Transversity in SIDIS and Drell Yan $k_\perp$ Dependence and "T-Odd" Effects in TSSAs & Azimuthal Asymmetries

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- Remarks Transverse Spin and Azimuthal Asymmetries in QCD
- ⋆ Reaction Mechanisms: Colinear ETQS-Twist Three, Beyond Co-lineararity BHS- FSI Twist Two
- ⋆ Unintegrated PDF and FFs-ISI/FSI: "T-odd" TMDs Distribution and Fragmentation Functions: Correlations btwn intrinsic $k_\perp$, transverse spin S_T
- ⋆ TSSA and Estimates of the Collins and Sivers Functions
- ⋆ Double T-odd $\cos 2\phi$ asymmetry: SIDIS & DRELL-YAN
- Conclusions

* G. R. Goldstein, Tufts, K.A. Oganessyan *Financial District NYC*, and D.S. Hwang Sejong, Andreas Metz, Marc Schlegel Bochum

Transverse SSA (TSSA) and AZIMUTHAL ASYMMETRIES (AA)

⋆ Co-linear approximation of QCD PREDICTS
vanishingly small TSSA at large scales and leading order α_s

- Generically,

$$|\perp/\top\rangle = \frac{1}{\sqrt{2}}(\,|+\rangle \pm i|-\rangle\,) \Rightarrow A_N = \frac{d\hat{\sigma}^{\perp} - d\hat{\sigma}^{\top}}{d\hat{\sigma}^{\perp} + d\hat{\sigma}^{\top}} \sim \frac{2\,Im\,f^{*\,+}f^{-}}{|f^{+}|^2 + |f^{-}|^2}$$

⋆ Requires *helicity flip* as well as relative phase btwn helicity amps

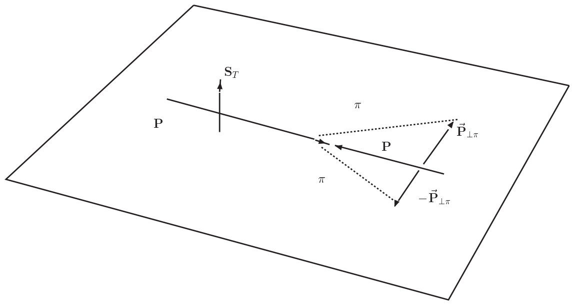
- Massless QCD conserves helicity & Born amplitudes are real!

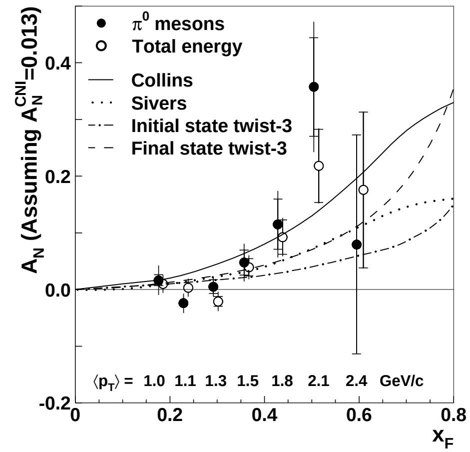
⋆ Incorporating Interference btwn loops-tree level Kane, Repko, PRL:1978
yield $A_N \sim m_q \alpha_s / \sqrt{s}$

Transverse SSA (TSSA) and AZIMUTHAL ASYMMETRIES (AA)

- LARGE TSSAS OBSERVED: E704-Fermi Lab, STAR & PHENIX

$$A_N = \frac{d\sigma^{p^{\uparrow}p\to\pi\ X} - d\sigma^{p^{\downarrow}p\to\pi\ X}}{d\sigma^{p^{\uparrow}p\to\pi\ X} + d\sigma^{p^{\downarrow}p\to\pi\ X}}$$





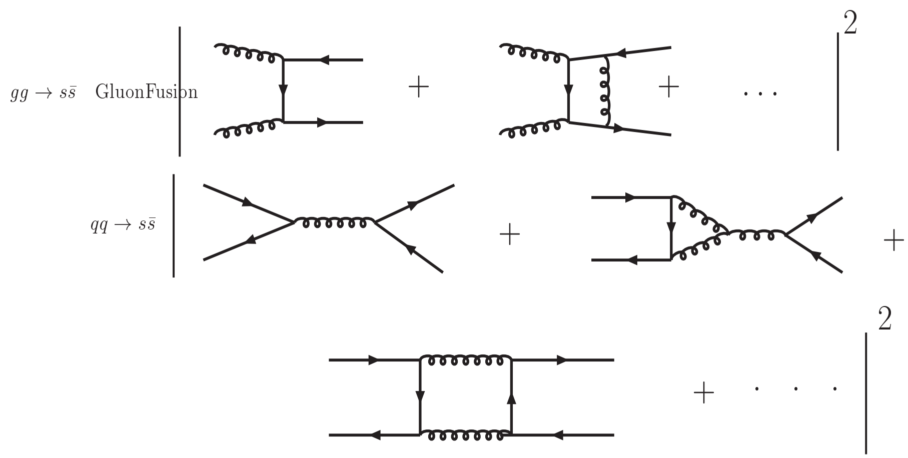
L-R asymmetry of π production and A_N for π_0 production at STAR : PRL 2004

Inclusive Λ Production From PQCD ($pp \to \Lambda^{\uparrow} X$)

PQCD contributions calculated: Dharmartna & Goldstein PRD 1990

$$P_{\Lambda} = \frac{d\sigma^{pp\to\Lambda^{\uparrow}X} - d\sigma^{pp\to\Lambda^{\downarrow}X}}{d\sigma^{pp\to\Lambda^{\uparrow}X} + d\sigma^{pp\to\Lambda^{\downarrow}X}}$$

- Need a strange quark to Polarize a Λ ($pp \to \Lambda^{\uparrow} X$)

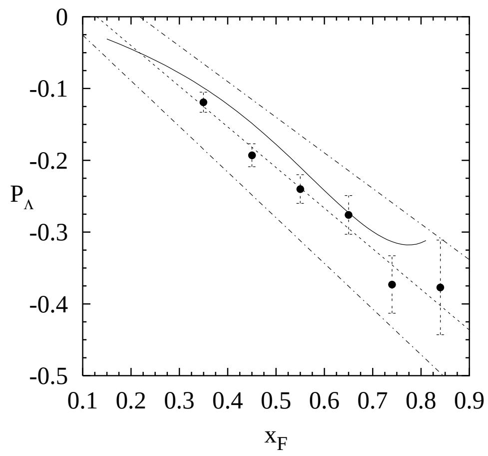


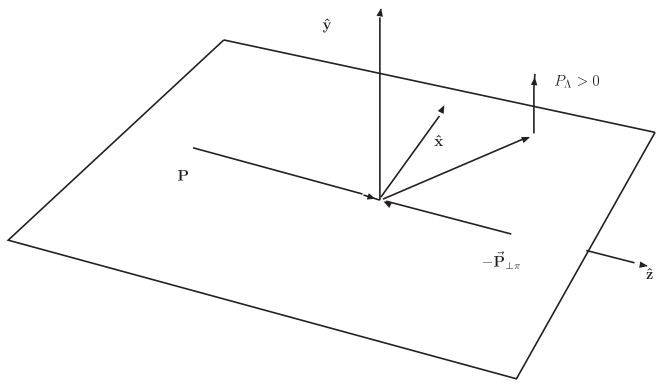
- $P_{\Lambda} \sim m_q \alpha_s/\sqrt{s}$: Effect is twist 3 & small $\approx 5\%$ as predicted on general grounds, m_q is the strange quark mass.

- Experiment *glaringly at odd with this result.*

P_Λ in p-p scattering from Fermi Lab

$$P_\Lambda = \frac{d\sigma^{pp\to\Lambda^\uparrow X} - d\sigma^{pp\to\Lambda^\downarrow X}}{d\sigma^{pp\to\Lambda^\uparrow X} + d\sigma^{pp\to\Lambda^\downarrow X}}$$

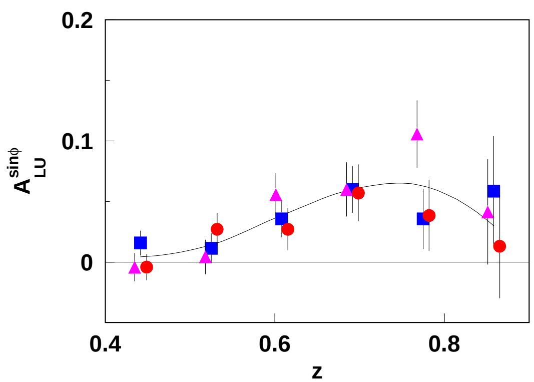


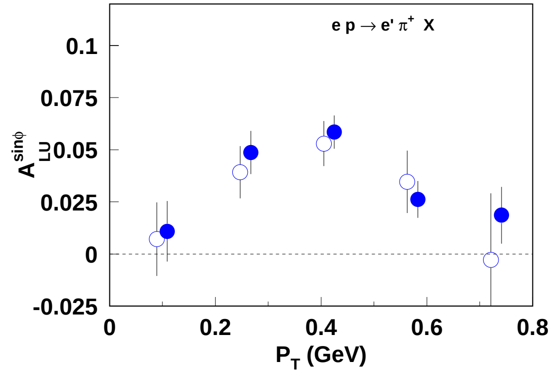


Heller,...,Bunce PRL:1983 PRL: 1983: Up-down asymmetry depicted for Λ production in p-p COM-frame.

Beam Spin Asymmetry HERMES and CLAS $\sin\phi \quad \to g^{\perp}$?

PRD-2004 CLAS   Bacchetta et al. PRD 2004, Afanesev & Carlson hep-ph/0603269, L.G., Hwang, Metz, Schlegel hep-ph/0604022



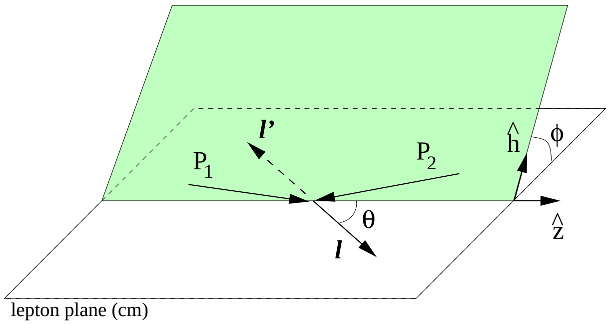


σ_{LU} specify the beam and target polarizations, respectively azimuthal angle ϕ is defined by a triple product:

$$\sin\phi = \frac{[\vec{k}_1 \times \vec{k}_2] \cdot \vec{P}_\perp}{|\vec{k}_1 \times \vec{k}_2||\vec{P}_\perp|} \sim \boldsymbol{S}_{e^-} \cdot (\boldsymbol{q}_\gamma \times \boldsymbol{p})$$

$$\frac{d\sigma_{LU}}{dx_B dy\, dz_h d^2P_\perp} \propto \lambda_e \sqrt{y^2+\gamma^2}\sqrt{1-y-\frac{1}{4}\gamma^2}\ \sin\phi\ \mathcal{H}'_{LT} \to \frac{g^\perp}{f_1}\ ?$$

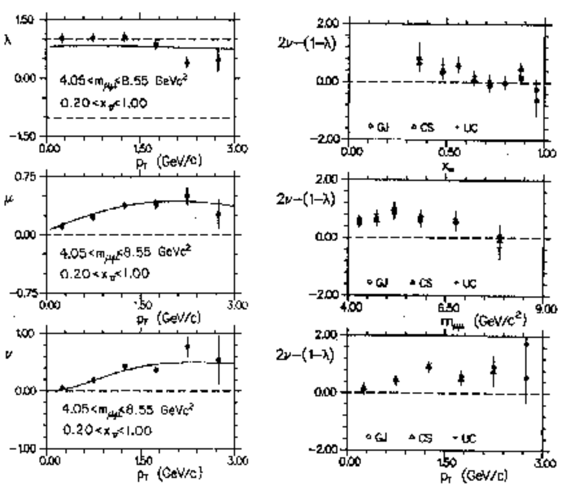
Unpolarized DRELL YAN $\cos 2\phi$



$$\bar{p} + p \to \mu^- \mu^+ + X$$

$$\frac{dN}{d\Omega} = (\frac{d\sigma}{d^4q})^{-1} \frac{d\sigma}{d^4q d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left(1 + \lambda \cos^2 \theta + \mu \sin^2 \theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right) \quad (1)$$

λ, μ, ν, **depend on** $s, x, m^2_{\mu\mu}, q_T$: **QCD-parton model NLO and NNLO predict Lam-Tung relation** $1 - \lambda - 2\nu = 0$



Measurements of $\pi + p \to \mu^+ + \mu^- + X$ discovered unexpectedly large values of these asymmetries [E615,NA10] compared to parton-model expectations resulting in violation

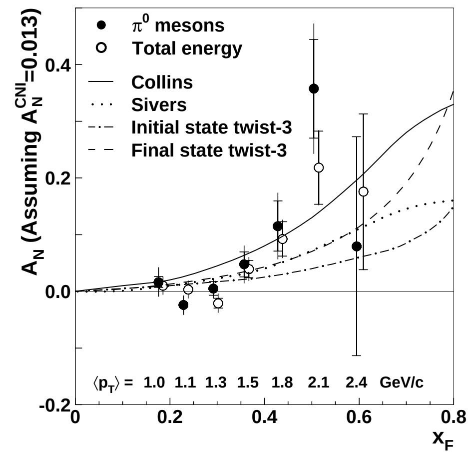
of the Lam-Tung relation

ETQS-Twist Three Mechanism (see talk Feng Yuan)

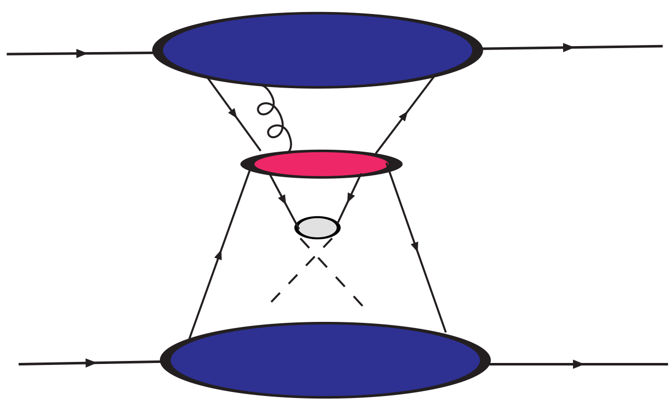
can describe TSSAs $p\,p^{\perp} \rightarrow \pi X$

@ Lg. P_T, A_N twist three but phases can be generated in co-linear QCD from gluonic and fermionic poles in propagator of hard parton subprocess

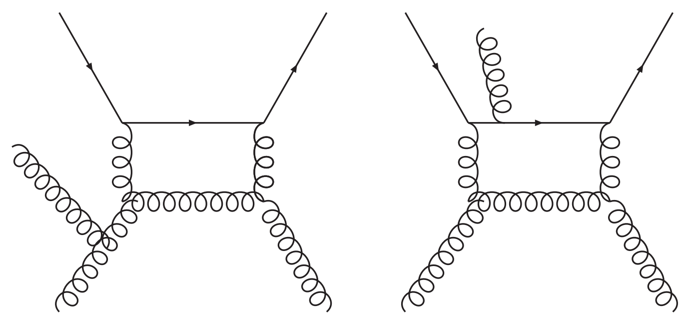
$$\frac{1}{x+s\pm i\epsilon} = P\frac{1}{x-s} \mp i\pi\delta(x-s)$$



- Qiu & Sterman :PLB 1991, 1999 & Koike & Kanazawa:PLB 2000 at Large $P_T > \Lambda_{qcd}$get helicity flip and phases

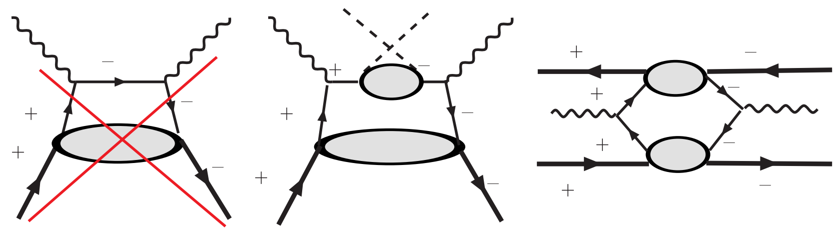


$\Rightarrow$



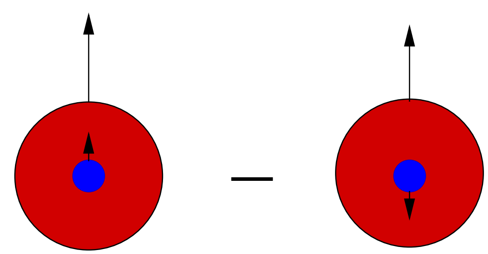
Helicity Flips Accommodated in Hard Scattering, from "Transversity" Distributions

Drell-Yan $p_\perp \, p_\perp \Rightarrow l^+ l^- \; X$ (2 in the initial)
SIDIS $l \; p_\perp \Rightarrow l' \, h \; X$ (1 in initial 1 in final)



⋆ DY:Ralston and Soper NPB:1979 encountered double transverse spin asymmetry

$$A_{TT}^{DY} = \frac{2\sin^2\theta\cos(\phi_1+\phi_2)}{1+\cos^2\theta}\frac{\sum_a e_a^2 h_1^a(x)\overline{h}_1^a(x)}{\sum_a e_a^2 f_1^a(x)\overline{f}_1^a(x)}$$

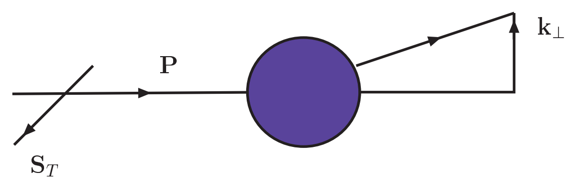


$h_1(x)$ **probability to find quark with spin polarized along transverse spin direction minus oppositely polarized case**

$k_\perp \sim \Lambda_{\rm qcd}$ "Naive-T-Odd" Correlations through TMD PDFs and FFs

- Sensitivity to $\boldsymbol{k}_\perp$ intrinsic quark momenta, associated non-perturbative transverse momentum distribution functions **TMD**
  Soper, PRL:1979: $\int d\boldsymbol{k}_\perp \mathcal{P}(\boldsymbol{k}_\perp, x) = f(x)$

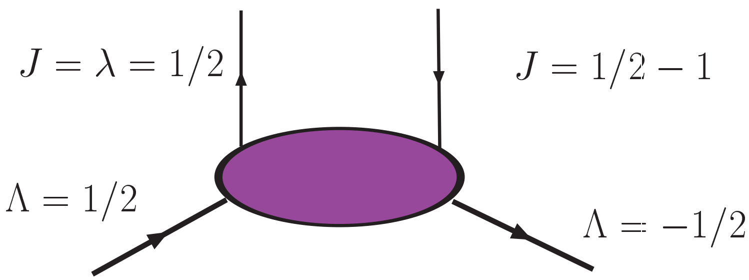
- TSSA indicative "T-odd" correlations among *transverse* spin and momenta
  Sivers: PRD 1990 e.g. $P\, P^\perp \to \pi\, X$ $\quad \boldsymbol{S}_T \cdot (\boldsymbol{P} \times \boldsymbol{k}_\perp) \to f_{1T}^\perp(x, \boldsymbol{k}_\perp)$



- Correlation accounts for left-right TSSA in inclusive π production (Sivers: PRD 1990, Anselmino & Murgia PLB: 1995 ...)

- Collins NPB 1993 T-odd correlation of transversely polarized fragmenting quark: TSSA in lepto-production $\ell\, \vec{p} \to \ell'\, \pi\, X$ $\quad s_T \cdot (\boldsymbol{p} \times \boldsymbol{P}_{h\perp})$, $\to H_1^\perp(x, \boldsymbol{p}_\perp)$
  s_T spin of fragmenting quark, $\boldsymbol{p}$ quark momentum and $\boldsymbol{P}_{h\perp}$ transverse momentum produced pion

Importance of transversity properties and intrinsic $k_\perp$:
Sivers Function furnishes Hadron helicity flip through orbital angular momentum i.e. quarks have $\boldsymbol{k}_\perp$



- Intriguing connection of Sivers function $f_{1T}^{\perp(q)} \sim -\kappa^q$ with anomolus magnetic moment of quark-q through the impact parameter space representation of the spin flip chirally-even GPD $\mathcal{E}(x, \boldsymbol{b}\perp)$: serves to fix sign of Sivers function

- As well $k_T^q \to h_1^{\perp q}$ through $\tilde{H}(x, 0, -\Delta_\perp)$ and $E_T(x, 0, -\Delta)$ (chirally odd transversity GPDs) where κ_T governs the transverse spin-flavor dipole moment in an unpolarized target. Burkardt, Diehl, Hägler et. al

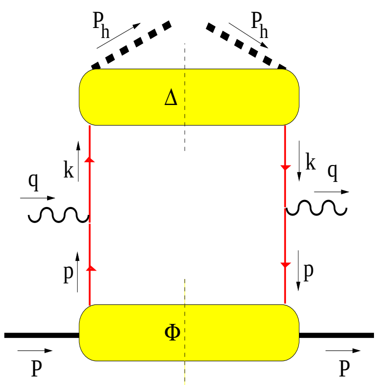
⋆ The study of Transversity properties of quarks-hadrons goes well beyond h_1.

Beyond Co-linear QCD: T-Odd Correlations

Recent Times Boer & Mulders and Co. incorporated $\boldsymbol{k}_\perp$ T-odd PDFs and FFs: Relevant to hard scattering QCD at leading twist. Adopted Factorized Description

Ellis, Furmanski, Petronzio NPB: 1982, Collins *et al. PQCD...* : 82 , J. Qui PRD: 1990, Levelt & Mulders, Mulders & Tangerman, NPB: 1994, 1996

$$\frac{d\sigma^{\ell N\to\ell' hX}}{dxdydzd^2P_{h\perp}} = \frac{M\pi\alpha^2 y}{2Q^4 z} L_{\mu\nu}\mathcal{W}^{\mu\nu}$$

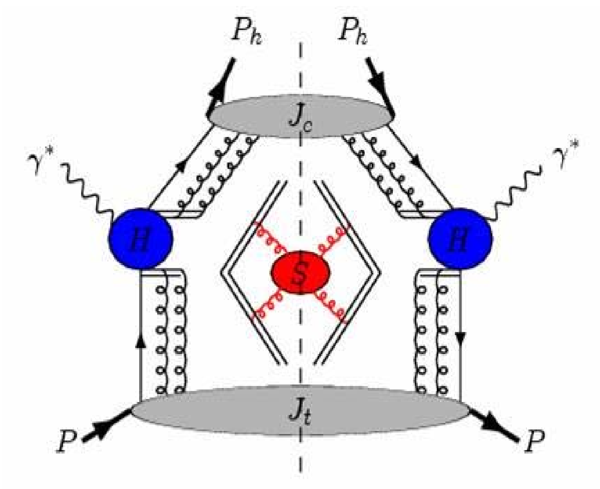


Hadronic Tensor

$$2M\mathcal{W}^{\mu\nu}(q,P,P_h) = \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^2(\boldsymbol{p}_T+\boldsymbol{q}_T-\boldsymbol{k}_T)\,\mathrm{Tr}[\Phi(x_B,\boldsymbol{p}_T)\gamma^\mu\Delta(z_h,\boldsymbol{k}_T)\gamma^\nu] + (q\leftrightarrow -q\ ,\ \mu\leftrightarrow\nu)$$

Factorization Demonstrated For TMD PDF and FF and Hard and Soft Parts

More recently Ji, Ma, Yuan: 2004 building on work of Collins-Soper NPB: 81, extended factorization theorems to 1-loop and beyond

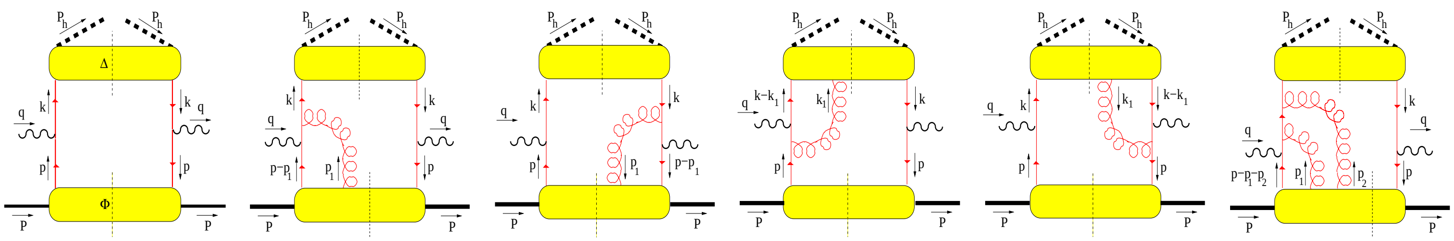


Suggestions of Universality & Collins and Metz: 2005

T-Odd Effects to QCD Processes Naturally Built into Color Gauge Invariant Factorized QCD at ''leading twist'' thru-Wilson Line &

- Gauge Invariant Distribution and Fragmentation Functions

  Boer, Mulder: NPB 2000, Ji, Yuan & Belitsky PLB: 2002, NPB 2003, Boer, Mulder, Pijlman NPB 2003



etc . . .

Sub-class of loops in eikonal limit (soft gluons) sum up to yield color gauge invariant hadronic tensor factorized into the distribution Φ and fragmentation Δ operators

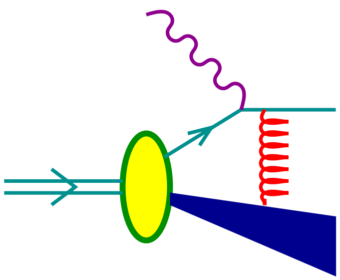
$$\Phi(p,P) = \int \frac{d^3\xi}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(\xi^-,\xi_\perp)\mathcal{G}^\dagger_{[\xi^-,\infty]}|X\rangle\langle X|\mathcal{G}_{[0,\infty]}\psi(0)|P\rangle|_{\xi^+=0}$$

$$\Delta(k,P_h) = \int \frac{d^3\xi}{4z(2\pi)^3}\, e^{ik\cdot\xi}\, \langle 0\,|\mathcal{G}_{[\xi^+,-\infty]}\psi(\xi)|X;P_h\rangle\langle X;P_h|\overline{\psi}(0)\mathcal{G}^\dagger_{[0,-\infty]}|0\rangle|_{\xi^-=0}$$

$$\mathcal{G}_{[\xi,\infty]} = \mathcal{G}_{[\xi_T,\infty]}\mathcal{G}_{[\xi^-,\infty]}, \quad \text{where} \quad \mathcal{G}_{[\xi^-,\infty]} = \mathcal{P}exp(-ig\int_{\xi^-}^{\infty} d\xi^- A^+)$$

Rescattering-ISI/FSI T-Odd Contributions to Asymmetries

PLB: 2002 Brodsky, Hwang, and Schmidt demonstrate rescattering of a gluon could produce the necessary phase leading to nonzero SSAs at *Leading Twist*



Initial-Final state effect: $\boldsymbol{S}_T \cdot (\boldsymbol{P} \times \boldsymbol{k}_\perp)$

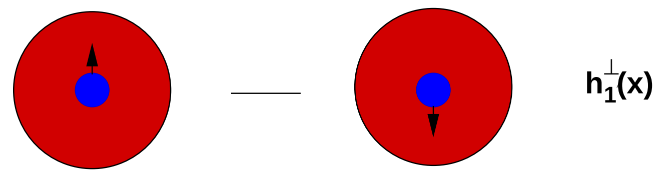
- Ji, Yuan PLB: 2002 describe effect in terms of gauge invariant distribution functions
- Demonstrates that BHS calculated Sivers Function $f_{1T}^{\perp}(x, k_\perp)|_{\rm SDIS}$
  In Singular gauge, $A^+ = 0$, effect remains
- Collins, PLB: 2002, modifies earlier claim of trivial Sivers Effect
  $f_{1T}^{\perp}(x, k_\perp)|_{\rm SDIS} = -f_{1T}^{\perp}(x, k_\perp)|_{\rm DY}$

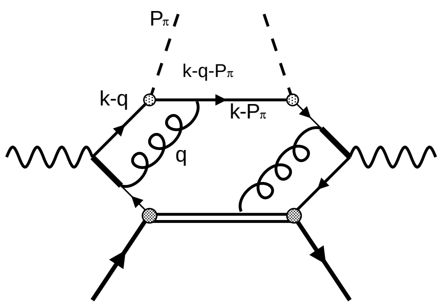
FSI Mechanism can Generate Boer-Mulders-$h_1^\perp$

Goldstein, L.G.–ICHEP-proc-hep-ph/0209085 (2002), L.G., Goldstein, Oganessyan PRD 2003

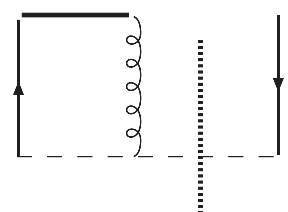
- $h_1^\perp$ "Naturally" defined from Color G.I. TMD: Convoluted with $H_1^\perp$ enters $\cos 2\phi$
- "Eikonal Feynman rules" to calculate Collins Soper: NPB: 1982

$$\Phi^{[\sigma^{\perp+}\gamma_5]}_{[h_1^\perp]}(x,k_\perp) = \frac{1}{2}\int dp^- \,\mathrm{Tr}\left(i\sigma^{+\perp}\gamma_5\Phi\right) = \frac{\varepsilon_{+-\perp j}k_{\perp j}}{M} h_1^\perp(x,k_\perp)$$





\+ h.c.



$+ \quad H.C..$

$$\Phi^{[\Gamma]}(x,k_\perp) = \sum_X \int \frac{d\xi^- d^2\xi_\perp}{2(2\pi)^3} e^{-i\xi\cdot\vec{k}_\perp} \langle P|\overline{\psi}(\xi)\mathcal{G}^\dagger_{[\xi^-,\infty]}|X\rangle\langle X|\mathcal{G}_{[0,\infty]}\Gamma\psi(0)|P\rangle|_{\xi^+=0}$$

$h_1^\perp(x,k_\perp)$, represents, number density transversely polarized quarks in an unpolarized nucleons nucleons complementary to $f_{1T}^\perp(x,k_\perp)$

Provide source of T-Odd Contributions to TSSA and AA

- **Enter the *leading twist* distribution and fragmentation correlators "T-odd" Distribution Functions: Transversity Properties of quarks in Hadrons**

Boer, Mulder: PRD 1998

$$\Delta(z, \boldsymbol{k}_\perp) = \frac{1}{4}\{D_1(z, z\boldsymbol{k}_\perp)\, \not{n}_- + H_1^\perp(z, z\boldsymbol{k}_\perp)\frac{\sigma^{\alpha\beta}k_{\perp\alpha}n_{-\beta}}{M_h} + D_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_-^\nu k_\perp^\rho S_{hT}^\sigma}{M_h} + \cdots\},$$

$$\Phi(x, \boldsymbol{p}_\perp) = \frac{1}{2}\{f_1(x, \boldsymbol{p}_\perp)\, \not{n}_+ + h_1^\perp(x, \boldsymbol{p}_\perp)\frac{\sigma^{\alpha\beta}p_{T\alpha}n_{+\beta}}{M} + f_{1T}^\perp(x, \boldsymbol{p}_\perp)\frac{\epsilon^{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_\perp^\rho S_T^\sigma}{M}\cdots\}$$

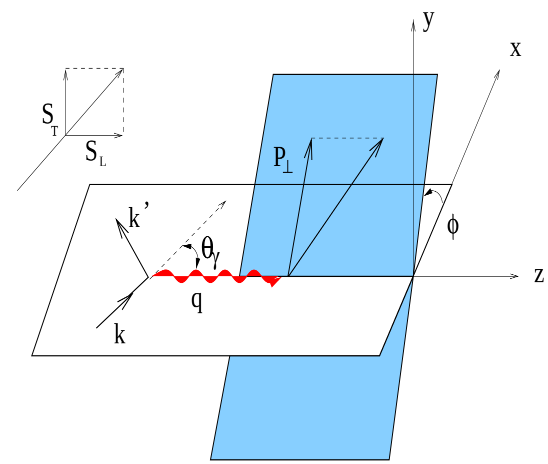
SIDIS cross section

$$\begin{aligned} d\sigma^{\ell N\to\ell\pi X}_{\{\lambda,\Lambda\}} \quad &\propto \quad f_1 \otimes d\hat{\sigma}^{\ell q\to\ell q} \otimes D_1 + \frac{k_\perp}{Q} f_1 \otimes d\hat{\sigma}^{\ell q\to\ell q} \otimes D_1 \cdot \cos\phi \\ &+ \quad \left[\frac{k_\perp^2}{Q^2} f_1 \otimes d\hat{\sigma}^{\ell q\to\ell q} \otimes D_1 + h_1^\perp \otimes d\hat{\sigma}^{\ell q\to\ell q} \otimes H_1^\perp\right] \cdot \cos 2\phi \\ &+ \quad |S_T| \cdot h_1 \otimes d\hat{\sigma}^{\ell q\to\ell q} \otimes H_1^\perp \cdot \sin(\phi + \phi_S) \quad \text{Collins} \\ &+ \quad |S_T| \cdot f_{1T}^\perp \otimes d\hat{\sigma}^{\ell q\to\ell q} \otimes D_1 \cdot \sin(\phi - \phi_S) \quad \text{Sivers} \\ &+ \quad \cdots \end{aligned}$$

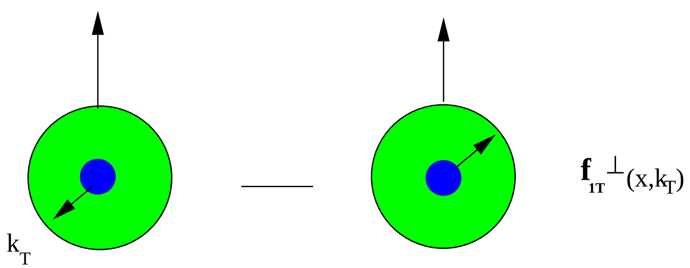
SIDIS-Transversity Properties at Leading Twist

- Collins NPB:1993, Kotzinian NPB:1995, Mulders, Tangerman PLB:1995

$$\langle \frac{P_{h\perp}}{M_\pi} = \sin(\phi+\phi_s)\rangle_{UT} \frac{\int d\phi_s \int d^2P_{h\perp} \frac{P_{h\perp}}{M_\pi} \sin(\phi+\phi_s)\left(d\sigma^{\uparrow} - d\sigma^{\downarrow}\right)}{\int d\phi_s \int d^2P_{h\perp}\left(d\sigma^{\uparrow}+d\sigma^{\downarrow}\right)} = |S_T| \frac{2(1-y)\sum_q e_q^2 h_1(x) z H_1^{\perp(1)}(z)}{(1+(1-y)^2)\sum_q e_q^2 f_1(x) D_1(z)}$$



- Sivers PRD: 1990, Anselmino & Murgia PLB: 1995 ...

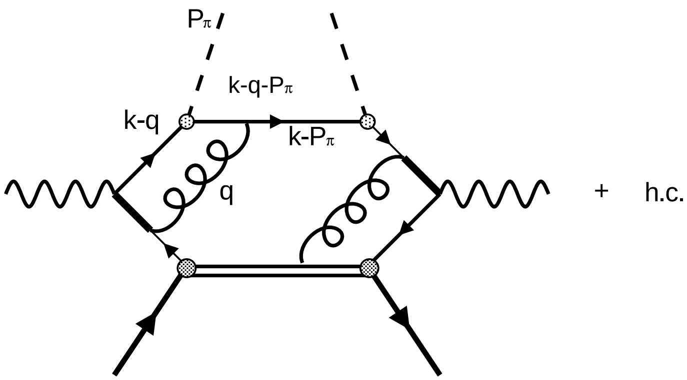


$$\langle \frac{|P_{h\perp}|}{M} \sin(\phi - \phi_S)\rangle_{UT} \quad = \quad |\boldsymbol{S}_T| \frac{(1+(1-y)^2)\sum_q e_q^2 f_{1T}^{\perp(1)}(x) z D_1^q(z)}{(1+(1-y)^2)\sum_q e_q^2 f_1(x) D_1(z)},$$

- Probes the probability for a transversely polarized target, pions are produced asymmetrically about pion production plane

$\cos 2\phi$ **Asymmetry** Generated by ISI & FSI thru Gauge link

Goldstein, L.G.–ICHEP-Amsterdam: 2002, hep-ph/0209085, G,G, & Oganessyan PRD:2003



$$A_{UU}^{\cos(2\phi)} = \langle \frac{|P_{h\perp}^2|}{MM_h} \cos 2\phi \rangle_{UU} = \frac{8(1-y)\sum_q e_q^2 h_1^{\perp(1)}(x,Q^2) z^2 H_1^{\perp(1)q}(z,Q^2)}{(1+(1-y)^2)\sum_q e_q^2 f_1^q(x,Q^2) D_1^q(z,Q^2)}$$

$$\frac{d\sigma}{dx\,dy\,dz\,d^2P_\perp} \propto f_1 \otimes D_1 + \frac{k_T}{Q} f_1 \otimes D_1 \cdot \cos\phi + \left[\frac{k_T^2}{Q^2} f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp\right] \cdot \cos 2\phi$$

Leading Twist Contribution from T-Odd D. Boer, P. Mulders, PRD: 1998

Estimates of T-odd Contribution in Drell Yan (GSI, JPARC)

$$\frac{d\Delta\sigma^{\uparrow}}{d\Omega dx_1 dx_2 d\mathbf{q}_T} \propto \sum_f e_f^2 \, |\mathbf{S}_{2_T}| \Big\{ - B(y) \, \sin(\phi + \phi_{S_2}) \, F \left[\hat{\mathbf{h}} \cdot \mathbf{p}_{1_T} \frac{\bar{h}_1^{\perp f} \, h_1^f}{M_1} \right]$$
$$+ A(y) \, \sin(\phi - \phi_{S_2}) \, F \left[\hat{\mathbf{h}} \cdot \mathbf{p}_{2_T} \frac{\bar{f}_1^f \, f_{1T}^{\perp f}}{M_2} \right] \, \ldots \Big\} \, , \qquad (2)$$

Estimates of T-odd Contribution in SIDIS (HERMES, JLAB 6& 12 GeV program)

$\cos 2\phi$ Asymmetry in SIDIS: Boer Mulders Effects Competes with Cahn Effect

⋆ The spectator model used in previous rescattering calculations assumes point-like nucleon-quark-diquark vertex, leads to logarithmically divergent, asymmetries
Goldstein, L.G., ICHEP 2002; hep-ph/0209085,
L.G., Goldstein, Oganessyan PRD 2003; Boer, Brodsky, Hwang, PRD: 2003(Drell-Yan)

$$\begin{aligned} h_1^{\perp(s)}(x,k_\perp) &= f_{1T}^{\perp(s)}(x,k_\perp) \\ &= \frac{g^2 e_1 e_2}{4\pi(2\pi)^3}\frac{(1-x)(m+xM)}{\Lambda(k_\perp^2)}\frac{M}{k_\perp^2}\ln\frac{\Lambda(k_\perp^2)}{\Lambda(0)} \end{aligned}$$

$$\Lambda(k_\perp^2) = k_\perp^2 + x(1-x)\left(-M^2 + \frac{m^2}{x} + \frac{\mu^2}{1-x}\right)$$

- Asymmetry involves weighted function

$$h_1^{(1)\perp}(x) \equiv \int d^2k_\perp \frac{k_\perp^2}{2M^2} h_1^\perp(x,k_\perp^2) \quad \textit{diverges}$$

Gaussian Distribution in $k_\perp$

Log divergence addressed by approximating the transverse momentum dependence of the quark-nucleon-vertex by a Gaussian distribution in $k_\perp^2$,

L.G., Goldstein, Oganessyan, PRD 67 (2003)

$$\langle n|\psi(0)|P\rangle = \left(\frac{i}{\not{k} - m}\right)\Upsilon(k_\perp^2)U(P,S), \quad b \equiv \frac{1}{\langle k_\perp^2\rangle}$$

where $\Upsilon(k_\perp^2) = \mathcal{N} e^{-bk_\perp^2}.$

$U(P,S)$ nucleon spinor, and quark propagator comes from untruncated quark line

$$h_1^\perp(x,k_\perp) \quad = \quad \frac{e_1 e_2 g^2}{2(2\pi)^4}\frac{b^2}{\pi^2}\frac{(m+xM)(1-x)}{\Lambda(k_\perp^2)}\frac{1}{k_\perp^2}\mathcal{R}(k_\perp^2,x) \tag{3}$$

with

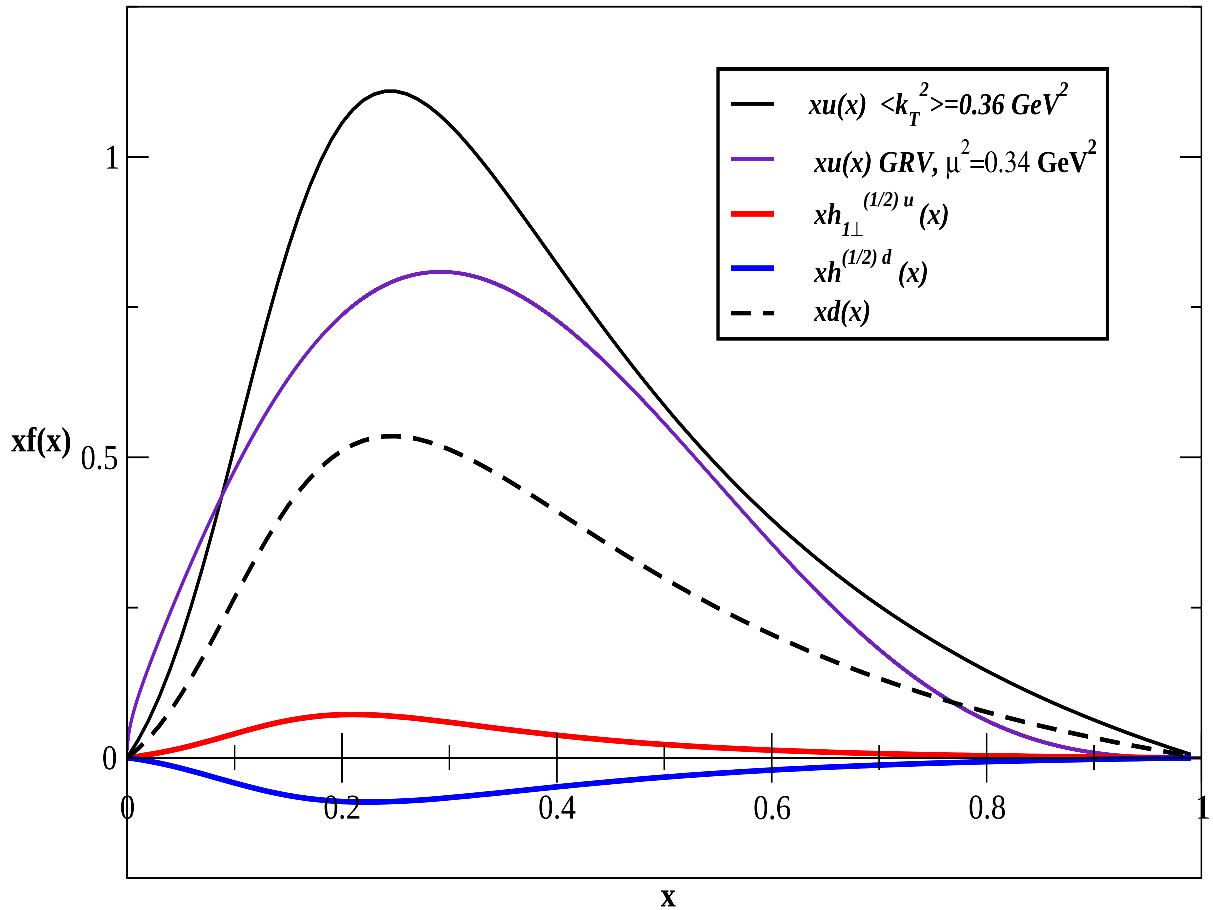
$$\mathcal{R}(k_\perp^2,x) = \exp^{-2b(k_\perp^2-\Lambda(0))}\Big(\Gamma(0,2b\Lambda(0)) - \Gamma(0,2b\Lambda(k_\perp^2))\Big)$$

- $\lim < k_\perp^2 > \to \infty$ width goes to infinity, regain *log* result

INPUTS: Boer-Mulders $h_1^{\perp(1/2)}$ and Unpolarized Structure Function $f_1(x)$

$$f_1(x) = \frac{g^2}{(2\pi)^2}\frac{b^2}{\pi^2}(1-x)\cdot\left\{\frac{(m+xM)^2-\Lambda(0)}{\Lambda(0)} - \left[2b\left((m+xM)^2-\Lambda(0)\right)-1\right]e^{2b\Lambda(0)}\Gamma(0,2b\Lambda(0))\right\}$$

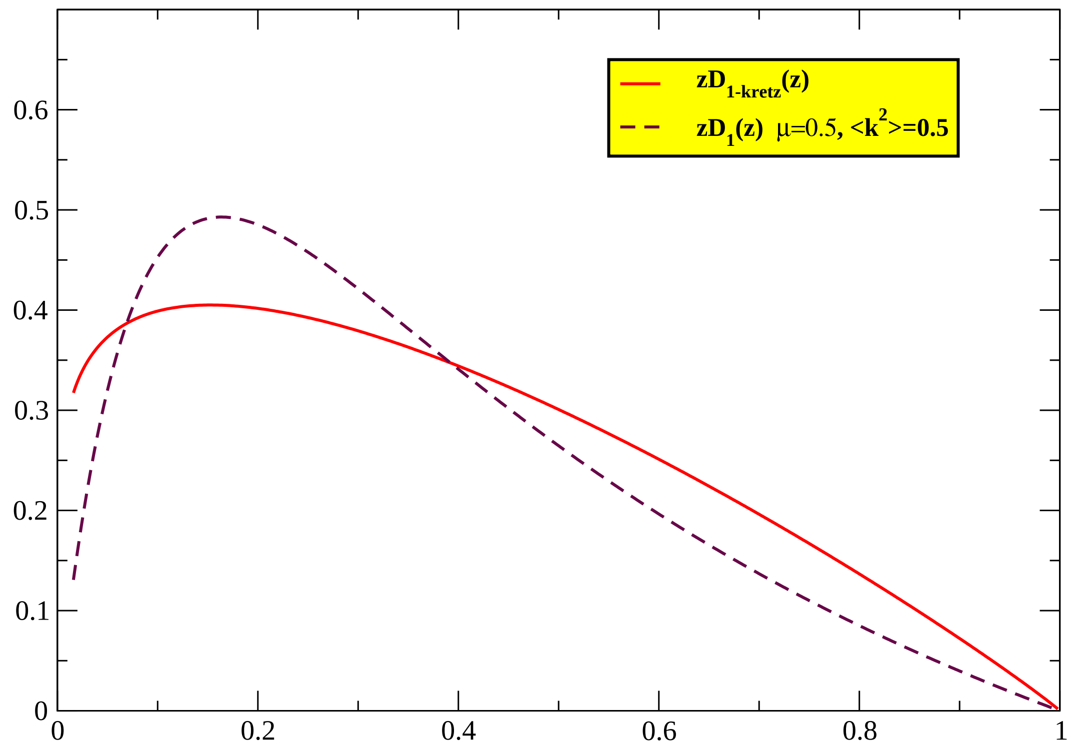
- ⋆ Normalization, $\int_0^1 u(x) = 2$
  $\int_0^1 d(x) = 1$
- Black curve- $xu(x)$
- Purple curve - $xu(x)$ GRV
- Red curve $xh_1^{\perp(1/2)(u)}$
- axial vector diquark coupling
  Jakob, Mulders, Rodrigues
  NPB:1997,
  $\gamma_5(\gamma^\mu + P^\mu/M)$



Pion Fragmentation Function

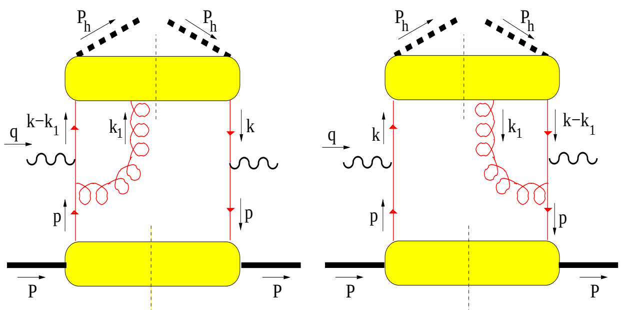
$$D_1(z) \quad = \quad \frac{{N'}^2 f_{q q\pi}^2}{4(2\pi)^2}\frac{1}{z}\frac{(1-z)}{z}\Big\{\frac{m^2-\Lambda'(0)}{\Lambda'(0)} - \Big[2b'\Big(m^2-\Lambda'(0)\Big)-1\Big]\, e^{2b'\Lambda'(0)}\Gamma(0,2b'\Lambda'(0))\Big\},$$

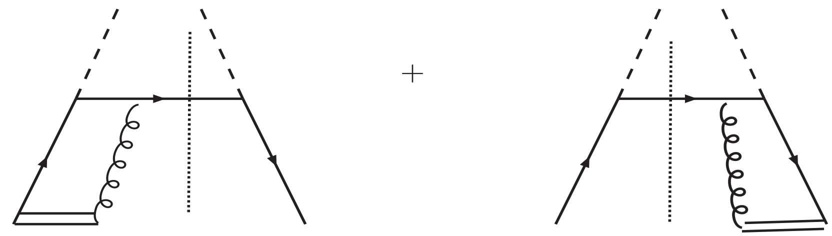
which, multiplied by z at $< k_\perp^2 >= (0.5)^2$ GeV^2 and $\mu = m$, estimates the distribution of Kretzer, PRD: 2000



Gauge Link-Pole Contribution to T-Odd Collins Function

L.G.,Goldstein,Oganessyan PRD68,2003 $\Delta^{[\sigma^{\perp -}\gamma_5]}(z,k_\perp) = \frac{1}{4z}\int dk^+ Tr(\gamma^-\gamma^\perp\gamma_5\Delta)|_{k^-=P_\pi^-/z}$

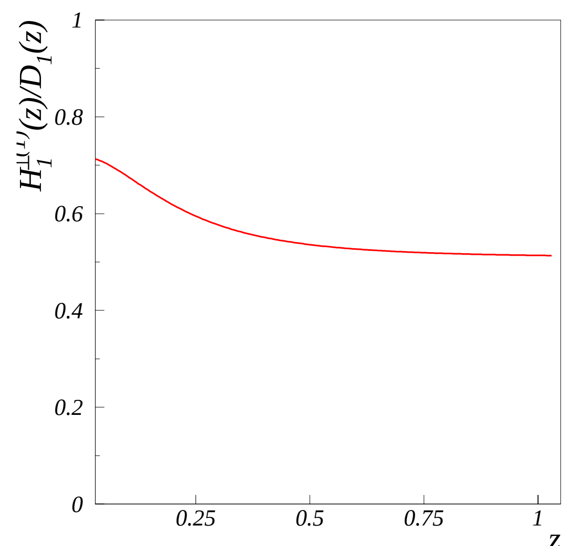




$\frac{i}{\ell^- - i\epsilon}$ *Eikonal Feynman Rules Collins '82'* $\frac{-i}{\ell^- + i\epsilon}$

Motivation:color gauge .inv frag. correlator "pole contribution"
leading twist T-odd pion fragmentation

$$H_1^\perp(z,k_\perp) = \frac{N'^2 f^2 g^2}{(2\pi)^4}\frac{1}{4z}\frac{(1-z)}{z}\frac{\mu}{\Lambda'(k_\perp^2)}\frac{M_\pi}{k_\perp^2}\mathcal{R}(z,\boldsymbol{k}_\perp^2)$$



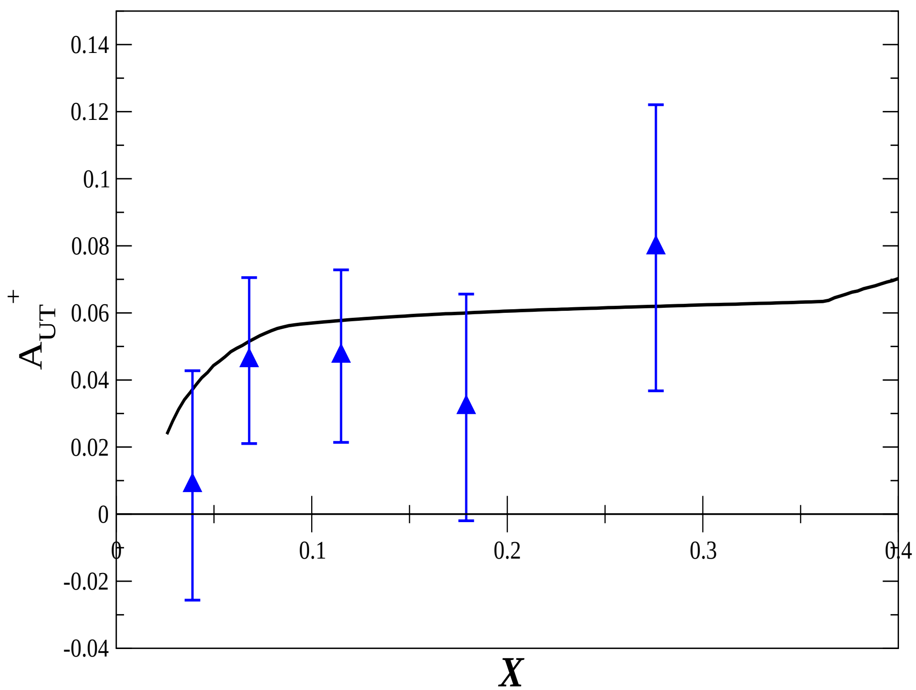
Collins Asymmetry

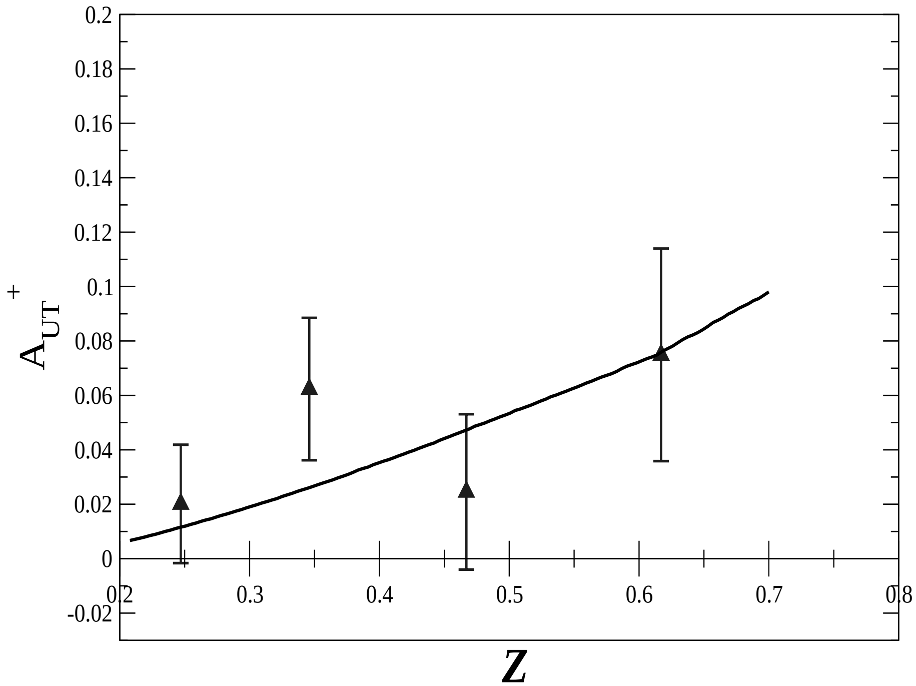
L.G., Goldstein, Oganessyan PRD 2003: updated For the HERMES kinematics

1 GeV$^2 \leq Q^2 \leq 15$ GeV^2, 4.5 GeV $\leq E_\pi \leq 13.5$ GeV, $0.2 \leq x \leq 0.41$, $0.2 \leq z \leq 0.7$, $0.2 \leq y \leq 0.8$, $< P_{h\perp}^2 >= 0.25$ GeV^2

$$\langle \frac{P_{h\perp}}{M_\pi} \sin(\phi + \phi_s) \rangle_{UT} = |S_T| \frac{2(1-y) \sum_q e_q^2 h_1(x) z H_1^{\perp(1)}(z)}{(1 + (1-y)^2) \sum_q e_q^2 f_1(x) D_1(z)}.$$

Data from A. Airapetian et al. PRL94,2005

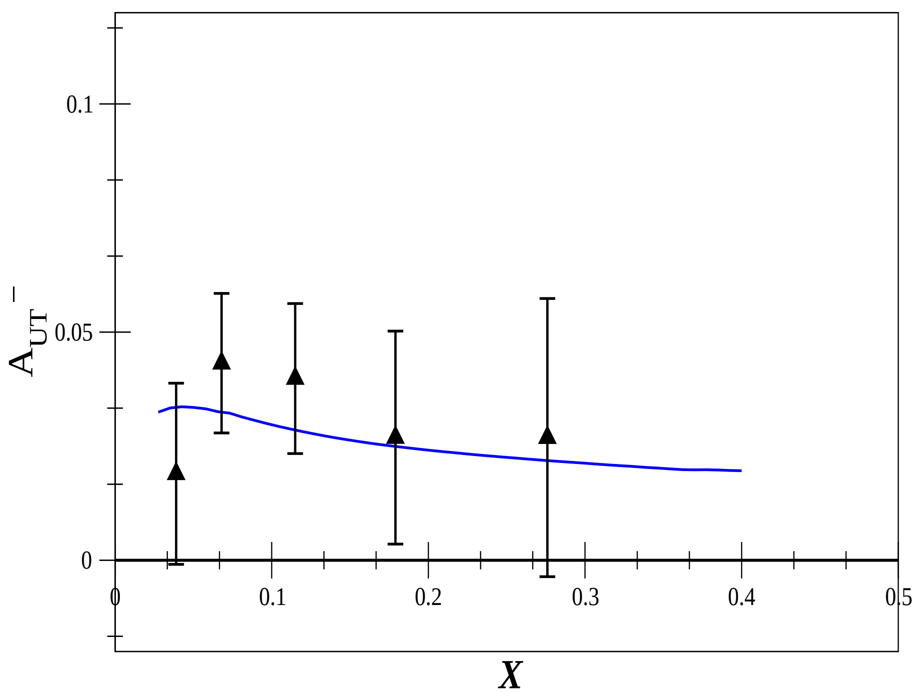


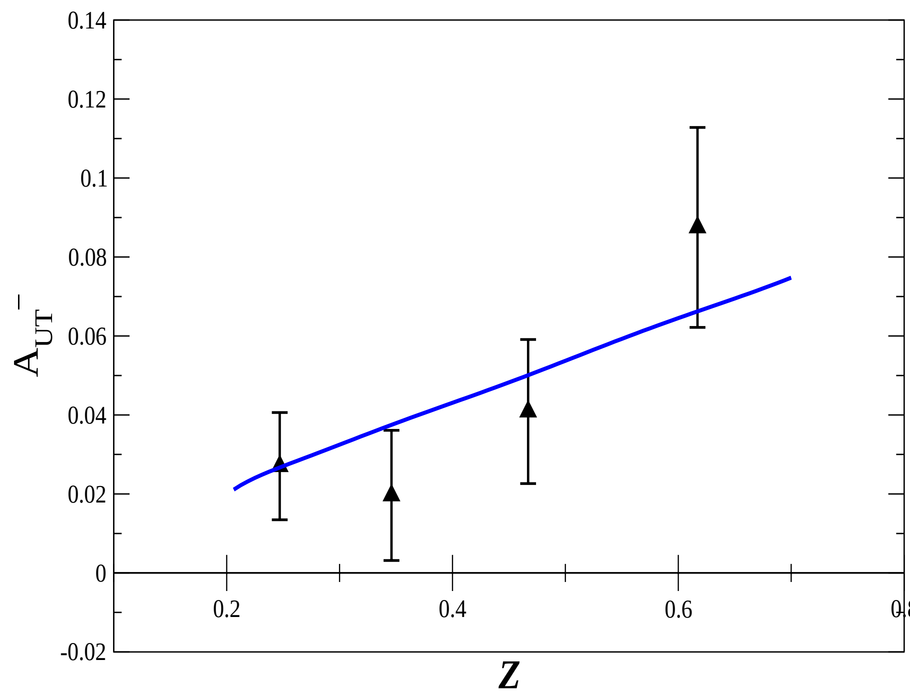


Estimates for Sivers Asymmetry

Data from A. Airapetian et al. PRL94,2005

$$\langle \frac{|P_{h\perp}|}{M} \sin(\phi - \phi_S) \rangle_{UT} = \frac{\int d^2 P_{h\perp} \frac{|P_{h\perp}|}{M} \sin(\phi - \phi_S)\, d\sigma}{\int d^2 P_{h\perp}\, d\sigma} = \frac{(1 + (1-y)^2) \sum_q e_q^2 f_{1T}^{\perp(1)}(x) z D_1^q(z)}{(1 + (1-y)^2) \sum_q e_q^2 f_1(x) D_1(z)},$$

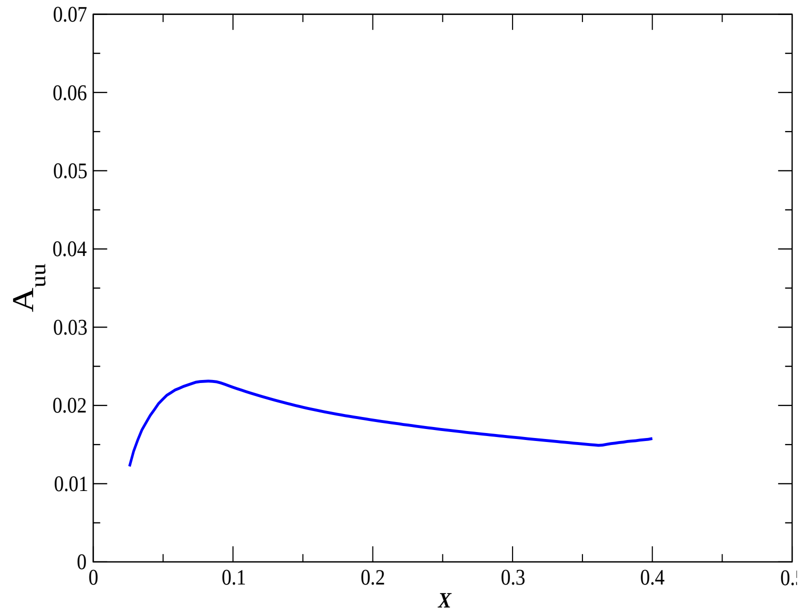


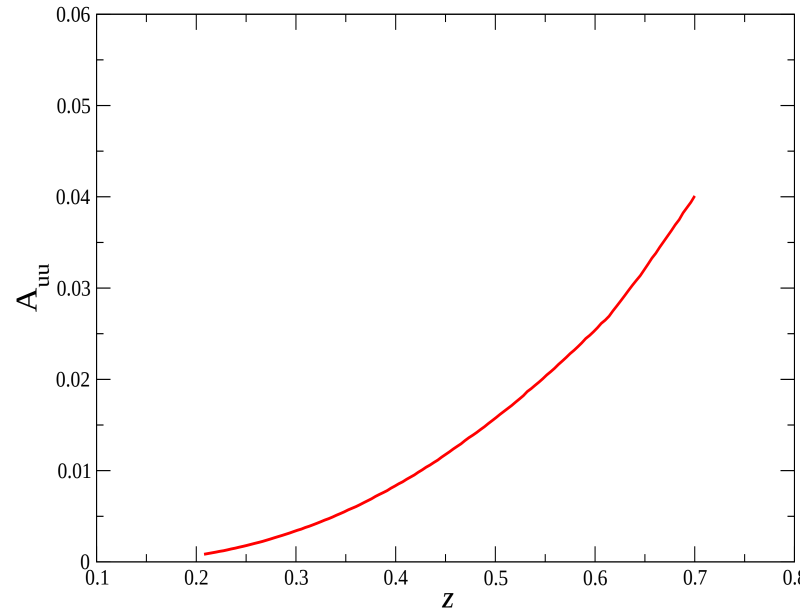


Double T-odd $\cos 2\phi$ asymmetry

Transversity of quarks inside an unpolarized hadron, and $\cos 2\phi$ asymmetries in unpolarized semi-inclusive DIS

$$\langle \frac{|P_{h\perp}^2|}{MM_h} \cos 2\phi \rangle_{UU} = \frac{\int d^2 P_{h\perp} \frac{|P_{h\perp}^2|}{MM_h} \cos 2\phi \, d\sigma}{\int d^2 P_{h\perp} \, d\sigma} = \frac{8(1-y) \sum_q e_q^2 h_1^{\perp(1)}(x) z^2 H_1^{\perp(1)}(z)}{(1+(1-y)^2) \sum_q e_q^2 f_1(x) D_1(z)}$$

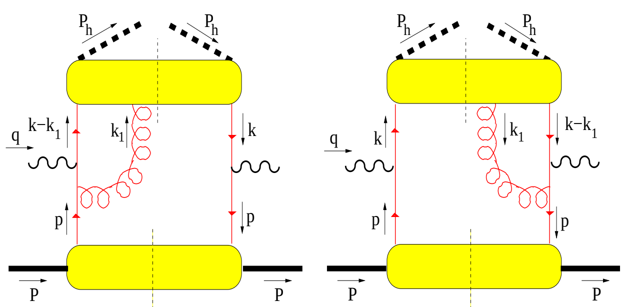


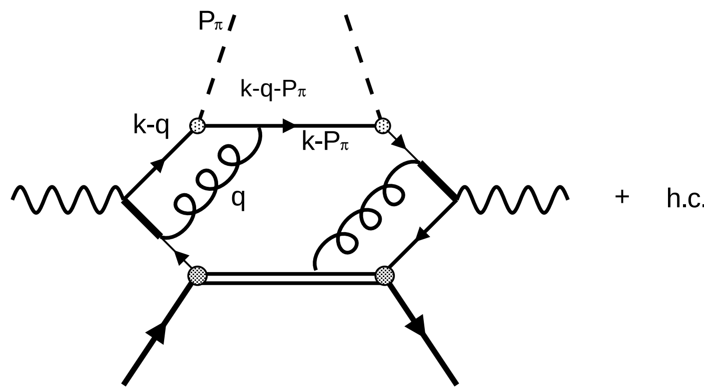


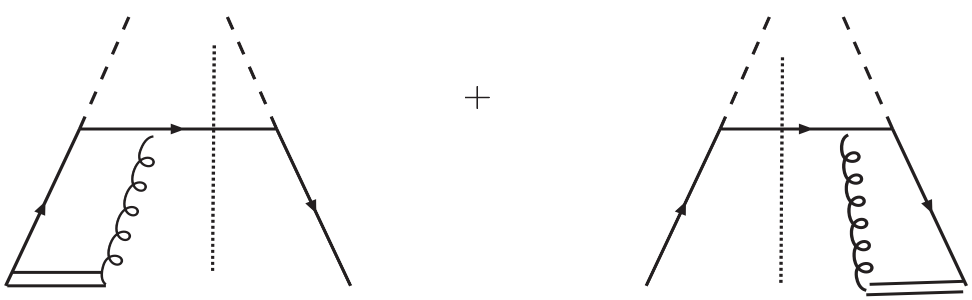
Gauge Link Contribution to Collins Function

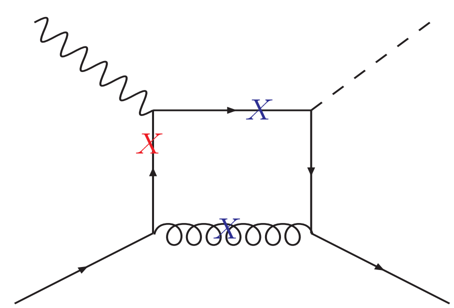
Metz: PBL 2002, L.G., Goldstein, Oganessyan PRD: 2003: Bacchetta, Metz, Jang: PLB: 2003, Amrath, Bacchetta, Metz: PRD 2005, L.G., Goldstein in progress

$\Delta^{[\sigma^{\perp -}\gamma_5]}(z,k_\perp) = \frac{1}{4z}\int dk^+ \mathrm{Tr}(\gamma^-\gamma^\perp\gamma_5\Delta)\Big|_{k^-=P_\pi^-/z}$ Boer, Pijlman, Muders: NPB 2003









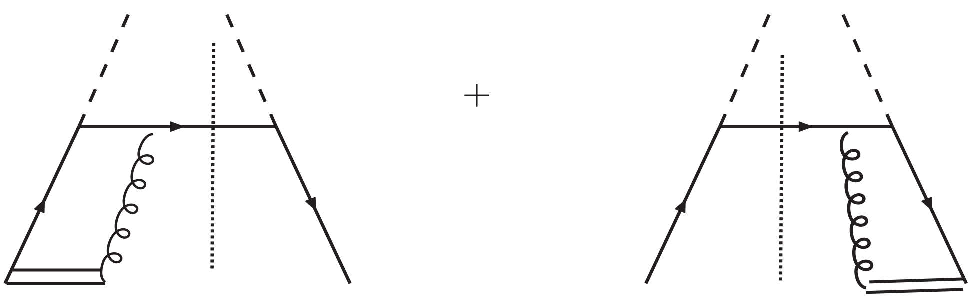
$\frac{i}{\ell^- - i\epsilon}$ *Eikonal Feynman Rules Collins '82'* $\frac{-i}{\ell^- + i\epsilon}$

On Issues of Process Dependence: Gauge Link Contribution to Collins Function

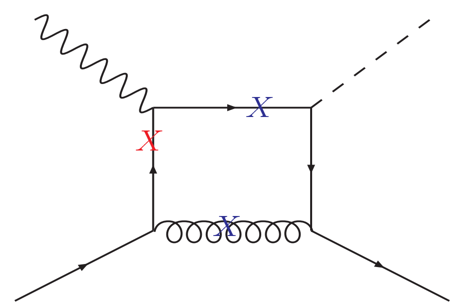
L.G., Goldstein, Oganessyan PRD: 2003: Bacchetta, Metz, Jang: PLB: 2003, Amrath, Bacchetta, Metz: PRD 2005,

L.G., Goldstein in progress

- Boer Piljman and Mulders: Two contributions to the Collins function.
  - ⋆ Gluonic Poles
  - ⋆ FSI
- The Bottom Line: Is the eikonal pole in physical regime of the Collins function Correlator? Off shell $\gamma + q \to \pi + q'$? Are these the same "objects"?



$\frac{i}{\ell^- - i\epsilon}$ $Eikonal\, Feynman\, Rules\, Collins$ $'82'$ $\frac{-i}{\ell^- + i\epsilon}$



- Explore Pole Structure of Box Diagram and the correlator side by side:
  - ⋆ Using Cauchy's theorem to evaluate the Color Gauge invariant Correlator $\Delta^{[\sigma^{\perp-}\gamma_5]}(z, k_\perp)$ eikonal pole exists.
  - Is it in the physical region?
  - What is the physical region? $\hat{s} > 0$ and or $\hat{t} > \mu^2$?

- Analysis of pole structure indicates a *endpoint singularities in loop integral-looks like a lightcone divergence*: $\delta(\ell^-)\theta(\ell^-)$

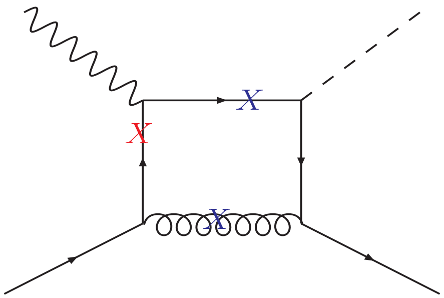
⋆ Regulate it keep n off light cone $n \cdot A$, $n = (n^-, n^+)$ (see Ji, Yuan, Ma PLB: 2004)

⋆ Pick up pole contributions in both channels

- Fragmenting quark and gluon ⇒ equivalent to cut in S-channel of box.
- Fragmenting eikonal and spectator ⇒ equivalent to cut in t-channel of box.

⋆ ? Consistent with Correlator definition? Yes "maybe"....

⋆ If this survives scrutiny in this framework, *suggests* Collins Function processes dependent between e^+e^- and SIDIS.



$$H_1^{\perp}(z, k_\perp) \quad = \quad \mathcal{N}''\alpha_s \frac{M_\pi}{4z}(1-z)\frac{\mathcal{I}_1(z, P_\perp^2) + \mathcal{I}_2(z, P_\perp^2)}{\Lambda'(P_\perp^2)P_\perp^2},$$

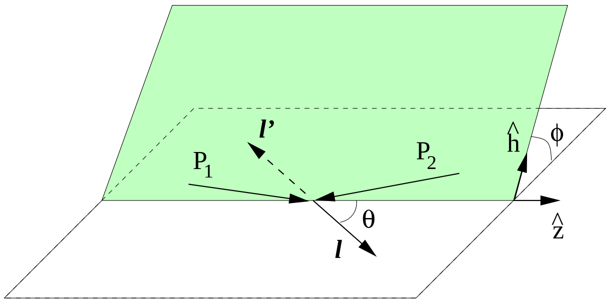
where

$$\mathcal{I}_1 \quad = \quad \pi(\mu - m(1-z))\frac{E_\pi + P\cos\theta}{P + E_\pi\cos\theta}\left[\ln\frac{(P + E_\pi\cos\theta)^2}{\mu^2} - \cos\theta\ln\frac{4P^2}{\mu^2}\right]$$

$$\mathcal{I}_2 \quad = \quad \pi\, z\, m\,\frac{P\sin^2\theta}{E_\pi - P\cos\theta}\ln\frac{4P^2}{\mu^2},$$

$P \equiv |\mathbf{P}_h|$ and $P_\perp^2 = k_\perp^2/z^2$. As in the case of the "gluonic pole" contribution, this survives the limit that incoming quark mass $m \rightarrow 0$. Both results depend the non-perturbative correlator mass μ.

Boer-Mulders Effect in Unpolarized DRELL YAN $\cos 2\phi$



$$\bar{p} + p \to \mu^- \mu^+ + X$$

$$\frac{dN}{d\Omega} = (\frac{d\sigma}{d^4 q})^{-} 1 \frac{d\sigma}{d^4 q d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left(1 + \lambda \cos^2 \theta + \mu \sin^2 \theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right) \tag{4}$$

Angles refer to the lepton pair orientation in their COM frame relative and the initial hadron's plane. Asymmetry parameters, λ, μ, ν, depend on $s, x, m^2_{\mu\mu}, q_T$

Boer PRD: 1999, Boer, Brodsky, Hwang PRD: 2003 Collins SoperPRD: 1977 subleading twist

- Leading twist $\cos 2\phi$ azimuthal asymmetry depends on T-odd distribution $h_1^\perp$.

$$\nu = \frac{2 \sum_a e_a^2 \mathcal{F} \left[(2 \boldsymbol{p}_\perp \cdot \boldsymbol{k}_\perp - \boldsymbol{p}_\perp \cdot \boldsymbol{k}_\perp) \frac{h_1^\perp(x, \boldsymbol{k}_T) \bar{h}_1^\perp(\bar{x}, \boldsymbol{p}_T)}{M_1 M_2} \right]}{\sum_{a,\bar{a}} e_a^2 \mathcal{F}[f_1 \bar{f}_1]} \tag{5}$$

Higher twist comes in

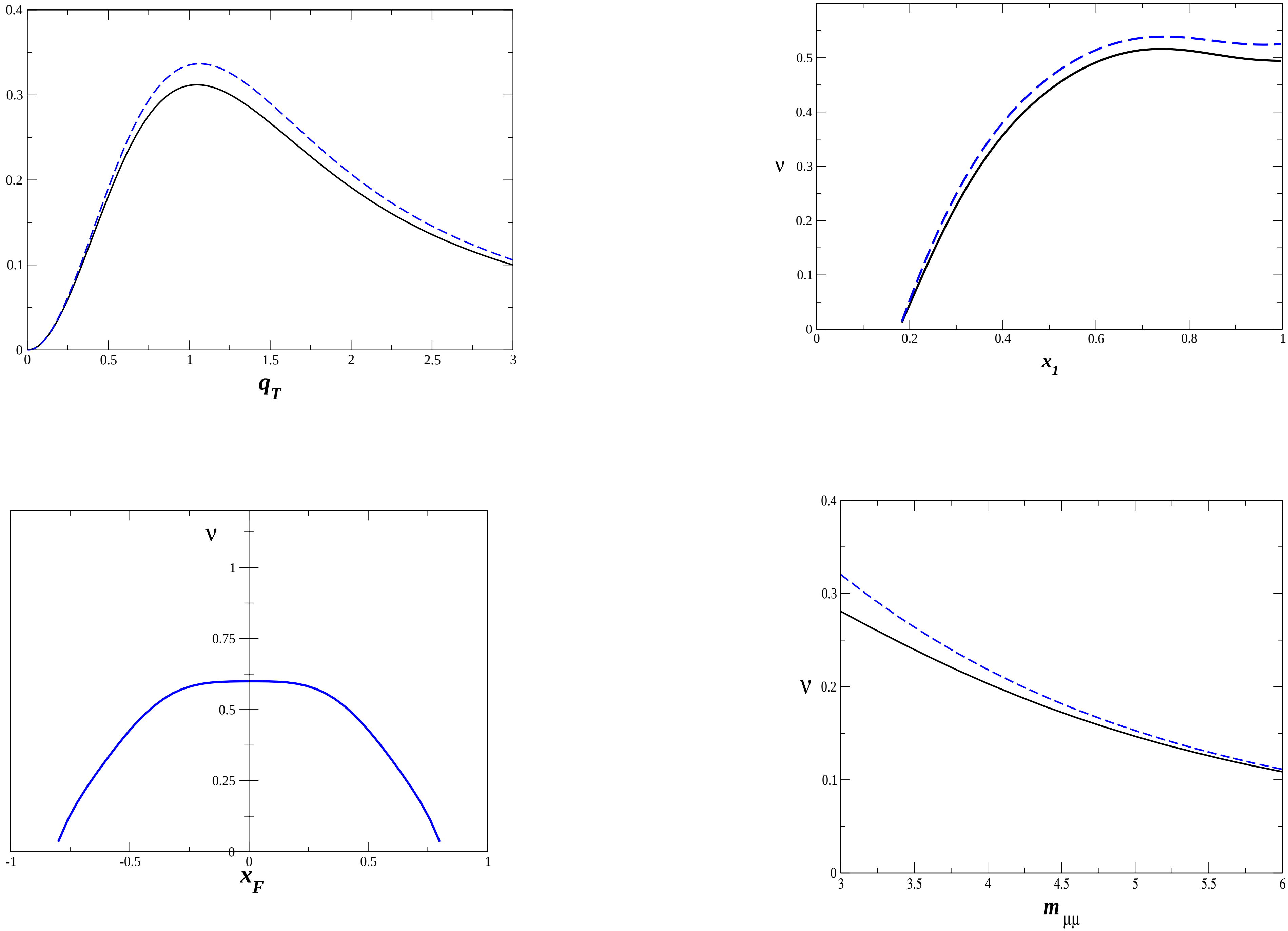
$$\nu = \frac{2\sum_a e_a^2 \mathcal{F}\left[(2\boldsymbol{p}_\perp\cdot\boldsymbol{k}_\perp - \boldsymbol{p}_\perp\cdot\boldsymbol{k}_\perp)\frac{h_1^\perp(x,\boldsymbol{k}_T^2)\bar{h}_1^\perp(\bar{x},\boldsymbol{p}_T)}{M_1 M_2}\right] + \nu_4[w_4 f_1\bar{f}_1]}{\sum_{a,\bar{a}} e_a^2 \mathcal{F}[f_1\bar{f}_1]}$$

$$\nu_4 = \frac{\frac{1}{Q^2}\sum_a e_a^2 \mathcal{F}\left[w_4\, f_1(x,\boldsymbol{k}_\perp)\bar{f}_1(\bar{x},\boldsymbol{p}_\perp)\right]}{\sum_a e_a^2 \mathcal{F}\left(f_1(x,\boldsymbol{k}_\perp)\bar{f}_1(\bar{x},\boldsymbol{p}_\perp)\right)},$$

Weight

$$w_4 = 2\left(\hat{\boldsymbol{h}}\cdot(\boldsymbol{k}_\perp - \boldsymbol{p}_\perp)\right)^2 - (\boldsymbol{k}_\perp - \boldsymbol{p}_\perp)^2$$

Convolution integral $\mathcal{F} \equiv \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{k}_\perp \delta^2(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp - \boldsymbol{q}_\perp) f^a(x,\boldsymbol{p}_\perp)\bar{f}^a(\bar{x},\boldsymbol{k}_\perp)$



L.G., Goldstein hep-ph/0506127 $s = 50\, GeV^2$, $x = [0.2 - 1.0]$, and $q = [3.0 - 6.0]\ GeV$, $\boldsymbol{q_T} = 0 - 3.0\, GeV$

SUMMARY

- Going beyond the collinear approximation in PQCD recent progress has been achieved characterizing transverse SSA and azimuthal asymmetries in terms of absorptive scattering.
- Central to this understanding is the role that transversity properties of quarks and hadrons assume in terms of correlations between transverse momentum and transverse spin in QCD hard scattering.
- These asymmetries provide a window to explore novel quark distribution and fragmentation functions which constitute essential information about the spin, transversity and generalized momentum structure of hadrons.
- Along with the chiral odd transversity T-even distribution function, existence of T-odd distribution and fragmentation functions can provide an explanation for the substantial asymmetries that have been observed in inclusive and semi-inclusive scattering reactions.
- We should consider the angular correlations in SDIS at 12 GeV for $\cos 2\phi$ from the standpoint of "rescattering" mechanism which generate T-odd, intrinsic transverse momentum, $k_\perp$, dependent *distribution and fragmentation* functions at leading twist
- Addressing issues of Universality of Collins Function

⋆ Azimuthal asymmetries in Drell Yan and SSA measured at HERMES and COMPASS, JLAB, Belle, GSI-PAX, JPARC *may* reveal the extent to which these leading twist T-odd effects are generating the data