$\hat{\sigma}(N) = \int_0^\infty ds_4 e^{-Ns_4/M^2} \hat{\sigma}(s_4)$$

Soft corrections

$$\mathcal{D}_l(s_4) \equiv \left[\frac{\ln^l(s_4/M^2)}{s_4} \right]_+ \rightarrow \frac{(-1)^{l+1}}{l+1} \ln^{l+1} N + \dots$$

We can formally resum these logarithms to all orders in α_s :
factorize soft gluons from the hard scattering

Invert back to momentum space

At NLO, $\mathcal{D}_1(s_4)$ and $\mathcal{D}_0(s_4)$ terms

LL **NLL**

At NNLO, $\mathcal{D}_3(s_4)$, $\mathcal{D}_2(s_4)$, $\mathcal{D}_1(s_4)$, and $\mathcal{D}_0(s_4)$ terms

LL **NLL** **NNLL** **NNNLL**

At NNNLO, $\mathcal{D}_5(s_4)$, $\mathcal{D}_4(s_4)$, $\mathcal{D}_3(s_4)$, $\mathcal{D}_2(s_4)$, $\mathcal{D}_1(s_4)$, and $\mathcal{D}_0(s_4)$ terms

LL **NLL** **NNLL** ...

Threshold resummation formalism has been applied to:

- Top quark pair hadroproduction
- Beauty and charm production
- Jet production
- Direct photon production
- Large- Q_T W production
- FCNC top production
- Charged Higgs production

Numerical results:

Soft corrections a good approximation of full NLO result

Higher-order corrections are sizable

Dramatic decrease of scale dependence

Soft-gluon resummation

A unified approach

$$\hat{\sigma}^{res}(N) = \exp \left[\sum_i E^{fi}(N_i) \right] \exp \left[\sum_j E'^{fj}(N_j) \right] \exp \left[\sum_i 2 \int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu} \gamma_{ili}(\alpha_s(\mu)) \right] \exp \left[2d_{\alpha_s} \int_{\mu_R}^{\sqrt{s}} \frac{d\mu}{\mu} \beta(\alpha_s(\mu)) \right] \\ \times \text{Tr} \left\{ H^{fifj}(\alpha_s(\mu_R)) \exp \left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_j} \frac{d\mu}{\mu} \Gamma_S^{\dagger fifj}(\alpha_s(\mu)) \right] \tilde{S}^{fifj}(\alpha_s(\sqrt{s}/\tilde{N}_j)) \exp \left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_j} \frac{d\mu}{\mu} \Gamma_S^{fifj}(\alpha_s(\mu)) \right] \right\}$$

In the $\overline{\text{MS}}$ scheme

$$E^{fi}(N_i) = - \int_0^1 dz \frac{z^{N_i-1} - 1}{1-z} \left\{ \int_{(1-z)^2}^1 \frac{d\lambda}{\lambda} A_i(\alpha_s(\lambda s)) + v_i[\alpha_s((1-z)^2 s)] \right\}$$

with $A_i(\alpha_s) = C_i[\alpha_s/\pi + (\alpha_s/\pi)^2 K/2] + A_i^{(3)} + \dots$ and $v_i = (\alpha_s/\pi)C_i + (\alpha_s/\pi)^2 v_i^{(2)} + (\alpha_s/\pi)^3 v_i^{(3)} + \dots$
and (for any massless final-state partons at lowest order)

$$E'^{fj}(N_j) = \int_0^1 dz \frac{z^{N_j-1} - 1}{1-z} \left\{ \int_{(1-z)^2}^{1-z} \frac{d\lambda}{\lambda} A_j(\alpha_s(\lambda s)) - B_j[\alpha_s((1-z)s)] - v_j[\alpha_s((1-z)^2 s)] \right\}$$

where $B_j = (\alpha_s/\pi)B_j^{(1)} + (\alpha_s/\pi)^2 B_j^{(2)} + (\alpha_s/\pi)^3 B_j^{(3)}$ with $B_q^{(1)} = 3C_F/4$ and $B_g^{(1)} = \beta_0/4$
 H are hard scattering matrices; S are soft matrices (noncollinear soft-gluon emission);
 Γ_S are soft anomalous dimension matrices

NLO master formula

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu_R^2)}{\pi} \{c_3 \mathcal{D}_1(s_4) + c_2 \mathcal{D}_0(s_4) + c_1 \delta(s_4)\} + \frac{\alpha_s^{d_{\alpha_s}+1}(\mu_R^2)}{\pi} [A^c \mathcal{D}_0(s_4) + T_1^c \delta(s_4)]$$

Here $c_3 = \sum_i 2 C_i - \sum_j C_j$ **For quarks** $C_F = (N_c^2 - 1)/(2N_c)$ **For gluons** $C_A = N_c$

and $c_2 = c_2^\mu + T_2$ with $c_2^\mu = -\sum_i C_i \ln(\mu_F^2/M^2)$ and

$$T_2 = -\sum_i \left[C_i + 2 C_i \ln\left(\frac{-t_i}{M^2}\right) + C_i \ln\left(\frac{M^2}{s}\right) \right] - \sum_j \left[B_j^{(1)} + C_j + C_j \ln\left(\frac{M^2}{s}\right) \right]$$

For quarks $B_q^{(1)} = \gamma_q^{(1)} = 3C_F/4$ **For gluons** $B_g^{(1)} = \gamma_g^{(1)} = \beta_0/4$

Also $c_1 = c_1^\mu + T_1$, with

$$c_1^\mu = \sum_i \left[C_i \ln\left(\frac{-t_i}{M^2}\right) - \gamma_i^{(1)} \right] \ln\left(\frac{\mu_F^2}{M^2}\right) + d_{\alpha_s} \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{M^2}\right)$$

Matrix term $A^c = \text{tr} \left(H^{(0)} \Gamma'_S{}^{(1)\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma'_S{}^{(1)} \right)$

NNLO master formula

$$\begin{aligned}
 \hat{\sigma}^{(2)} = & \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \frac{1}{2} c_3^2 \mathcal{D}_3(s_4) + \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ \frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + \sum_j C_j \frac{\beta_0}{8} \right\} \mathcal{D}_2(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \frac{3}{2} c_3 A^c \mathcal{D}_2(s_4) \\
 & + \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} C_{D_1}^{(2)} \mathcal{D}_1(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \left\{ \left(2c_2 - \frac{\beta_0}{2} \right) A^c + c_3 T_1^c + F^c \right\} \mathcal{D}_1(s_4) \\
 & + \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} C_{D_0}^{(2)} \mathcal{D}_0(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \left\{ \left[c_1 - \zeta_2 c_3 + \frac{\beta_0}{4} \ln \left(\frac{\mu_R^2}{M^2} \right) + \frac{\beta_0}{4} \ln \left(\frac{M^2}{s} \right) \right] A^c \right. \\
 & \quad \left. + \left(c_2 - \frac{\beta_0}{2} \right) T_1^c + F^c \ln \left(\frac{M^2}{s} \right) + G^c \right\} \mathcal{D}_0(s_4)
 \end{aligned}$$

Here $C_{D_1}^{(2)} = c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln \left(\frac{\mu_R^2}{M^2} \right) + c_3 \frac{K}{2} - \sum_j \frac{\beta_0}{4} B_j^{(1)}$

$$\begin{aligned}
 C_{D_0}^{(2)} = & c_2 c_1 - \zeta_2 c_3 c_2 + \zeta_3 c_3^2 - \frac{\beta_0}{2} T_1 + \frac{\beta_0}{4} c_2 \ln \left(\frac{\mu_R^2}{M^2} \right) + d_{\alpha_s} \frac{\beta_0^2}{8} \ln \left(\frac{M^2}{s} \right) - \sum_i v_i^{(2)} - \frac{\beta_0}{2} \sum_i \gamma_i^{(1)} \ln \left(\frac{M^2}{s} \right) \\
 & + \sum_i C_i \frac{\beta_0}{8} \left[\ln^2 \left(\frac{\mu_F^2}{M^2} \right) - \ln^2 \left(\frac{M^2}{s} \right) - 2 \ln \left(\frac{M^2}{s} \right) \right] - \sum_i C_i \frac{K}{2} \left[\ln \left(\frac{\mu_F^2}{M^2} \right) + 2 \ln \left(\frac{-t_i}{M^2} \right) + \ln \left(\frac{M^2}{s} \right) \right] \\
 & - \sum_j \left(B_j^{(2)} + v_j^{(2)} \right) + \sum_j C_j \left[-\frac{\beta_0}{8} \ln^2 \left(\frac{M^2}{s} \right) - \frac{\beta_0}{4} \ln \left(\frac{M^2}{s} \right) - \frac{K}{2} \ln \left(\frac{M^2}{s} \right) \right] - \sum_j \frac{\beta_0}{2} B_j^{(1)} \ln \left(\frac{M^2}{s} \right)
 \end{aligned}$$

$$F^c = \text{tr} \left[H^{(0)} \left(\Gamma_S^{(1)\dagger} \right)^2 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma_S^{(1)} \right)^2 + 2H^{(0)} \Gamma_S^{(1)\dagger} S^{(0)} \Gamma_S^{(1)} \right]$$

$$G^c = \text{tr} \left[H^{(1)} \Gamma_S^{(1)\dagger} S^{(0)} + H^{(1)} S^{(0)} \Gamma_S^{(1)} + H^{(0)} \Gamma_S^{(1)\dagger} S^{(1)} + H^{(0)} S^{(1)} \Gamma_S^{(1)} + H^{(0)} \Gamma_S^{(2)\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma_S^{(2)} \right]$$

NNNLO master formula

$$\begin{aligned}
 \hat{\sigma}^{(3)} = & \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \frac{1}{8} c_3^3 \mathcal{D}_5(s_4) + \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{ \frac{5}{8} c_3^2 c_2 - \frac{5}{2} c_3 X_3 \right\} \mathcal{D}_4(s_4) + \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \frac{5}{8} c_3^2 A^c \mathcal{D}_4(s_4) \\
 & + \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{ c_3 c_2^2 + \frac{c_3^2}{2} c_1 - \zeta_2 c_3^3 + (\beta_0 - 4c_2) X_3 + 2c_3 X_2 - \sum_j C_j \frac{\beta_0^2}{48} \right\} \mathcal{D}_3(s_4) \\
 & + \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \left\{ \frac{1}{2} c_3^2 T_1^c + \left[2c_3 c_2 - \frac{\beta_0}{2} c_3 - 4X_3 \right] A^c + c_3 F^c \right\} \mathcal{D}_3(s_4) \\
 & + \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{ \frac{3}{2} c_3 c_2 c_1 + \frac{1}{2} c_2^3 - 3\zeta_2 c_3^2 c_2 + \frac{5}{2} \zeta_3 c_3^3 + \left(-3c_1 + \frac{27}{2} \zeta_2 c_3 \right) X_3 + (3c_2 - \beta_0) X_2 - \frac{3}{2} c_3 X_1 \right. \\
 & \quad \left. - \sum_i C_i \frac{\beta_1}{8} + \sum_j \frac{\beta_0^2}{16} B'_j{}^{(1)} + \sum_j \frac{3}{32} C_j \beta_1 + \sum_j C_j \frac{\beta_0}{16} \left[\beta_0 \ln \left(\frac{\mu_R^2}{M^2} \right) + 2K \right] \right\} \mathcal{D}_2(s_4) \\
 & + \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \left\{ \left(\frac{3}{2} c_3 c_2 - 3X_3 \right) T_1^c + \frac{3}{2} \left[c_2 + c_3 \ln \left(\frac{M^2}{s} \right) \right] F^c + \frac{3}{2} c_3 G^c + \frac{1}{2} K_3^c \right. \\
 & \quad \left. + \left[\frac{3}{2} c_2^2 + \frac{3}{2} c_3 c_1 - 3\zeta_2 c_3^2 + 3X_2 + \frac{\beta_0^2}{4} - \frac{3}{4} \beta_0 \left(c_2 - \frac{c_3}{2} \ln \left(\frac{\mu_R^2}{M^2} \right) \right) - \frac{3\beta_0}{8} c_3 \ln \left(\frac{M^2}{s} \right) \right] A^c \right\} \mathcal{D}_2(s_4) \\
 & + \dots
 \end{aligned}$$

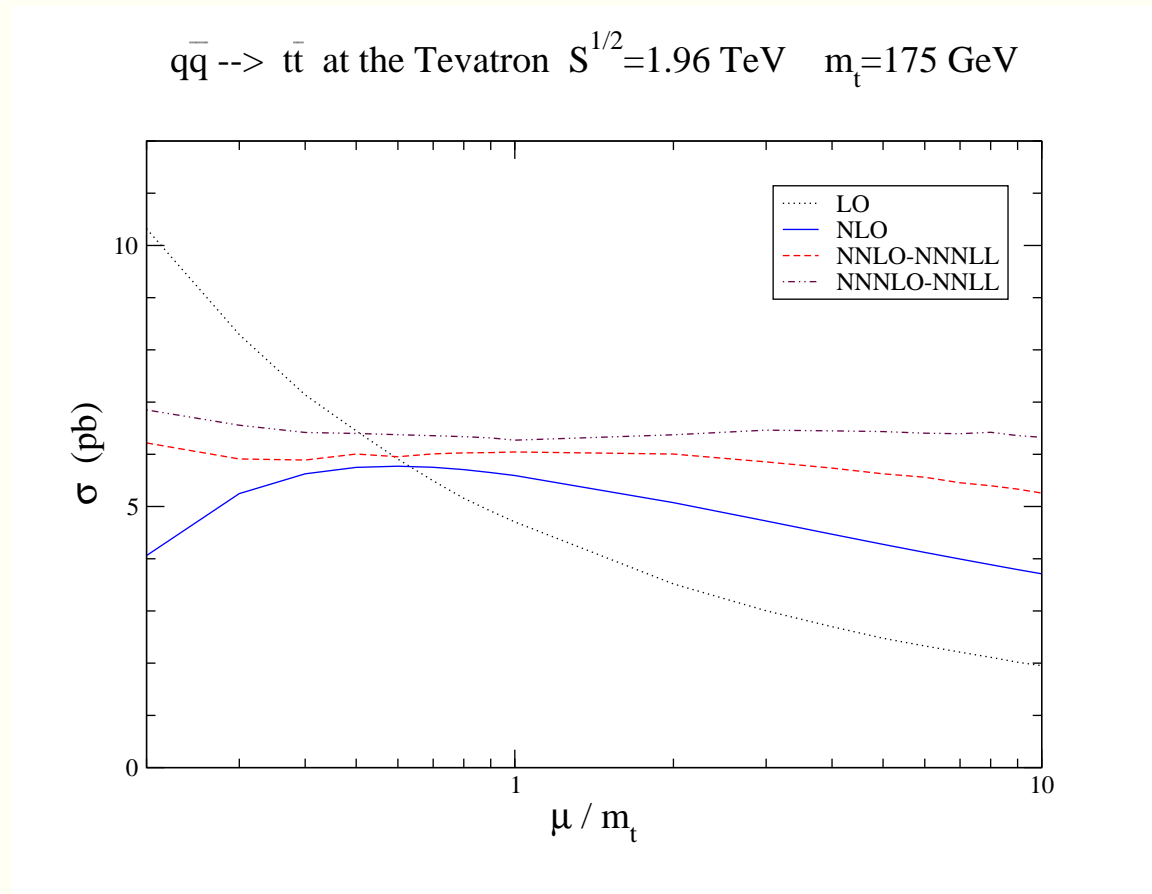
Here $X_3 = (\beta_0/12)c_3 - \sum_j C_j \beta_0/24$ $X_2 = -(\beta_0/4)T_2 + (\beta_0/8)c_3 \ln(\mu_R^2/M^2) + c_3 K/4 - \sum_j (\beta_0/8)B_j^{(1)}$

$X_1 = c_2 c_1 - \zeta_2 c_3 c_2 + \zeta_3 c_3^2 + (\beta_0/4) \zeta_2 c_3 - \sum_j C_j (\beta_0/8) \zeta_2 - C_{D_0}^{(2)}$

$K_3^c = \text{tr} \left[H^{(0)} \left(\Gamma_S^{(1)\dagger} \right)^3 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma_S^{(1)} \right)^3 + 3 H^{(0)} \left(\Gamma_S^{(1)\dagger} \right)^2 S^{(0)} \Gamma_S^{(1)} + 3 H^{(0)} \Gamma_S^{(1)\dagger} S^{(0)} \left(\Gamma_S^{(1)} \right)^2 \right]$

NNNLO soft-gluon corrections for top quark production

Top pair production in the $q\bar{q}$ channel at the Tevatron



Higher-order corrections greatly reduce the scale dependence

Summary

- **Soft-gluon resummation**
- **Soft-gluon threshold corrections are sizable**
- **NLO, NNLO, and NNNLO soft-gluon expansions**
- **Important for greater theoretical accuracy**
- **Significant contributions to K -factors**
- **Dramatic decrease of the scale dependence**