Multi-Jet Processes in the High Energy Limit of QCD

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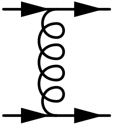
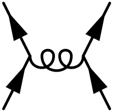
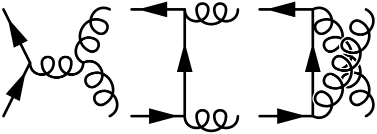
DIS2006, April 22

Outline

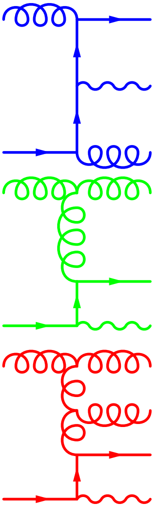
The High Energy Limit of Fixed Order Matrix Elements

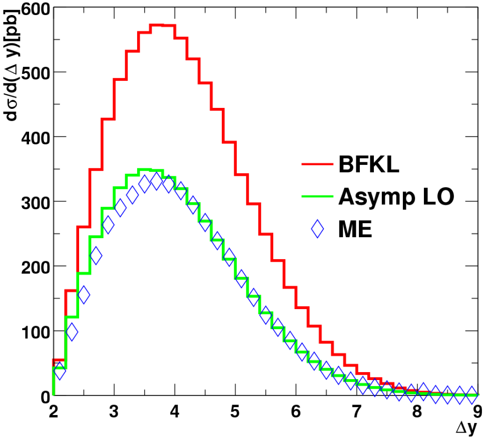
Process	Diagrams	$\overline{\sum}\lvert\mathcal{M}\rvert^2/g^4$
$qq' \to qq'$		$\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$
$q\bar{q} \to q'\bar{q}'$		$\frac{4}{9}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$
$q\bar{q} \to gg$		$\frac{32}{27}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$

High Energy Limit: $\lvert\hat{t}\rvert$ fixed, $\hat{s} \to \infty$

t–channel dominance

Example: W+n-jet production at the LHC





$$\Delta y = y_{j_2} - y_{j_1}, y_W, y_{j_2} \geq 1, y_{j_1} \leq -1$$

Observations

- In the limit of large rapidity spans, the fixed order matrix elements are dominated by contributions from diagrams with a *t*-channel gluon exchange
- This limit will be called The High Energy Limit and is generally characterised by the following phase space configuration of the final state particles

$$y_0 > y_1 > \cdots > y_n > y_{n+1}, \quad |k_0| \sim |k_i| \sim |k_{n+1}|$$

  i.e. multiple, isolated, hard parton production (multiple jets)
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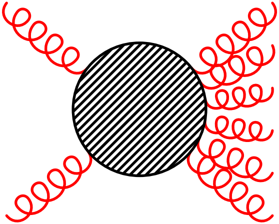
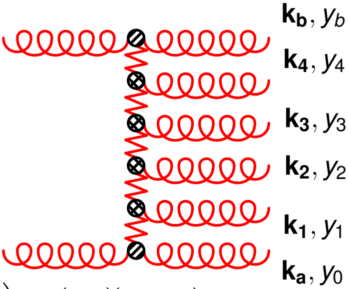
The Possibility for Prediction of n-jet Rates
The Power of Reggeisation



High Energy Limit
$$|\hat{t}| \quad \text{fixed}, \quad \hat{s} \to \infty$$



$$\mathcal{A}^R_{2\to 2+n} = \Gamma_{A'A} \left(\prod_{i=1}^{n} \frac{e^{\omega(q_i)(y_{i-1}-y_i)}}{q_i^2} V^{J_i}(q_i, q_{i+1}) \right) \frac{e^{\omega(q_{n+1})(y_n - y_{n+1})}}{q_{n+1}^2} \Gamma_{B'B}$$

$$\mathbf{q}_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$$

At LL only gluon production; at NLL also quark–anti-quark pairs produced.

Prediction of any-jet rate possible.

Reggeisation and the BFKL Equation

The **evolution of the reggeised gluon** is described by the BFKL equation

$$\omega f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right) = \delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right) + \int d^{2+2\epsilon}\mathbf{k}'\mathcal{K}_\epsilon\left(\mathbf{k}_a,\mathbf{k}'\right) f_\omega\left(\mathbf{k}',\mathbf{k}_b\right)$$

ω: Mellin conjugated variable to the rapidity y along the evolution.

- The kernel $\mathcal{K}_\epsilon$ consists of the **virtual** corrections of the trajectory and the **real** corrections from the Lipatov vertices.
- The BFKL equation provides a very convenient framework for organising the divergences in the factorised form of the $|\mathcal{M}|^2$ on the previous slide.

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Energy and Momentum Conservation in an Inclusive Framework

One of the benefits of BFKL : Fully inclusive any-jet cross sections can be calculated analytically $(p'_a, p'_b \to p_a, \{p_i\}, p_b)$

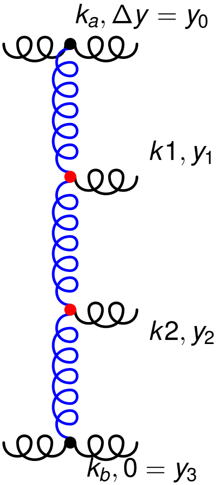
$$\mathrm{d}\hat{\sigma}(p_a, p_b) = \Gamma_a(\mathbf{p}_a)\ f(\mathbf{p}_a, -\mathbf{p}_b, \Delta)\ \Gamma_b(\mathbf{p}_b)$$

However, we need the total energy in order to calculate predictions at any collider. No constrain on the initial state $\to$ need full final state information[1]!

[1] Not resummation of soft, collinear radiation: large contribution to energy

Iteration at Next to Leading Logarithmic Accuracy

$$
\begin{aligned}
f(\mathbf{k}_a,\mathbf{k}_b,\Delta) \quad = \quad & \exp\left(\omega_0\left(\mathbf{k}_a^2,\lambda^2,\mu\right)\Delta\right)\delta^{(2)}(\mathbf{k}_a-\mathbf{k}_b) \\
+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\int d^2\mathbf{k}_i\int_0^{y_{i-1}} dy_i & \left[V\left(\mathbf{k}_i,\mathbf{k}_a+\sum_{l=0}^{i-1}\mathbf{k}_l,\mu\right)\right] \\
\times\ \exp\left[\omega_0\left(\left(\mathbf{k}_a+\sum_{l=1}^{i-1}\mathbf{k}_l\right)^2,\lambda^2,\mu\right)(y_{i-1}-y_i)\right] & \\
\times\exp\left[\omega_0\left(\left(\mathbf{k}_a+\sum_{l=1}^{n}\mathbf{k}_l\right)^2,\lambda^2,\mu\right)(y_n-0)\right] & \\
\times\delta^{(2)}\left(\sum_{l=1}^{n}\mathbf{k}_l+\mathbf{k}_a-\mathbf{k}_b\right) &
\end{aligned}
$$



Direct BFKL Evolution @ LL&NLL

Solution to the BFKL equation at fixed Δ at both LL and NLL:

$$f(\mathbf{k}_a, \mathbf{k}_b, \Delta) = \sum_{n=0}^{\infty} \int \mathrm{d}\mathcal{P}_n \, \mathcal{F}_n,$$

$$\int \mathrm{d}\mathcal{P}_n = \left(\int \prod_{i=1}^{n} \mathrm{d}\mathbf{k}_i \int_0^{y_0} \mathrm{d}y_1 \int_0^{y_1} \mathrm{d}y_2 \cdots \int_0^{y_{n-1}} \mathrm{d}y_n \right) \delta^{(2)} \left(\mathbf{k}_a + \sum_{l=1}^{n} \mathbf{k}_l - \mathbf{k}_b \right)$$

$$\mathcal{F}_n = \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)(y_{i-1}-y_i)} \, V(\mathbf{q}_i, \mathbf{q}_{i+1}) \right) e^{\omega(\mathbf{q}_{n+1})(y_n - y_{n+1})}$$

$$\int_0^{y_0} \mathrm{d}y_1 \int_0^{\mathrm{d}y_1} \mathrm{d}y_2 \cdots \int_0^{\mathrm{d}y_{n-1}} \mathrm{d}y_n \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)(y_{i-1}-y_i)} \right) e^{\omega(\mathbf{q}_{n+1})(y_n - y_{n+1})}$$

$$= \int_0^{\Delta} \mathrm{d}\delta y_n \int_0^{\Delta - y_n} \mathrm{d}\delta y_{n-1} \cdots \int_0^{\Delta - y_n - \cdots - y_2} \mathrm{d}\delta y_1 \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)\delta y_i} \right) e^{\omega(\mathbf{q}_{n+1})\delta y_{n+1}}$$

$$= \int_0^{\infty} \mathrm{d}\delta y_{n+1} \int_0^{\infty} \mathrm{d}\delta y_n \cdots \int_0^{\infty} \mathrm{d}\delta y_1 \; \delta(\Delta - \sum_{i=1}^{n+1} \delta y_i) \prod_{i=1}^{n+1} e^{\omega(\mathbf{q}_i)\delta y_i}$$

Direct BFKL Evolution, 2

$$f(\mathbf{k}_a, \mathbf{k}_b, \Delta) = \sum_{n=0}^{\infty} \int \mathrm{d}\mathcal{P}_n \, \mathcal{F}_n,$$

$$\int \mathrm{d}\mathcal{P}_n = \left(\prod_{i=1}^{n} \int \mathrm{d}\mathbf{k}_i \int_0^{\infty} \mathrm{d}\delta y_i \right) \int_0^{\infty} \mathrm{d}\delta y_{n+1} \; \delta^{(2)} \left(\mathbf{k}_a + \sum_{l=1}^{n} \mathbf{k}_l - \mathbf{k}_b \right) \delta \left(\Delta - \sum_{i=1}^{n+1} \delta y_i \right)$$

$$\mathcal{F}_n = \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)\delta y_i} \; V(\mathbf{q}_i, \mathbf{q}_{i+1}) \right) \; e^{\omega(\mathbf{q}_{n+1})\delta y_{n+1}}$$

$f(\mathbf{k}_a, \mathbf{k}_b, \Delta)$: the value at $\Delta \equiv \sum_{i=1}^{n+1} \delta y_i$ of the product of vertices $V(\mathbf{q}_i, \mathbf{q}_{i+1})$ at rapidity $y_i = \sum_{j=1}^{i} \delta y_j$ connected with Regge factors $e^{\omega(\mathbf{q}_i)\delta y_i}$ describing the probability of no (resolved) emission between two adjacent (in rapidity) vertices

Direct BFKL Evolution, 3

1. Choose a random number of vertices for the evolution, $n \geq 0$
2. Generate a set $\{\mathbf{k}_i\}_{i=1,\dots,n}$ of transverse momenta (the outgoing momenta are $\{-\mathbf{k}_i\}_{i=1,\dots,n}$)
3. Calculate the corresponding set of trajectories $\{\omega(\mathbf{q}_i)\}_{i=1,\dots,n+1}$, and vertex factors $\{V(\mathbf{q}_i, \mathbf{q}_{i+1})\}_{i=1,\dots,n}$, $\mathbf{q}_i = k_a + \sum_{l=1}^{i-1} \mathbf{k}_l$
4. Generate the inter-vertex rapidity separations $\{\delta y_i\}$ according to the distributions $e^{\omega(\mathbf{q}_i)\delta y_i}$
5. Calculate the corresponding $\Delta = \sum_{i=1}^{n+1} \delta y_i$ and return $\prod_{i=1}^{n} V(\mathbf{q}_i, \mathbf{q}_{i+1})$

Have constructed full final state[2]! Trivial to impose energy and momentum conservation and do proper jet studies.

[2] See later

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Observation

1. Imposing Energy and Momentum conservation (i.e. restricting phase space integral to that assessable at a given energy) is completely unrelated to NLL corrections to the evolution.
2. To calculate an observable to full NLL accuracy, three ingredients are necessary:
   - NLL Impact Factors
   - NLL Evolution
   - Energy and Momentum Conservation

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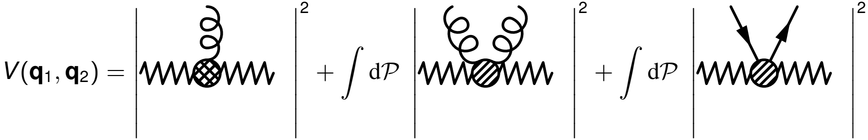
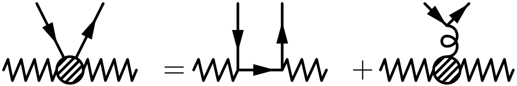
The Ingredients of the NLL Vertex

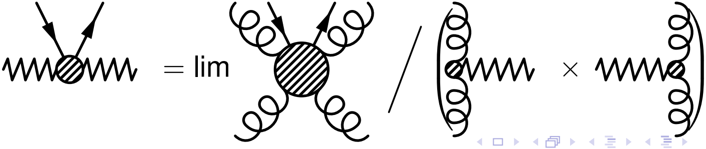


Two methods for obtaining the vertices at NLL:

- Fadin & Lipatov:

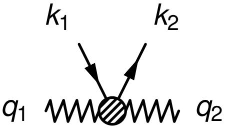


- V. Del Duca:



Divergences and Strategy

Divergences of the quark contribution to the NLL vertex:



Divergences separate into two categories:

$\Delta = \mathbf{q}_1 - \mathbf{q}_2 = 0$: Regulated by the NLL Trajectory

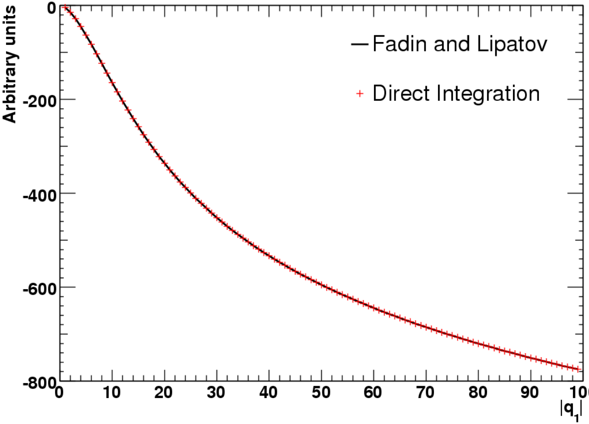
$\mathbf{k}_1 \to x\Delta$: Regulated by quark contribution to one-loop corrections to one-gluon production vertex (x the light-cone momentum fraction of the quarks)

Strategy:

1. Implement Lipatov Vertices and perform integration, while having access to full final state information. Only possibility of combining energy and momentum conservation with NLL evolution.
2. Check that the numerical integration over full phase space agrees with the result of Fadin & Lipatov (or Camici & Ciafaloni)

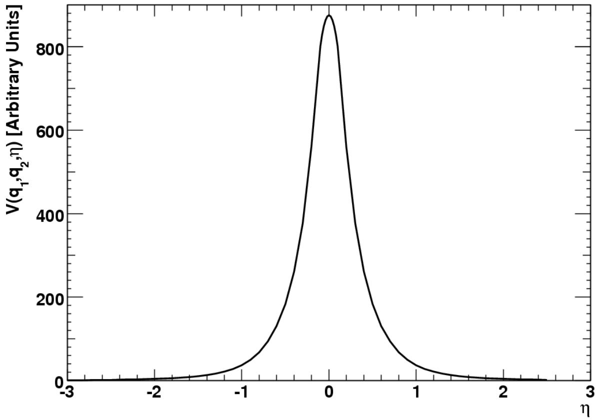
First Check. . .

Check of finite part



$1/N_c^2$ suppressed terms: 100% agreement.
Calculation under control.

Study of Rapidity Distribution within the $q\bar{q}$-Vertex



Antiquark: $(\mathbf{k_1}, \eta)$, quark: $(\mathbf{k_2}, -\eta)$

$q_1 = (20, 0)\mathrm{GeV}, q_2 = (15, 1)\mathrm{GeV}$

Summary and Conclusions

- Have constructed a very efficient method for obtaining the BFKL evolution as an approximation to multi-leg processes
  Also applicable to small-x studies etc.
- Have started the program to obtain fully exclusive final state information of the NLL BFKL Evolution necessary for energy and momentum conservation and thus full NLL accuracy
- Conclusion from the study of the exclusive NLL quark–anti-quark vertex:
  Exclusive information absolutely crucial for realistic phenomenology, since the $q\bar{q}$-vertex gets contributions from relatively large invariant masses of the $q\bar{q}$-pair.
  Cannot assign a single rapidity to the pair.
- `http://www.hep.phy.cam.ac.uk/~andersen/BFKL`