CEM vs. NRQCD
in CHARMONIUM PRODUCTION

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Contents

- By imposing the CEM assumptions on the NRQCD factorization formulas for hadrons and for perturbative free quarks, we obtain constraints on the NRQCD matrix elements.
- The constraints are at odds with phenomenological determinations of the matrix elements and violate the NRQCD v-scaling rules.
- Direct comparison of CEM and NRQCD predictions with the CDF data for charmonium cross section at order α_s^3 (NLO $2 \to 1$ + LO $2 \to 2$) is provided.
- Multiple gluon emission effect is studied using phenomenological k_T smearing model of Gaussian form.
- k_T smearing is essential to obtain a reasonable p_T spectrum in the CEM. It is essential in NRQCD factorization in the P-wave case, which is constrained by decay data.

NRQCD Factorization Formalism

- NRQCD Factorization Formula

$$\sigma[AB \to H + X] = \sum_n c_n^{AB}(\Lambda)\langle \mathcal{O}_n^H(\Lambda)\rangle.$$

- Cross section is expressed in a linear combination of long-distance NRQCD matrix elements which are classified based on velocity scaling rules. The series can be truncated at a given order in v.

$$\begin{aligned} \mathcal{O}_n^H &= \chi^\dagger \kappa_n \psi \mathcal{P}^H(\Lambda)\psi^\dagger \kappa_n' \chi, \\ \mathcal{P}^H(\Lambda) &= \sum_X |H + X\rangle\langle H + X|. \end{aligned}$$

- Short-distance coefficients $c_n^{AB}(\Lambda)$ are IR insensitive and perturbatively calculable.

Color-evaporation Model(CEM)

- CEM formula for inclusive quarkonium production

$$\sigma_{\rm CEM}[AB \to H + X] = F_H \int_{4m^2}^{4m_M^2} dm^2_{Q\bar{Q}} \frac{d\sigma}{dm^2_{Q\bar{Q}}}[AB \to Q\bar{Q} + X],$$

- M is the lowest-lying meson containing a heavy quark Q.
- In the formula one sums over color and spin of the final-state quarks.
- $Q\bar{Q}$ pair is assumed to neutralize its color by interaction.
- In some versions of the CEM, color neutralization process is also assumed to randomize the spins of the Q and $\bar{Q}$.
- The F_H are constants that must be determined phenomenologically for each quarkonium state H.

Available Theoretical Calculations

- Predictions including NLO $2 \longrightarrow 1$ and LO $2 \longrightarrow 2$ subprocesses.
  - CEM: R. Vogt(2005) based on Mangano, Nason, Ridolfi, NPB 405, 507 (1993).
  - NRQCD: modified version of a code by Maltoni, Mangano, Petrelli.

  to be compared with the CDF data for charmonium production.

- NLO $2 \longrightarrow 1$ contribution to $d\sigma/dp_T^2$ includes singular terms $\sim \delta(p_T^2)$ and $1/p_T^2$, but but the integral of the cross section from $p_T = 0$ up to a given p_T is well behaved. (The separate integrals of the singularities are not well behaved.)

- Effects of soft-gluon emission make the curve smooth at low p_T.
  - We use a phenomenological k_T smearing model.
  - Proper way : include multiple gluon emission by resummation of logarithmic corrections to all orders in α_s.

- The CEM calculations that we quote fix the parameters m and F_H by using the fixed-target data.
- In the k_T-smeared CEM calculations that we quote, the amount of k_T smearing is adjusted to provide the best fit to the Tevatron J/ψ data.
- In the k_T-smeared NRQCD calculations, we do not adjust the amount of k_T smearing, but simply take the amount that was used in the CEM calculations.

What happens to NRQCD under CEM?

- If we assume CEM, NRQCD matrix elements are simplified.

$$\langle \mathcal{O}_n^H \rangle = \frac{1}{2\pi^2} F_H \int_0^{k_{\rm max}} k^2 dk \, \langle \mathcal{O}_n^{Q\bar{Q}}(k) \rangle .$$

$$\mathcal{O}_n^{Q\bar{Q}}(k) = \chi^\dagger \kappa_n \psi \left(\int \frac{d\Omega_k}{4\pi} \sum_{\rm spins} \sum_{\rm colors} \mathcal{P}^{Q(+\boldsymbol{k})\bar{Q}(-\boldsymbol{k})} \right) \psi^\dagger \kappa_n' \chi ,$$

  Above equations embody the CEM assumptions.

- Any NRQCD matrix element reduces into a single matrix element upto multiplicative scaling factors that are completely determined.

$$\langle \mathcal{O}_n^H \rangle = \frac{3(2j+1)}{(2l+1)(2l+3)} C_n k_{\rm max}^{2l} \langle \mathcal{O}_1^H({}^1S_0) \rangle ,$$

$$k_{\rm max}^2 = m_M^2 - m_c^2, \quad C_1 = 1, \quad C_8 = C_F = \frac{4}{3}.$$

What happens to NRQCD under CEM?

- The CEM assumptions violate the v-scaling rules of NRQCD. Only powers of v that arise from covariant derivatives in operators are taken into account properly in the CEM.

	$1,{}^1S_0$	$1,{}^3S_1$	$8,{}^1S_0$	$8,{}^3S_1$	$1,{}^1P_1$	$1,{}^3P_0$	$1,{}^3P_1$	$1,{}^3P_2$	$8,{}^1P_1$	$8,{}^3P_0$	$8,{}^3P_1$	$8,{}^3P_2$
						NRQCD Factorization						
η_c	1		v^4	v^3					v^4			
J/ψ		1	v^3	v^4						v^4	v^4	v^4
h_c			v^2		v^2							
χ_{c0}				v^2		v^2						
χ_{c1}				v^2			v^2					
χ_{c2}				v^2				v^2				
						Color-Evaporation Model						
H	1	1	1	1	v^2	v^2	v^2	v^2	v^2	v^2	v^2	v^2

NRQCD ME for S-wave production

- In the S-wave charmonium ($H = J/\psi,\ \psi(2S)$) production at the Tevatron with $p_T > 5$ GeV, the most important NRQCD ME are $\langle \mathcal{O}_8^H(^3S_1)\rangle$ and

$$M_r^H = (r/m^2)\langle \mathcal{O}_8^H(^3P_0)\rangle + \langle \mathcal{O}_8^H(^1S_0)\rangle,\ \ r \approx 3 \qquad (1)$$

- If we enforce the CEM assumptions, then the ratio of the two ME's differs from the predictions of NRQCD v-scaling.

$$R^H = \frac{M_r^H}{\langle \mathcal{O}_8^H(^3S_1)\rangle} \sim v^0 \gg R^H_{\mathrm{CEM}} = \frac{r}{15}\frac{k^2_{\max}}{m^2} + \frac{1}{3}.$$

- Since $^3S_1^{(8)}$ is important at large p_T and M_r is important at low p_T, in S-wave production we expect

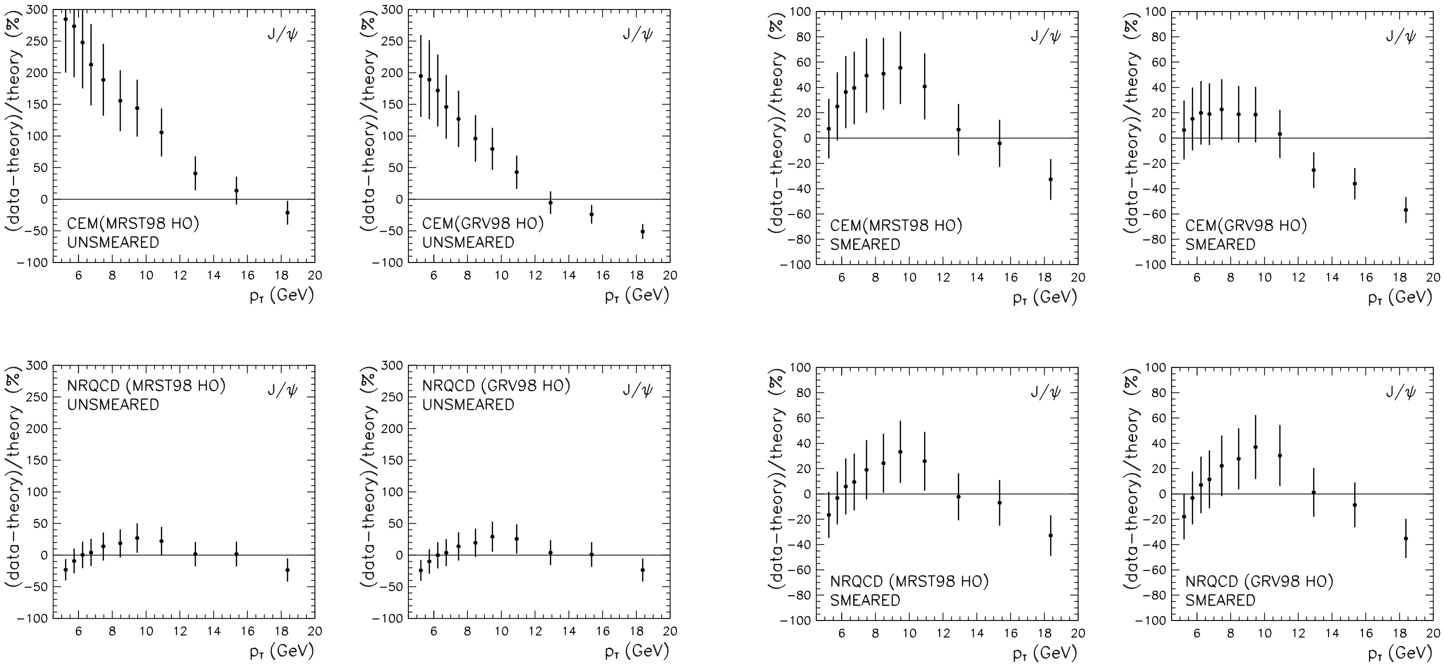
$$\sigma_{\mathrm{CEM}} < \sigma_{\mathrm{NRQCD}} : \mathrm{at\ \ low}\ p_T$$
$$\sigma_{\mathrm{CEM}} > \sigma_{\mathrm{NRQCD}} : \mathrm{at\ \ large}\ p_T$$

Comparison with phenomenological fits

Reference	PDF	$R^{J/\psi}$	$R^{J/\psi}_{\mathrm{CEM}}$	r	m_c (GeV)	$\langle k_T \rangle$ (GeV)
LO collinear factorization						
[27]	MRS(D0) [28]	10 ± 4	0.44	3	1.48	
[30]	CTEQ4L [29]	$4.1 \pm 1.2\ ^{+3.6}_{-1.3}$	0.46	3.5	1.5	
	GRV-LO(94) [31]	$3.5 \pm 1.1\ ^{+1.6}_{-0.9}$				
	MRS(R2) [32]	$7.8 \pm 1.9\ ^{+8.0}_{-2.8}$				
[33]	MRST-LO(98) [34]	20 ± 4	0.46	3.4	1.5	
	CTEQ5L [35]	17 ± 4				
parton-shower radiation						
[37]	CTEQ2L [36]	1.4 ± 0.3	0.44	3	1.48	
	MRS(D0) [28]	1.9 ± 0.6				
	GRV-HO(94) [31]	0.49 ± 0.11				
[38]	CTEQ4M [29]	2.1 ± 0.8	0.45	3.5	1.55	
k_T smearing						
[39]	CTEQ4M [29]	5.7 ± 1.6	0.46	3.5	1.5	1.0
		2.6 ± 0.9				1.5
[40]	MRS(D'_-) [28]	6.3 ± 1.7	≈ 0.44	3	≈ 1.5	0.7
		4.7 ± 1.2				1.0

- As expected, $R^H \gg R^H_{\mathrm{CEM}}$.

Comparison with Tevatron data (J/ψ)

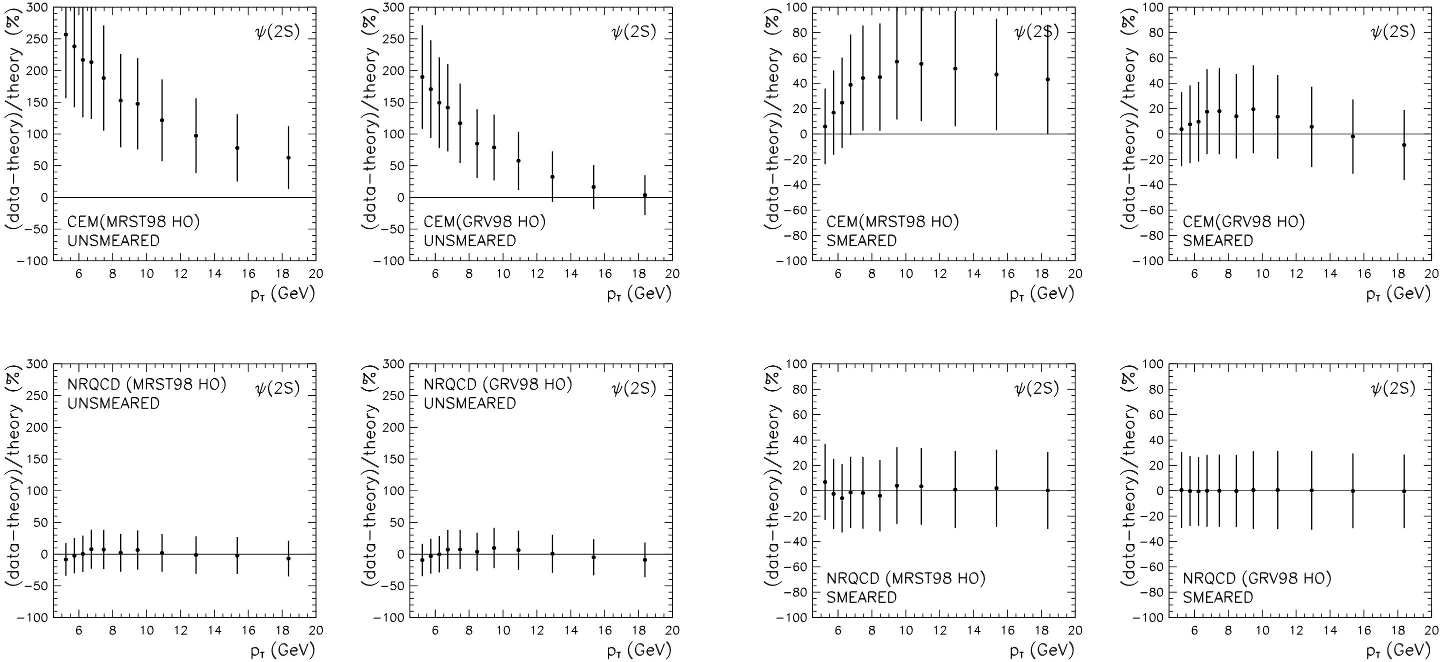


- CEM underestimates the x-section at low p_T as expected.
- After including smearing effect, χ^2/d.o.f. decreases.
- NRQCD prediction is better in either case.
- NRQCD factorization has more free parameters than the CEM, but the CEM does not yield a satisfactory fit to the data.

Comparison with Tevatron data (J/ψ)

PDF	$\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle$ (GeV$^3 \times 10^{-2}$)	$M_{3.5}^{J/\psi}$ (GeV$^3 \times 10^{-2}$)	$R^{J/\psi}$	χ^2/d.o.f.
NRQCD Factorization				
MRST98 HO	1.00 ± 0.22	8.83 ± 1.24	8.83 ± 2.27	7.16/(11−2)=0.80
GRV98 HO	1.02 ± 0.23	10.6 ± 1.42	10.4 ± 2.76	7.98/(11−2)=0.89
MRST98 HO (smeared)	1.41 ± 0.13	0.41 ± 0.15	0.29 ± 0.11	10.28/(11−2)=1.14
GRV98 HO (smeared)	1.54 ± 0.14	0.49 ± 0.16	0.32 ± 0.11	12.69/(11−2)=1.41
Color-Evaporation Model				
MRST98 HO				89.18/11=8.11
GRV98 HO				80.86/11=7.35
MRST98 HO (smeared)				20.78/(11−1)=2.08
GRV98 HO (smeared)				45.70/(11−1)=4.57

Comparison with Tevatron data [$\psi(2S)$]



- CEM underestimates the x-section at low p_T as expected.
- After including smearing effect, χ^2/d.o.f. decreases.
- NRQCD prediction is better in either case.
- NRQCD factorization has more free parameters than the CEM, but the CEM does not yield a satisfactory fit to the data.

Comparison with Tevatron data [$\psi(2S)$]

TABLE V: Values of matrix elements, $R^{\psi(2S)}$, and χ^2/d.o.f. from the NRQCD factorization and CEM fits to the $\psi(2S)$ data. In the NRQCD factorization fits, we set $\langle\mathcal{O}_1^{\psi(2S)}(^3S_1)\rangle$=0.76 GeV^3 and give the fitted values of $\langle\mathcal{O}_8^{\psi(2S)}(^3S_1)\rangle$ and $M_{3.5}^{\psi(2S)}$.

PDF	$\langle\mathcal{O}_8^{\psi(2S)}(^3S_1)\rangle$ (GeV$^3 \times 10^{-3}$)	$M_{3.5}^{\psi(2S)}$ (GeV$^3 \times 10^{-4}$)	$R^{\psi(2S)}$	χ^2/d.o.f.
NRQCD Factorization				
MRST98 HO	2.34 ± 0.47	44.0 ± 19.2	18.83 ± 9.08	0.35/(11−2)=0.04
GRV98 HO	2.51 ± 0.52	55.4 ± 22.2	22.02 ± 9.93	0.55/(11−2)=0.06
MRST98 HO (smeared)	2.12 ± 0.26	-6.77 ± 2.20	-3.19 ± 1.11	0.17/(11−2)=0.02
GRV98 HO (smeared)	2.34 ± 0.29	-6.80 ± 2.39	-2.90 ± 1.08	0.22/(11−2)=0.02
Color-Evaporation Model				
MRST98 HO				47.72/11=4.34
GRV98 HO				29.85/11=2.71
MRST98 HO (smeared)				10.43/11=0.95
GRV98 HO (smeared)				1.49/11=0.14

NRQCD ME for P-wave production

- The most important ME for χ_{cj} production

$$\langle \mathcal{O}_8^{\chi_{cj}}(^3S_1)\rangle, \quad \langle \mathcal{O}_{1,8}^{\chi_{cj}}(^3P_j)\rangle = (2j+1)\langle \mathcal{O}_{1,8}^{\chi_{c0}}(^3P_0)\rangle,$$

- The ratio is a useful phenomenological variable.

$$R^{\chi_c} = \frac{\langle \mathcal{O}_8^{\chi_{c0}}(^3S_1)\rangle}{\langle \mathcal{O}_1^{\chi_{c0}}(^3P_0)\rangle/m^2} \sim \frac{v^0}{2N_c} \sim 0.17.$$

- The CEM assumptions lead to a different prediction for the ratio than the v-scaling rules of NRQCD.

$$R_{\text{CEM}}^{\chi_c} = 15C_F\frac{m^2}{k_{\text{max}}^2} \sim \frac{1}{v^2} \gg R^{\chi_c}.$$

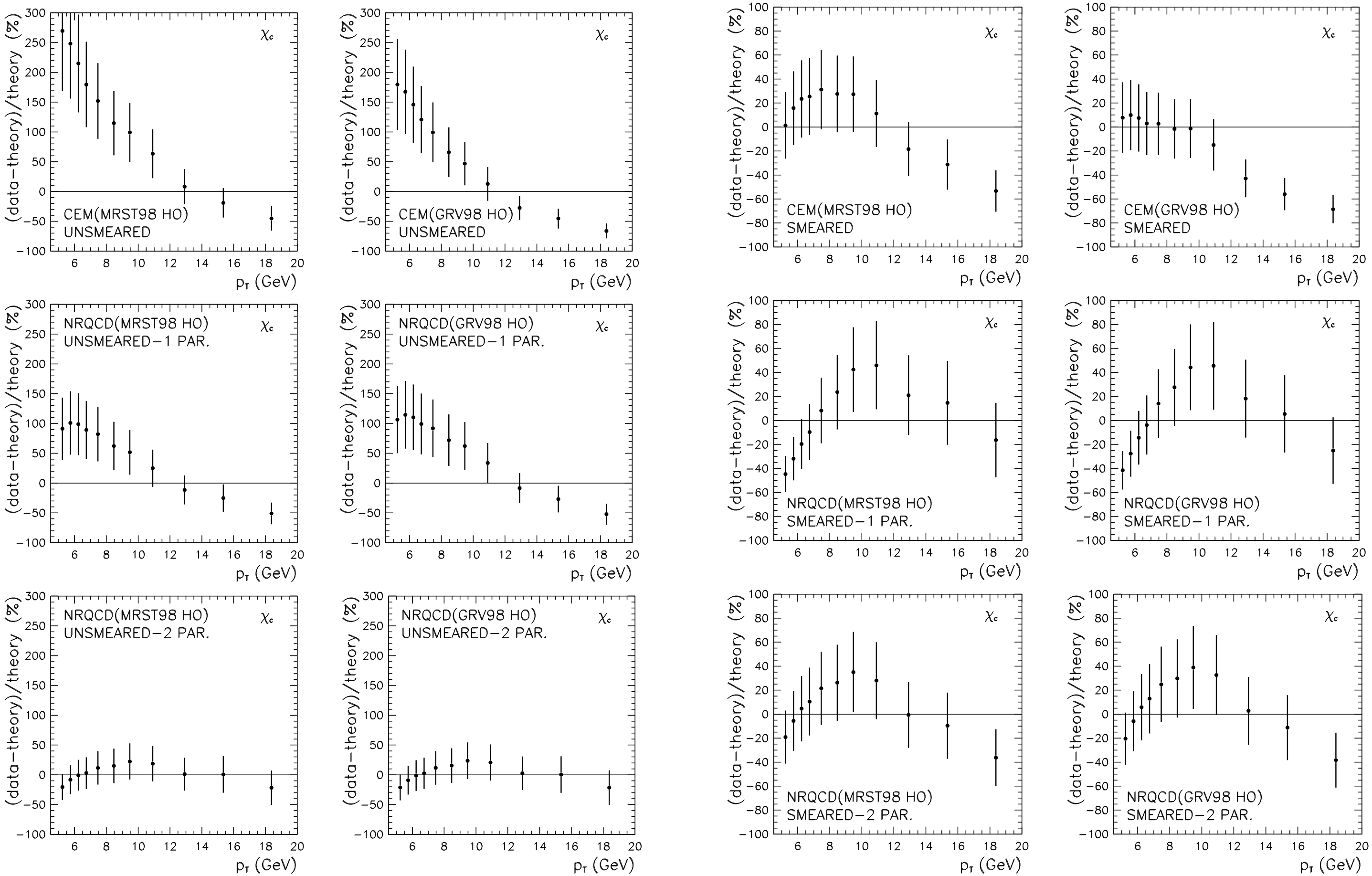
- $R^{\chi_c} \ll R^{\chi_c}_{\rm CEM}$ as expected.

Reference	PDF	R^{χ_c}	$R^{\chi_c}_{\rm CEM}$	m_c (GeV)
LO collinear factorization				
[27]	MRS(D0) [28]	$(6.6 \pm 0.8)\times 10^{-2}$	36	1.48
[38]	CTEQ4L [29]	$(0.71 \pm 0.21)\times 10^{-2}$	40	1.55
[33]	MRST-LO(98) [34]	$(5.8 \pm 1.1)\times 10^{-2}$	37	1.5
	CTEQ5L [35]	$(4.7 \pm 0.8)\times 10^{-2}$		

- Since ${}^3P_0^{(8)}$ is important at low p_T and ${}^3S_1^{(8)}$ dominates at large p_T, we expect

$$\sigma_{\rm NRQCD}(\chi_{cj}) > \sigma_{\rm CEM}(\chi_{cj}) \quad \text{at} \quad \text{low} \quad p_T$$
$$\sigma_{\rm NRQCD}(\chi_{cj}) < \sigma_{\rm CEM}(\chi_{cj}) \quad \text{at} \quad \text{large} \quad p_T$$

Comparison with Tevatron data (χ_{cj})



Comparison with Tevatron data (χ_{cj})

TABLE VII: Values of matrix elements, R^{χ_c}, and χ^2/d.o.f. from the NRQCD factorization and CEM fits to the χ_c data. In the NRQCD factorization fits, the upper sets of parameters are for fits in which $\langle \mathcal{O}_1^{\chi_{c0}}(^3P_0) \rangle$ is fixed, as described in the text, while the lower sets of parameters are for fits in which $\langle \mathcal{O}_1^{\chi_{c0}}(^3P_0) \rangle$ is varied.

PDF	$\langle \mathcal{O}_1^{\chi_{c0}}(^3P_0) \rangle$ (GeV$^5 \times 10^{-2}$)	$\langle \mathcal{O}_8^{\chi_{c0}}(^3S_1) \rangle$ (GeV$^3 \times 10^{-3}$)	R^{χ_c} (10^{-2})	χ^2/d.o.f.
NRQCD Factorization				
MRST98 HO	7.2 (input)	3.59 ± 0.39	11.23 ± 1.23	31.0/(11−1)=3.10
GRV98 HO	7.2 (input)	3.94 ± 0.43	12.30 ± 1.35	35.5/(11−1)=3.55
MRST98 HO (smeared)	7.2 (input)	1.71 ± 0.29	5.36 ± 0.89	17.4/(11−1)=1.74
GRV98 HO (smeared)	7.2 (input)	2.08 ± 0.32	6.50 ± 0.99	14.5/(11−1)=1.45
MRST98 HO	40.8 ± 6.3	1.20 ± 0.60	0.66 ± 0.35	2.97/(11−2)=0.33
GRV98 HO	48.7 ± 7.3	1.17 ± 0.65	0.54 ± 0.31	3.19/(11−2)=0.35
MRST98 HO (smeared)	3.88 ± 1.00	2.43 ± 0.36	14.12 ± 4.21	6.40/(11−2)=0.71
GRV98 HO (smeared)	4.39 ± 1.09	2.67 ± 0.39	13.66 ± 3.93	7.88/(11−2)=0.88
Color-Evaporation Model				
MRST98 HO				50.20/11=4.56
GRV98 HO				66.30/11=6.03
MRST98 HO (smeared)				16.15/11=1.47
GRV98 HO (smeared)				63.69/11=5.79

Comparison with Tevatron data (χ_{cj})

- CEM underestimates the x-section at low p_T as expected.
- After including smearing effect, χ^2/d.o.f. decreases.
- NRQCD factorization has more free parameters than the CEM, but the CEM does not represent the data adequately.
- The ${}^3P_0^{(1)}$ matrix element is constrained by data from P-wave decays.
- In order to obtain a fit in NRQCD factorization that is compatible with this constraint, we must include some k_T smearing.
- The optimal amount of smearing is less for NRQCD factorization than for the CEM.

Conclusion

- We compared CEM and NRQCD predictions for charmonium production with the CDF data.
  - NLO $2 \rightarrow 1$ parton processes are included.
  - Multiple gluon emission effect is included using k_T-smearing.
- CEM
  - not satisfactory in both normalization and slope.
  - k_T smearing improves CEM prediction but still unsatisfactory.
- NRQCD
  - NRQCD factorization has more free parameters than the CEM, but it gives a satisfactory fit to the data.
  - In the P-wave case, which is constrained by decay data, k_T smearing is essential to obtain a satisfactory fit.
- Proper inclusion of effects of multiple soft-gluon emission could provide a stringent test of NRQCD factorization in the P-wave case.