Multiparton Cross Sections at Hadron Colliders –

HELAC – Monte Carlo generator for hard multiparton processes

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Plan

- Matrix elements calculation using an iterative algorithm based on Dyson-Schwinger equations - without Feynman Diagrams !!!
- Monte Carlo summation instead of summation over all possible colour and helicity configurations
- Much improvement in computational efficiency $\sim 3^n$
- Checks & Results
- Summary & Outlook

In collaboration with: C. G. Papadopoulos (INP NCSR "$\Delta\eta\mu o\kappa\varrho\iota\tau o\varsigma$" Athens)

Process Calculation

Given process: $gg \rightarrow u\bar{u}u\bar{u}gggg$

With some cuts:

$$p_T = \sqrt{p_x^2 + p_y^2} \qquad \Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2} \qquad y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

we ask to compute the cross section for a given CMS energy

<u>Steps:</u>

- Find the number of Feynman graphs
- Write them down
- Compute them to get the amplitude
- Sum over all color and helicity configurations
- Square the amplitude
- Integrate over the phase space

Number of Graphs

P. Draggiotis, R. Kleiss Eur. Phys. J. C23, 701 (2002)

QCD with 1 fermion pairs

N=8	N=9	N=10	N=11	N=12	N=13
15495	231280	4016775	79603720	1773172275	43864374400

QCD with 3 (identical) fermion pairs

N=8	N=9	N=10	N=11	N=12	N=13
4362	59424	946050	17258640	355273170	8151299520

roughly grows like **n!**

Dyson-Schwinger Equations

- Starting point Dyson-Schwinger equations
- Give recursively n-point Green functions in terms of 1-, 2- ... (n-1)-point functions

$$\langle 0|T\phi(x_1)\dots\phi(x_n)|0\rangle = \frac{\int d\phi\ (\phi(x_1)\dots\phi(x_n))e^{iS}}{\int d\phi e^{iS}} \qquad S=\int d^4x\mathcal{L}$$

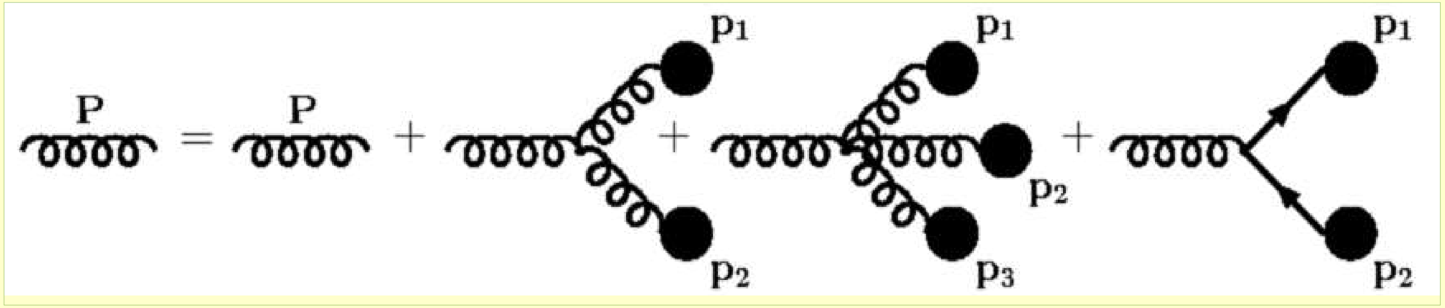
- Quantum equations of motion for Green's functions = DS equations

$$\langle 0|\left(\frac{\delta}{\delta\phi(x)}\int d^4x'\ \mathcal{L}\right)T\phi(x_1)\dots\phi(x_n)|0\rangle=\sum_{i=1}^{n}\langle 0|T\phi(x_1)\dots(i\delta(x-x_i))\dots\phi(x_n)|0\rangle$$

Recursion Equation

- Diagrammatically:

A. Kanaki, C. G. Papadopoulos Comp. Phys. Commun. 132, 306 (2000)
P. Draggiotis, R. Kleiss, C. G. Papadopoulos Eur. Phys. J. C24,447 (2002)



gluon self-interaction

fermion-antifermion vertex

- Subamplitude with an off-shell gluon of momentum P
- Contributions from 3-, 4-gluon vertex and quark-antiquark vertex
- Blobs denote subamplitudes with the same structure

Recursion Equation

- Gluon (suppressing colour):

$$A^{\mu}(P)=\sum_{i=1}^{n}\delta(P-p_i)A^{\mu}(p_i) \;+\; ig_s\sum_{p_1+p_2}\Pi^{\mu}_{\nu}\bar{\psi}(p_1)\gamma^{\nu}\psi(p_2)\sigma(p_1,p_2)$$

fermion-antifermion vertex

$$+\; ig_s\sum_{p_1+p_2}\Pi^{\mu}_{\varrho}V^{\varrho\nu\lambda}(P,p_1,p_2)A_{\nu}(p_1)A_{\lambda}(p_2)\sigma(p_1,p_2)$$

three-gluon vertex

$$(ig_s)^2\sum_{p_1+p_2+p_3}\Pi^{\mu}_{\sigma}G^{\sigma\nu\lambda\varrho}(P,p_1,p_2,p_3)A_{\nu}(p_1)A_{\lambda}(p_2)A_{\varrho}(p_3)\sigma(p_1,p_2+p_3)$$

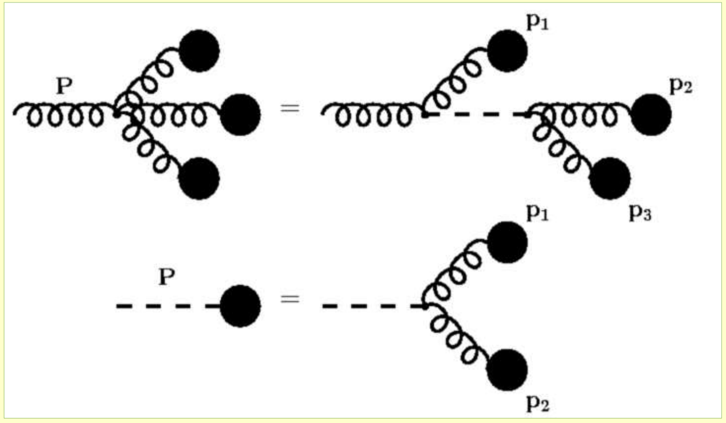
four-gluon vertex

with gluon propagator and sign function

$$\Pi^{\mu}_{\nu} = -\frac{ig^{\mu}_{\nu}}{P^2}$$

Auxiliary Field

- Auxiliary field $H_{\mu\nu}$ - reduce computational complexity down to 3^n !!!



- Elimination of the 4-gluon vertex

$$A^{\mu}(P) = ig_s \sum_{p_1+p_2} \Pi^{\mu}_{\sigma} X^{\sigma\nu\lambda\varrho} A_{\nu}(p_1) H_{\lambda\varrho}(p_2) \sigma(p_1,p_2)$$

- New equation for auxiliary field and new H-gluon-gluon vertex

$$H_{\mu\nu}(P) = ig_s \sum_{p_1+p_2} X^{\mu\nu\lambda\varrho} A_{\lambda}(p_1) A_{\varrho}(p_2) \sigma(p_1,p_2)$$

$$X^{\mu\nu\lambda\varrho} = g^{\mu\lambda} g^{\nu\varrho} - g^{\nu\lambda} g^{\mu\varrho}$$

Building Amplitude

- Off-shell fields – building blocks of any process
- Used iteratively, at each step two momenta are combined
- Initial conditions for the external particles:
  - Gluon: $A^{\mu}(p_i) = \epsilon^{\mu}_{\lambda}(p_i)$
  - Quarks: $\psi(p_i) = u_{\lambda}(p_i)$ $\quad \psi(p_i) = \bar{u}_{\lambda}(p_i)$
  - Antiquarks: $\bar{\psi}(p_i) = \bar{u}_{\lambda}(p_i)$ $\quad \bar{\psi}(p_i) = u_{\lambda}(p_i)$
- After n-1 steps:

$$\mathcal{A}(p_1, p_2, \ldots, p_n) = \begin{cases} \hat{A}^{\mu}(P_i) A_{\mu}(p_i) \\ \hat{\bar{\psi}}(P_i) \psi(p_i) \\ \bar{\psi}(p_i) \hat{\psi}(P_i) \end{cases}$$

Colour and Helicity

- Sum over color and helicity configurations

$8^{n_g} \times 3^{n_q} \times 3^{n_{\bar{q}}}$ color configurations

$2^{n_g} \times 2^{n_q} \times 2^{n_{\bar{q}}}$ helicity configurations

$n_g, n_q, n_{\bar{q}}$ number of gluons, quarks and antiquarks

- Many colour and helicity configurations lead to zero amplitude

> - Any representation can be used as long as we end up with the correct sums on the average

Color Treatment

P. Draggiotis, R. Kleiss, C. G. Papadopoulos Phys. Lett. B439, 157 (1998)

- Simplification for gluon fields

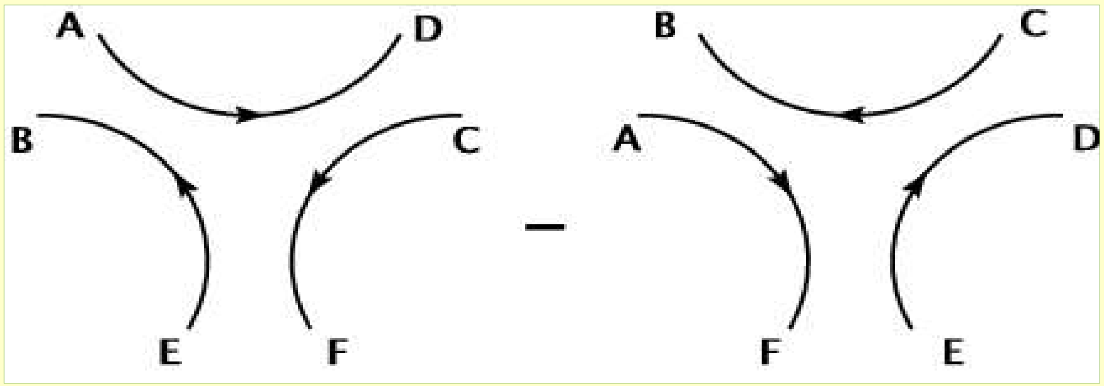
$$A_{AB} \equiv \sum_{a=1}^{8} t^a_{AB} A^a \qquad A, B = 1,2,3$$

- New objects traceless 3x3 matrices in colour space
- Diagonalization of the colour structure of 3-gluon vertex

$$f^{abc} t^a_{AB} t^b_{CD} t^c_{EF} = -\frac{i}{4}(\delta_{AD}\delta_{CF}\delta_{EB} - \delta_{AF}\delta_{CB}\delta_{ED})$$

Color Treatment

- 3-gluon vertex in new representation



- Shows the color flow in the real physical process
- Gluon represented by $q\bar{q}$ states in colour space
- Colour remains unchanged on an uninterrupted colour line

Color Treatment

- Quarks and anti-quarks already in this representation
- Additionally representation independent identity is used

$$\sum_{a=1}^{8} t^{a}_{ij} t^{a}_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{3}\delta_{ij}\delta_{kl}\right)$$

$i, j, k, l = 1,2,3$

- Recursion equations modified to reflect the new colour structure

Next step - make the computation of the colour part of an amplitude more efficient !

Monte Carlo Summation

- Produce a cross section with sufficient speed

Sum over colour $3^{n_q+n_{\bar{q}}}$ $\longrightarrow$ Monte Carlo techniques

- Choosing via MC particular colour and anti-colour assignment for quark, antiquark and gluon ($q\bar{q}$ pair)
- Necessary condition for non-vanishing colour assignment

$$N_C \,\mathrm{Type}\, I_i = N_{\bar{C}} \,\mathrm{Type}\, I_i \qquad i=1,2,3$$

$$3^{n_q+n_{\bar{q}}} \longrightarrow \sum_{i=0}^{n_q}\sum_{j=0}^{n_q-i}\sum_{k=0}^{n_q-i-j}\left(\frac{n_q!}{i!\,j!\,k!}\right)^2 \delta(i+j+k=n_q)$$

Number of Colour Configurations

C. G. Papadopoulos, M. Worek hep-ph/0512150

Process	N_{cc}^{ALL}	N_{cc}	N_{cc}/N_{cc}^{ALL}	N_{cc}^{F} (%)
$gg \to 2g$	6561	639	0.0974	59.1
$gg \to 3g$	59049	4653	0.0788	68.4
$gg \to 4g$	531441	35169	0.0662	77.4
$gg \to 5g$	4782969	272835	0.0570	85.0
$gg \to 6g$	43046721	2157759	0.0501	90.4
$gg \to 7g$	387420489	17319837	0.0447	94.0
$gg \to 8g$	3486784401	140668065	0.0403	96.4

- $N_{CC}^{ALL} = 3^{n_q + n_{\bar{q}}}$ - All possible colour configurations
- N_{CC}^{F} - Number of non vanishing colour configurations evaluated by MC inside N_{CC}

Helicity Treatment

- Summation over helicity configurations of the external partons

  $\longrightarrow$ MC integration over a phase variable

- Polarization vector for gluons

$$\epsilon^{\mu}_{\phi}(p)=e^{i\phi}\epsilon^{\mu}(p,+)+e^{-i\phi}\epsilon^{\mu}(p,-)$$

- Sum over helicities by integrating over ϕ

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi\, \epsilon^{\mu}_{\phi}(p)(\epsilon^{\nu}_{\phi}(p))^{*}=\sum_{\lambda=\pm}\epsilon^{\mu}(p,\lambda)(\epsilon^{\nu}(p,\lambda))^{*}$$

- ϕ - random number $\phi\in(0,2\pi)$

Cross Check

C. G. Papadopoulos, M. Worek hep-ph/0512150

- CTEQ6 PDF
- Fixed strong coupling constant

  $\alpha_s = 0.13$
- Cuts:

  $p_T > 60\,\text{GeV},\ |y| < 2.5,\ \Delta R > 1.0$
- CMS: $\sqrt{s} = 14\,\text{TeV}$
- 10^6 - MC points passing cuts

Process	$\sigma_{\rm EXACT} \pm \epsilon$ (nb)	ϵ (%)	$\sigma_{\rm MC} \pm \epsilon$ (nb)	ϵ (%)
$gg \to 2g$	$(0.46572 \pm 0.00258)\times10^4$	0.5	$(0.46849 \pm 0.00308)\times10^4$	0.6
$gg \to 3g$	$(0.15040 \pm 0.00159)\times10^3$	1.0	$(0.15127 \pm 0.00110)\times10^3$	0.7
$gg \to 4g$	$(0.11873 \pm 0.00224)\times10^2$	1.9	$(0.12116 \pm 0.00134)\times10^2$	1.1
$gg \to 5g$	$(0.10082 \pm 0.00198)\times10^1$	1.9	$(0.09719 \pm 0.00142)\times10^1$	1.5
$gg \to 6g$	$(0.74717 \pm 0.01490)\times10^{-1}$	2.0	$(0.76652 \pm 0.01862)\times10^{-1}$	2.4
$gg \to u\bar{u}$	$(0.36435 \pm 0.00199)\times10^2$	0.5	$(0.36619 \pm 0.00132)\times10^2$	0.4
$gg \to gu\bar{u}$	$(0.35768 \pm 0.00459)\times10^1$	1.3	$(0.35466 \pm 0.00291)\times10^1$	0.8
$gg \to 2gu\bar{u}$	$(0.49721 \pm 0.00758)\times10^0$	1.5	$(0.50053 \pm 0.00725)\times10^0$	1.4
$gg \to 3gu\bar{u}$	$(0.50598 \pm 0.01441)\times10^{-1}$	2.8	$(0.52908 \pm 0.01264)\times10^{-1}$	2.4
$gg \to 4gu\bar{u}$	$(0.51549 \pm 0.02017)\times10^{-2}$	3.9	$(0.51581 \pm 0.01245)\times10^{-2}$	2.4
$gg \to c\bar{c}c\bar{c}$	$(0.25190 \pm 0.00528)\times10^{-2}$	2.1	$(0.24903 \pm 0.00373)\times10^{-2}$	1.5
$gg \to gc\bar{c}c\bar{c}$	$(0.60196 \pm 0.01908)\times10^{-3}$	3.2	$(0.58817 \pm 0.00926)\times10^{-3}$	1.6
$gg \to 2gc\bar{c}c\bar{c}$	$(0.95682 \pm 0.03441)\times10^{-4}$	3.6	$(0.92212 \pm 0.02485)\times10^{-4}$	2.7

Timing

C. G. Papadopoulos, M. Worek hep-ph/0512150

- Comparison of the computational time for $|M|^2$ - exact summation with colour flow approach versus MC summation over colour configurations

Process	t^{CF}_{EXACT}/t_{MC}
$gg \to 2g$	0.7033
$gg \to 3g$	2.8914
$gg \to 4g$	8.2288
$gg \to 5g$	53.8389
$gg \to 6g$	158.9916

Results

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- Results for the total cross section for processes with gluons, $Z, W^{\pm}$ and quarks with up to 2 $q\bar{q}$ pairs for large number of external particles

Process	$\sigma_{MC} \pm \varepsilon$ (nb)	ε (%)
$gg \to 7g$	$(0.53185 \pm 0.01149)\times 10^{-2}$	2.1
$gg \to 8g$	$(0.33330 \pm 0.00804)\times 10^{-3}$	2.4
$gg \to 9g$	$(0.13875 \pm 0.00430)\times 10^{-4}$	3.1
$gg \to 5gu\bar{u}$	$(0.38044 \pm 0.01096)\times 10^{-3}$	2.8
$gg \to 3gc\bar{c}c\bar{c}$	$(0.95109 \pm 0.02456)\times 10^{-5}$	2.6
$gg \to 4gc\bar{c}c\bar{c}$	$(0.81400 \pm 0.02583)\times 10^{-6}$	3.2
$gg \to Zu\bar{u}gg$	$(0.18948 \pm 0.00344)\times 10^{-3}$	1.8
$gg \to W^+\bar{u}dgg$	$(0.62704 \pm 0.01458)\times 10^{-3}$	2.3
$gg \to ZZu\bar{u}gg$	$(0.16217 \pm 0.00420)\times 10^{-6}$	2.6
$gg \to W^+W^-u\bar{u}gg$	$(0.27526 \pm 0.00752)\times 10^{-5}$	2.7
$d\bar{d} \to Zu\bar{u}gg$	$(0.38811 \pm 0.00569)\times 10^{-5}$	1.5
$d\bar{d} \to W^+\bar{c}sgg$	$(0.18765 \pm 0.00453)\times 10^{-5}$	2.4
$d\bar{d} \to ZZgggg$	$(0.99763 \pm 0.02976)\times 10^{-7}$	2.9
$d\bar{d} \to W^+W^-gggg$	$(0.52355 \pm 0.01509)\times 10^{-6}$	2.9

Results

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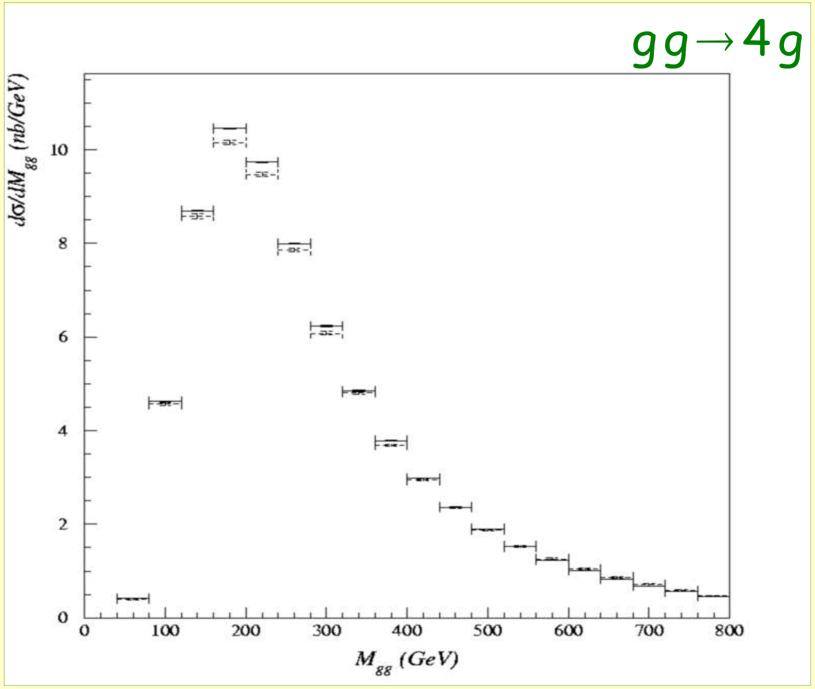
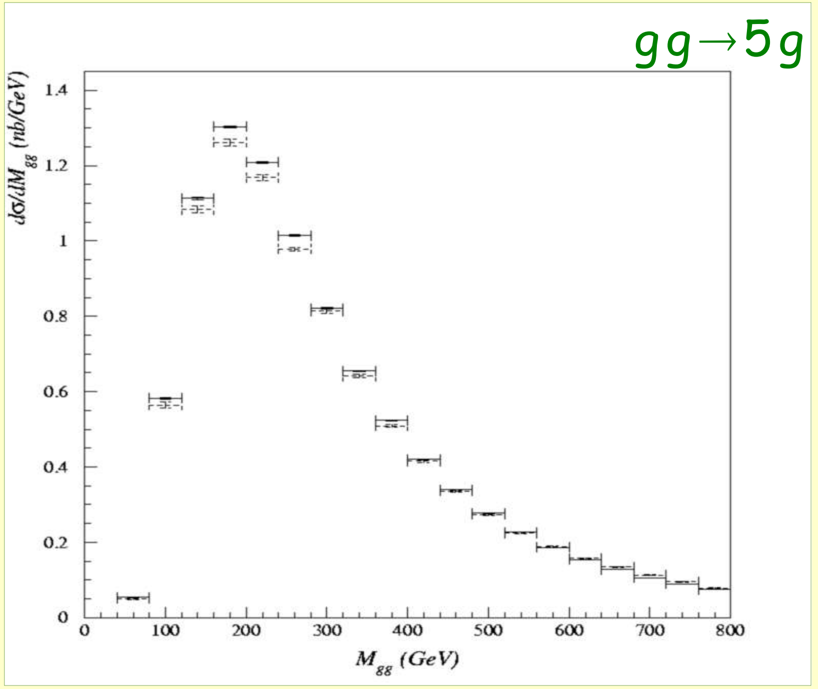
- Invariant mass distribution of 2 gluons





Results

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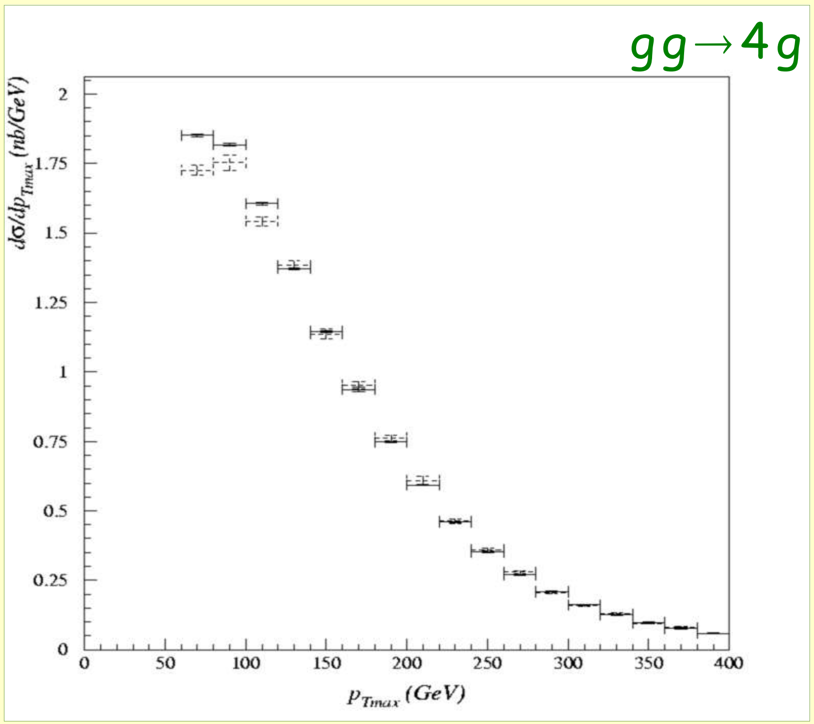
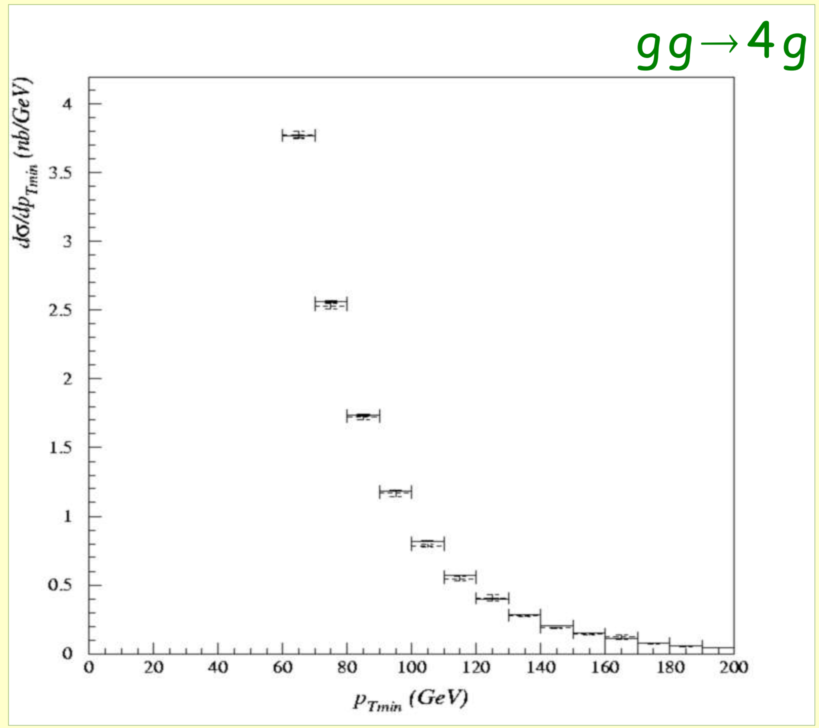
- Transverse momentum distribution of the most (left) and the less (right) energetic gluon





Results

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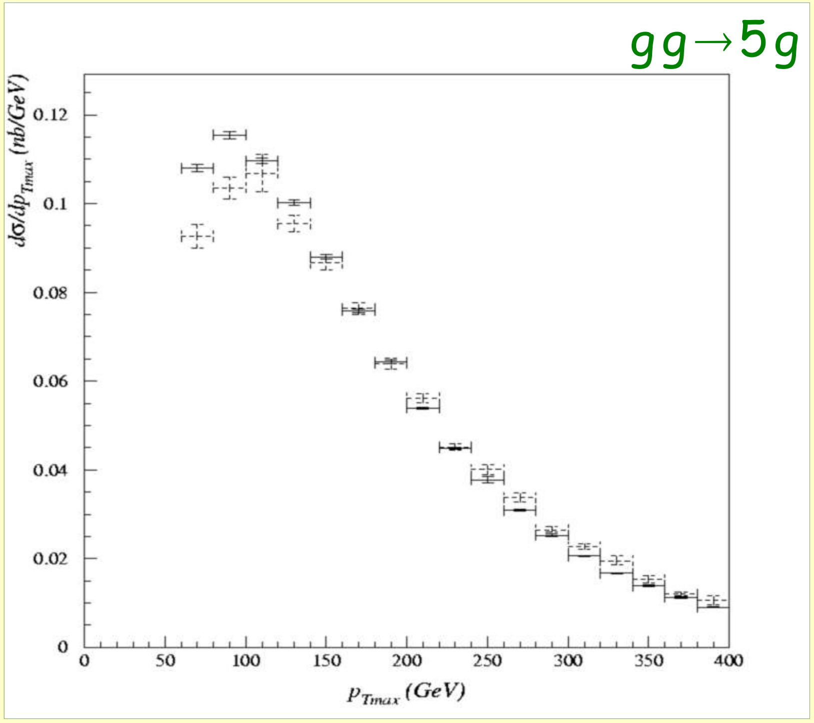

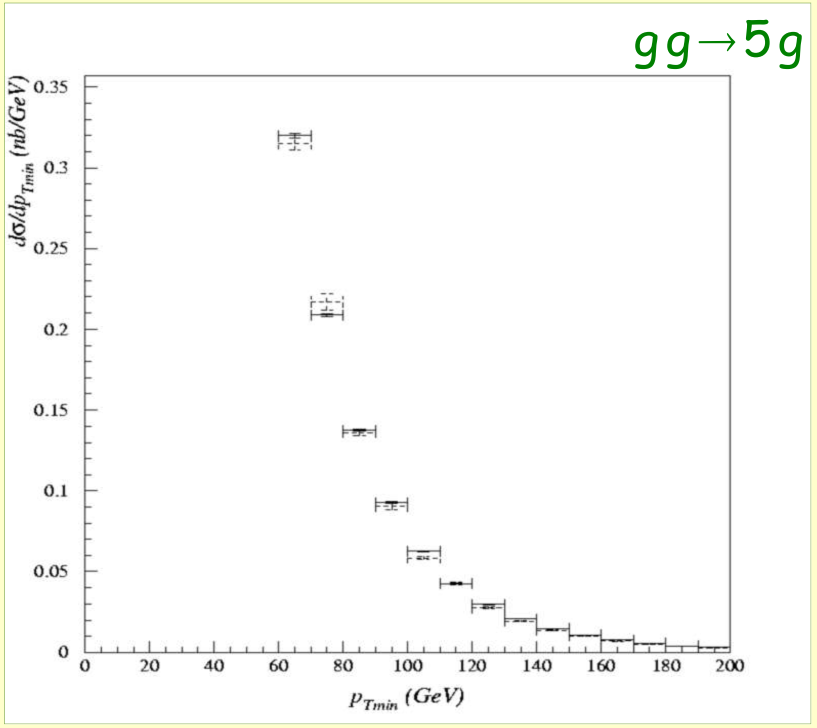

Results

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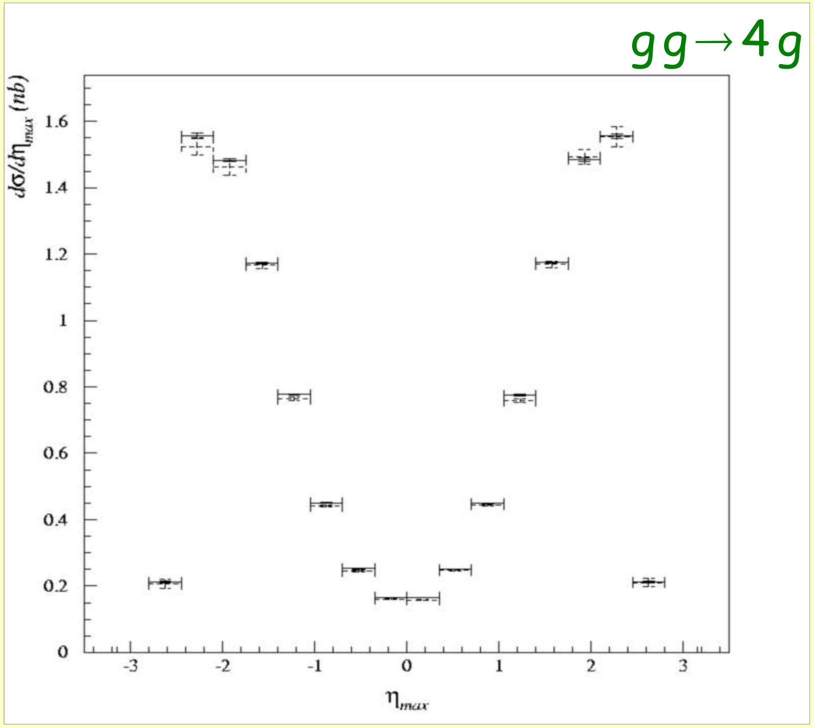
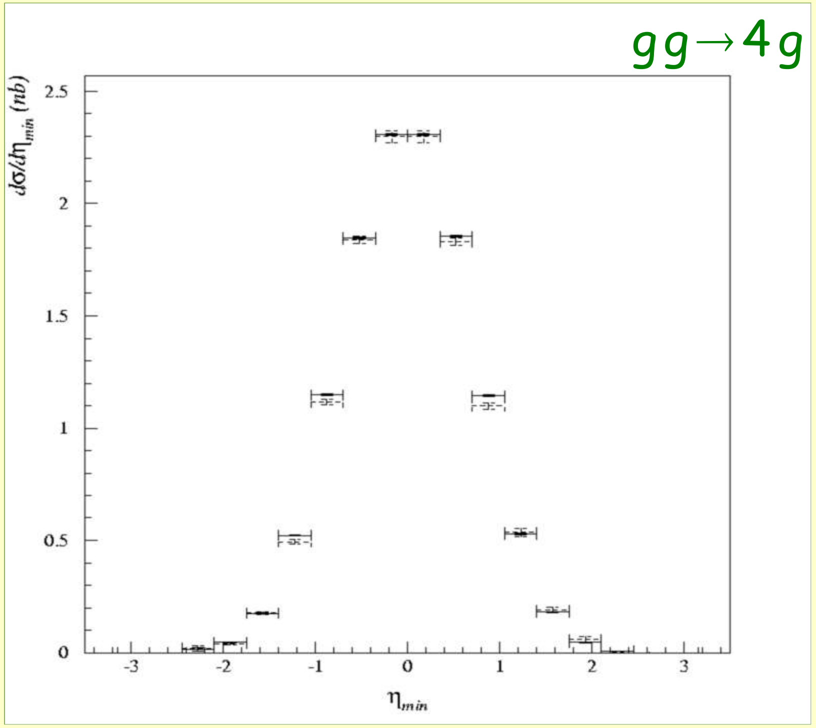
- Rapidity distribution of the most (left) and the less (right) energetic gluon





Results

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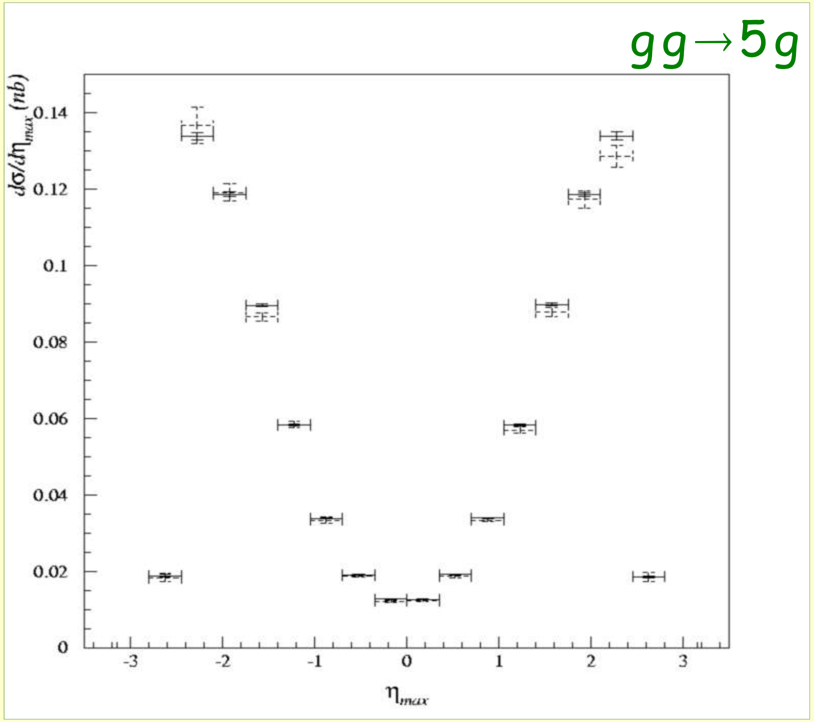
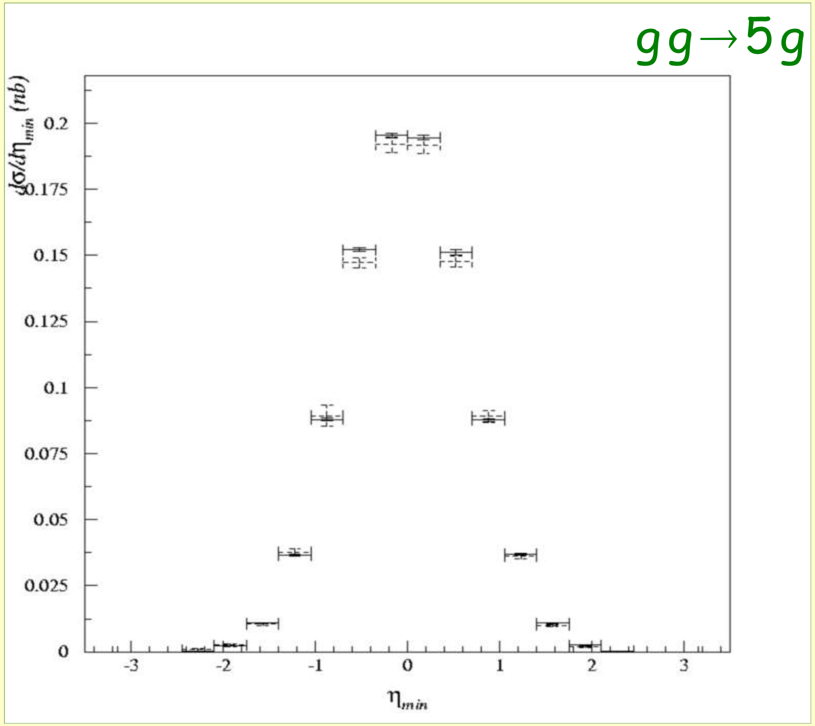
Rapidity distribution of the most (left) and the less (right) energetic gluon





Status Report

- HELAC
  - Exact LO matrix element and cross section calculation for any process in the Standard Model (EW/QCD interference) and for any number of external particles
  - Computational efficiency 3^n
  - Free from the task of calculating Feynman diagrams
  - Parton level event generation (weighted/unweighted)
  - Mass effects and PDF set
  - Code available in F95
  - Selection of kinematical cuts

Status Report

- Evolution of the parton level final state through parton shower and hadronization phases done by PYTHIA (new p_T- ordered parton shower)

  → Les Houches Accord interfacing routines

- Unweight events, generate colour flow information, write events to file

  → For particular colour configuration coresponding colour flows are found

  F. Maltoni et al. Phys. Rev. D67, 014026 (2003)

  → Colour flow assigned to each event on the basis of the relative probabilities of all possible colour flows

Extra Slides

Systematic Approach

- Systematic approach to build the amplitude from the initial momenta (binary representation)
- Process with n external particles with momenta p_i^μ
- Define momentum

$$P^\mu = \sum_{i=1}^{n} m_i p_i^\mu \qquad m_i = 0,1$$

- Binary vector $\vec{m} = (m_1, \ldots, m_n)$ can be uniquely represented by the integer

$$m = \sum_{i=1}^{n} 2^{i-1} m_i \qquad 0 \leqslant m \leqslant 2^n - 1$$

Systematic Approach

- All momenta replaced by the corresponding integers

$$J^{\mu}(P) \to J^{\mu}(m)$$

- Ordering of integers in binary representation by levels $l = \sum_{i=1}^{n} m_i$
- All external momenta are of level 1
- Amplitude corresponds to the unique level n number $2^n - 1$

$$\mathcal{A} = J(1) \cdot J(2^n - 2)$$

Ordering dictates path of the computation
– from level-1 to level-2 and so on up to the amplitude

Cost Function

- How many operations are needed to compute the n-point amplitude
- Basic steps:
  - How many subamplitudes at each level k
  - How many ways of splitting k to two numbers of levels k_1, k_2
  - Sum over all levels

$$O(n) = \sum_{k=1}^{n-1} \binom{n}{k} \sum_{l=1}^{k-1} \binom{k}{l} = \sum_{k=1}^{n-1} \binom{n}{k} (2^k - 2) = 3^n - 3 \cdot 2^n + 3$$

- Asymptotically number of operation grows like 3^n instead of $n!$