D-meson production in the GM-VFN scheme

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Outline

1. Overview and Motivation
2. Heavy Flavour Schemes
   - Fixed Order Perturbation Theory (FFNS)
   - Massless Variable Flavour Number Scheme (ZM-VFNS)
   - Massive Variable Flavour Number Scheme (GM-VFNS)
3. Hard scattering coefficients with heavy-quark masses
   - Massless Limit
   - Mass Factorization
4. Numerical Results
   - Fragmentation Functions
   - Hadroproduction
   - Electroproduction
5. Summary

Overview

Subject of this talk:

- One-particle inclusive production of heavy hadrons $H = D, B, \Lambda_c, \ldots$
- General-Mass Variable Flavour Number Scheme (GM-VFNS): [1]
  - Collinear logarithms of the heavy-quark mass $\ln \mu/m_h$ are subtracted and resummed
  - Finite non-logarithmic m_h/Q terms are kept in the hard part/taken into account
  - Scheme guided by the factorization theorem of Collins with heavy quarks [2]

Ongoing effort to compute all relevant processes in the GM-VFNS at NLO:

- Available:
  - $\gamma + \gamma \to D^{\star +} + X$: direct process [3]
  - $\gamma + \gamma \to D^{\star +} + X$: single-resolved process [4]
  - $\gamma + p \to D^{\star +} + X$: direct process [5]
  - $p + \bar{p} \to (D^0, D^{\star +}, D^+, D_s^+, \Lambda_c^+) + X$ [1,6]

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[1] B.K.,Kramer,Schienbein,Spiesberger, PRD71(2005)014018; EPJC41(2005)199
[2] J. Collins, PRD58(1998)094002
[3] Kramer,Spiesberger, EPJC22(2001)289; [4] EPJC28(2003)495; [5] EPJC38(2004)309

[6] B.K.,Kramer,Schienbein,Spiesberger, PRL96(2006)012001

Overview -continued-

Input for the computation: Fragmentation Functions (FFs) into heavy hadrons H

- FFs from fits to e^+e^- data from LEP (OPAL) directly in x-space
- Use new fits with initial scale $\mu_0 = m_h$ (instead of $\mu_0 = 2m_h$)
  $\rightarrow$ consistency with PDFs

Work in progress:

- Extraction of FFs in the GM-VFNS (→ include low-energy data) — T. Kneesch, Univ. Hamburg

Goal:

- Test pQCD formalism/universality of FFs in as many processes as possible

Motivation

Why heavy flavour production?

Heavy Quarks: $h = c, b, t$

. . . are interesting:

- $m_h \gg \Lambda_{\mathrm{QCD}} \Rightarrow \alpha_s(m_h^2) \propto \ln^{-1}(\frac{m_h^2}{\Lambda_{\mathrm{QCD}}^2}) \ll 1$ (asymptotic freedom)
- m_h sets hard scale; acts as long distance cut-off

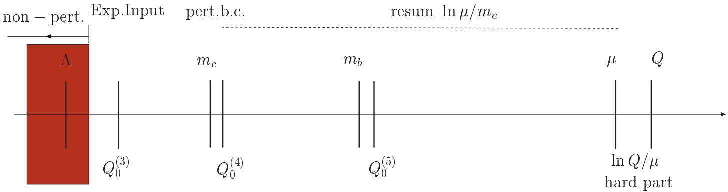
$\Rightarrow$ Perturbation Theory (pQCD) applicable!

Heavy quark production processes are:

- Fundamental elementary particle processes
- Important background to New Physics searches at the LHC

Fixed Order Perturbation Theory:

- finite collinear logs $\ln Q/m_c$ arise $\rightarrow$ can be kept in hard part
  Of course need exp. Input for u, d, s, g PDFs at scale $Q_0^{(3)}$

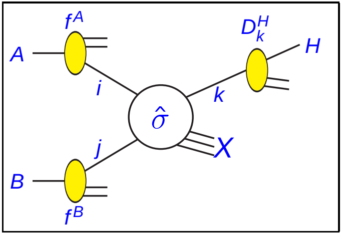


Variable Flavour Number Scheme (VFNS):

- often large ratios of scales involved: multi-scale problems
  For $Q \gg m_c$: write $\ln Q/m_c = \ln Q/\mu + \ln \mu/m_c$, subtract $\ln \mu/m_c$ and resum $\ln \mu/m_c$
  by introducing charm PDF at $Q_0^{(4)} \simeq m_c$ using a perturbative boundary condition

$$d\sigma = \sum_{i,j,k} f_i^A(x_1) \otimes f_j^B(x_2) \otimes d\hat{\sigma}(ij \to kX) \otimes D_k^H(z)$$

Sum over all possible subprocesses $i + j \to k + X$



- $d\hat{\sigma}(\mu_F, \mu_F', \alpha_s(\mu_R), [\frac{m_h}{p_T}])$: hard scattering cross sections, perturbatively computable, free of long-distance physics $\to m_h$ can be kept
- PDFs $f_i^A(x_1, \mu_F)$, $f_j^B(x_2, \mu_F)$: universal, non-perturbative input
  - Direct Photon: $i, j = \gamma$, $f_\gamma^\gamma(x, \mu_F) = \delta(1 - x)$
  - Resolved Photon/Proton: $i, j = g, q, \ldots$ $[q = u, d, s]$
- Fragmentation functions $D_k^H(z, [\mu_F'])$: universal, non-perturbative input $k = h, \ldots$
- Error:
  - light hadrons: $\mathcal{O}((\Lambda/p_T)^p)$ with p_T hard scale, Λ hadronic scale, $p = 1, 2$
  - heavy hadrons: $\mathcal{O}((m_h/p_T)^p)$ if m_h neglected in $d\hat{\sigma}$

Details (which subprocesses, PDFs, FFs; mass terms) depend on the Heavy Flavour Scheme

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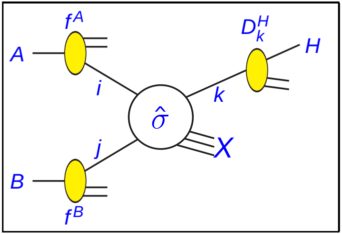

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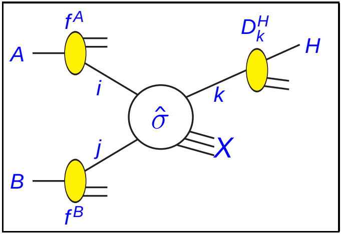

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Theoretical Basis: Factorization Formulae for $A + B \to H + X$

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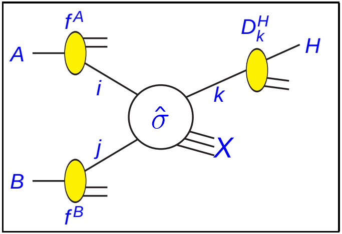

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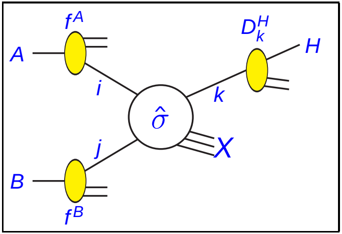

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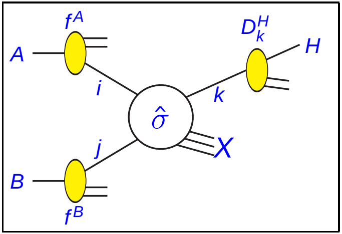

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Heavy Flavour Schemes

▸▸ Skip Details

Two basic approaches:

- Fixed Order Perturbation Theory (FFNS)
- Parton Model (ZM-VFNS)

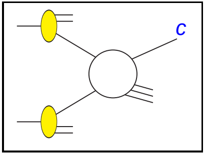
Interpolating scheme combining the good features:

- Parton Model with quark masses (GM-VFNS, ACOT)

Glossary:

- ZM: Zero Mass
- GM: General Mass
- VFNS: Variable Flavour Number Scheme
- FFNS: Fixed Flavour Number Scheme

- $m_c \neq 0, \quad n_f = 3$ fixed
- Partons: g, u, d, s
  [NO charm parton; Charm (only) in final state]
- collinear logarithms $\ln \frac{s}{m_c^2}$ finite
  $\rightarrow$ No factorization; no conceptual necessity for FFs
  $\rightarrow$ fixed order perturbation theory; no resummation
- Usually c treated in on-shell scheme ($\overline{\mathrm{MS}}_m$)



Pro and Contra:

+ $(\frac{m_c}{p_T})^n$ terms included; correct threshold suppression
  $\Rightarrow$ valid for $0 \leq p_T^2 \lesssim m_c^2$ $\Rightarrow$ σ_{tot} calculable
– fixed order logarithms $\ln \frac{p_T^2}{m_c^2}$ large for $p_T^2 \gg m_c^2$;
  resummation of these large logarithms necessary
  $\Rightarrow$ breaks down for $p_T^2 \gg m_c^2$
– non-perturbative function $D_c^H(z)$, describing the hadronisation $c \rightarrow H$ needed to match data;
  $\rightarrow$ not based on factorization theorem (no AP evolution)
  $\rightarrow$ universal?

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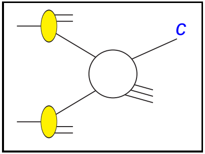

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- $m_c = 0 \to$ 'Zero Mass'
- Number of active partons depends on scale $\mu_F \to$ 'VFNS'
  - $\mu_F < m_c$: $n_f = 3$, Partons: g, u, d, s
  - $m_c \le \mu_F < m_b$: $n_f = 4$, Partons: g, u, d, s, c
  - $m_b \le \mu_F$: $n_f = 5$, Partons: g, u, d, s, c, b
- Matching conditions at transition scale $Q_0 = m_c$ (similar at m_b): $n_f = 3 \to n_f = 4$

$$\left.\begin{array}{l} \alpha_s^{(3)} \to \alpha_s^{(4)} = \alpha_s^{(3)} + \mathcal{O}(\alpha_s^3) \\ f_i^{(3)} \to f_i^{(4)} = f_i^{(3)} + \mathcal{O}(\alpha_s^2) \end{array}\right\} @\, Q_0 = m_c \qquad \boxed{f_c^{(4)}(x, Q_0^2 = m_c^2) = 0} \text{ pert. b.c.}$$

- Collinear divergences related to c lines factorized into non-perturbative PDFs and FFs

Pro and Contra:

+ large collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ resummed in evolved $f_c(x, \mu^2)$ and $D_c^{D^\star}(x, \mu^2)$ to LL and NLL accuracy

  $\Rightarrow$ $\boxed{\text{good for large } \mu^2 \simeq p_T^2 \gg m_c^2}$

+ Universality of PDFs and FFs guaranteed by factorization theorem $\to$ predictive power, global data analysis

− $(\frac{m_c}{p_T})^n$ terms neglected in the hard part

  $\Rightarrow$ $\boxed{\text{breaks down for } p_T^2 \lesssim m_c^2}$ $\Rightarrow$ No σ_{tot}

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- VFNS with $m_c \neq 0$
- Partons: g, u, d, s, c (∃ charm parton: $f_c \neq 0$)
- collinear $\ln \frac{\mu^2}{m_c^2}$ terms:
  subtracted from hard part (avoid double counting!) and
  resummed by AP evolution equations ($\rightarrow f_c \neq 0$)
- $D_c^{D^\star}(z, {\mu'_F}^2)$ evolved

Pro and Contra:

- technically more involved:
  - calculation with $m_c \neq 0$
  - subtraction of collinear parts ↔ 'IR-safe' hard parts
    Mass factorization with massive regularization
  - kinematics: factorization with massive partons → 'ACOT-χ' in DIS
- \+ large collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ resummed in evolved $f_c(x, \mu^2)$ and $D_c^{D^\star}(x, \mu^2)$
- \+ $(\frac{m_c}{p_T})^n$ included

⇒ good for all p_T: $0 \leq p_T^2 \lesssim m_c^2$ and $p_T^2 \gg m_c^2$

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Two ways to derive them:

- Massless limit of fixed order calculation
- Mass factorization with massive regularization

Factorization Formula: [1]

$$d\sigma(p\bar{p} \to D^\star X) = \sum_{i,j,k} \int dx_1\, dx_2\, dz\, f_i^p(x_1)\, f_j^{\bar{p}}(x_2) \times$$
$$\mathrm{d}\hat{\sigma}(ij \to kX)\, D_k^{D^\star}(z) + \mathcal{O}(\alpha_s^{n+1}, (\tfrac{\Lambda}{Q})^p)$$

Q: hard scale, $p = 1, 2$

- $\mathrm{d}\hat{\sigma}(\mu_F, \mu_F', \alpha_s(\mu_R), \frac{m_h}{p_T})$: hard scattering cross sections
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- FFs $D_k^{D^\star}(z, \mu_F')$: $k = g, q, c$

$\Rightarrow$ need short distance coefficients including heavy quark masses

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[1] J. Collins, 'Hard-scattering factorization with heavy quarks: A general treatment', PRD58(1998)094002

List of Subprocesses: GM-VFNS

	Only light lines	Heavy quark initiated ($m_Q = 0$)	Mass effects: $m_Q \neq 0$
1	$gg \to qX$	-	$gg \to QX$
2	$gg \to gX$	-	-
3	$qg \to gX$	$Qg \to gX$	-
4	$qg \to qX$	$Qg \to QX$	-
5	$q\bar{q} \to gX$	$Q\bar{Q} \to gX$	-
6	$q\bar{q} \to qX$	$Q\bar{Q} \to QX$	-
7	$qg \to \bar{q}X$	$Qg \to \bar{Q}X$	-
8	$qg \to \bar{q}'X$	$Qg \to \bar{q}X$	$qg \to \bar{Q}X$
9	$qg \to q'X$	$Qg \to qX$	$qg \to QX$
10	$qq \to gX$	$QQ \to gX$	-
11	$qq \to qX$	$QQ \to QX$	-
12	$q\bar{q} \to q'X$	$Q\bar{Q} \to qX$	$q\bar{q} \to QX$
13	$q\bar{q}' \to gX$	$Q\bar{q} \to gX$, $q\bar{Q} \to gX$	-
14	$q\bar{q}' \to qX$	$Q\bar{q} \to QX$, $q\bar{Q} \to qX$	-
15	$qq' \to gX$	$Qq \to gX$, $qQ \to gX$	-
16	$qq' \to qX$	$Qq \to QX$, $qQ \to qX$	-

$\oplus$ charge conjugated processes

- Calculate $m \to 0$ limit of massive 3-FFNS calculation (at partonic level) [1]
  Only keep m as regulator in $\ln \frac{m^2}{s}$

  Partonic subprocesses in 3-FFNS:

  - Leading Order (LO):
    1. $gg \to c\bar{c}$
    2. $q\bar{q} \to c\bar{c} \quad (q = u, d, s)$
  - Next-To-Leading Order (NLO):
    1. $gg \to c\bar{c}g$
    2. $q\bar{q} \to c\bar{c}g$
    3. $gq \to c\bar{c}q$ New@NLO

  Limiting procedure non-trivial:
  - Map from PS-slicing to Subtraction method
  - care needed to recover $\delta(1-w)$, $(\frac{1}{1-w})_+$, $(\frac{\ln(1-w)}{1-w})_+$

  Checks:
  - Compare Abelian parts with results in [2]
  - Numerical tests

[1] Bojak, Stratmann, PRD67(2003)034010; FORTRAN code provided by I. Bojak

[2] Kramer, Spiesberger, EPJC22(2001)289; EPJC28(2003)495

- Compare $m \to 0$ limit of massive calculation with massless $\overline{\text{MS}}$ calculation [1]

$$\lim_{m\to 0} \mathrm{d}\sigma(m) = \mathrm{d}\hat{\sigma}(\overline{\text{MS}}) + \Delta \mathrm{d}\sigma$$

$\Rightarrow$ Subtraction terms

$$\mathrm{d}\sigma_{\text{SUB}} \equiv \Delta \mathrm{d}\sigma = \lim_{m\to 0} \mathrm{d}\sigma(m) - \mathrm{d}\hat{\sigma}(\overline{\text{MS}})$$

- Subtract $\mathrm{d}\sigma_{\text{SUB}}$ from massive partonic cross section while keeping mass terms

$$\mathrm{d}\hat{\sigma}(m) = \mathrm{d}\sigma(m) - \mathrm{d}\sigma_{\text{SUB}}$$

$\longrightarrow$ $\mathrm{d}\hat{\sigma}(m)$ short distance coefficient including m

$\longrightarrow$ allows to use PDFs and FFs with $\overline{\text{MS}}$ factorization $\oplus$ massive short dist. cross sections

- Treat contributions with charm in the initial state with $m_c = 0$;
  $\rightsquigarrow$ scheme choice of practical importance; tiny effect in DIS [2]

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105
[2] Kretzer, Schienbein, PRD58(1998)094035

- Compare $m \to 0$ limit of massive calculation with massless $\overline{\text{MS}}$ calculation [1]

$$\lim_{m\to 0} d\sigma(m) = d\hat{\sigma}(\overline{\text{MS}}) + \Delta d\sigma$$

$\Rightarrow$ Subtraction terms

$$d\sigma_{\text{SUB}} \equiv \Delta d\sigma = \lim_{m\to 0} d\sigma(m) - d\hat{\sigma}(\overline{\text{MS}})$$

- Subtract $d\sigma_{\text{SUB}}$ from massive partonic cross section while keeping mass terms

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[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105
[2] Kretzer, Schienbein, PRD58(1998)094035

Sketch of kinematics:

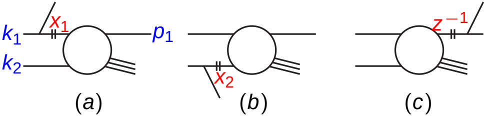


Fig. (a):

$$d\sigma^{\rm sub}(ab \to QX) = \int_0^1 dx_1\, f^{(1)}_{a\to i}(x_1,\mu_F^2)\, d\hat\sigma^{(0)}(ib \to QX)[x_1k_1,k_2,p_1] \equiv f^{(1)}_{a\to i}(x_1) \otimes d\hat\sigma^{(0)}(ib \to QX)$$

Fig. (b):

$$d\sigma^{\rm sub}(ab \to QX) = \int_0^1 dx_2\, f^{(1)}_{b\to j}(x_2,\mu_F^2)\, d\hat\sigma^{(0)}(aj \to QX)[k_1,x_2k_2,p_1] \equiv f^{(1)}_{b\to j}(x_2) \otimes d\hat\sigma^{(0)}(aj \to QX)$$

Fig. (c):

$$d\sigma^{\rm sub}(ab \to QX) = \int_0^1 dz\, d\hat\sigma^{(0)}(ab \to kX)[k_1,k_2,z^{-1}p_1]\, d^{(1)}_{k\to Q}(z,\mu_F'^{\,2}) \equiv d\hat\sigma^{(0)}(ab \to kX) \otimes d^{(1)}_{k\to Q}(z)$$

---

[1] B.K., Kramer, Schienbein, Spiesberger, EPJC41(2005)199

1. initial state:

$$f^{(1)}_{g\to Q}(x,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} P^{(0)}_{g\to q}(x) \ln\frac{\mu^2}{m^2}$$

$$f^{(1)}_{Q\to Q}(x,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} C_F \left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu^2}{m^2} - 2\ln(1-z) - 1\right)\right]_+ \qquad [2]$$

$$f^{(1)}_{g\to g}(x,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi}\frac{1}{3}\ln\frac{\mu^2}{m^2}\delta(1-x)$$

2. final state:

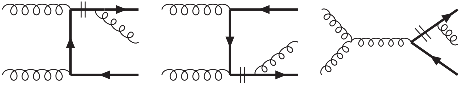
$$d^{(1)}_{g\to Q}(z,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} P^{(0)}_{g\to q}(z) \ln\frac{\mu^2}{m^2}$$

$$d^{(1)}_{Q\to Q}(z,\mu^2) = C_F \frac{\alpha_s(\mu)}{2\pi}\left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu^2}{m^2} - 2\ln(1-z) - 1\right)\right]_+ \qquad [1,2,3]$$

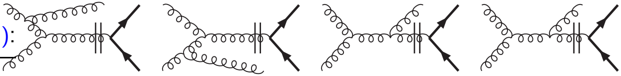
- Other distributions are zero to this order in α_s
- Analogous for photon splitting: $g \to \gamma$, $\alpha_s \to \alpha$, color factors

[1] Mele, Nason, NPB361(1991)626; Ma, NPB506(1997)329 [$\gamma^\star \to c\bar{c}g$]
[2] Kretzer, Schienbein, PRD58(1998)094035;D59(1999)054004 [$c\gamma^\star \to cg$]
[3] Melnikov, Mitov, PRD70(2004)034027; Mitov, PRD71(2005)054021 [$\mathcal{O}(\alpha_s^2)$]

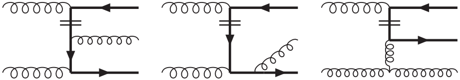
$d\hat{\sigma}^{(0)}(gg \to Q\bar{Q}) \otimes d^{(1)}_{Q \to Q}(z)$:



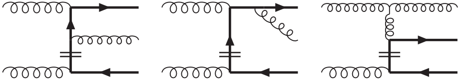
$d\hat{\sigma}^{(0)}(gg \to gg) \otimes d^{(1)}_{g \to Q}(z)$:



$f^{(1)}_{g \to Q}(x_1) \otimes d\hat{\sigma}^{(0)}(Qg \to Qg)$:

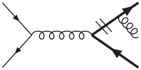


$f^{(1)}_{g \to Q}(x_2) \otimes d\hat{\sigma}^{(0)}(gQ \to Qg)$:

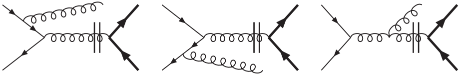


GRAPHICAL REPRESENTATION OF SUBTRACTION TERMS FOR $q\bar{q} \to Q\bar{Q}g$ AND $gq \to Q\bar{Q}q$

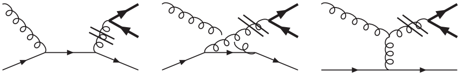
$\mathrm{d}\hat{\sigma}^{(0)}(q\bar{q} \to Q\bar{Q}) \otimes d^{(1)}_{Q\to Q}(z)$:



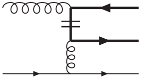
$\mathrm{d}\hat{\sigma}^{(0)}(q\bar{q} \to gg) \otimes d^{(1)}_{g\to Q}(z)$:



$\mathrm{d}\hat{\sigma}^{(0)}(gq \to gq) \otimes d^{(1)}_{g\to Q}(z)$:



$f^{(1)}_{g\to Q}(x_1) \otimes \mathrm{d}\hat{\sigma}^{(0)}(Qq \to Qq)$:



Existing calculations: (based on DGLAP evolution)

- ZM-VFNS: Binnewies et al.; Cacciari et al.
- FFNS: Frixione et al.
- GM-VFNS:
  - direct part: Kramer, Spiesberger
  - resolved part: B.K., Kramer, Schienbein, Spiesberger (from hadroproduction)

Status:

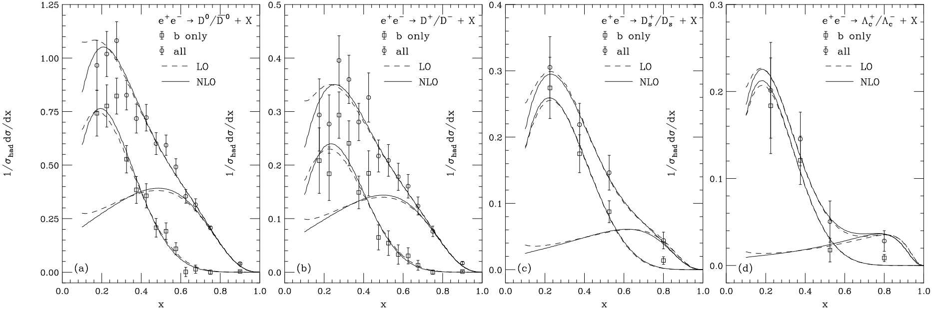
- For comparison of GM-VFNS with preliminary ZEUS data (p_T, y, W, z distributions) see Kramer, Spiesberger, EPJC38(2004)309
- Including resolved part in GM-VFNS will help to improve results at small p_T
- Use new FFs with initial scale $\mu_0 = m_h$

NUMERICAL RESULTS

Input parameters:

- $\alpha_s(M_Z) = 0.1181$
- $m_c = 1.5\ \mathrm{GeV}, m_b = 5\ \mathrm{GeV}$
- PDFs: CTEQ6M (NLO)
- FFs: NLO FFs from fits to LEP-OPAL data,
  initial scale for evolution: $\mu_0 = m_c$ (D-mesons) resp. $\mu_0 = m_b$ (B-mesons)
- Default scale choice: $\mu_R = \mu_F = \mu_F' = m_T$ where $m_T = \sqrt{p_T^2 + m^2}$

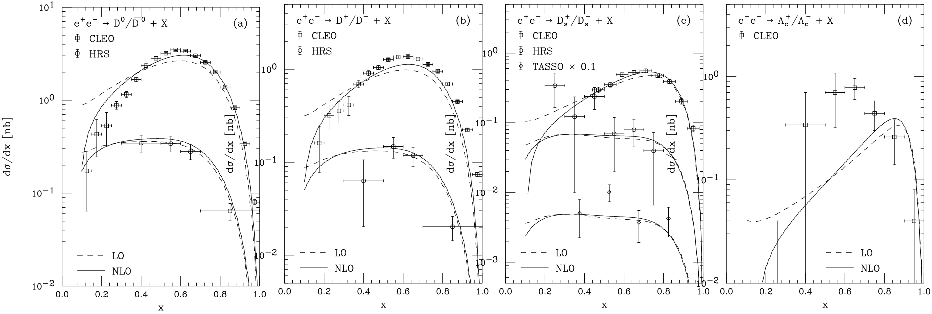
- $(1/\sigma_{\text{tot}})d\sigma/dx$, ZM-VFNS
- $\mu_R, \mu'_F = \sqrt{S}$
- Fit to LEP1 data from OPAL [2]



[1] B.K., Kramer, PRD71(2005)094013

[2] Alexander et al, ZPC72(1996)1

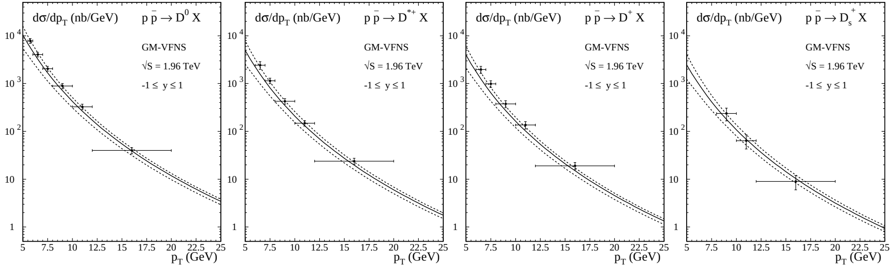
- $d\sigma/dx$, ZM-VFNS
- Comparison with CLEO [2], HRS [3], and TASSO [4]



[1] B.K., Kramer, PRD71(2005)094013
[2] CLEO, PRD37(1988)1719, PRD62(2000)072003, PRD70(2004)112001
[3] HRS, PRL54(1985)2568, PLB206(1988)551

[4] TASSO, PLB136(1984)130

- $d\sigma/dp_T$ (nb/GeV), $|y| \leq 1$, GM-VFNS
- Uncertainty band: independent variation of $\mu_R, \mu_F, \mu'_F = \xi m_T$, $\xi \in [1/2, 2]$



- Prompt charm data (no secondary charm from B decay) from CDF in run II [2]
- Data described by QCD within errors
- Predictions for Λ_c^+ available

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[1] B.K.,Kramer,Schienbein,Spiesberger,PRL96(2006)012001; [2] Acosta et al, PRL91(2003)241804

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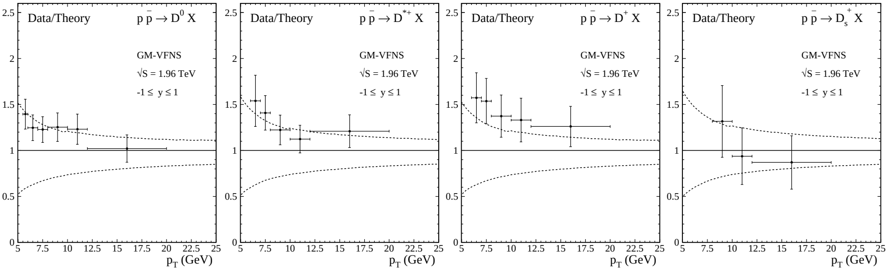
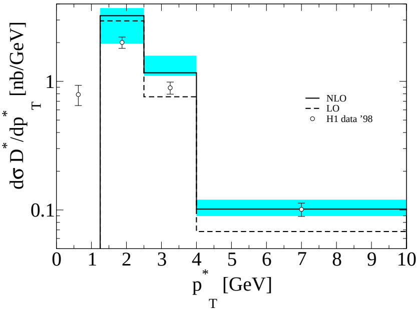


- Prompt charm data (no secondary charm from B decay) from CDF in run II
- Data and Theory compatible within errors
- Central values: Data/Theory $\simeq 1.5 - 1.2$

[1] Kniehl,Kramer,I.S.,Spiesberger,hep-ph/0508129; [2] Acosta et al, PRL91(2003)241804

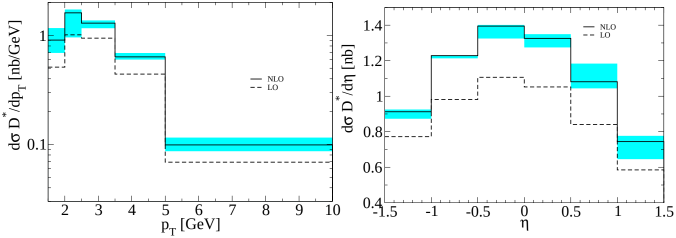
- Phase Space: $2 < Q^2 < 100\ \mathrm{GeV}^2$, $0.05 < y < 0.7$,
  $1.5 \le p_{T,Lab}(D^\star) \le 15\ \mathrm{GeV}$, $|\eta_{Lab}(D^\star)| < 1.5$,



- ${\mu_R}^2 = {\mu_F}^2 = {\mu'_F}^2 = \xi \frac{Q^2+(p_T^*)^2}{2}$
- Uncertainty band: variation of $\xi \in [1/2, 2]$

[1] B.K., Kramer, Maniatis, NPB711(2005)345

- Phase Space: $2 < Q^2 < 100\ \mathrm{GeV}^2$, $0.05 < y < 0.7$,
  $1.5 \le p_{T,Lab}(D^\star) \le 15\ \mathrm{GeV}$, $|\eta_{Lab}(D^\star)| < 1.5$,
- Additional Cut: $p_T^*(D^*) > 2\ \mathrm{GeV}$ ($\gamma^* p$-CMS)



- $\mu_R{}^2 = \mu_F{}^2 = \mu_F'{}^2 = \xi \frac{Q^2 + (p_T^*)^2}{2}$
- Uncertainty band: variation of $\xi \in [1/2, 2]$

More distributions

[1] B.K., Kramer, Maniatis, NPB711(2005)345

- In this talk:
  Discussion of one-particle inclusive production of heavy quarks in the GM-VFNS
- Available at NLO in the GM-VFNS:
  - $\gamma\gamma \to HX$
  - $\gamma p \to HX$
  - $p\bar{p} \to HX$
- General expectation:
  - Improvement at $p_T \gg m_h$ due to updated FFs (and PDFs)
  - Mass effects: Improve agreement with (HERA) data for $p_T \gtrsim m_h$
  - Mass effects: Reduced factorization scale dependence

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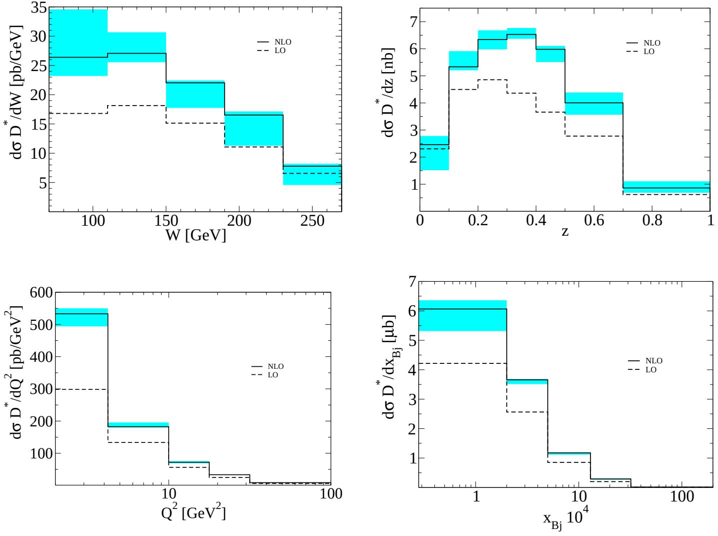
Backup Slides

PREDICTIONS FOR $ep \to D^{\star +}X$ IN THE ZM-VFNS (PROVIDED BY MARKOS MANIATIS)



◂ go back

$$\mathrm{FONLL} = \mathrm{FO} + (\mathrm{RS} - \mathrm{FOM0})\mathrm{G}(\mathrm{m}, \mathrm{p_T})$$

FO: Fixed Order; FOM0: Massless limit of FO; RS: Resummed

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2}$$

$$\Rightarrow \mathrm{FONLL} = \begin{cases} \mathrm{FO} & : \quad \mathrm{p_T} \lesssim 5\mathrm{m} \\ \mathrm{RS} & : \quad \mathrm{p_T} \gtrsim 5\mathrm{m} \end{cases}$$

back to schemes

[1] Cacciari, Greco, Nason, JHEP05(1998)007

$$\mathrm{FONLL} = \mathrm{FO} + (\mathrm{RS} - \mathrm{FOM0})\mathrm{G}(\mathrm{m}, \mathrm{p_T})$$ with

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2}$$

$$\text{GM-VFNS} = \mathrm{FO} + (\mathrm{RS} - \mathrm{FOM0})\tilde{\mathrm{G}}(\mathrm{m}, \mathrm{p_T})$$ with

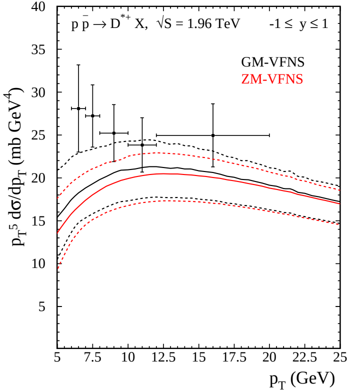
$$\tilde{G}(m, p_T) = 1$$

FO: Fixed Order; FOM0: Massless limit of FO; RS $\equiv$ ZM-VFNS: Resummed

- Both approaches interpolate between FO and ZM-VFNS
  - FONLL: obvious;
  - GM-VFNS: matching with FO at quark level (see Olness,Scalise,Tung,PRD59(1998)014506)
- Factor $\tilde{G}(m, p_T)$ follows from calculation; $\tilde{G}(m, p_T) = 1 \leftrightarrow$ S-ACOT scheme
- Different point-of-view: GM-VFNS finally needs PDFs and FFs in this scheme.
- Numerical comparisons interesting and should be done!

$p\bar{p} \to D^{\star +}X$

- Results with old FFs with initial scale $\mu_0 = 2m_c$
- Uncertainty band: independent variation of $\mu_R, \mu_F, \mu'_F = \xi m_T, \xi \in [1/2, 2]$



- In this example still $p_T > 3m_c$
- Mass effects bigger for small μ_R (large $\alpha_s(\mu_R)$)

STRONG COUPLING CONSTANT

- PDG'04: $\alpha_s(M_Z) = 0.1187 \pm 0.0020$
- CTEQ6M PDFs: $\alpha_s(M_Z) = 0.118$; MRST03 $\alpha_s(M_Z) = 0.1165$;
- $d\sigma(p\bar{p} \to DX) \propto \alpha_s^2(1 + \alpha_s K)$