Recent developments in perturbative QCD

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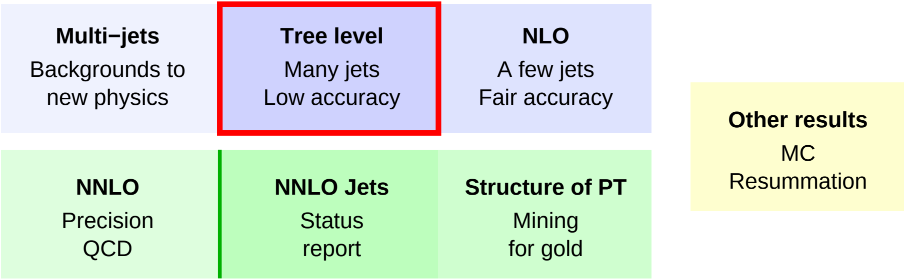
Introduction

Concentrate on the effort to 'get QCD in shape for LHC era':

- Predicting multi-jet final-states:
  [because new-particle signatures involve many jets]
  - New tree level techniques
  - NLO and progress in 1-loop calculations
- Aiming for accuracy
  [because NLO theory is often far behind HERA/LEP precision]
  - NNLO jets: status & progress report
  - What NNLO is teaching us about QCD itself
- Other developments (mostly 'all-order' QCD)

Some recently very active fields, not covered:

- Small-x saturation ➥ talk by Iancu
- Generalised parton distributions – a field in its own right
  ➥ talk by Diehl, + hep-ph/0512201



Multi−jets
Backgrounds to
new physics

Tree level
Many jets
Low accuracy

NLO
A few jets
Fair accuracy

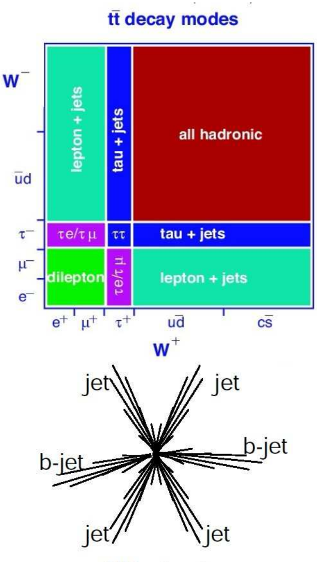
NNLO
Precision
QCD

NNLO Jets
Status
report

Structure of PT
Mining
for gold

Other results
MC
Resummation

Multi-jet final-states



All-hadronic
(BR~46%, huge bckg)

Juste – Lepton Photon '05

Heavy objects: multi-jet final-states

- Need to understand QCD multi-jet production (background)
- Max # jets: tree level ≤ 8 jets

MadEvent,AlpGen,Helac/Phegas
CompHEP,Grace,Amegic

# jets	# events for $10\,\mathrm{fb}^{-1}$
3	$9 \cdot 10^8$
4	$7 \cdot 10^7$
5	$6 \cdot 10^6$
6	$3 \cdot 10^5$
7	$2 \cdot 10^4$
8	$2 \cdot 10^3$

$p_t(\mathrm{jet}) > 60$ GeV, $\theta_{ij} > 30\,\mathrm{deg}$, $|y_{ij}| < 3$
Draggiotis, Kleiss & Papadopoulos '02

Tree level history

$$\mathcal{A}^{\text{tree}}(1,2,\ldots,n) = g^{n-2} \sum_{perms} \underbrace{\text{Tr}(T_1 T_2 \ldots T_n)}_{\text{colour struct.}} \underbrace{A^{\text{tree}}(1,2,\ldots,n)}_{\text{colour ordered amp.}}$$

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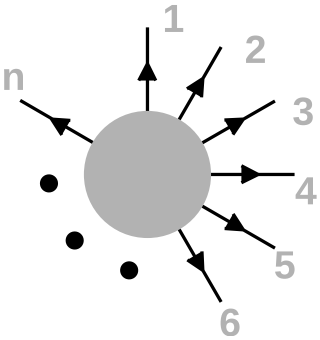

n	# diags	# col-ord diags
4	4	3
5	25	10
6	220	36
7	2485	133
8	34300	501
9	559405	1991
10	10525900	7335
	non-planar	*planar*

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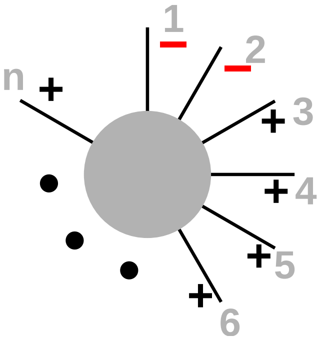

Maximal Helicity Violating (MHV)

Helicity amplitude: simplifies!

$$A^{\text{tree}}(--++\dots) = \frac{i\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\dots\langle n1\rangle}$$

Parke & Taylor, Kunszt '85
Berends & Giele '88

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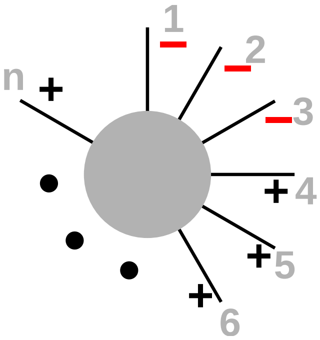

$$\begin{aligned}
&A_n(- + \cdots + - -) = \\
&= \frac{i}{\langle 1\,2\rangle \langle 2\,3\rangle \cdots \langle (n-2)\,(n-1)\rangle\, [(n-2)\,(n-1)]\, [(n-1)\,n]\, [n\,1]\, [1\,2]} \\
&\times \Bigg(\frac{(n-1\,n)\,\langle 1\,2\rangle\, [(n-1)\,(n-2)]\, \langle (n-1)^- |\, K_- \,|2^-\rangle^2}{S_{3,n-1}} + \frac{(1\,n)\,\langle (n-1)\,(n-2)\rangle\, [1\,2]\, \langle 1^- |\, K_- \,|(n-2)^-\rangle^2}{S_{1,n-3}} \\
&\quad + \frac{\langle n\,(n-1)\rangle \langle 1\,(n-1)\rangle \langle 1\,(n-2)\rangle\, [1\,n]\, [1\,2]\, [(n-1)\,(n-2)]\, \langle 1^- |\, K_- \,|(n-2)^-\rangle}{S_{1,n-3}} \\
&\quad + \frac{\langle n\,1\rangle \langle (n-1)\,1\rangle \langle (n-1)\,2\rangle\, [(n-1)\,n]\, [1\,2]\, [(n-1)\,(n-2)]\, \langle (n-1)^- |\, K_- \,|2^-\rangle}{S_{3,n-1}} \\
&\quad - \langle 1\,(n-1)\rangle^2 S_{2,n-2}\, [1\,2]\, [(n-1)\,(n-2)] - \frac{(n-1\,n)\,(1\,n)\,\langle 1\,(n-1)\rangle \langle 1^- |\, K_- \,|(n-2)^-\rangle\, [1\,2]}{S_{1,n-3}} \\
&\quad - [n\,1]\, [n\,(n-1)]\, [1\,2]\, [(n-1)\,(n-2)] \\
&\qquad \times \sum_{l=3}^{n-3} \Bigg[\frac{\langle n\,(n-1)\rangle^2 \langle (n-1)\,1\rangle \langle 1^- |\, \not{K}_{1,l-1} \not{k}_l \,|1^+\rangle}{S_{1,l-1} S_{1,l}} \\
&\qquad\quad + \frac{\langle n\,1\rangle^2 \langle 1\,(n-1)\rangle \langle (n-1)^- |\, \not{K}_{l+1,n-1} \not{k}_l \,|(n-1)^+\rangle}{S_{l+1,n-1} S_{l,n-1}} \\
&\qquad\quad - \frac{\langle n\,1\rangle \langle n\,(n-1)\rangle \langle (n-1)\,1\rangle \langle (n-1)^- |\, \not{K}_{l+1,n} \not{k}_l \,|1^+\rangle}{S_{1,l} S_{l,n-1}} \\
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\end{aligned} \tag{5.2}$$

NEXT to Maximal Helicity Violating (NMHV)

Kosower, '90

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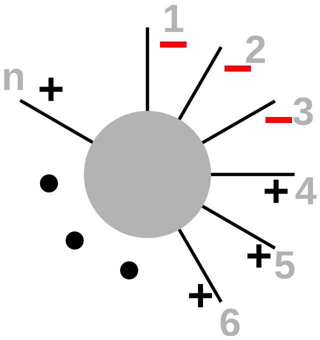


Helicity amplitude: simplifies!

$$A^{\text{tree}}(- - - + + \dots) =$$

$$\frac{1}{F_{3,1}} \sum_{j=4}^{n-1} \frac{\langle 1|P_{2,j} P_{j+1,2}|3\rangle}{P_{2,j}^2 P_{j+1,2}^2} \times$$

$$\frac{\langle j{+}1\; j\rangle}{[2|P_{2,j}|\,j{+}1\rangle\langle j|P_{j+1,2}|2]}$$

Britto et al., hep-th/0503198

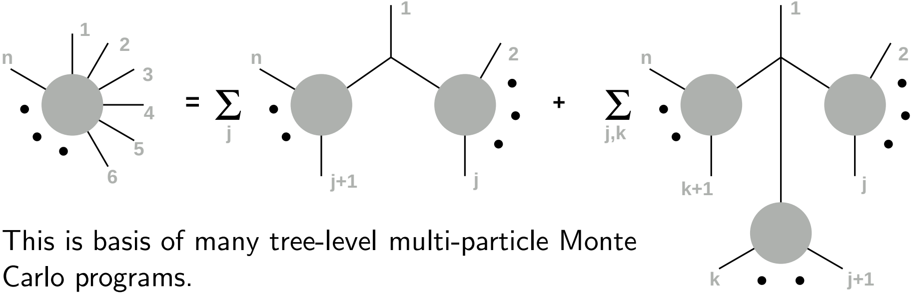
NEXT to Maximal Helicity Violating (NMHV)

Just one of vast array of results obtained with new recursion *(Twistor)* techniques.

Old recursion relations

Build multi-leg amplitudes by joining sub-amplitudes.

Berends Giele (1988): Join smaller off-shell amplitudes through a (colour-stripped) three or four-gluon vertex:



This is basis of many tree-level multi-particle Monte Carlo programs.

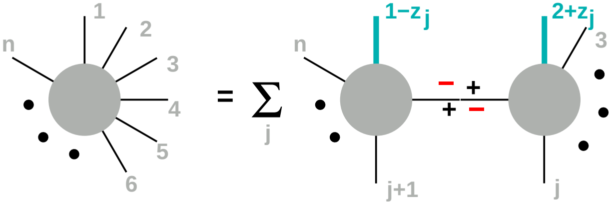
Why powerful?

Sub-amplitudes can be simplified *before* joining them together.

Feynman diagrams, in contrast, can only be simplified after full calculation.

New recursion relations (twistors)

Britto-Cachazo-Feng (BCF): Join smaller sub-amplitudes by a *propagator*. Sub-amplitudes made *on-shell* by analytic continuation ($\pm \mathbf{z_j}$) of two reference momenta:



Britto, Cachazo & Feng hep-th/0412265; *idem.* + Witten hep-th/0501052
Earlier (related) rules: Cachazo, Svrcek & Witten hep-th/0403047

Proof based on analytic structure of tree-graphs (they are a sum of poles in complex plane) — *very general.*

Simplicity lies in on-shellness of sub-amplitudes and the need for just a scalar propagator to join them.

Recursion tree-level highlights

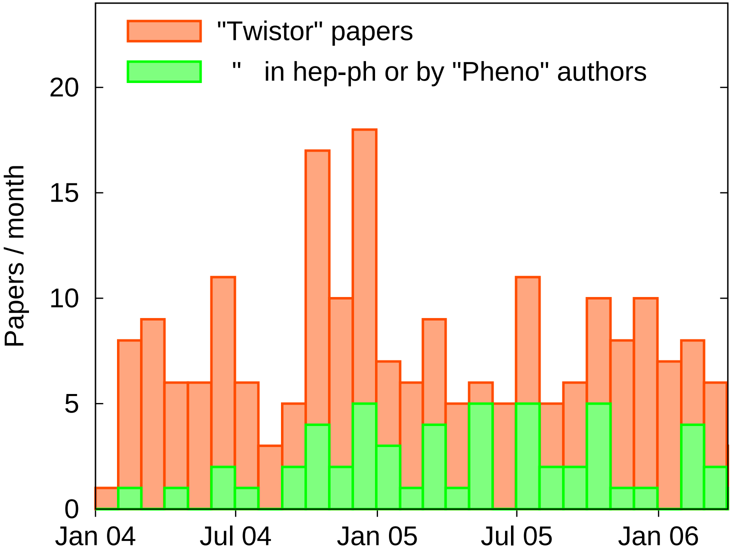
Very active field: 200 articles in 2 years ($\sim$ 50 by 'QCD people')

Tree level

- Specific compact results, including NNMHV — Kosower '04;Roiban et al '04
  - Hints of yet deeper simplifications — Luo & Wen '05; Britto et al '05
- Efficient (recursive) formulations — Bena, Bern, Kosower '04
  - NB: recall ∃ 'standard' numerical methods for tree-level calculations: Berends-Giele ('88); 'Alpha' ('95)
- Massless quarks, gluinos — Georgiou, Glover & Khoze '04; Wu & Zhu '04
- External Higgs boson — Dixon/Badger, Glover & Khoze '04
- External weak boson (& fermions) — Bern, Forde, Kosower & Mastrolia '04
- Collinear limits — Birthwright et al '05
- Massive quarks, scalars — Forde & Kosower '05; Schwinn & Weinzierl '06; Ferrario, Rodrigo & Talavera '06; Ozeren & Stirling '06

Amazing progress in short time. . .

Recursion (Twistor) Papers



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Structure of PT
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Wanted at NLO

Currently available

NLOJET++, MCFM, PHOX, ...

`http://www.cedar.ac.uk/hepcode/`

Experimenters' priorities

Les Houches '05

1. pp → W W + jet
2. pp → H + 2 jets
   - Background to VBF Higgs production
3. pp → $t\bar{t}b\bar{b}$
4. pp → $t\bar{t}$ + 2 jets
   - Background to $t\bar{t}$H
5. pp → W W $b\bar{b}$
6. pp → V V + 2 jets
   - Background to $W\,W \to H \to W\,W$
7. pp → V + 3 jets
   - General background to new physics
8. pp → V V V + jet
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Theorist's list (G. Heinrich)

- 2 → 3 (OK for a good student!)
  - pp → W W + jet
  - pp → V V V
  - pp → H + 2 jets
- 2 → 4 (Beyond today's means)
  - pp → 4 jets
  - pp → $t\bar{t}$ + 2 jets
  - pp → $t\bar{t}b\bar{b}$
  - pp → V + 3 jets
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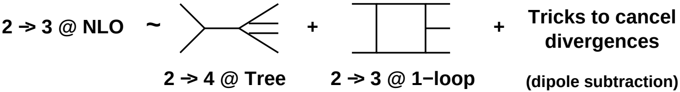
NLO bottleneck: loop calc.



2 -> 3 @ NLO ~ 2 -> 4 @ Tree + 2 -> 3 @ 1−loop + Tricks to cancel divergences (dipole subtraction)

Traditionally: 1-loop for 2 → 3 proc. takes 1–2 years

Two ways of doing this more efficiently:

- Understand field theory better

  Enormous progress on this in past two years: ∼ 200 articles

- Get a computer to do most of the work for you

  First full 2 → 4 (6-leg) result obtained this way

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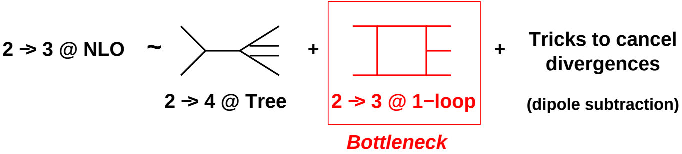

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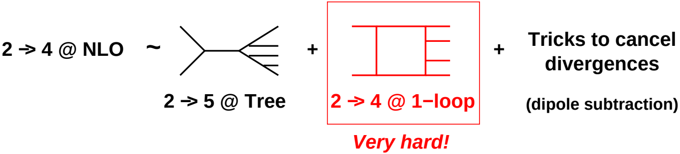

2 -> 4 @ NLO ~ **2 -> 5 @ Tree** + **2 -> 4 @ 1−loop** + **Tricks to cancel divergences** **(dipole subtraction)**

Very hard!

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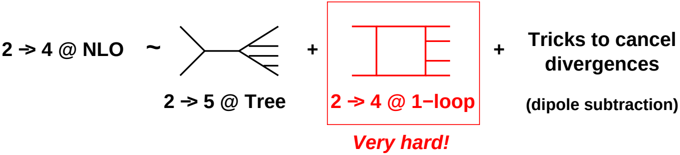

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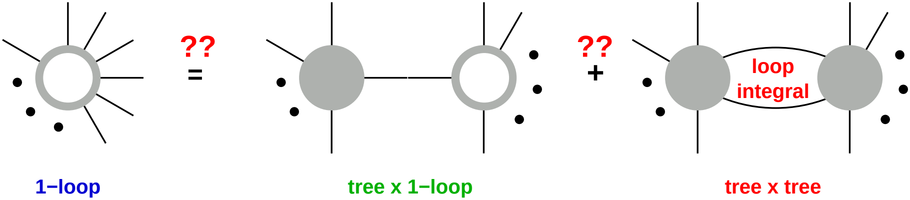
Recursion for loops?

Would like a relation that avoids need for loop integrations. *Various kinds of recursion possible*



Technically: loop diagrams have more complex analytic properties than trees (*cuts* as well as poles), so BCFW does not apply.

Complex problem, much progress made, many people involved.

Bedford, Bena, Bern, Bidder, Bjerrum-Bohr, Brandhuber, Britto, Cachazo, Del Duca, Dixon, Dunbar, Feng, Forde, Ita, Kosower, McNamara, Mastrolia, Perkins, Roiban, Spence, Travaglini, [. . .]

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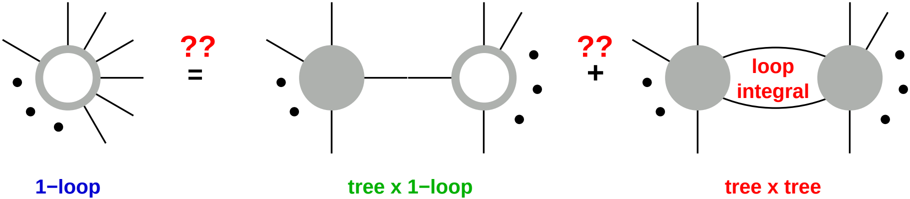

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6-gluon status via recursion

One ingredient of *one* of the "priority processes" ($pp \to 4$ jets) is the 6-gluon 1-loop amplitude:

$$\mathcal{A}_g = \underbrace{(\mathcal{A}_g + 4\mathcal{A}_f + 3\mathcal{A}_s)}_{\mathcal{N}=4\text{ SUSY}} - \underbrace{4(\mathcal{A}_f + \mathcal{A}_s)}_{\mathcal{N}=1\text{ chiral SUSY}} + \underbrace{\mathcal{A}_s}_{\text{scalar}}$$

	$\mathcal{N}=4$	$\mathcal{N}=1$	$S\,(c,d,e)$	$S\,(R)$
$A(--++++)$	BDDK94a	BDDK94b	BDDK94b	BDK05
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$A(---+++)$	BDDK94b	BBDD04	BBDI05	Dixon05
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Table adapted from hep-ph/0603187; NB: many results go beyond 6 gluons

Promising + much progress made! But QCD loops are still far from having simplicity of the tree-level results. . .

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Loops from Feynman diagrams

Automation of loop calculations with Feynman diagram techniques:

- Get expressions for all Feynman graphs (QGRAF, FeynArts). This gives answer in terms of a set of *loop integrals*
- Use *recursion relations* to reexpress each loop integral in terms of a basis set of known standard integrals NB: recursion for integrals, not amplitudes!
  - Analytically with algebraic manipulation programs Binoth, Guillet, Heinrich, Pilon, Schubert '05; + others
  - Semi-numerically, "on the fly", Ellis, Giele, Glover, Zanderighi '04-05
  - Results unstable at special phase-space points (e.g. co-planar momenta): use dedicated strategies there.
- Alternative integration techniques: e.g. subtract out divergences *before integrating*, do rest numerically. Nagy, Soper '03

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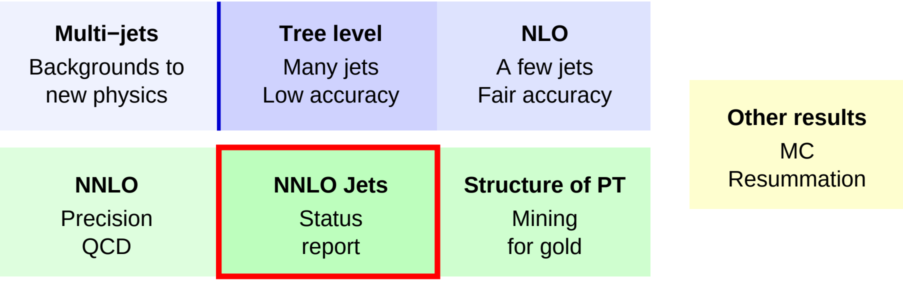
Automated loops: applications

- **Full 6-gluon 1-loop amplitude!** Ellis, Giele, Zanderighi '06
  Only fully known $2 \longrightarrow 4$ 1-loop amplitude in QCD
- $pp \longrightarrow H + 2$ jets: amplitudes done, implementation into MCFM in progress Ellis, Campbell, Giele, Zanderighi, '05-06
- $gg \longrightarrow WW$ via quark loop Binoth, Ciccolini, Kauer, Krämer '05
- Similar techniques in EW: $e^+e^- \longrightarrow 4$ fermions
  Denner, Dittmaier, Roth, Wieders '05

Automated techniques have advantage of flexibility
But: speed can be issue in numerical variants.

NB: more 'traditional' NLO methods still important, ➥talk by Oleari



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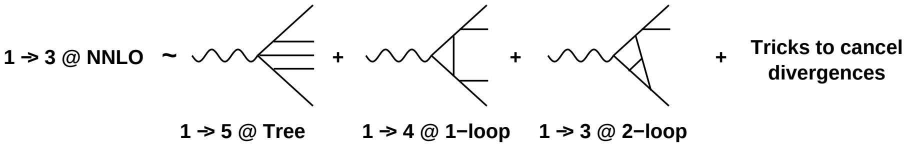
NNLO status & road-map

- Processes **with two QCD partons** @ LO are mostly done
  - $e^+e^- \to$ hadrons, $\tau \to \nu +$ hadrons
  - DIS coeff. fns., sum rules
  - $pp \to W, Z, \gamma^*, H, WH, ZH$ (many including spin correl.)
- **Next in line:** $e^+e^- \to 3\,$jets?
  - simplest!
  - α_s & other measurements at LEP are theory limited

    theory uncertainty $\sim 3 - 4\times$ exp. error
  - useful for studying perturbative/ non-perturbative interface.
- Then DIS $\to 2+1$ and $pp \to 2\,$jets. . .

NNLO bottleneck

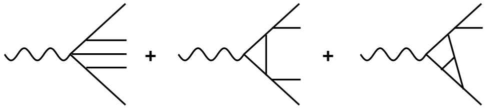


NNLO bottleneck

$4+2\varepsilon$ dim:
J is observable

$\int d\Phi_5\ J(p_{1..5})$ $\int d\Phi_4\ \varepsilon^{-2} J(p_{1..4})$ $\int d\Phi_3\ \varepsilon^{-4} J(p_{1..3})$

1 -> 3 @ NNLO ~



+ Tricks to cancel divergences

1 -> 5 @ Tree **1 -> 4 @ 1−loop** **1 -> 3 @ 2−loop**

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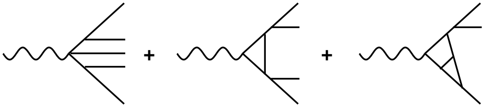

$$\mathbf{1 \to 3\ @\ NNLO} \sim \int d\Phi_5\ J(p_{1..5}) + \int d\Phi_4\ \varepsilon^{-2} J(p_{1..4}) + \int d\Phi_3\ \varepsilon^{-4} J(p_{1..3}) + \boxed{\text{Tricks to cancel divergences}}$$

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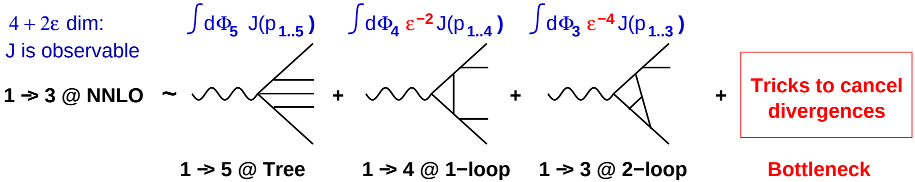

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"You have to do the integral, but you don't know the integrand"

Anastasiou (KITP LoopFest III)

- **Subtraction:** Catani, Seymour '96 + earlier authors
  - find an integrable function with same divergences as amplitudes
  - subtract it from real
  - add integrated version to virtuals.
- **Sector decomposition:** Binoth, Heinrich '00
  - split phase space into regions with at most one divergence each
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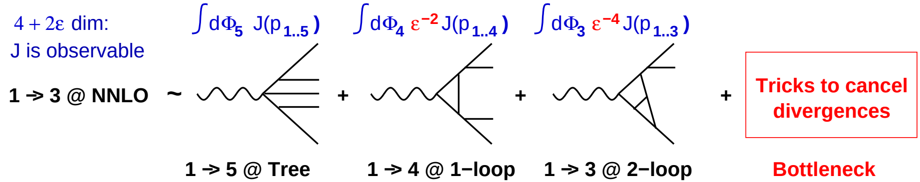

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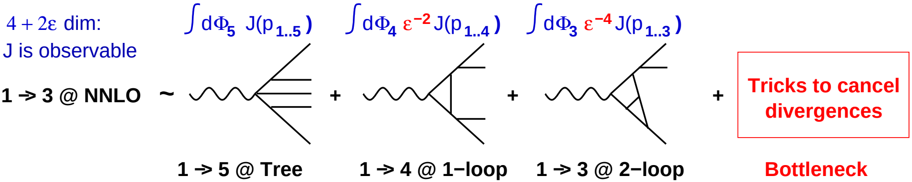
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NNLO progress

Subtraction

[Standard approach @ NLO]

- Applied to C_F^3 colour part of $e^+e^- \longrightarrow 3\,\text{jets}$

$$(\alpha_s C_F/2\pi)^3 \text{ piece of } \langle 1 - T \rangle = -20.4 \pm 4$$

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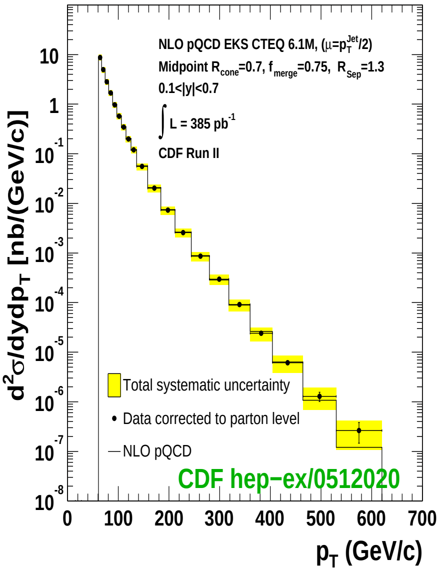

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- Jet-algo is not IR safe
  - CDF has *modified* midpoint cone
  - New 'search-cone step' IR unsafe [discovered by Wobisch]
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- NB: 'NNLO-NLL' – rough apprx. of NNLO, ignorant of jet-algo

Good news:

- CDF also has k_t-algo result
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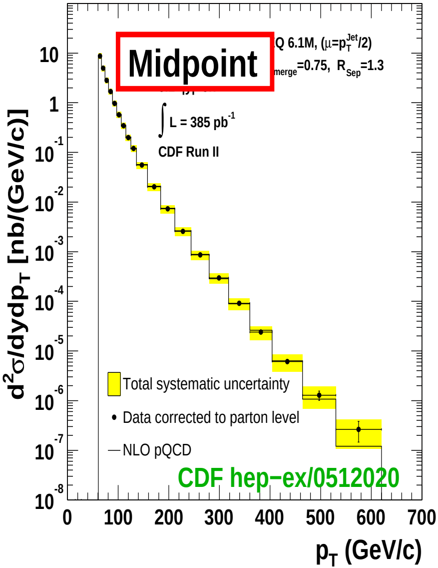

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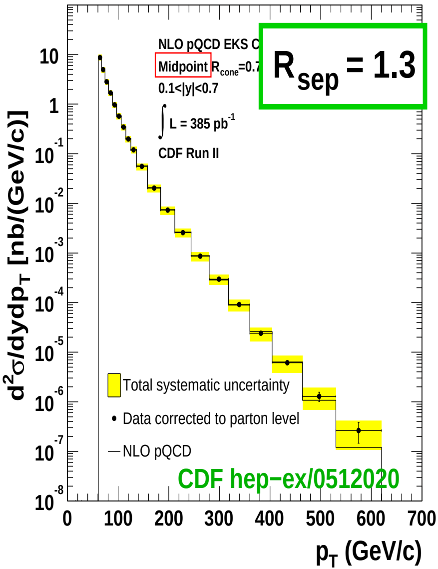

CDF hep−ex/0512020

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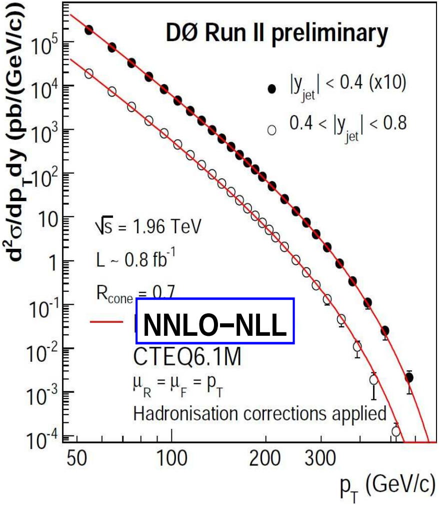

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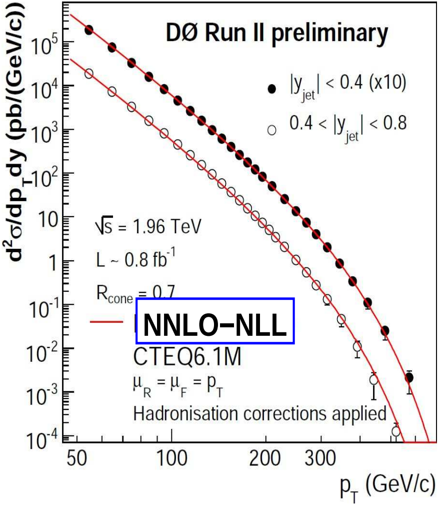

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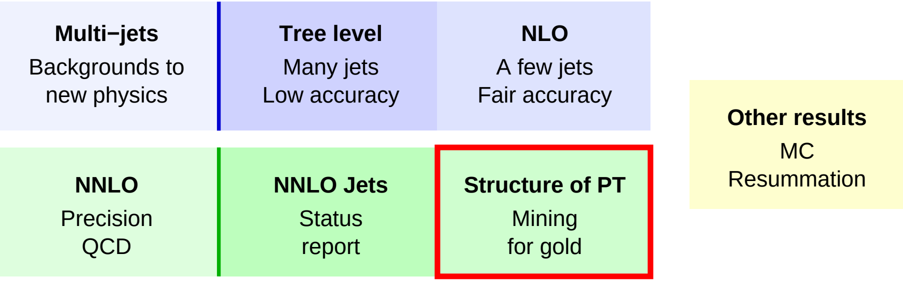

Multi−jets
Backgrounds to
new physics

Tree level
Many jets
Low accuracy

NLO
A few jets
Fair accuracy

NNLO
Precision
QCD

NNLO Jets
Status
report

Structure of PT
Mining
for gold

Other results
MC
Resummation

Year 2 after MVV NNLO P_{ij}

1. Next-to-Next-to-Leading Order Evolution of Non-Singlet Fragmentation Functions. By A. Mitov, S. Moch, A. Vogt. [hep-ph/0604053] DESY-06-036 (Apr 2006) 10p.
2. Higher-order soft corrections to lepton pair and Higgs boson production. By S. Moch & A. Vogt. Phys.Lett.B631:48-57,2005. [hep-ph/0508265]
3. Three-loop results for quark and gluon form-factors. By S. Moch, J.A.M. Vermaseren, A. Vogt. Phys.Lett.B625:245-252,2005. [hep-ph/0508055]
4. The Quark form-factor at higher orders. By S. Moch, J.A.M. Vermaseren, A. Vogt. JHEP 0508:049,2005. [hep-ph/0507039]
5. Higher-order corrections in threshold resummation. By S. Moch, J.A.M. Vermaseren, A. Vogt. Nucl.Phys.B726:317-335,2005. [hep-ph/0506288]
6. The Third-order QCD corrections to deep-inelastic scattering by photon exchange. By J.A.M. Vermaseren, A. Vogt, S. Moch. Nucl.Phys.B724:3-182,2005. [hep-ph/0504242]
7. The Longitudinal structure function at the third order. By S. Moch, J.A.M. Vermaseren, A. Vogt. Phys.Lett.B606:123-129,2005. [hep-ph/0411112]
8. Three loop universal anomalous dimension of the Wilson operators in N=4 SUSY Yang-Mills model. By A.V. Kotikov, L.N. Lipatov, A.I. Onishchenko, V.N. Velizhanin. Phys.Lett.B595:521-529,2004, Erratum-ibid.B632:754-756,2006. [hep-th/0404092]

Hidden structure

Various unexpected structures in MVV results. E.g. at large x, can write

$$P_{ij}(x) = \frac{A}{(1-x)_+} + B\,\delta(1-x) + C\ln(1-x) + \mathcal{O}(1)\;, \quad A = \sum A_n(\alpha_s/4\pi)^n\,,\ etc.$$

Remarkably, different coefficients seem to be interrelated:

- $C_2 = A_1^2$ Curci, Furmanski, Petronzio '80
- $C_3 = 2A_1A_2$ MVV '04

∃ a proposal that there is a more fundamental evolution equation with *a universal splitting function* appendix of Dokshitzer, Khoze, Troian '96

$$\partial_{\ln Q^2} D(x,Q^2) = \int_0^1 \frac{dz}{z}\mathcal{P}(x,\alpha_s(\tfrac{Q^2}{z}))\,D(\tfrac{x}{z}, z^\sigma Q^2) \quad \begin{cases} \sigma = 1: & \text{time-like} \\ \sigma = -1: & \text{space-like}\end{cases}$$

Postulate new universal splitting function $\mathcal{P}$ to be classical at large x ⇒

$C = A^2$ at all orders; get most of NNLL $\mathcal{O}(1)$ term too!

Dokshitzer, Marchesini, GPS '05

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Time and space-like splitting

Original aim of Dokshitzer was to understand difference between time-like ($\sigma = +1$) and space-like ($\sigma = -1$) splitting functions.

i.e. fragmentation function and splitting function evolution

Normally related at order n via:

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More structure?

- Many other "goodies" in the MVV papers (even more in supersymmetric limit)…

- In $\mathcal{N} = 4$ SUSY Yang-Mills amplitudes, planar *n*-loop seems to be reducible just to powers of 1-loop:

$$M_{n-leg}^{(2-loop)} = \frac{1}{2}(M_n^{(1)})^2 + f(\epsilon) M_n^{(1)}(2\epsilon) - \frac{\pi^4}{72} + \mathcal{O}\left(()\,\epsilon\right)$$

4-legs: Anastasiou, Bern, Dixon, Kosowoer '03

5-legs: Cachazo, Spradlin, Volovich '06; Bern et al '06

NB: numerical loop calcs: Anastasiou & Daleo [➥talk] '05 ; Czakon '05

$$M_{n-leg}^{(3-loop)} = -\frac{1}{3}(M_n^{(1)})^3 + M_n^{(1)}(\epsilon) M_n^{(2)}(\epsilon) + f^{(3)}(\epsilon) M_n^{(1)}(3\epsilon) + C^{(3)} + \mathcal{O}\left(\epsilon\right)$$

4-legs: Bern, Dixon, Smirnov '05

- In large-angle soft-colour resummation ('fifth form factor') for 2 → 2 scattering, symmetry in exch. of kinematic variables and # of colours:

$$\frac{\ln\frac{s^2}{ut} - 2\pi i}{\ln\frac{u}{t}} \Longleftrightarrow N_C$$

Dokshitzer & Marchesini '05 (see also Seymour '05)

Multi−jets
Backgrounds to
new physics

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Many jets
Low accuracy

NLO
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Fair accuracy

NNLO
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NNLO Jets
Status
report

Structure of PT
Mining
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Other results
MC
Resummation

Some other results of note

Fixed order calculations

- 4-loop decoupling relations for α_s (i.e. heavy-quark thresholds)
  Schroder & Steinhauser '05; Chetyrkin, Kuhn, Sturm, '05
- IR safety for jet flavour — Banfi, GPS [➥ talk], Zanderighi '06

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- Herwig++ → adolescence ($pp \to$ DY), Ariadne++ — ➥ Lönnblad's talk
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  alternative methods: ➥Soper's, Nason's talks
- Using NNLL and NNLO for reweighting of event generators
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Analytical resummations:

- Collinear region (and threshold): MVV ⇒ $\alpha_s^n L^{n-2}$
- Generic large angle region, even $\alpha_s^n L^n$ *much less well understood*
  - Gaps-between-jets phenomenology — Forshaw, Kyrieleis, Seymour '05-'06
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  Some phenomenologists diverted into strings

- Automated 1-loop calculations are important complementary development. More flexible; crucial for cross-checks

- $e^+e^- \to 3$ jets at NNLO is on final stretch

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EXTRA SLIDES

Practical impact of twistors?

CPU time in seconds for the computation of the *n* gluon amplitude on a standard PC (2 GHz Pentium IV), summed over all helicities.

n	4	5	6	7	8	9	10	11	12
Berends-Giele	0.00005	0.00023	0.0009	0.003	0.011	*0.030*	*0.09*	*0.27*	*0.7*
CSW	*0.00001*	0.00040	0.0042	0.033	0.24	1.77	13	81	—
BCF	*0.00001*	*0.00007*	*0.0003*	*0.001*	*0.006*	0.037	0.19	0.97	5.5

Dinsdale, Ternick & Weinzierl '06

Gain a factor of ~ 4 for moderate *n* — useful, not overwhelming.

Slowly making it into phenomenological work

NB: trees in MadEvent, ALPGEN, HELAC/PHEGAS, CompHEP, GRACE, Amegic

➥ talk by Worek

But: real progress here is in discovery of new analytical structures in field theory (helpful also elsewhere, e.g. loops).

1-loop from twistors/recursion

Supersymmetric decomposition (allow gluons, fermions and scalars in loops)

$$\mathcal{A}_g = \underbrace{(\mathcal{A}_g + 4\mathcal{A}_f + 3\mathcal{A}_s)}_{\mathcal{N}=4\text{ SUSY}} - \underbrace{4(\mathcal{A}_f + \mathcal{A}_s)}_{\mathcal{N}=1\text{ chiral SUSY}} + \underbrace{\mathcal{A}_s}_{\text{scalar}}$$

SUSY gives many cancellations. Most difficult piece is *scalar*.

Analytical structure involves coefficients (c, d, e) of standard boxes (I_4), triangles (I_3) and bubbles (I_2), and rational terms (R):

$$\mathcal{A}_s = \sum_i c_i I_4^i + \sum_i d_i I_3^i + \sum_i e_i I_2^i + R$$

- coefficients (c, d, e) can be (i) read off by merging trees (*cut constructibility*) (ii) obtained recursively (à la BCFW)
- rational parts can be obtained recursively

1-loop from integral recursion

Example of Giele-Glover method

$$\int \frac{d^D\ell \;\ell^{\mu_1}\ell^{\mu_2}}{(\ell+q_1)^2(\ell+q_2)^2(\ell+q_3)^2(\ell+q_4)^2}$$
$$= \tfrac{1}{2}g^{\mu_1\mu_2} I(D+2;1,1,1,1) \; + 2q_1^{\mu_1} 2q_1^{\mu_2} I_4(D+4;3,1,1,1) + \dots$$

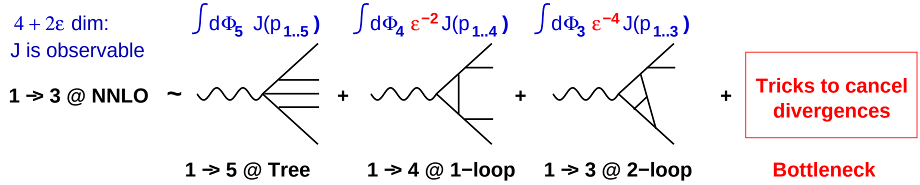
Then

$$\begin{aligned} 2I_4(8;3,1,1,1) = -2\left(\textstyle\sum_i S_{1i}^{-1}\right) I_4(8;2,1,1,1) \\ - S_{11}^{-1} I_4(6;1,1,1,1) - S_{12}^{-1} I_3(6;1,0,1,1) \\ - S_{13}^{-1} I_3(6;1,1,0,1) - S_{14}^{-1} I_3(6;1,1,1,0) \end{aligned}$$

The $I_n(D;1,1,1,1)$ etc. are the basis integrals. S_{ij} is kinematical matrix, $S_{ij} = (q_i - q_j)^2$.

Reduction procedure done numerically for each kinematic configuration.

Cancelling NNLO divergences



$4+2\varepsilon$ dim:
J is observable

1 -> 3 @ NNLO ~ $\int d\Phi_5\, J(p_{1..5})$ + $\int d\Phi_4\, \varepsilon^{-2} J(p_{1..4})$ + $\int d\Phi_3\, \varepsilon^{-4} J(p_{1..3})$ + **Tricks to cancel divergences**

1 -> 5 @ Tree | **1 -> 4 @ 1-loop** | **1 -> 3 @ 2-loop** | **Bottleneck**

How to get cancellations?

1. Subtraction method:

$$\int d^D\Phi_5 M_5 J(p_{1..5}) + \int d^D\Phi_4 M_4 J(p_{1..4}) + \dots$$

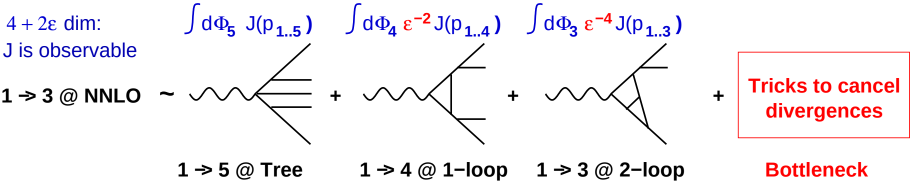
Applied to $e^+e^- \to 2$ jets and C_F^3 colour part of $e^+e^- \to 3$ jets:

$(\alpha_s C_F/2\pi)^3$ piece of $\langle 1-T\rangle = -20.4 \pm 4$

Gehrmann-De Ridder, Gehrmann & Glover '04

In principle all $e^+e^- \to 3$ jet 'antenna' subtraction pieces are ready — 'just' need to be coded! *idem.* '05

Cancelling NNLO divergences



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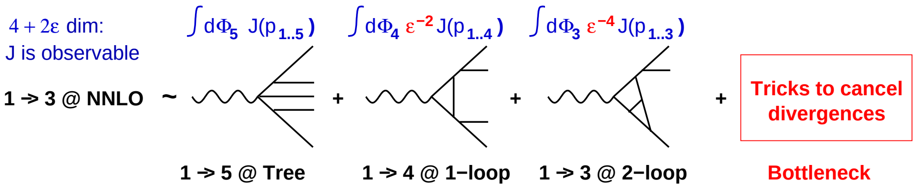

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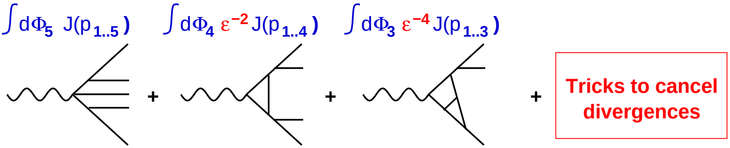

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How to get cancellations?

2. Sector decomposition for isolating divergences Binoth & Heinrich '00

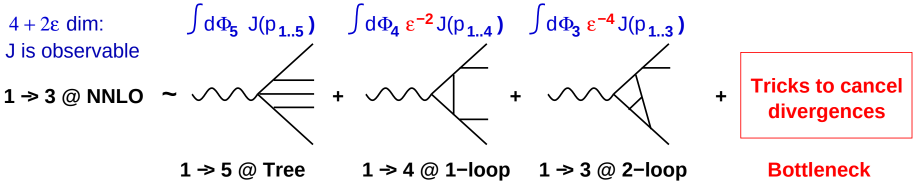
$$\int d^D\Phi_5 M_5 J(p_{1..5}) = \epsilon^{-4}\int d^4\Phi_5 f_{-4} M_5 J(p_{1..5}) + \cdots + \int d^4\Phi_5 f_0 M_5 J(p_{1..5})$$

The f_{-i} involve plus-distributions of kinematic invariants. Each integral finite.

Applied to

- $e^+e^- \to 2\,\text{jets}$ Binoth & Heinrich '04; Anastasiou, Melnikov & Petriello '04
- $e^+e^- \to 3\,\text{jets}$ (partial) Heinrich '06
- $pp \to W, Z, H$ (fully exclusive) Anastasiou, Dixon, Melnikov & Petriello '04-06

Cancelling NNLO divergences



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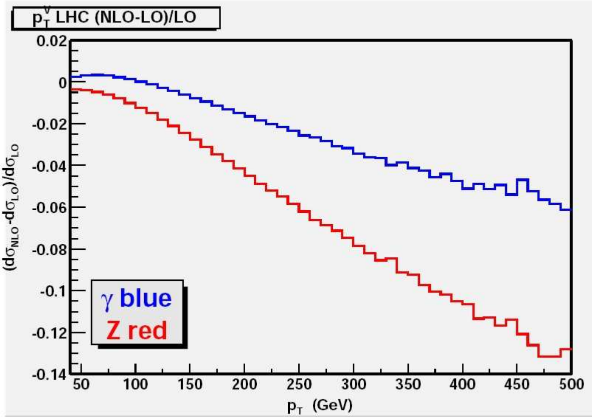
EW is not so weak



Widely discussed for ILC. How about *pp*?

e.g. NLO EW corrections to $pp \to Z+$jet

These are *significant* (even NNLO ∼ few %)

Maina Moretti Ross '04

Kulesza et al '04

QED effects $\lesssim 1\%$

Martin et al.

Glosser et al

MC status & plans

Fortran

- Matching to multi-parton LO matrix elements now widespread CKKW
- New, better shower in Pythia ($k_\perp$ ordered)
- Underlying event models much improved / more practical Pythia, Jimmy (Herwig)
- Reaching end of line soon!

C++

based on ThePEG			Independent	
Herwig++	Pythia 7	Ariadne/LDC	Pythia 8	Sherpa
ready for e^+e^- $pp \to DY$ ready	*cancelled*	see talk by Lönnblad	being coded	ready for e^+e^- and pp

Includes new, improved angular-ordered shower

New player! Dresden group

Resummation ingredients summary

	Soft + collinear		Hard Collinear		Soft large angle		
order	incl.	hadr.	incl.	hadr.	incl.	global	NG
$\alpha_s^n L^{n+1}$	✓	[✓]+BSZ04	N.A.		N.A.		
$\alpha_s^n L^n$	✓	[✓]+BSZ04	✓	✓	✓	[✓]	[$N_c \to \infty$]
$\alpha_s^n L^{n-1}$	MVV04	[FG04]	BCFG03	[FG04]	CFG+HK01	—	—
$\alpha_s^n L^{n-2}$	—	—	MVV05	—	MVV05	—	—

Large angle global	$2 \to 2$ BKOS89–98; generic: BCMN03; $2 \to 3$ [partial] KS05
Large angle NG	hemisph./patch: DS01–02; k_t algo: AS02, BD05

✓≡ historical results/techniques (< '01)   [. . .] ≡ only for special cases
hadr. ≡ anything measuring hadrons   NG ≡ non-global

AS: Appleby & Seymour; BCFG: Bozzi & CFG; BCMN: Bonciani, Catani, Mangano, Nason; BD: Banfi & Dasgupta; BKOS: Botts, Kidonakis, Oderda, Sterman; BSZ: Banfi, GPS & Zanderighi; CFG: Catani & FG; DS: Dasgupta & GPS; FG: de Florian & Grazzini; KS: Kyrieleis & Seymour; HK: Harlander & Kilgore.

NNLL resummations

Best accuracies (NNLL) for most inclusive observables

Higgs transverse-mom. distribution.

First differential NNLL resummation

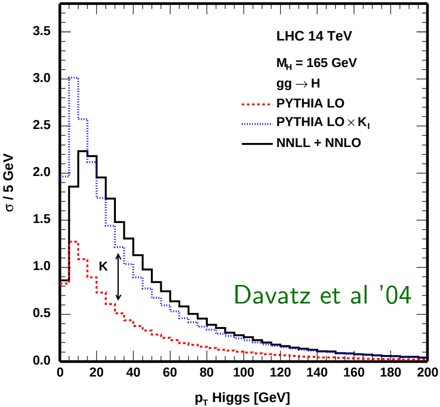
Resums $L \simeq \ln \frac{M_H}{Q_t}$ (for $gg \to H$)

$$\exp[\alpha_s^n L^{n+1} + \alpha_s^n L^n + \alpha_s^n L^{n-1}]$$

Bozzi et al '03

- NNLL uncertainty $\sim 7\%$ ($\sim$NLL/2)
- Shape quite different from plain parton showering (Pythia) — relevant for Higgs searches ($gg \to H \to WW \to \ell\nu\ell\nu$)?

Davatz et al '04

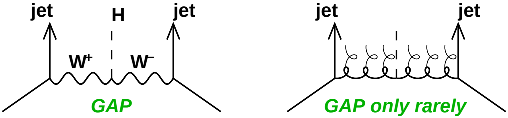


Davatz et al '04

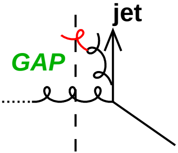
Gap resummations

How rare are gaps in
$pp \rightarrow 2\,\text{jets}$ with big ΔY?

Answer needs advanced tools



Non-global logarithms



- Appear for measurements of *part* of phase space

  Also e.g. dijet properties, Banfi & Dasgupta '03
- Only in large-N_c limit! Not automated!

  Connections to BFKL: Marchesini-Mueller '03; Weigert '03

Multi-jet structure

- Stony Brook soft-colour evolution
- Breakdown of 'probabilistic radiation'

Are Monte Carlos up to the job?
Unknown. . .