

# New tests for the $k_T$ -factorization: beauty quark production at HERA

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## O U T L I N E

1. Motivation
2. Ingredients of the  $k_T$ -factorization approach
3. Numerical results
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# 1. Motivation

The *beauty* production at H.E. is the subject of intensive studies from theor. and exp. points of view.

The first measured of  $b$ -quark c.s. at HERA was significantly higher than the NLO QCD predictions:

C. Adloff et al. (H1 Coll.), PL B467 (1999) 156;  
Erratum: B518 (2001) 331.

Similar observations were made in  $pp$ -collisions at Tevatron and in  $\gamma\gamma$  interactions at LEP2.

In  $\gamma\gamma$  case the NLO QCD predictions are more than three standard deviations below the exp. data.

At Tevatron the description of exp. data in the pQCD was achieved by adopting the non-perturbative fragmentation function of the  $b$ -quark into the  $B$ -meson:

M. Cacciari, N. Nason, PRL 89 (2002) 122003;  
M. Cacciari et al., JHEP 0407 (2004) 033.

Recently **H1** and **ZEUS** Collaborations have reported the exp. data on **b-photoproduction** (inclusive and associated with hadron jets):

**ZEUS Coll., EPJ C18 (2001) 625,  
PR D70 (2004) 012008;  
H1 Coll., EPJ C41 (2005) 453,**

and the **beauty** production in **DIS**:

**H1 Coll., hep-ex/0411046, 0507081;  
ZEUS Coll., PL B599 (2004) 173.**

First measurement of SF  $F_b^2$  at small  $x$  and high  $Q^2$  have been presented also.

All HERA **b-photoproduction** data are in a reasonable agreement with the NLO QCD predictions or somewhat higher except the muon and/or associated jet transverse momenta at low values.

The large excess of the first measurements over NLO QCD, reported by the H1 Coll., is not confirmed.

For the **beauty** production in **DIS** the NLO QCD predictions at low values of  $Q^2, x$ , muon tr. momentum and high values of jet tr. energy and muon pseudo-rapidity is about two standard deviation below the data.

We use the **semi-hard (SHA)**

L. Gribov, E. Levin, M. Ryskin (1983),  
Phys. Rep. 100 (1983) 1;

E. Levin, M. Ryskin, Y. Shabelski, A. Shuvaev,  
Sov. J. Nucl. Phys. 53 (1991) 657,

or the  **$k_T$ -factorization** QCD approach

S. Catani, M. Ciafaloni, F. Hautmann,  
Nucl. Phys. B366 (1991) 135;

J. Collins, R. Ellis, Nucl. Phys. B360 (1991) 3,

since the beauty production at HERA is dominated by the photon-gluon or gluon-gluon fusion (direct and resolved photon contributions, respectively) and therefore sensitive to the gluon densities in a proton and in a photon at small values of  $x$ .

The  $k_T$ -factorization approach is based on the Balitsky-Fadin-Kuraev-Lipatov (BFKL)

E.A. Kuraev, L.N. Lipatov, V.S. Fadin,  
Sov. Phys. JETP 44 (1976) 443, 45 (1977) 199;

Y.Y. Balitskii, L.N. Lipatov,  
Sov. J. Nucl. Phys. 28 (1978) 822,

or Ciafaloni-Catani-Fiorani-Marchesini (CCFM)

M. Ciafaloni, Nucl. Phys. B296 (1988) 49;

S. Catani, F. Fiorani, G. Marchesini,  
Nucl. Phys. B336 (1990) 18;

G. Marchesini, Nucl. Phys. B445 (1995) 49,

gluon evolution equations which sum up the large logarithmic terms proportional to  $\ln(1/x)$  or  $\ln(1/(1-x))$  in the LLA.

## 2. Ingredients of the $k_T$ -factorization (SHA)

- The basic dynamical quantity of the  $k_T$ -factorization approach is the unintegrated ( $\mathbf{k}_T$ -dependent) gluon distribution (UGD)  $\mathcal{A}(x, \mathbf{k}_T^2, \mu^2)$  obtained from the analytical or numerical solution of the BFKL or CCFM ev. eqs..

To calculate the cross sections of any physical process the UGD  $\mathcal{A}(x, \mathbf{k}_T^2, \mu^2)$  has to be convoluted with the relevant partonic cross section  $\hat{\sigma}$ :

$$\sigma = \int \frac{dz}{z} d\mathbf{k}_T^2 \hat{\sigma}(x/z, \mathbf{k}_T^2, \mu^2) \mathcal{A}(x, \mathbf{k}_T^2, \mu^2).$$

- The partonic cross section  $\hat{\sigma}$  has to be taken **off mass shell** ( $\mathbf{k}_T$ -dependent).
- It also assumes a modification of their **polarization density matrix**. It has to be taken in **BFKL** form:

$$\sum \epsilon^\mu \epsilon^{*\nu} = \frac{k_T^\mu k_T^\nu}{\mathbf{k}_T^2}.$$

The direct photon contribution to the differential cross section of  $\gamma p \rightarrow b\bar{b} + X$  process is given by

$$\frac{d\sigma^{(\text{dir})}(\gamma p \rightarrow b\bar{b} + X)}{dy_b d\mathbf{p}_{bT}^2} = \int \frac{|\bar{\mathcal{M}}|^2(\gamma g^* \rightarrow b\bar{b})}{16\pi(x_2 s)^2(1 - \alpha_1)} \times \\ \times \mathcal{A}(x_2, \mathbf{k}_{2T}^2, \mu^2) d\mathbf{k}_{2T}^2 \frac{d\phi_2}{2\pi} \frac{d\phi_b}{2\pi},$$

The formula for the resolved photon contribution has the form

$$\frac{d\sigma^{(\text{res})}(\gamma p \rightarrow b\bar{b} + X)}{dy_b d\mathbf{p}_{bT}^2} = \int \frac{|\bar{\mathcal{M}}|^2(g^* g^* \rightarrow b\bar{b})}{16\pi(x_1 x_2 s)^2} \times \\ \times \mathcal{A}_\gamma(x_1, \mathbf{k}_{1T}^2, \mu^2) \mathcal{A}(x_2, \mathbf{k}_{2T}^2, \mu^2) d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 dy_{\bar{b}} \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{d\phi_b}{2\pi}.$$

It is important that squared off-shell matrix element  $|\bar{\mathcal{M}}|^2(g^* g^* \rightarrow b\bar{b})$  depends on the both transverse momenta  $\mathbf{k}_{1T}^2$  and  $\mathbf{k}_{2T}^2$ . The analytic expressions for the  $|\bar{\mathcal{M}}|^2(\gamma g^* \rightarrow b\bar{b})$  and  $|\bar{\mathcal{M}}|^2(g^* g^* \rightarrow b\bar{b})$  have been derived in our previous papers.

If we average these expression over  $\mathbf{k}_{1T}$  and  $\mathbf{k}_{2T}$  and take the limit  $\mathbf{k}_{1T}^2 \rightarrow 0$  and  $\mathbf{k}_{2T}^2 \rightarrow 0$ , then we obtain well-known formulas corresponding to the standard LO QCD results.

## Unintegrated gluon distributions

- The **KMS** parametrization

J. Kwiecinski, A. Martin, A. Stasto,  
Phys. Rev. D56 (1997) 3991

was obtained from a unified **BFKL** and  
**DGLAP** description of  $F_2$  data

and includes the so called consistency constraint

J. Kwiecinski, A. Martin, A. Sutton,  
Phys. Rev. D52 (1995) 1445,  
Z.Phys. C71 (1996) 585.

The consistency constraint introduces a large correction to the **LO BFKL** equation: about **70%** of the full **NLO corrections** to the BFKL exponent  $\Delta$  are effectively included in this constraint ( $\Delta \approx 0.3$ , instead BFKL LO value 0.53)

J. Kwiecinski, A. Martin, J. Outhwaite,  
Eur. Phys. J. C9 (2001) 611.



- The **KMR** parametrization.

In Kimber-Martin-Ryskin approach the UPD is constructed from the known conventional PDF. The  $\mu$  dependence of the UPD  $\mathcal{A}(x, \mathbf{k}_T^2, \mu^2)$  enters at the last step of evolution. The parameter  $\mu$  plays a dual role: it acts as the factorization scale and also controls the angular ordering of radiated partons. Therefore single scale ev. eq. (DGLAP or unified DGLAP-BFKL in the KMS form) can be used up to last step:

M. Kimber, A. Martin, M. Ryskin,  
Phys. Rev. D63 (2001) 114027.

This result in a form similar to the differential form of the CCFM equation (with angular ordering) with the splitting function  $\mathbf{P}(\mathbf{z})$  is taken as the DGLAP or unified DGLAP-BFKL expression.

We use the last version of KMR UPD obtained from DGLAP eqs.:

G. Watt, A.D. Martin, M.G. Ryskin,  
Eur. Phys. C31 (2003) 73.

In this case ( $a(x, \mu^2) = xG$  or  $a(x, \mu^2) = xq$ ):

- The normalization condition

$$a(x, \mu^2) = \int_0^{\mu^2} f_a(x, \mathbf{k}_T^2, \mu^2) d\mathbf{k}_T^2,$$

is satisfied, if

$$f_a(x, \mathbf{k}_T^2, \mu^2)|_{\mathbf{k}_T^2 < \mu_0^2} = a(x, \mu_0^2) T_a(\mu_0^2, \mu^2),$$

where  $T_a(\mu_0^2, \mu^2)$  are the quark and gluon Sudakov form factors.

- The UPD  $f_a(x, \mathbf{k}_T^2, \mu^2)$  is defined in all  $\mathbf{k}_T^2$  region.

- The **J2003** parametrization.

The CCFM ev. eqs. have been solved numerically using a **Monte-Carlo** method:

H. Jung, hep-ph/9908497,

H. Jung, G. Salam, EPJ C19 (2001) 359.

According to the CCFM ev. eqs., the emission of gluons during the initial cascade is only allowed in an angular-ordered region of phase space.

The maximum allowed angle  $\Xi$  for any gluon emission sets the scale  $\mu$  and is defined by the hard scattering quark box.

The free parameters of the starting gluon distribution were fitted to the SF  $F_2(x, Q^2)$  (as in the KMS UPD) in the range  $x < 10^{-2}$  and  $Q^2 > 5 \text{ GeV}^2$ .

Here last versions of the UPD J2003 are used:

H. Jung, Mod. Phys. Lett A19 (2004) 1.

An advantage of the CCFM evolution, compared to the BFKL evolution, is that it is fairly well suited for implementation into an event generator program ( CASCADE and/or LDC), which makes quantitative comparison with data feasible also for non-inclusive observables.

## Numerical results

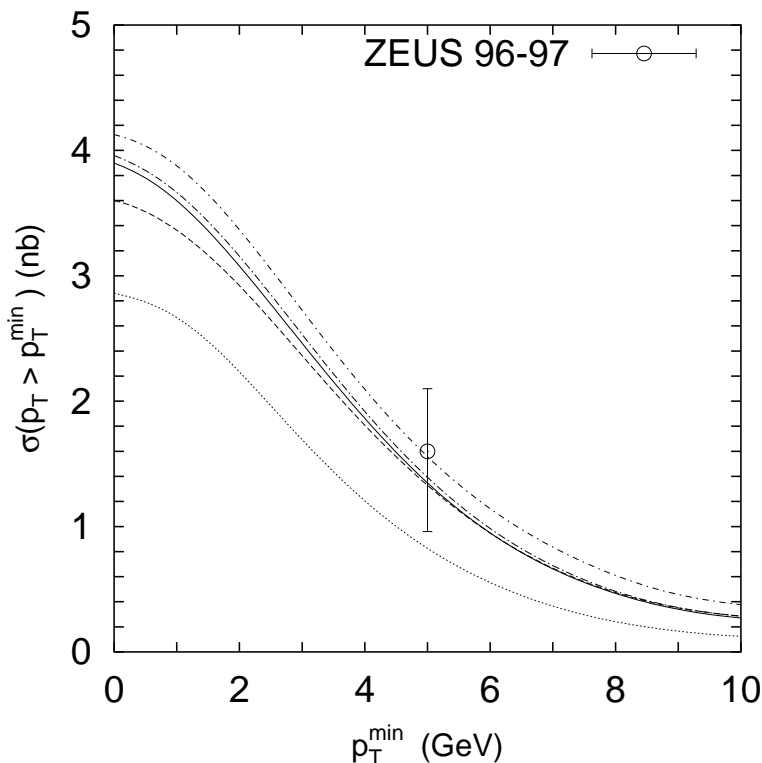
- Inclusive beauty photoproduction.

The parameters:

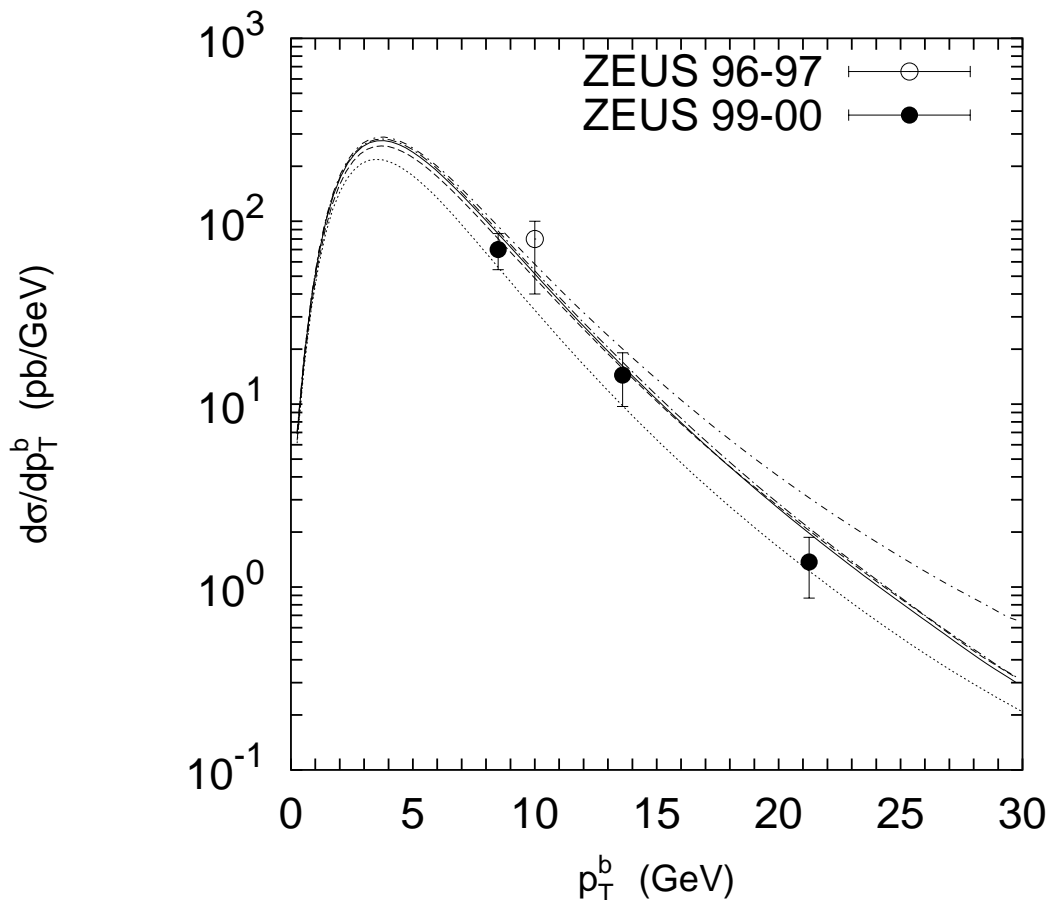
$$\mu_R = \mu_F = \mu = \sqrt{\mu_b^2 + \langle p_T^2 \rangle}, \mu_b = 4.75$$

**GeV**, LO formula for  $\alpha_s(\mu^2)$  with  $n_f = 4$

at  $\Lambda_{QCD} = 200 \text{ MeV}$ , so  $\alpha_s(m_Z^2) = 0.1232$ .



**Figure 1:** *The inclusive beauty c.s. as a function of  $p_T^{\min}$  at  $|\eta^b| < 2$ ,  $Q^2 < 1 \text{ GeV}^2$  and  $0.2 < y < 0.8$ . The short dash-dotted, dash-dotted, solid, dashed and dotted curves correspond to the *KMS*, *J2003 set 1-3*, *KMR u.p.d.*.*



**Figure 2:** *The differential c.s.  $d\sigma/dp_T^b$  of inclusive beauty photoproduction at  $|\eta^b| < 2$ ,  $Q^2 < 1 \text{ GeV}^2$  and  $0.2 < y < 0.8$ . All curves are the same as in Fig. 1.*

- J2003 and KMS results are close to each other
- KMR results are below the ZEUS exp. data and very similar to the NLO QCD predictions ( $\ln(1/x)$  effects are not included into the KMR formalism).

Source	$\sigma(ep \rightarrow e'b\bar{b} + X)$ [nb]
H1 measurement [1]	$14.8 \pm 1.3$ (stat.) $^{+3.3}_{-2.8}$ (sys.)
Cascade [26]	$5.2^{+1.1}_{-0.9}$
J2003 set 1	<b>6.78</b>
J2003 set 2	<b>6.62</b>
J2003 set 3	<b>7.16</b>
KMR	<b>3.91</b>
KMS	<b>7.57</b>

**Table 1:** *The total cross section of the inclusive beauty photoproduction in ep-collisions at  $Q^2 < 1 \text{ GeV}^2$ .*

The earlier H1 [1] data exceed our theor. estimations by a factor about 2. However, recent the ZEUS and H1 analysis

PR D70 (2004) 012008;

H1 Coll., EPJ C41 (2005) 453,

does not confirm the H1 results [1].

The c. s. for muon from  $b$  decays in dijet photoproduction events was found by H1 to be significantly lower than one in [1].

We can expect that the inclusive  $b$ -quark c. s. (which can be obtained after extrapolation of dijet and muon c. s. to the full phase space) will be reduced and agreement with our predictions will be significantly improved.

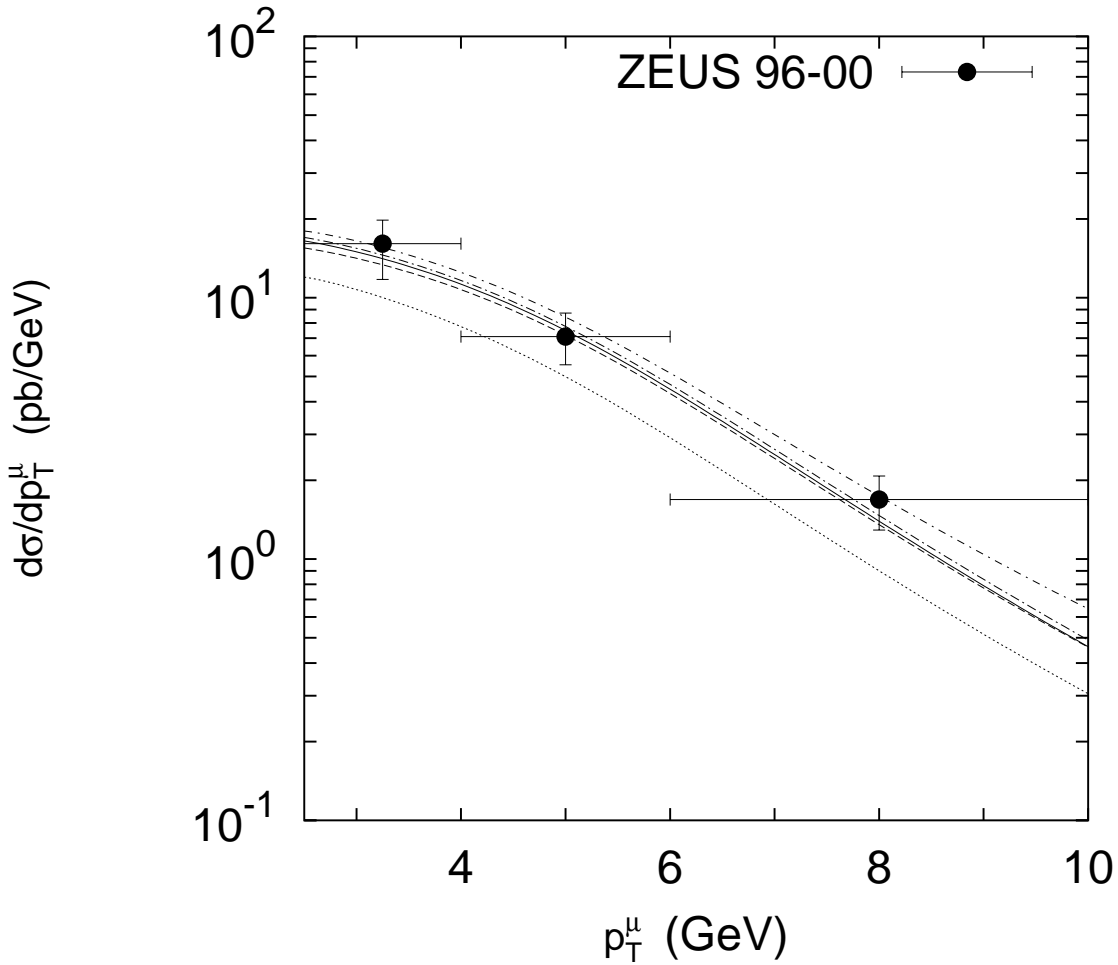
- So the c. s. of inclusive beauty photoproduction calculated in the  $k_T$ -factorization approach (supplemented with the CCFM or unified BFKL-DGLAP evolutions) are larger by 30 – 40% than ones calculated at NLO level of collinear QCD.
- Our results for the total and differential c. s. are in a better agreement with the H1 and ZEUS data than the NLO QCD predictions.
- The individual contributions from the photon-gluon and gluon-gluon fusion to the inclusive  $b$ -quark c. s. in the  $k_T$ -factorization approach is about 85 and 15%, respectively.

- Dijet associated **b**-photoproduction.

Transition  $b \rightarrow B$ : Peterson F.F. with

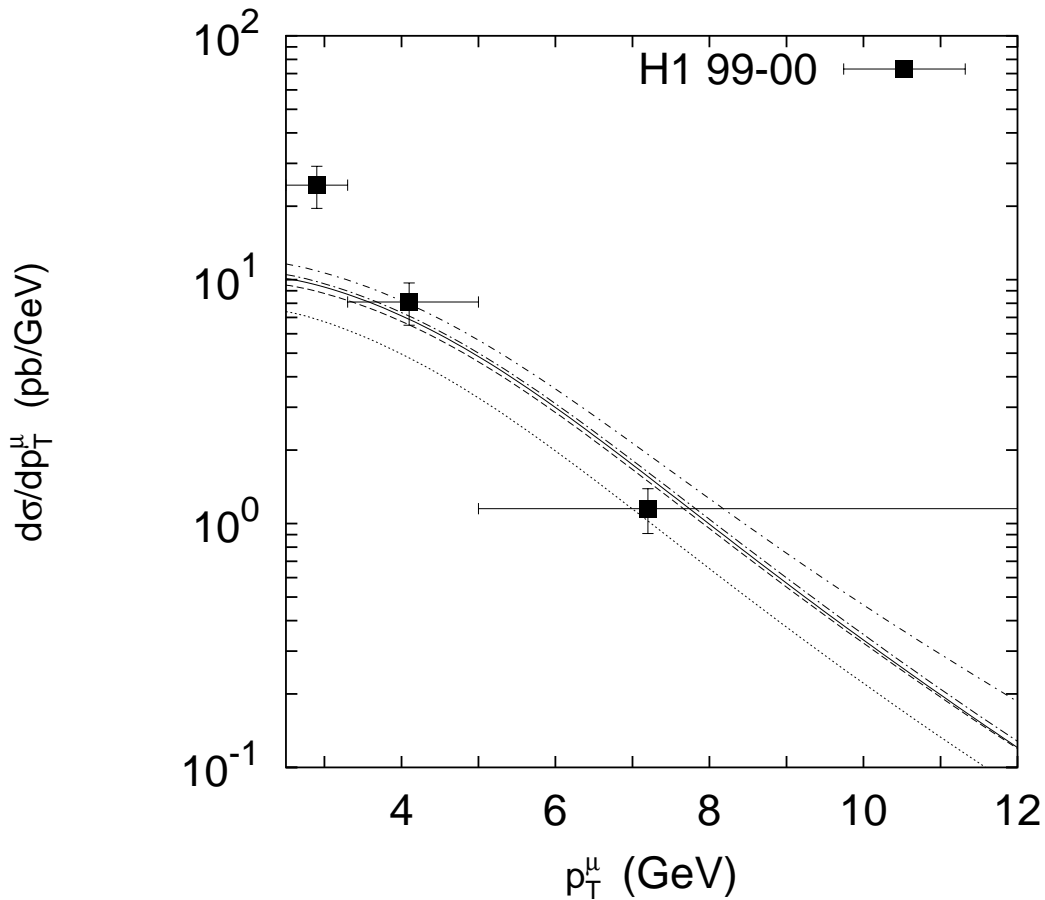
$\epsilon_b = 0.0035$ ,  $f_b(b \rightarrow \mu) = 10.8\%$ .

Transition  $B \rightarrow \mu$  - EW theory.



**Figure 3:** The differential c.s.  $d\sigma/dp_T^\mu$  of dijets with associated muon from  $b$ -decay at  $-1.6 < \eta^\mu < 2.3$ ,  $Q^2 < 1 \text{ GeV}^2$ ,  $0.2 < y < 0.8$ ,  $p_T^{jet1} > 7$ ,  $p_T^{jet2} > 6 \text{ GeV}$  and  $|\eta^{jet}| < 2.5$ . All curves are the same as in Fig. 1.

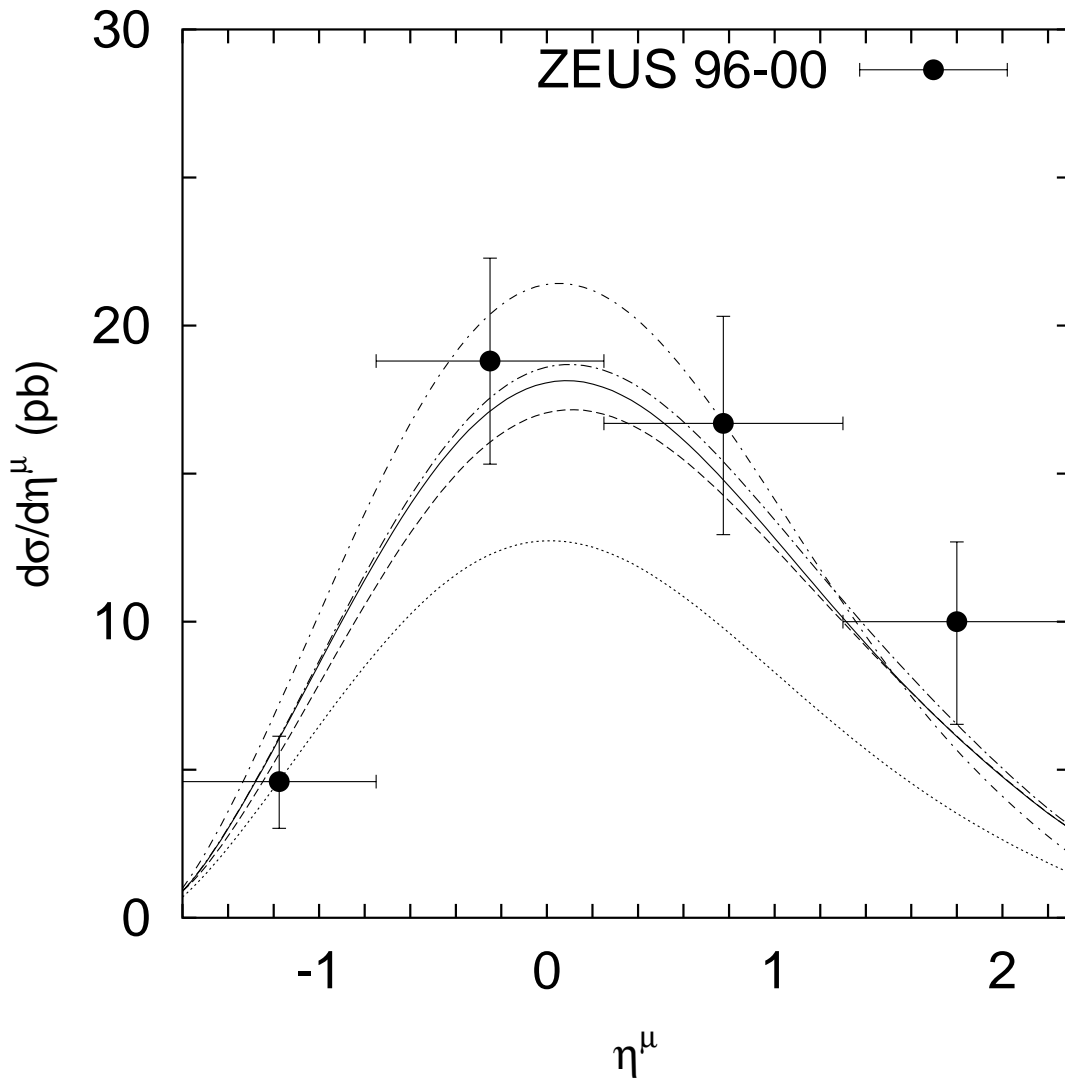




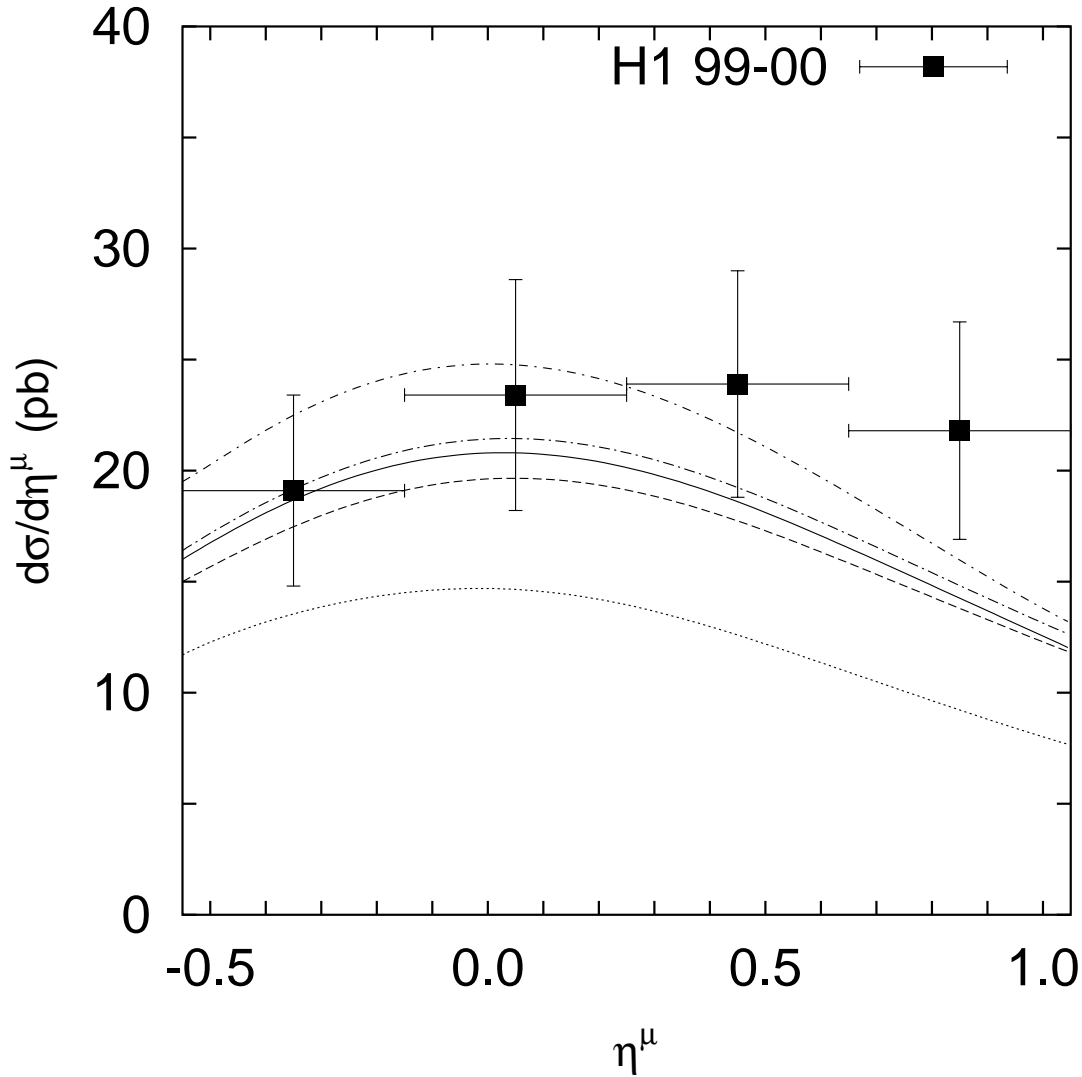
**Figure 4:** *The differential c.s.  $d\sigma/dp_T^\mu$  of dijets with associated muon from  $b$ -decay at  $-0.55 < \eta^\mu < 2.3, Q^2 < 1 \text{ GeV}^2, 0.2 < y < 0.8, p_T^{jet_1} > 7, p_T^{jet_2} > 6 \text{ GeV}$  and  $|\eta^{jet}| < 2.5$ . All curves are the same as in Fig. 1.*

- J2003 and KMS results agree well with the exp.data (except the low  $p_T^\mu$ ).

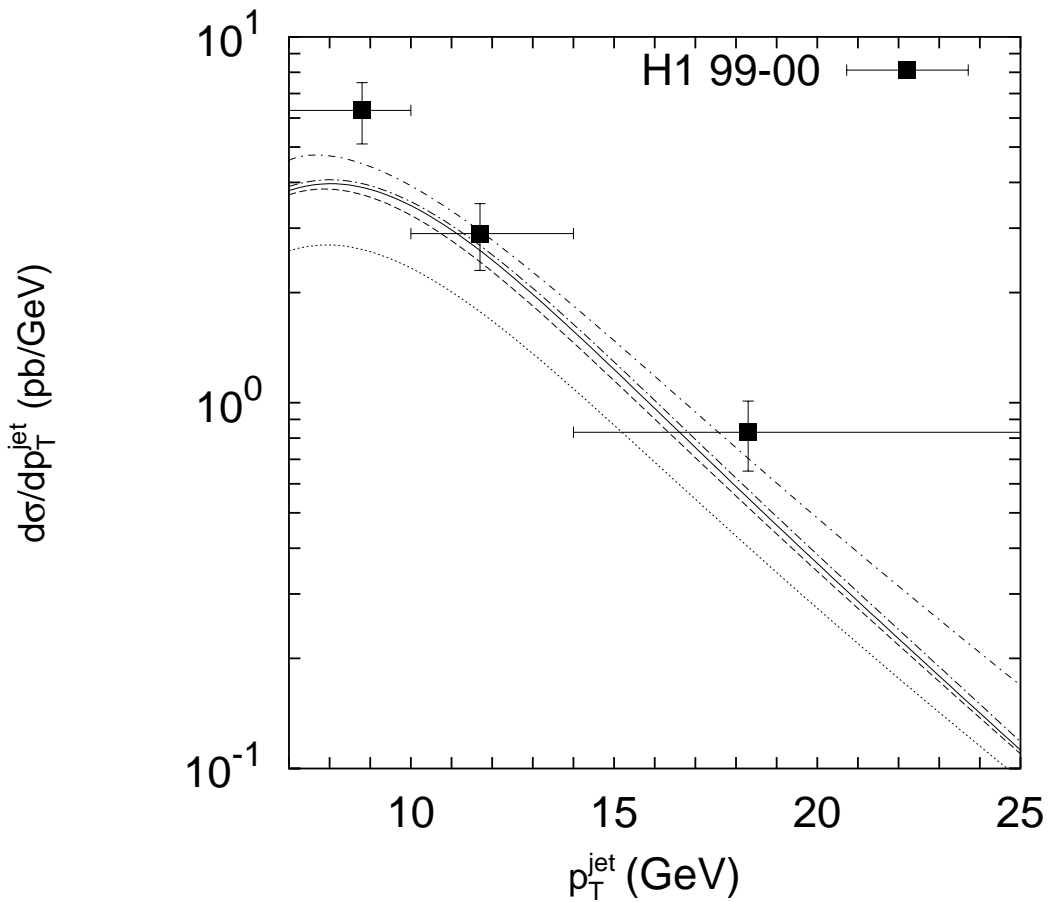
But H1 and ZEUS exp. data are very different in this region: H1  $p_T^\mu$  spectrum falls steeply.



**Figure 5:** *The differential c.s.  $d\sigma/d\eta^\mu$  of dijets with associated muon from  $b$ -decay at  $p_T^\mu > 2.5$  GeV,  $Q^2 < 1$  GeV<sup>2</sup>,  $0.2 < y < 0.8$ ,  $p_T^{jet_1} > 7$ ,  $p_T^{jet_2} > 6$  GeV and  $|\eta^{jet}| < 2.5$ .*



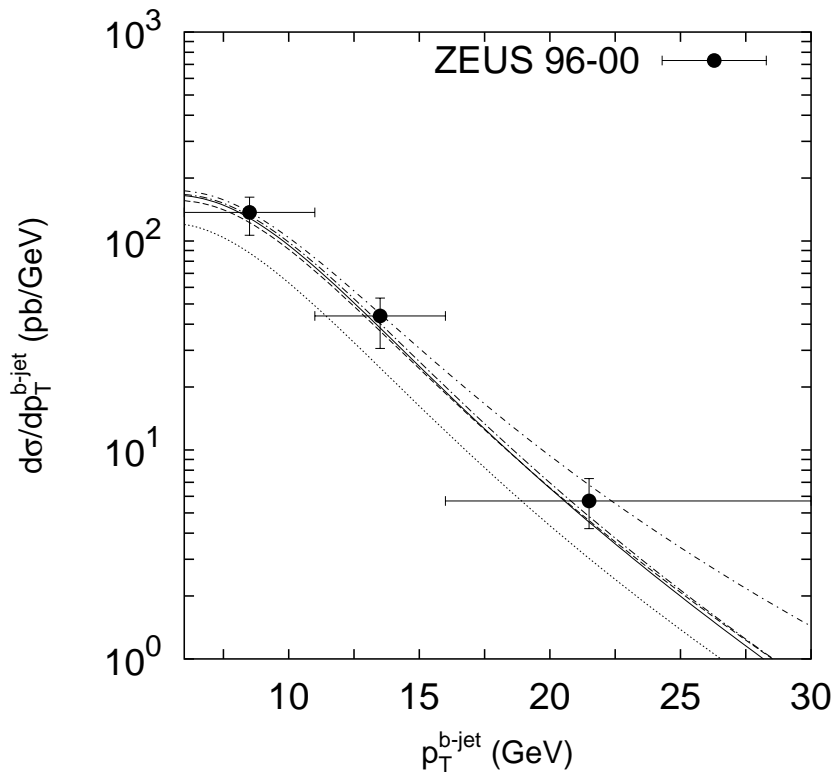
**Figure 6:** *The differential c.s.  $d\sigma/d\eta^\mu$  of dijets with associated muon from  $b$ -decay at  $p_T^\mu > 2.5$  GeV,  $Q^2 < 1$  GeV<sup>2</sup>,  $0.2 < y < 0.8$ ,  $p_T^{jet_1} > 7$ ,  $p_T^{jet_2} > 6$  GeV and  $|\eta^{jet}| < 2.5$ .*



**Figure 7:** The leading jet tr. mom. distr.  $d\sigma/dp_T^{jet}$  of dijets with associated muon from  $b$ -decay at  $-0.55 < \eta^\mu < 1.1, Q^2 < 1 \text{ GeV}^2, 0.2 < y < 0.8, p_T^{jet_1} > 7, p_T^{jet_2} > 6 \text{ GeV}$  and  $|\eta^{jet}| < 2.5$ .

- Our results are lower than the H1 data in the lowest momentum bin by factor of 2.5.

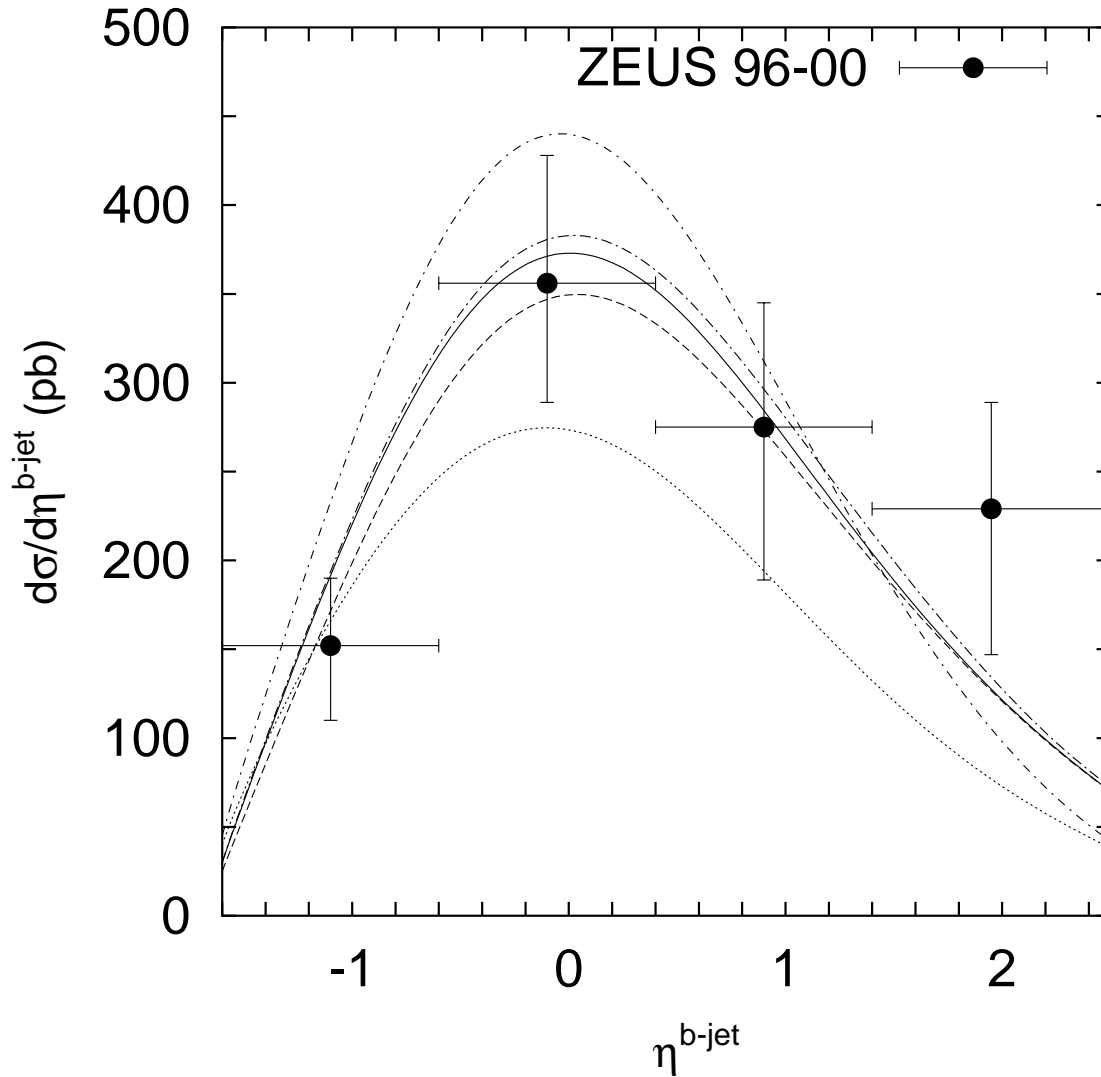
There is some inconsistency between the H1 and ZEUS data (?).



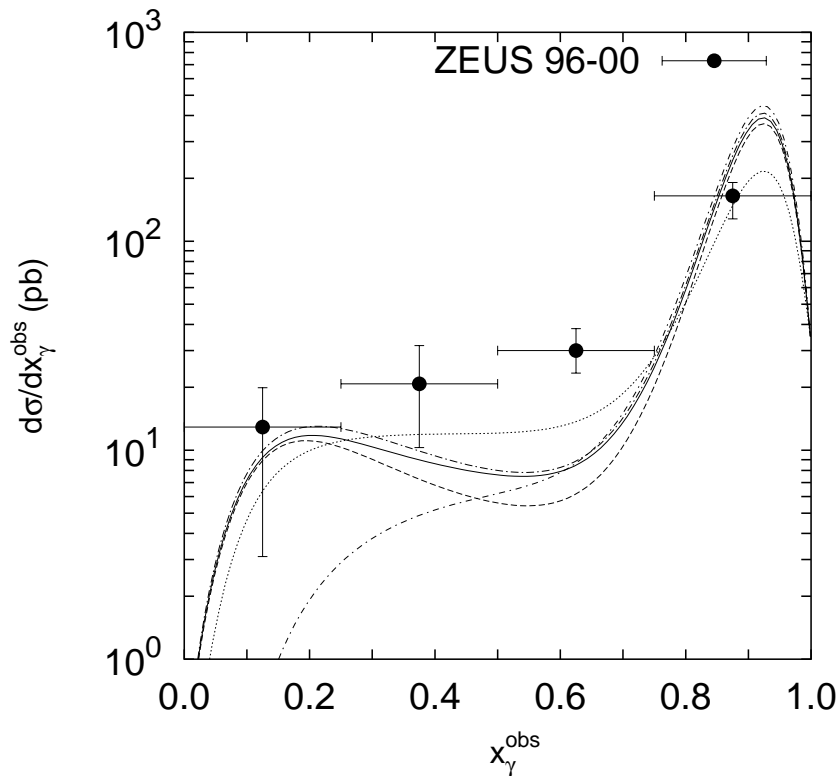
**Figure 8:** *The tr. mom. distr. of the jet containing B-hadron at  $-0.55 < \eta^\mu < 1.1$ ,  $p_T^\mu > 2.5$  GeV,  $Q^2 < 1$  GeV<sup>2</sup>,  $0.2 < y < 0.8$  and  $|\eta^{b-jet}| < 2.5$ .*

- KMS provides more hard spectrum
- KMR results in some underestimation of the c.s.
- The difference between KMS and J2003  $\sim 20\%$ .

But KMS c.s. are larger for  $D^* + 2$  jets.



**Figure 9:** *The pseudo-rapidity distr. of the jet containing B-hadron at  $-0.55 < \eta^\mu < 1.1, p_T^\mu > 2.5 \text{ GeV}, Q^2 < 1 \text{ GeV}^2, 0.2 < y < 0.8$  and  $|\eta^{b-jet}| < 2.5$ .*



**Figure 10:** *The differential c.s.  $d\sigma/dx_\gamma^{obs}$  of dijets with associated muon from  $b$ -decay at  $-1.6 < \eta^\mu < 2.3, p_T^\mu > 2.5$  GeV,  $Q^2 < 1$  GeV<sup>2</sup>,  $0.2 < y < 0.8, p_T^{jet1} > 7, p_T^{jet2} > 6$  GeV and  $|\eta^{jet}| < 2.5$ . The curves are obtained with account of direct and resolved photon contributions.*

- J2003 and KMR results describe the data (except some underestimation at middle region)
- KMS gives significant discrepancy (the  $gg$  fusion contribution from resolved photon is absent)
- $gg$  fusion contribution is important at low  $x_\gamma$ .

- The total c.s. of  $b + dijet$

Source	$\sigma(ep \rightarrow ebb + X \rightarrow ejj\mu + X')$
H1 measurement [7]	$38.4 \pm 3.4$ (stat.) $\pm 5.4$ (s)
NLO QCD (FMNR) [44]	$23.8^{+7.4}_{-5.1}$
Cascade (J2003 set 2)	<b>22.6</b>
Pythia [46]	<b>20.9</b>
J2003 set 1	<b>28.37</b>
J2003 set 2	<b>27.33</b>
J2003 set 3	<b>29.25</b>
KMR	<b>17.43</b>
KMS	<b>33.87</b>
KMS ( $m_b = 4.5$ GeV, $\Lambda_{\text{QCD}} = 250$ MeV)	<b>38.84</b>

**Table 2:** *The total cross section of the beauty and associated dijet photoproduction obtained in the kinematic range  $-0.55 < \eta^\mu < 1.1$ ,  $p_T^\mu > 2.5$  GeV,  $Q^2 < 1$  GeV<sup>2</sup>,  $0.2 < y < 0.8$ ,  $p_T^{\text{jet}_1} > 7$  GeV,  $p_T^{\text{jet}_2} > 6$  GeV and  $|\eta^{\text{jet}}| < 2.5$ .*

- Results of MC programs PYTIA, CASCADE and NLO QCD calculations agree with each other but are about 1.5 standard deviations below the data.
- Our predictions are somewhat higher but still below data too.



## Conclusions

- The  $k_T$ -factorization approach (with the J2003 and KMS UGD) reproduces very well the numerous HERA data on beauty photoproduction.
- The KMR formalism results in some underestimations of the cross sections.
- We have demonstrated the importance of gluon-gluon fusion contribution from resolved photon in the description of the exp. data at low values  $x_\gamma$ .