Soft Gluon Ressumation in Effective Field Theory

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References: Ildibi, Ji, Yuan, PLB 625, 253 (2005);
Ildibi, Ji, Ma, Yuan, PRD73, 077501 (2006).

Two Large Scales Generate Large Double Logs in pQCD

- For example, a differential cross section depends on Q_1, where $Q^2 \gg Q_1^2 \gg \Lambda_{QCD}^2$

$$\frac{d\sigma}{dQ_1^2} = \frac{1}{Q_1^2} f_1 \otimes f_2 \otimes \sum_i \alpha_s^i \ln^{2i} \frac{Q^2}{Q_1^2} + \cdots$$

- We have to resum these large logs to make reliable predictions
  - $Q_\perp$: Dokshitzer, Diakonov, Troian, 78; Parisi Petronzio, 79; Collins, Soper, Sterman, 85
  - Threshold: Sterman 87; Catani and Trentadue 89

Soft-Collinear Effective Theory

Bauer, Fleming, Pirjol, Stewart

- Effective Theory approach
  - Choose low-energy degree of freedom
    - Soft, Collinear fields (ξ_n, A_n, A_s, …)
  - Write down effective Lagrangian through gauge symmetry and power counting
- Applications:
  - Derive factorization theorems consistent with low energy degree of freedom
  - Derive twist expansion for complicated QCD processes ($B \rightarrow \pi$ decay, $B \rightarrow D$ decay, jet structure)
  - Resumming large logarithms

Two Steps Matching

- At scale Q, match the quark (gluon) current between full QCD and SCET
  - Derive the matching coefficient and the **anomalous dimension,** which controls the running
- At lower scale in SCET, match to the (product of) parton distribution, like a usual pQCD factorization for the cross section

$$\sigma = C_1(\mu_H) \times \text{Running} \times C_2(\mu_L) \times f(\mu_L)$$

Resummation in SCET

- DIS Structure function at large x, **Manohar, 03**
  - Two scales, Q^2, $(1\text{-}x)\,Q^2$
  - Extended to Drell-Yan, Idilbi, Ji, 05
- Applications to $Q_\perp$ resummation
  - Gao, Li, Liu, NLL, 05
  - Idilbi, Ji, Yuan, NLL, 05
- All order equivalence for threshold resummation
  - Idilbi, Ji, Ma, Yuan, 05

Matching at Q

- At scale Q, one can integrate out the fluctuations of order Q. Since all other scales are small, we can set them to zero, and the processes are very similar to elastic form factors. Therefore, the integration can be done by matching the full theory current to effective theory current, and only virtual diagrams contribute

Form Factors

- High order corrections to the current in the full theory is represented as on-shell quark (gluon) form factors,

$$F(Q^2,\varepsilon)=1+\alpha_s F^{(1)}+\alpha_s^{2} F^{(2)}+\cdots$$

- Similarly, we can also calculate the form factors in the effective theory, which has the same IR structure as the full theory form factor. So their matching can be expressed as

$$F(Q^2,\varepsilon)=C(Q^2/\mu^2)F_{eff}(Q^2/\mu^2,\varepsilon)$$

Matching coefficients at $\mu=M_H$

- The matching $C(M_H)$ can be expanded in terms of $\alpha_s(M_H)$

$$
\begin{aligned}
C_g^{(1)} &= 7C_A\zeta_2 \\
C_g^{(2)} &= C_A^2\left(\frac{5105}{162}+\frac{335}{6}\zeta_2-\frac{143}{9}\zeta_3+\frac{125}{10}\zeta_2^2\right) \\
&+ C_A n_f\left(-\frac{916}{81}-\frac{25}{3}\zeta_2-\frac{46}{9}\zeta_3\right) \\
&+ C_F n_f\left(-\frac{67}{6}+8\zeta_3\right)
\end{aligned}
$$

Anomalous dimension

- The anomalous dimension controls the running of C(μ)

$$\gamma_1 = \frac{d}{d \ln \mu} \ln \left[C(Q^2/\mu^2) \right]$$

- Using the current known quark (gluon) form factors up to three-loop (MVV 05)

$$\gamma_1^{(i)} = A^{(i)} \ln \frac{Q^2}{\mu^2} - 2B^{(i)} - f^{(i)} \qquad (i = 1, 2, 3)$$

  - $A^{(i)}$, the cusp anomalous dimension
  - $B^{(i)}$, the $\delta(1\text{-}x)$ coefficient in the splitting function
  - $f^{(i)}$, a universal structure, similar to $A^{(i)}$

Matching at μ_L

- Can be calculated from the cross section,

  $\sigma_{eff}(\mu)_L) = M_N(\mu)_L) \otimes f_1(\mu_L) \otimes f_2(\mu_L)$

- A detailed formulation in EFT is not necessary, rather we can use the result from the full QCD calculations in the soft-collinear limit,

$$
\begin{aligned}
\mathcal{M}_N^{(1)} &= 2C_A\zeta_2 \\
\mathcal{M}_N^{(2)} &= C_A^2\left[\frac{2428}{81} + \frac{67}{9}\zeta_2 - \frac{22}{9}\zeta_3 - 10\zeta_2^2\right] \\
&+ C_A N_F\left[-\frac{328}{81} - \frac{10}{9}\zeta_2 + \frac{4}{9}\zeta_3\right]
\end{aligned}
$$

  - M_N is universal, C_A->C_F will give the quark one

Final Result for the Threshold Resummation

- The cross section in the moment space,

$$\sigma_N(M_H) = |C(\alpha_s(M_H^2))|^2 e^{-\mathcal{S}(M_H^2,\ \mu_L^2,\mu_F^2)} \times \mathcal{M}_N(\alpha_s(\mu_L^2)) f_N(\mu_F) f'_N(\mu_F)$$

- $C(M_H)$ and $M_N(\mu_L)$ only depend on α_s, the large logs are contained in the exponential factor

$$\begin{aligned}\mathcal{S} &= I_1 + I_2 \\ &= \int_{\mu_L}^{M_H} \frac{d\mu}{\mu} 2\gamma_1(\mu^2, \alpha_s(\mu^2)) + \int_{\mu_F}^{\mu_L} \frac{d\mu}{\mu} 2\gamma_2(\mu^2, \alpha_s(\mu^2))\end{aligned}$$

Exponential Form Factor

- γ_1 controls running from M_H to $\mu_L = M_H/N$, γ_2 controls μ_L to μ_F

$$\begin{aligned} \gamma_1 &= A_1(\mu^2) \ln M_H^2/\mu^2 + B_1(\mu^2) \\ \gamma_2 &= A_2(\mu^2) \ln \overline{N}^2 + B_2(\mu^2) \end{aligned}$$

- A_1, A_2, B_1, B_2 are known and calculated up to 3-loop, and introduce the third integral,

$$I_3 = \int_{\mu_L^2}^{Q^2} \frac{d\mu^2}{\mu^2} \triangle B_1 \qquad \text{with}$$

$$\triangle B_1(\mu^2) = \frac{d \ln \mathcal{M}_N(\mu^2)}{d \ln \mu^2} = -\beta_s \frac{d \ln \mathcal{M}_N(\alpha_s)}{d \ln \alpha_s}$$

- Final result

$$\sigma_N(M_H) = |C(\alpha_s(M_H))|^2 \mathcal{M}_N(\alpha_s(M_H^2)) \times e^{-I_1-I_2-I_3}$$

- N-dependent terms are entirely in I_1, I_2, I_3

All Orders Equivalence

- In conventional resummation formalism

Sterman 87, Catani and Trentadue 89

$$I_\triangle = \int_0^1 dz \frac{z^{N-1}-1}{1-z}\left[2\int_{\mu^2}^{(1-z)^2Q^2}\frac{d\mu^2}{\mu^2}A(\mu^2)\right.$$
$$\left. +D(\alpha_s((1-z)^2Q^2))\right]$$

- All order relation between these two

$I_1+I_2+I_3=I_\triangle+\ln C_G$ with

$$A_1 = A_2 = A$$
$$2(B_1+\triangle B_1 - B_2) = D(\mu^2)$$
$$+\partial_{\alpha_s}\Gamma_2(\partial_{\alpha_s})\left[4A - \partial_{\alpha_s}D\right]$$

At three-loop

- We have calculated $D^{(2)}$ and $D^{(3)}$ from SCET formalism, agree with the results from full theory expansions (Vogt, et al., 05)

$$
\begin{aligned}
D_g^{(1)} &= 0 \\
D_g^{(2)} &= -2f_g^{(2)} + 4\beta_0\zeta_2 A_g^{(1)} - 2\beta_0\mathcal{M}_N^{(1)} \\
D_g^{(3)} &= -2f_g^{(3)} + 4\zeta_2\beta_1 A_g^{(1)} + 8\zeta_2\beta_0 A_g^{(2)} + \frac{32}{3}\zeta_3\beta_0^2 A_g^{(1)} \\
&\quad -2\beta_1\mathcal{M}_N^{(1)} - 2\beta_0\left[2\mathcal{M}_N^{(2)} - \left(\mathcal{M}_N^{(1)}\right)^2\right]
\end{aligned}
$$

Conclusion

- As a perfect tool, SCET has shown great ability to do resummation for the $Q_\perp$ and threshold cases
- SCET provide an intuitive way to understand the resummation, and it is much simpler to perform the calculations
- Further application of SCET to higher order corrections and other processes are desirable