Parton Distributions
What parton distributions are there?
What parton distributions do we need?

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How many different parton distributions are there?

One way of counting.

$u_V(x,Q^2)$ and $d_V(x,Q^2)$ – the traditional quark model valence partons. Only 50% of momentum.

$\bar{u}(x,Q^2)$ and $\bar{d}(x,Q^2)$ – certainly not the same (NA51, E866). Difference probably valencelike.

Gluons, $g(x,Q^2)$ – over 30% momentum.

$s(x,Q^2)$ and $\bar{s}(x,Q^2)$. Often related to $(\bar{u}(x,Q^2)+\bar{d}(x,Q^2))$ but normalization unknown. Also is $s(x,Q^2)=\bar{s}(x,Q^2)$ (NuTeV)?

$c(x,Q^2)$ and $b(x,Q^2)$ (and $(t(x,Q^2))$) perturbatively generated. Not independent. Intrinsic contributions (high x) for both.

$\gamma(x,Q^2)$ distribution if QED included.

Possible isospin violation $u^p(x,Q^2)\neq d^n(x,Q^2)$, $u^p(x,Q^2)\neq d^n(x,Q^2)$. Again demanded with QED.

So $6-16$ different parton distributions. Some very small. Often not needed.

Another way of counting.

LO, NLO, NNLO, possible resummation corrections or allowances for higher twist.

$\overline{MS}$, DIS factorization schemes.

Fixed-flavour number scheme FFNS, zero-mass variable-flavour number scheme ZM-VFNS, general-mass variable flavour number scheme VFNS (different versions of this.

CTEQ4A1, CTEQ4A2 .... CTEQ5HJ, ... CTEQ6...

MRST98 .... MRST03c, ... MRST04 QED, MRST04...

Alekhin00, Alekhin03, GRV98, Fermi02...

ZEUS, ZEUS-ZJ, H1, Botje...

Far, far more than number of independent parton sets.

Are all of these really necessary?

Complicated and controversial question.

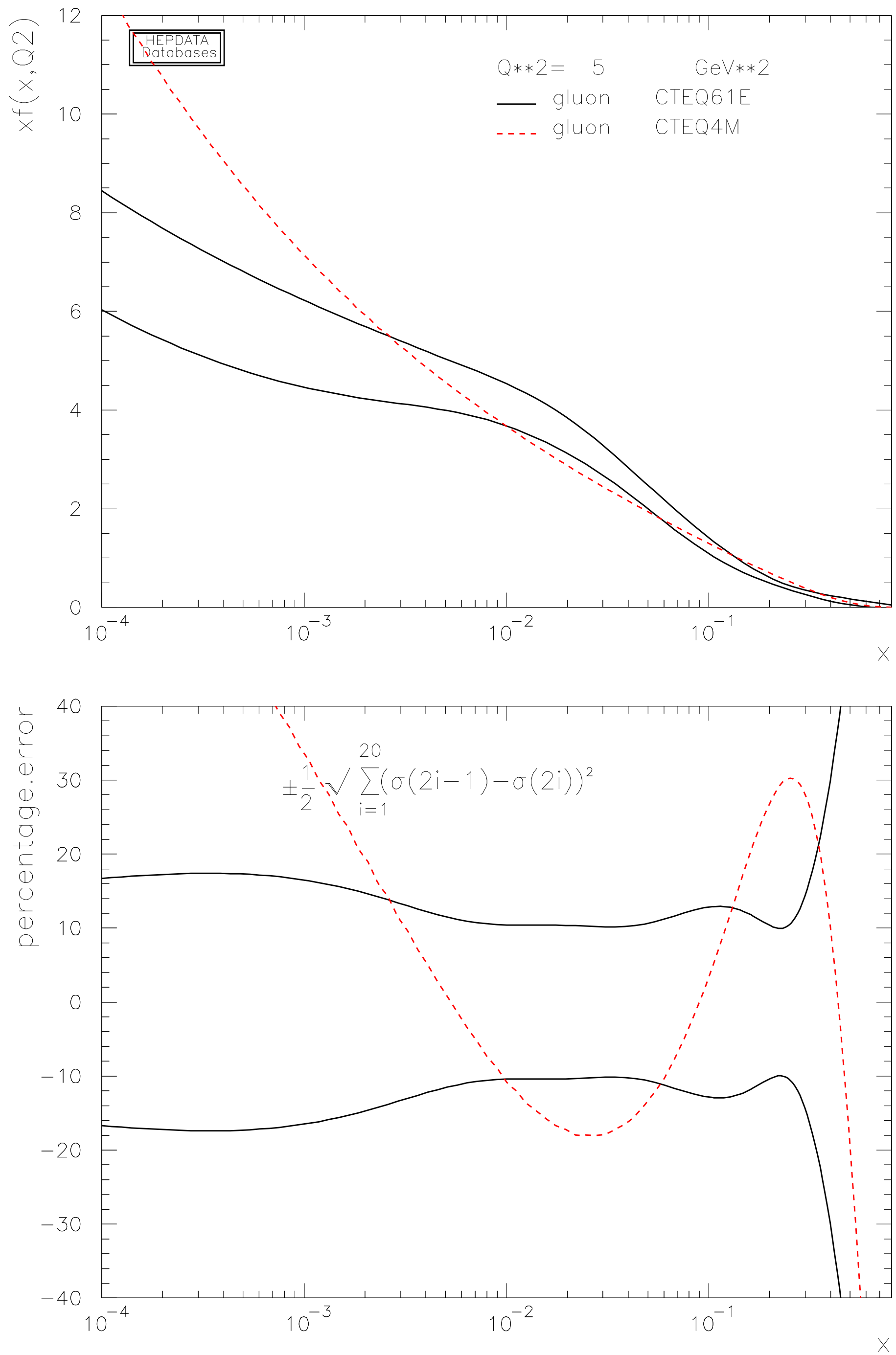
One restriction very easy to impose.

Many (still cited) partons are simply out of date!

Unless there is no (and I mean no) option do not use pre-2000 parton distributions.

Lots of much improved, particularly HERA and Tevatron jet data since then.

Some older partons have minor bugs (MRST98 – gluon, CTEQ4 – sea quarks).



Order of Partons

LO is simply not competitive in a global fit.

χ^2 far inferior to NLO and NNLO, particularly for HERA data – though good for Tevatron jet data.

Requires $\alpha_S(M_Z^2) = 0.130$ (if scale set at Q^2).

No recent updates (MRST01) since no real change at LO.

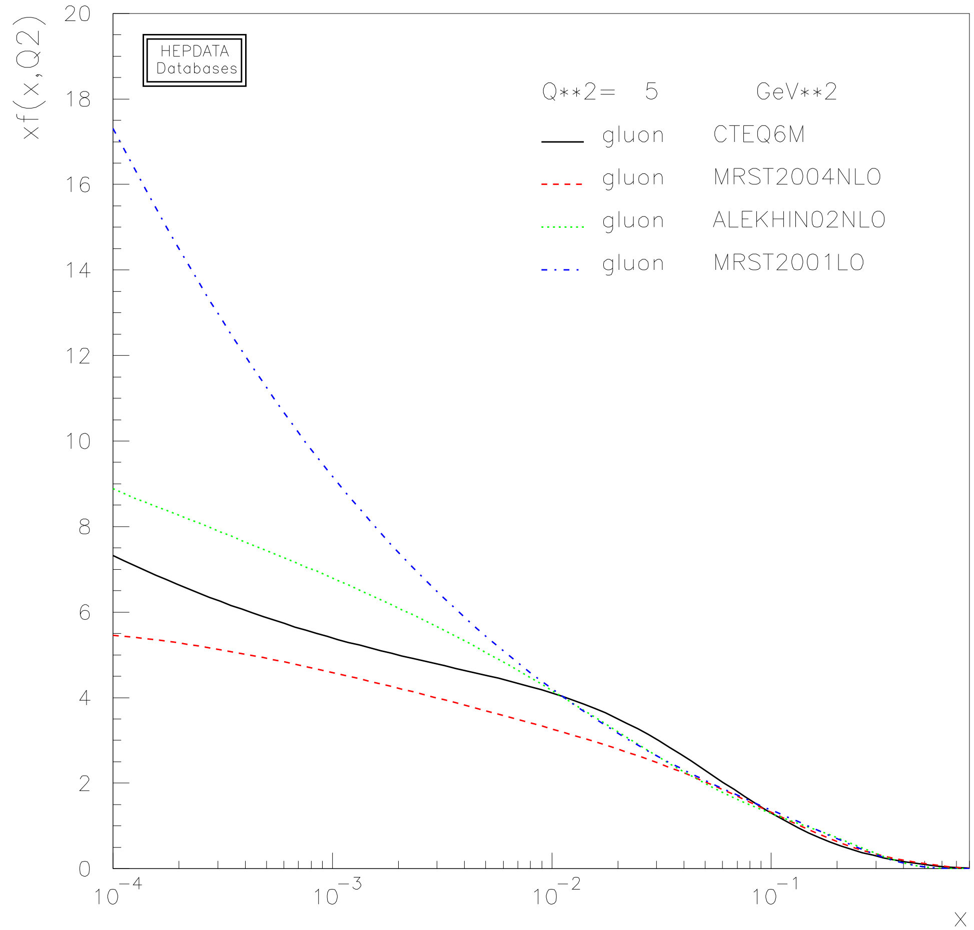
Data set	No. of data pts	LO	NLO	NNLO
H1 ep	400	493	382	385
ZEUS ep	272	296	254	264
BCDMS μp	179	179	193	173
BCDMS μd	155	206	218	231
NMC μp	126	131	134	121
NMC μd	126	98	100	88
SLAC ep	53	83	66	59
SLAC ed	54	98	56	59
E665 μp	53	50	51	55
E665 μd	53	62	61	62
CCFR $F_2^{\nu N}$	74	116	85	80
CCFR $F_3^{\nu N}$	105	126	107	110
NMC n/p	156	157	155	150
E605 DY	136	282	232	220
Tevatron Jets	113	123	170	186
Total	2097	2500	2328	2323

LO partons in some regions qualitatively different to all NLO and NNLO partons. Due to important missing NLO corrections in splitting functions.

Can lead to wrong conclusions on size of small-x gluon, and conclusions on shadowing *etc.*

Nevertheless, LO partons are the appropriate ones to use with many LO Monte Carlo programs.

All such results should be treated with care.



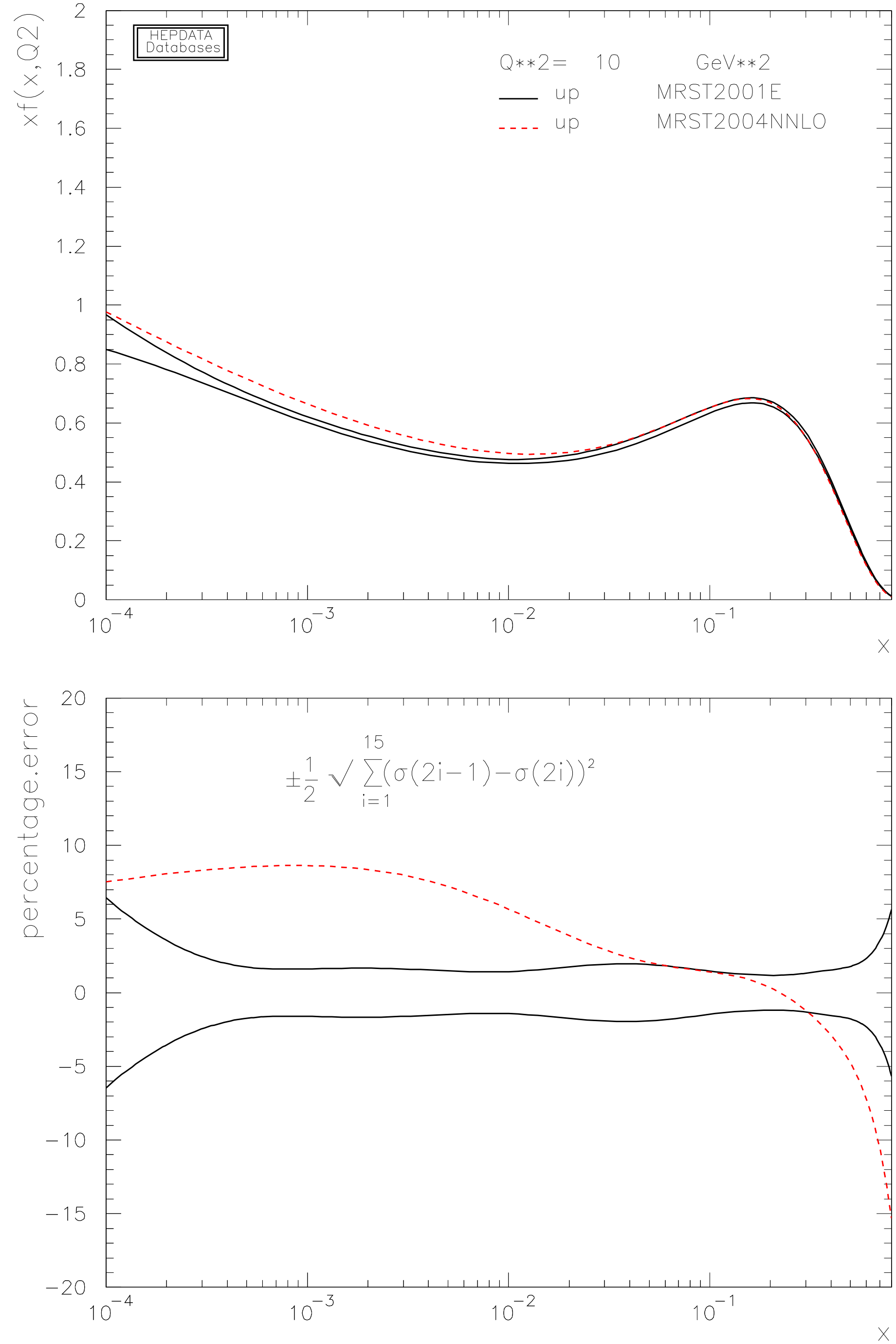
NNLO

Default has long been NLO. Essentially well understood. Now starting to go further.

NNLO coefficient functions for structure functions know for many years.

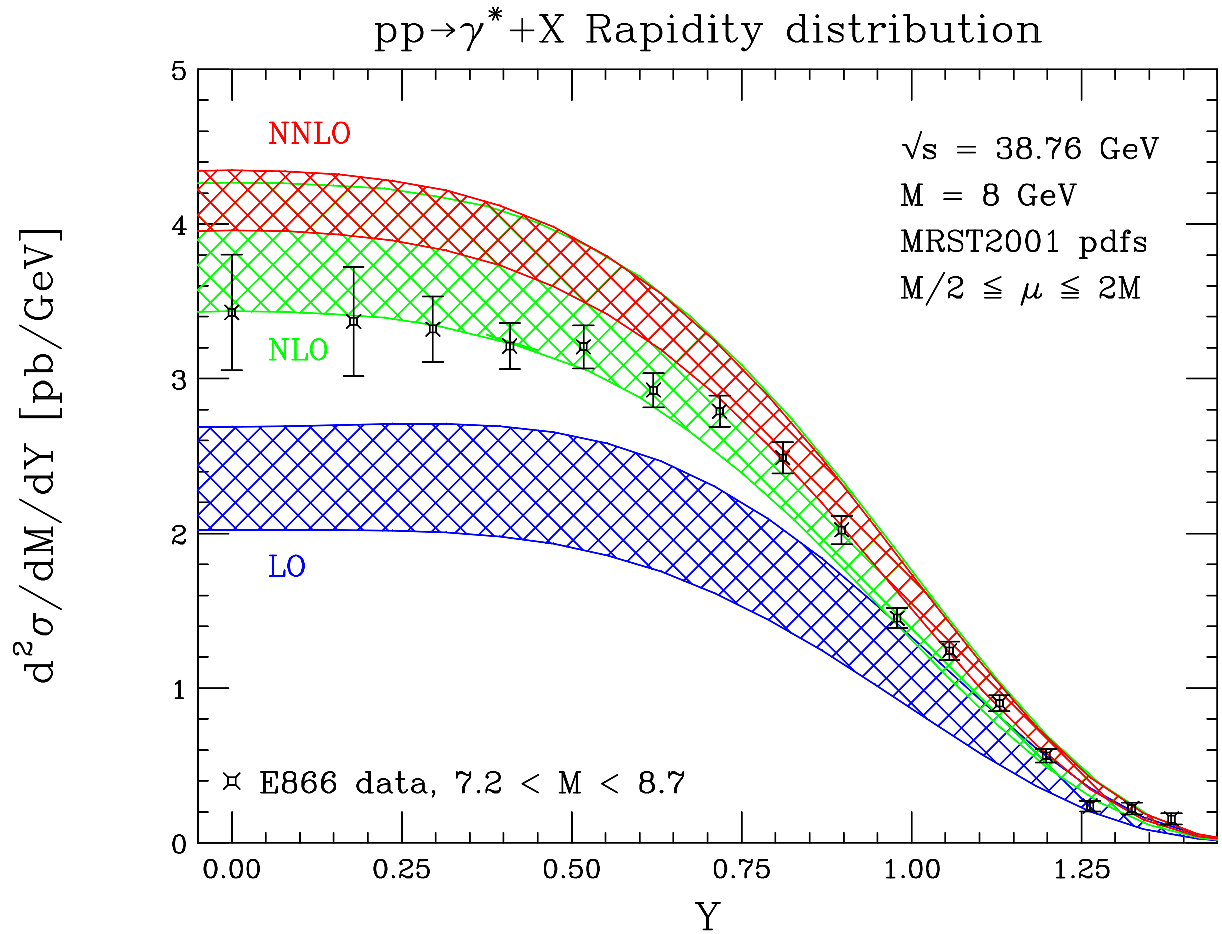
Splitting functions now complete. (Moch, Vermaseren and Vogt). Extremely similar to average of best estimates → no significant change in NNLO partons. Improve quality of fit very slightly (MRST), and reduces α_S.

Can be big change from NLO → NNLO



To do absolutely correct NNLO fit we need not only exact NNLO splitting functions.

NNLO differential Drell-Yan cross-sections recently calculated in terms of y by Anastasiou, Dixon, Melnikov and Petriello.

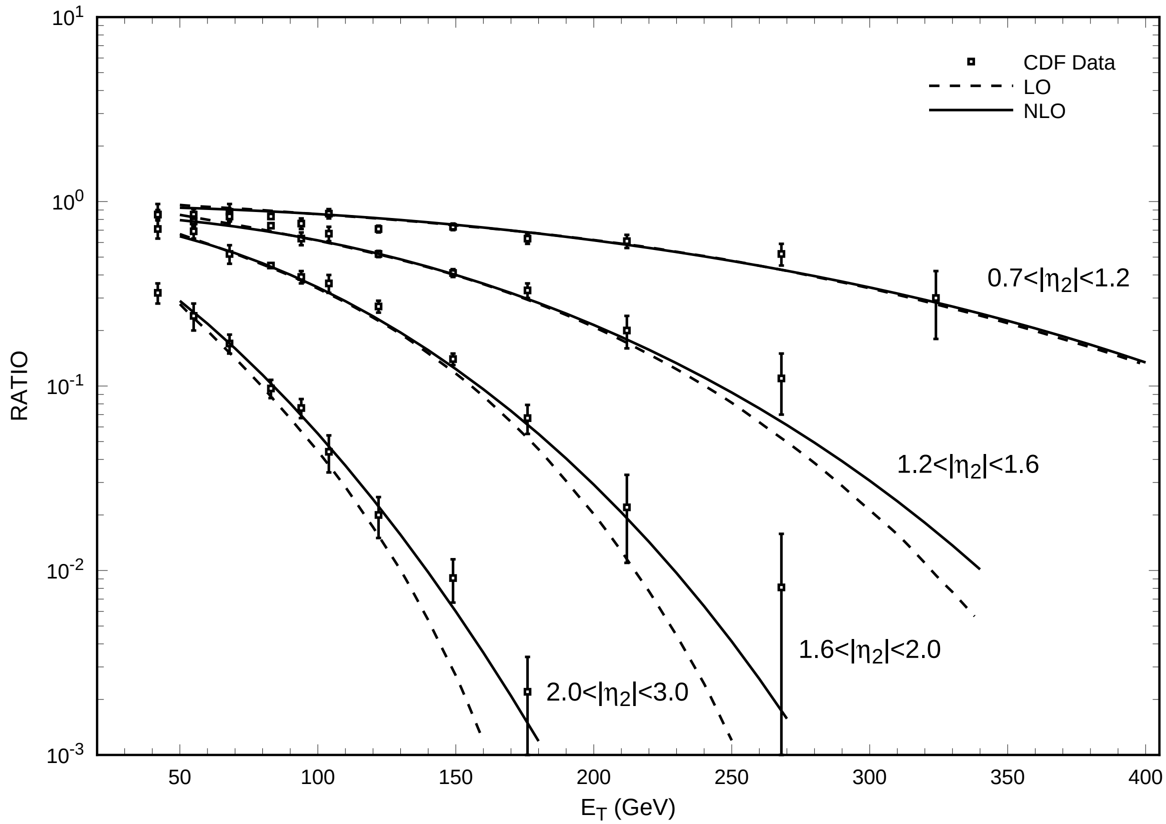


Decrease sea quarks. Implemented by Alekhin. Expected effect. Also want to use as function of x_F.

Do not know NNLO corrections to jet production in $pp(\bar{p})$ collisions. Stumbling block?

NLO corrections themselves not large, except at high rapidities.

At central rapidities $\leq 10\%$. Similar to correlated errors.



Also good NNLO estimates Kidonakis, Owens. Calculated threshold correction logarithms – should give decent indication.

$\rightarrow$ flat $3-4\%$ correction. Consistent with NLO indications. Smaller than systematics.

Also require rigorous treatment of heavy quark thresholds.

Will discuss below. Rather significant effect.

Essentially full NNLO determination of partons possible. Surely this is best, i.e. most accurate.

Yes, but ... only know some hard cross-sections at NNLO.

Processes with two strongly interacting particles largely completed

DIS coefficient functions and sum rules

$pp(\bar{p}) \rightarrow \gamma^\star, W, Z$ (including rapidity dist.), H, A^0, WH, ZH.

But for many other final states NNLO not known. NLO still more appropriate.

Resummations may be important even beyond NNLO in some regions, as may higher twist.

Factorization schemes.

In practice all hard cross-sections calculated in $\overline{MS}$ scheme in general.

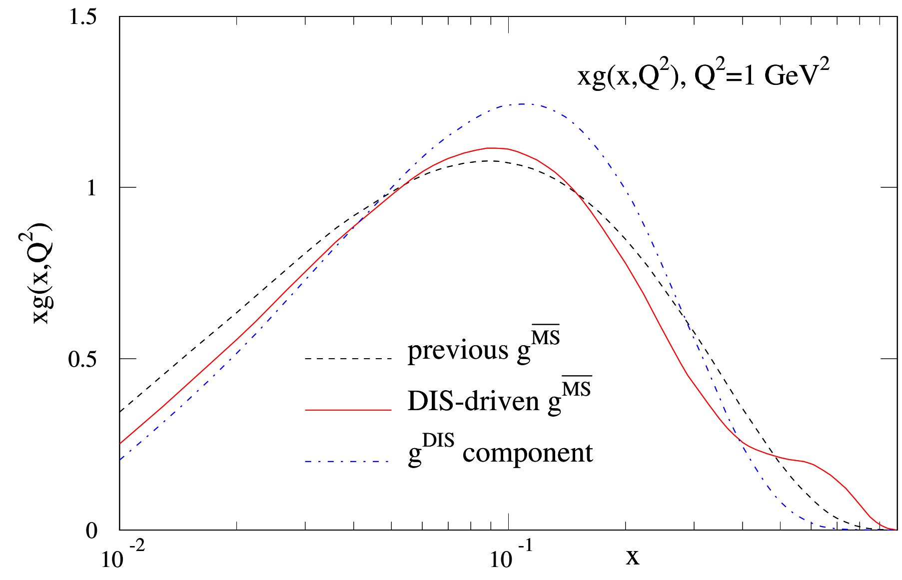
$\rightarrow$ everyone wishes to use $\overline{MS}$ parton distributions.

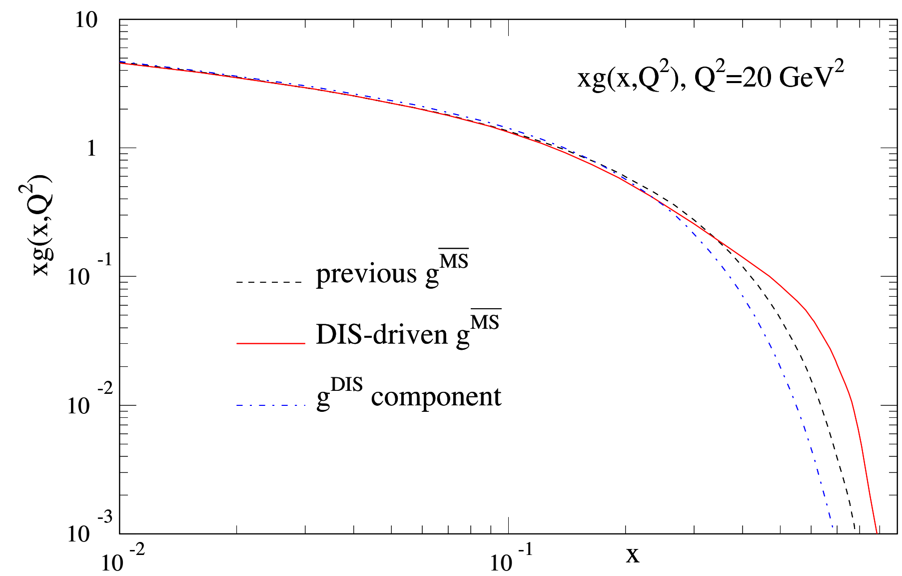
DIS-scheme partons can be more useful for relating partons to real physical results (especially at higher orders).

e.g. large high-x gluon required by Tevatron jets at NLO and NNLO in $\overline{MS}$ scheme is perfectly natural in DIS scheme. (Klasen, Kramer, sort of).

Implies high- behaviour can be calculated from scheme dependence (MRST).

Other schemes beyond $\overline{MS}$ most useful in this type of context.





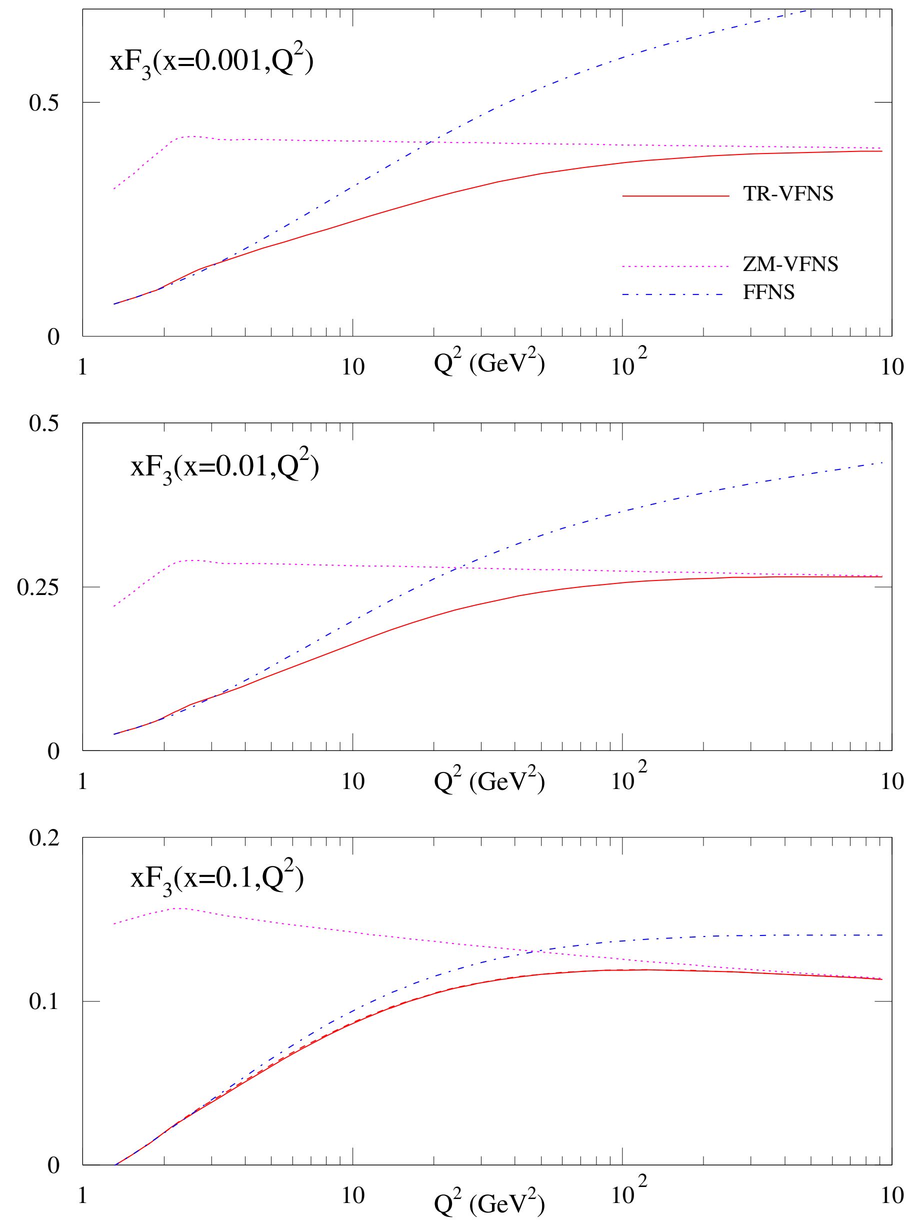
Flavour description.

Not too controversial. FFNS is in some senses intrinsically inferior to VFNS.

Does not sum $\ln Q^2/m_H^2$ terms in perturbative expansion correctly.

Sometimes leads to distinctly different results to inclusion of heavy partons at high scales (e.g. xF_3, measured using neutrino scattering at NuTeV).

Often very desirable to have heavy flavour partons due to lack of mass effects in calculated cross-sections (most high scale processes).



However FFNS partons sometimes needed because hard cross-sections only calculated with all heavy flavour generated in the final state.

HQVDIS for differential heavy flavour production in DIS, MC@NLO for heavy flavours, HERWIG for heavy flavour production (strictly needs LO partons), etc. Preferable to have VFNS descriptions. Not really on horizon.

However, FFNS must be done properly. Often wrong.

The NLO ($\mathcal{O}(\alpha_S^2)$) coefficient functions for heavy flavour in DIS calculated in scheme where the coupling α_S is fixed at **3** flavours. Partons have to be defined in same way, else double- (or zero) counting of $\alpha_S^2 \ln^2(Q^2/m_H^2)$ terms. $\rightarrow$ large error.

Also, no FFNS coefficient functions at NNLO (or even exact at NLO for charged current, or differential Drell-Yan).

Important since NNLO FFNS contains $\alpha_S^3 \ln^3(Q^2/m_H^2)$.

Consider ZM-VFNS *scheme*.

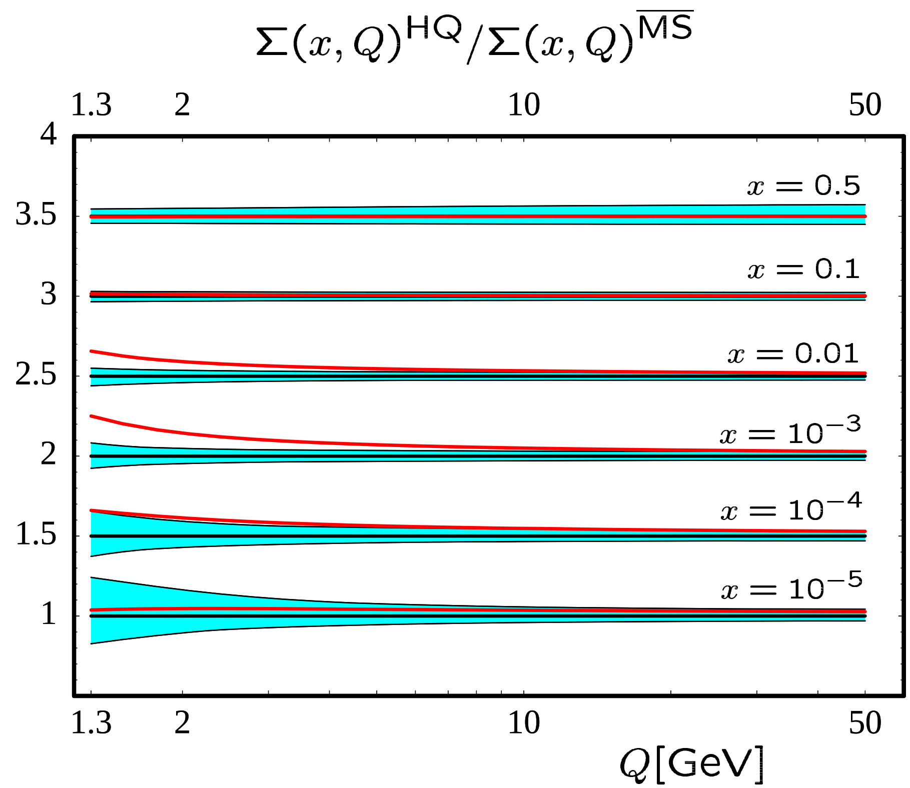
Terminology *scheme* misleading. Usually different way of arranging correct result.

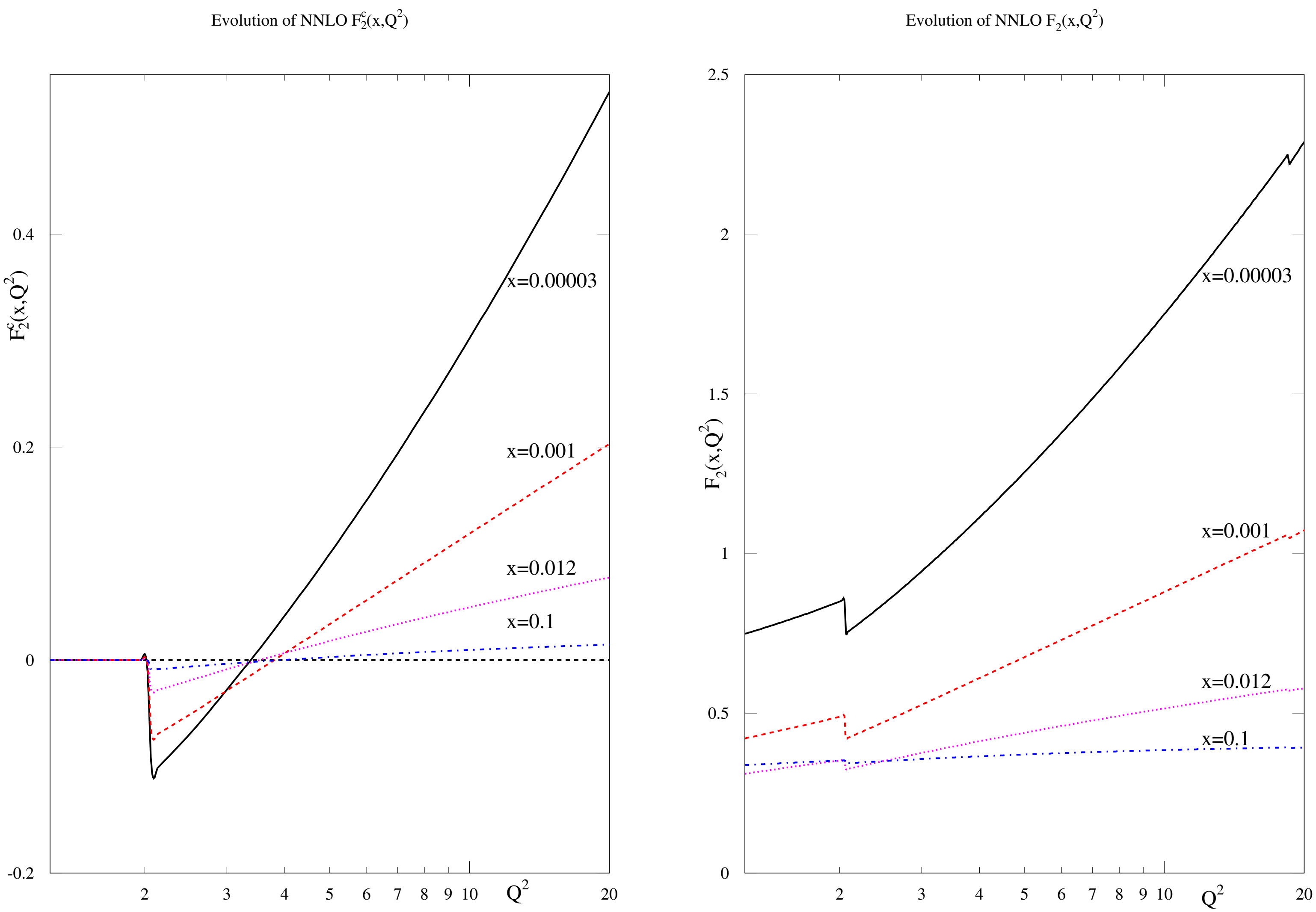
In this case simply an error of $\mathcal{O}(m_H^2/Q^2)$. Incorrect compared to general VFNS.

Can't see why it is useful. At high scales often in massless limit for c and b. VFNS reduces to this limit.

Partons obtained from fitting in region where $\mathcal{O}(m_H^2/Q^2)$. Ignoring these terms $\rightarrow$ incorrect partons at both high and low Q^2.

Difference between approaches for CTEQ compared to (conservative) uncertainties.





Also at NNLO partons discontinuous at transition points (NLO $\overline{MS}$-scheme lucky). $c(x,Q^2)$ at m_c^2 very negative – nothing to do with negative $g(x,Q^2)$.

Need a general (VFNS) for any sensible description with heavy quark partons .

If VFNS coefficient functions not known error of $\mathcal{O}(m_H^2/Q^2)$ from using VFNS. No worse than permanent error from using ZM-VFNS. Can also input at least general kinematic requirements.

Variety of different definitions of VFNS.

However, some convergence. Most variations perfectly good so long as they are based on definition in terms of parton distribution functions (not on extrapolation between structure function limits).

Each choice superior to ZM-VFNS and to FFNS in general terms.

However, most not really defined up to NNLO yet – at least not in detail.

Some disagreements (come to special joint Structure Functions/Heavy Flavours session).

Variety of Different Parton Analyses.

As well as all the partons defined in different ways by order, scheme, *etc.* we have a wide variety of choices for any given theoretical prescription for partons.

Are all of these necessary, or even useful?

Obvious that some competition is necessary – ZEUS, H1, Atlas, CMS.

Not all partons are really completely equal though.

Some are in some sense simply in error.

Variety of reasons – bugs in programs, incorrect theoretical approach (e.g. wrong coupling for flavour scheme), approximations to complete theoretical approach (region of applicability – MRST03c only suitable within region of cuts).

Effect sometimes small, but often of size of intrinsic uncertainty or greater. If so should not be used.

NNLO mainly still in the approximate, or occasionally *wrong* stage.

Other issues to consider.

Treatment of errors.

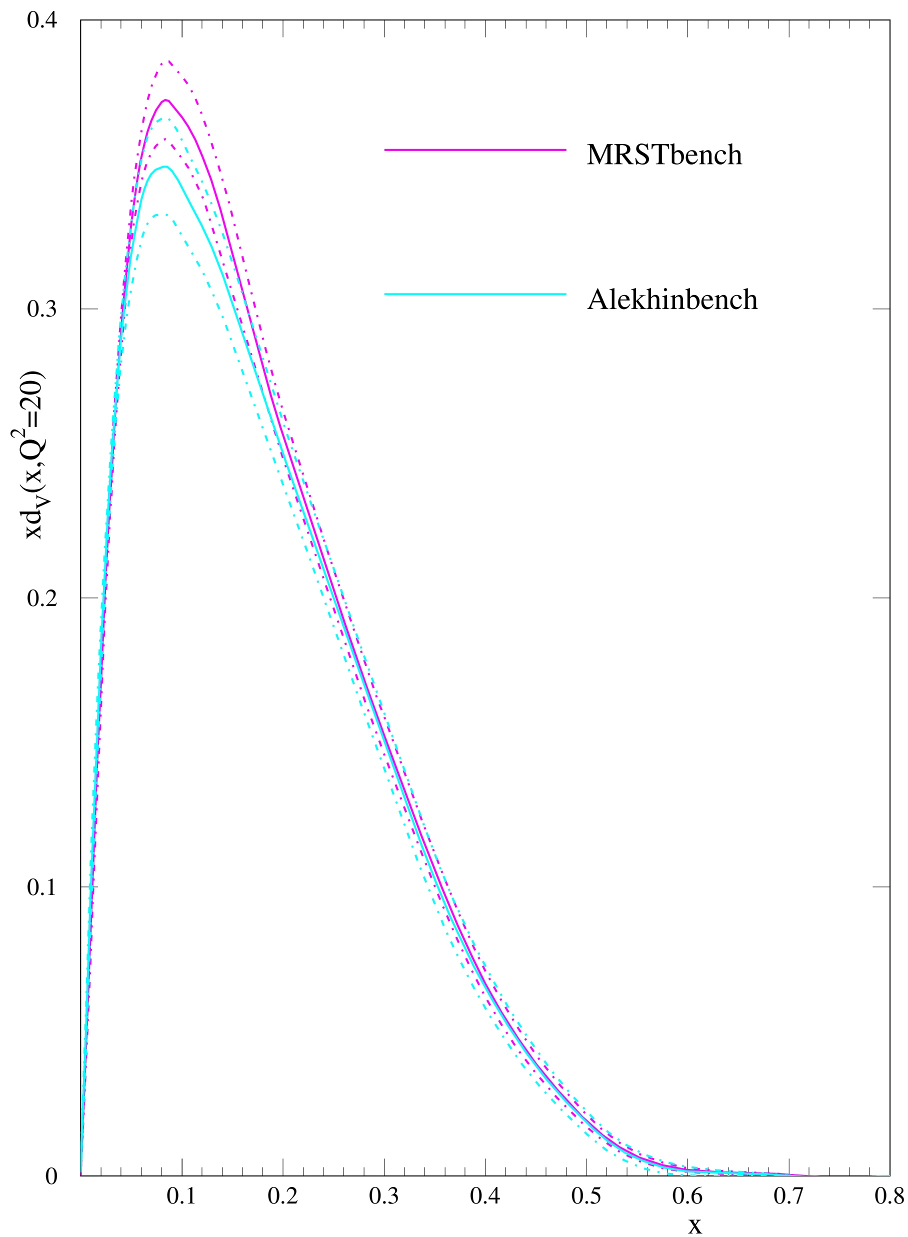
Exercise for $HERA-LHC$ meeting. Fit proton and deuteron structure function data from H1, ZEUS, NMC and BCDMS, for $Q^2 > 9\text{GeV}^2$ using $ZM-VFNS$ and same form of parton inputs at same $Q_0^2 = 1\text{GeV}^2$.

Very conservative fit.

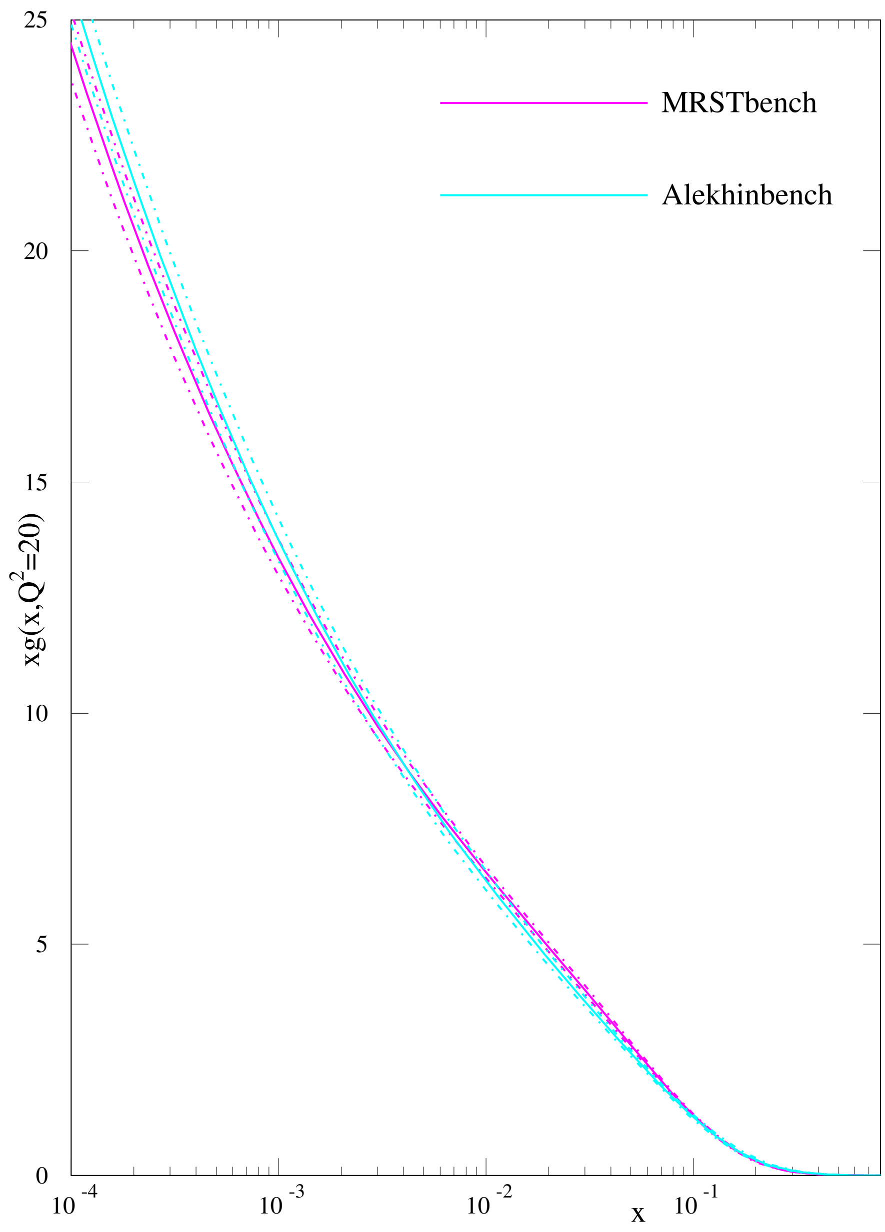
Compare rigorous treatment of all systematic errors (Alekhin) with simple quadratures approach (MRST), both with $\Delta\chi^2 = 1$.

$\rightarrow$ some difference in central values (other possible reasons) and similar errors.

Fairly consistent.



Same conclusion for all partons, e.g. gluon.



Surely standard treatment of systematic errors best.

Yes, but perhaps not without question. Consider joint H1-ZEUS data sets. Systematics of one reduced by comparison to the other Glazov.

Average of all HERA data

Changes in systematic uncertainties:

Fitted systematics:

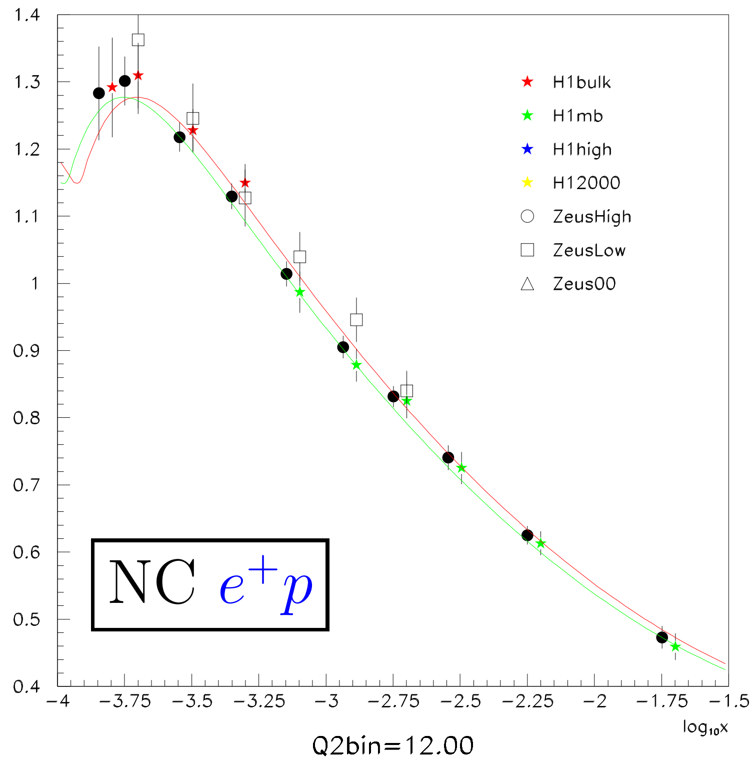
		shift	uncertainty
1	zlumi1_zncepl	-1.2841	0.5836
2	h2_Ee_Spacal	0.6440	0.3281
3	h3_Ee_Lar_00	-0.8265	0.4435
4	h4_ThetaE_spacal	-0.2569	0.6566
5	h5_ThetaE_94-97	-0.1756	0.7802
6	h6_ThetaE_00	-0.3027	0.5288
7	h7_H_Scale_Spacal	0.3750	0.4813
8	h8_H_Scale_Lar	-0.8554	0.5353
9	h9_Noise_Hcal	-0.6404	0.3591
10	h10_GP_BG_Spacal	-0.1805	0.8260
11	h11_GP_BG_LAr	1.0769	0.8560
12	h12_BG_CC_94-97	0.2680	0.7883
13	h13_BG_CC_98-00	-1.0295	0.8589
14	h14_ChargeAsym	0.0246	0.9993
15	hllumi1_SPACAL_bulk	-0.0696	0.5612
16	hllumi2_SPACAL_MB	1.0815	0.6271
17	h1lumi3_LAr_94-97_e+p	-2.7111	0.6103
18	h1lumi4_LAr_e-p	-0.6585	0.7737
19	h1lumi5_LAr_2000	-2.5156	0.5885

- Good global $\chi^2/ndf = 533.9/601$
- Most of the changes are within 1σ
- Several systematic sources are reduced by factor 2 and more

Joint data set → much more accurate data with small systematic errors.

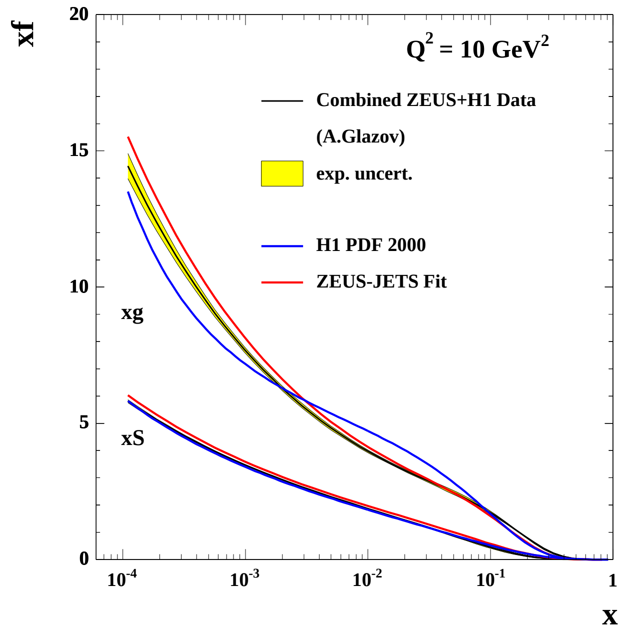
Average of all published HERA NC/CC data

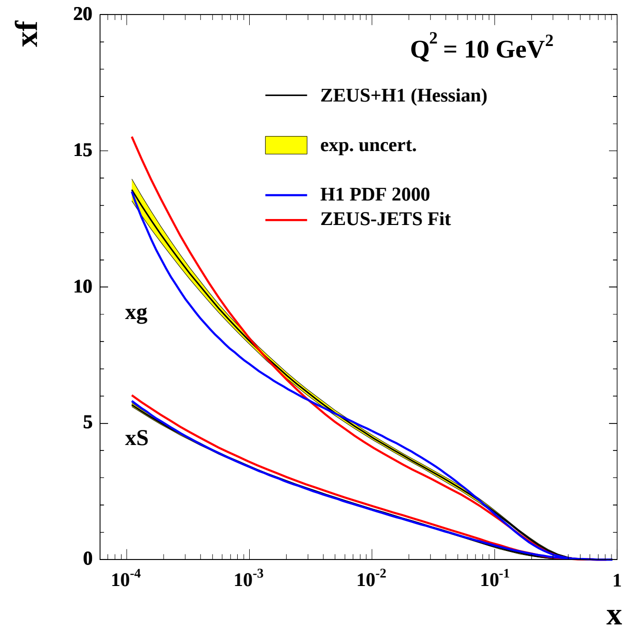
16 individual data sets of NC/CC data published by H1 and Zeus collaborations. Examples for some Q^2 bins:



Central bit fit to data moves.

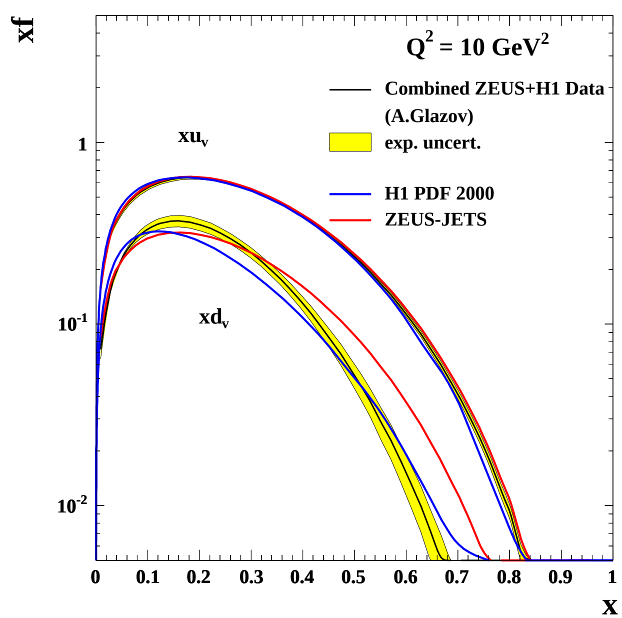
(Gwenlan and Cooper-Sarkar). More quantitative. Compare a fit using same procedure to both data sets compared to combined data set.

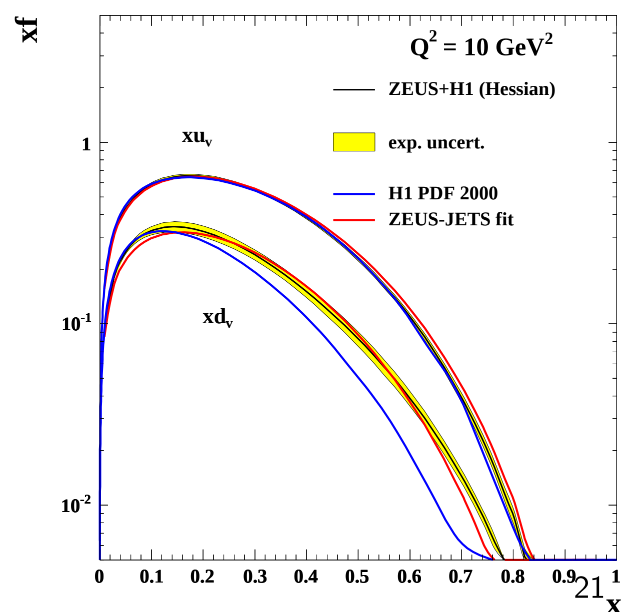




Partons from fit to combined data not the same as from fit to both sets, even within errors.

Systematics of joint set contain different (better) information than combination of sets independently.





Syatematic uncertainty s_λ	in PDF fit	in Theory-free fit
ZEUS electron efficiency	1.68	0.31
ZEUS electron angle	-1.26	-0.11
ZEUS electron energy scale	-1.04	0.97
ZEUS hadron calorimeter energy scale	1.05	-0.58
H1 electron energy scale	-0.51	0.61
H1 hadron energy scale	-0.26	-0.98
H1 calorimeter noise	1.00	-0.63
H1 photoproduction background	-0.36	0.97

Table 1: Systematic shifts for ZEUS and H1 data as determine by a joint pQCD PDF fit, and as determined by the theory-free data combination fit

Partons pulled in different direction by systematics of joint set compared to combined pull of independent sets.

Data and theory can move relative to each other due to systematics, but this may be due to failures in theory. We are not fitting to perfect theoretical model with unknown parameters, we are testing QCD at some order.

Always best to remember this and try to minimize systematic errors.

Joint H1/ZEUS data set very desirable. Easier to understand and trust dominant statistical errors.

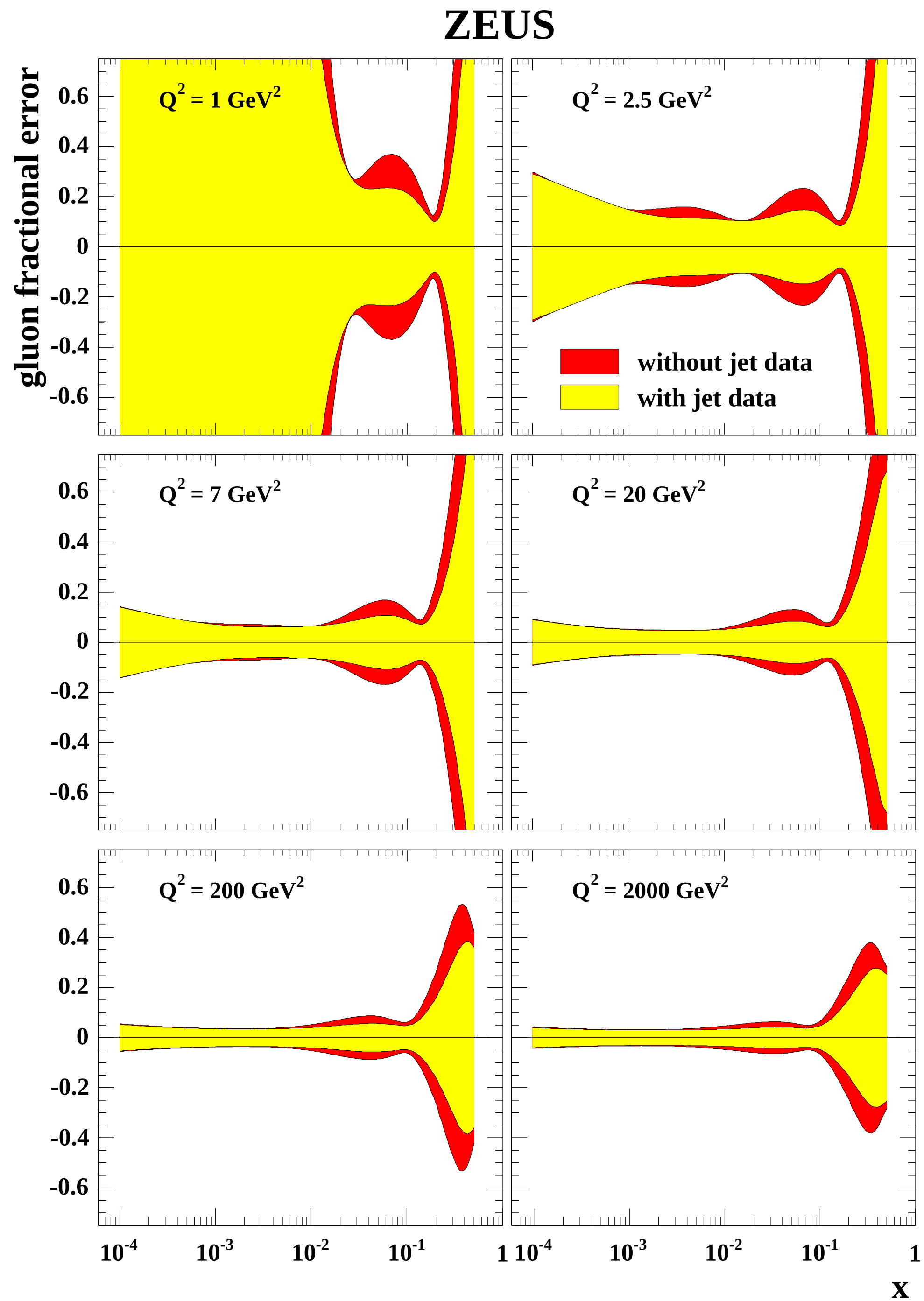
Data used.

In general want to consider as much data as possible.

Improvement of uncertainty in ZEUS extraction of partons when including their own improved charged current data (valence partons) and especially jet data (gluons).

Uncertainty improves significantly.

In principle central value can move. Not much in this case.



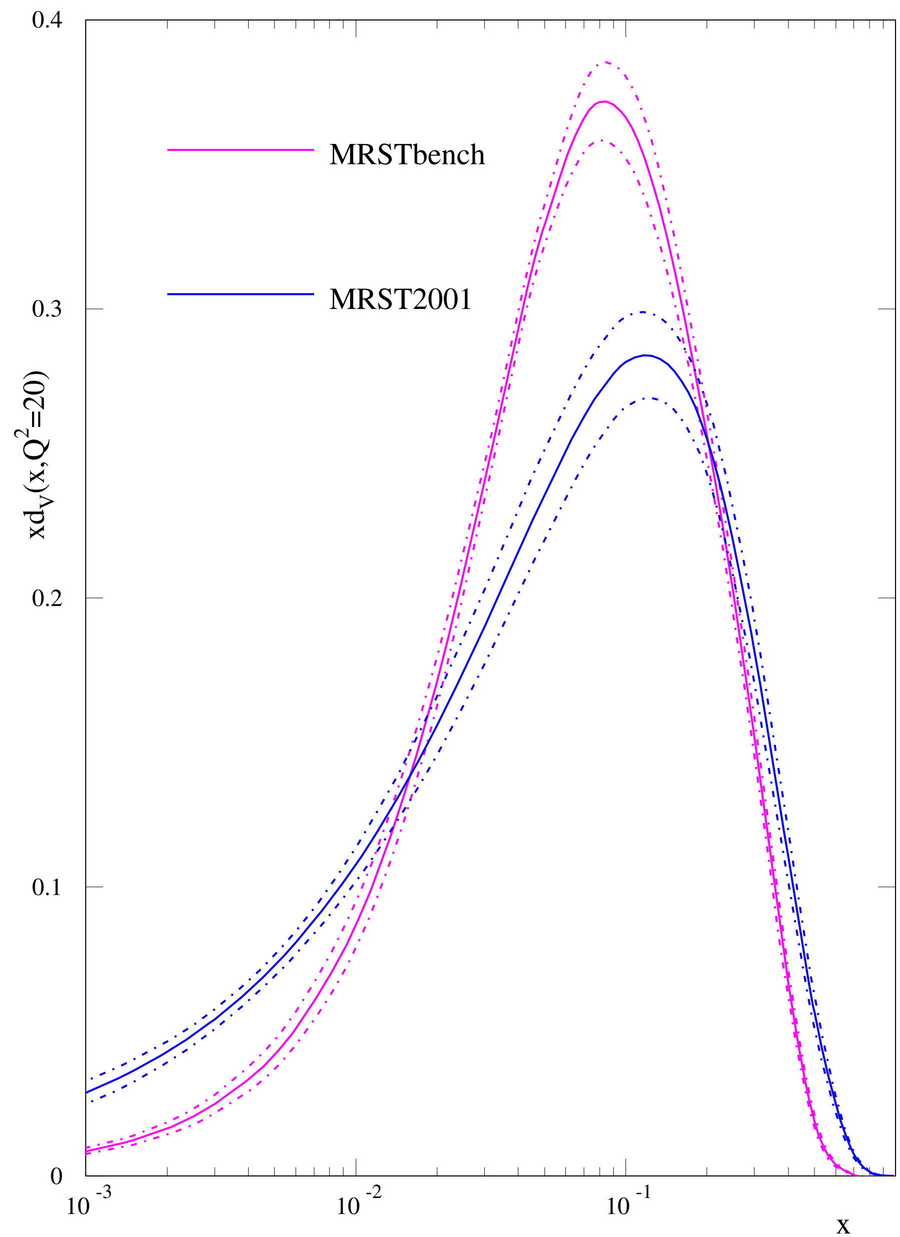
Back to HERA-LHC benchmark partons.

How do partons from very conservative, structure function only data compare to global partons?

Compare to MRST01 partons with uncertainty from $\Delta\chi^2 = 50$.

Enormous difference in central values.

Errors similar.

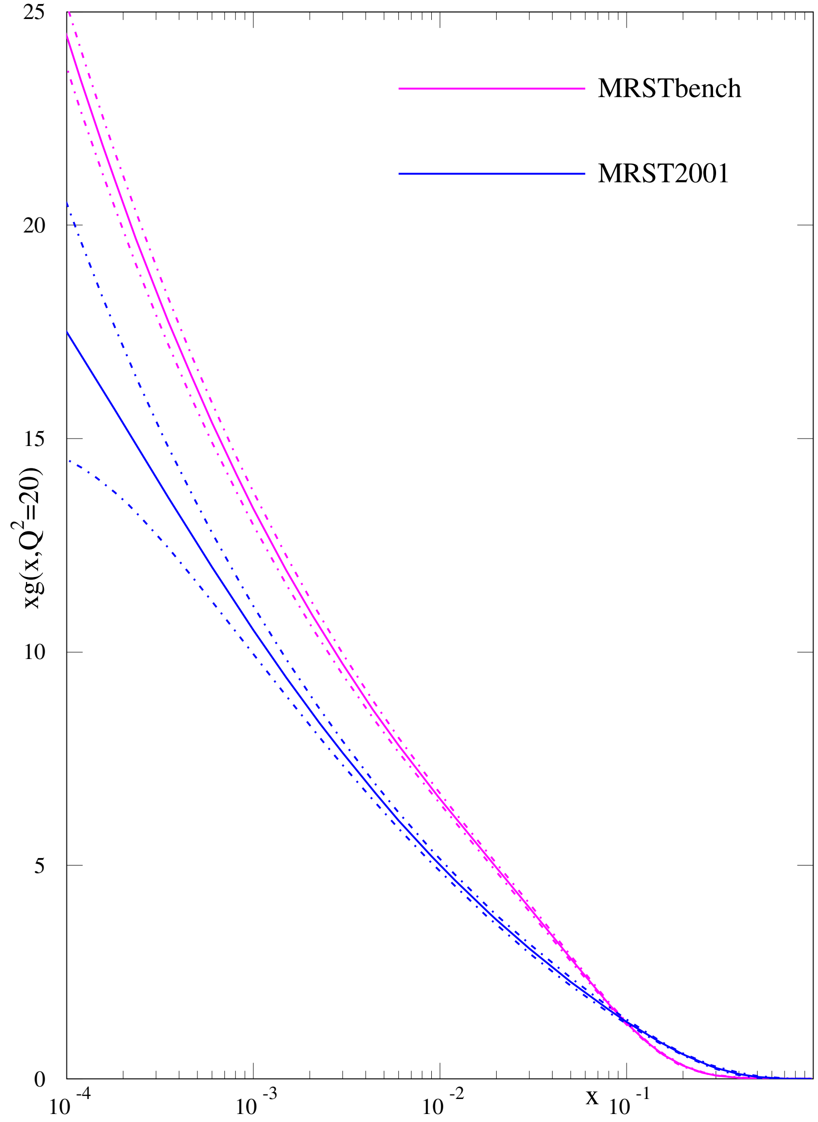


Similar for gluon.

Moreover $\alpha_S(M_Z^2) = 0.1110 \pm 0.0015$ compared to $\alpha_S(M_Z^2) = 0.119 \pm 0.002$.

Uncertainty of small-x gluon small due to valencelike input at low scale.

Gluon starts at zero. Only uncertainty in evolution.

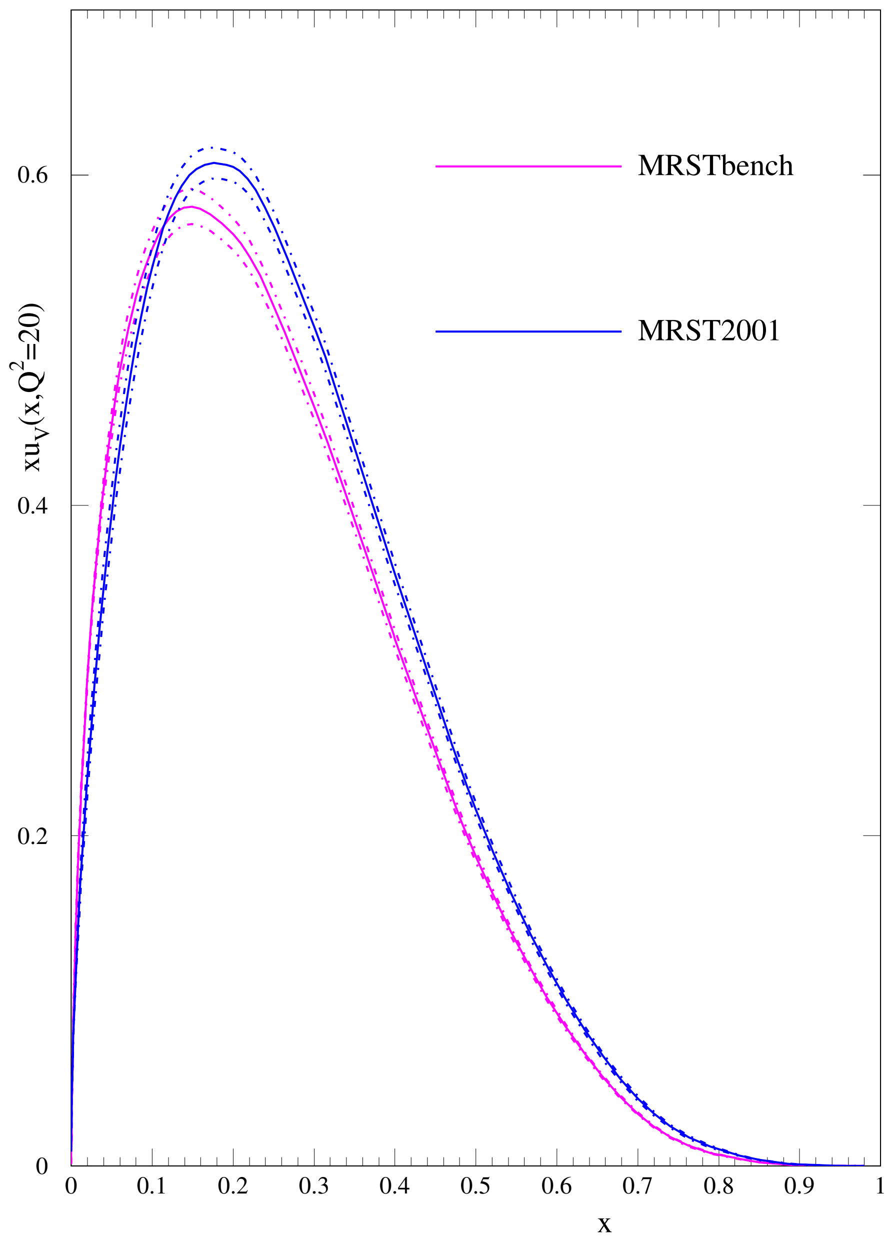


Even for u_V quark.

This, if anything should have been good in benchmark fit.

Sea large at high x – valence quarks too low.

Not consistent with Drell-Yan data.



Something is seriously wrong in one of these analyses.

Central values different by many σ.

Errors similar from $\Delta\chi^2 = 50$ compared to $\Delta\chi^2 = 1$ with only approx. twice the data.

Very confident benchmark fit is wrong. Fails when compared to pretty much all data not in the fit. Is all this data unreliable?

Also, rigorously defined error analysis of this data is wrong. It does not produce true uncertainty by some way.

Partons should be constrained by all possible reliable data. Benchmark fit extreme, but not so extreme. Some partons listed are similar in data used, but many input implicit constraints from elsewhere.

In global fit $\Delta\chi^2 = 1$ is not reliable due to strict incompatibility of different data sets.

Something better than $\Delta\chi^2 = 50(100)$ or offset method. Not sure what it is yet. See parallel sessions.

Why is standard approach so inconsistent?

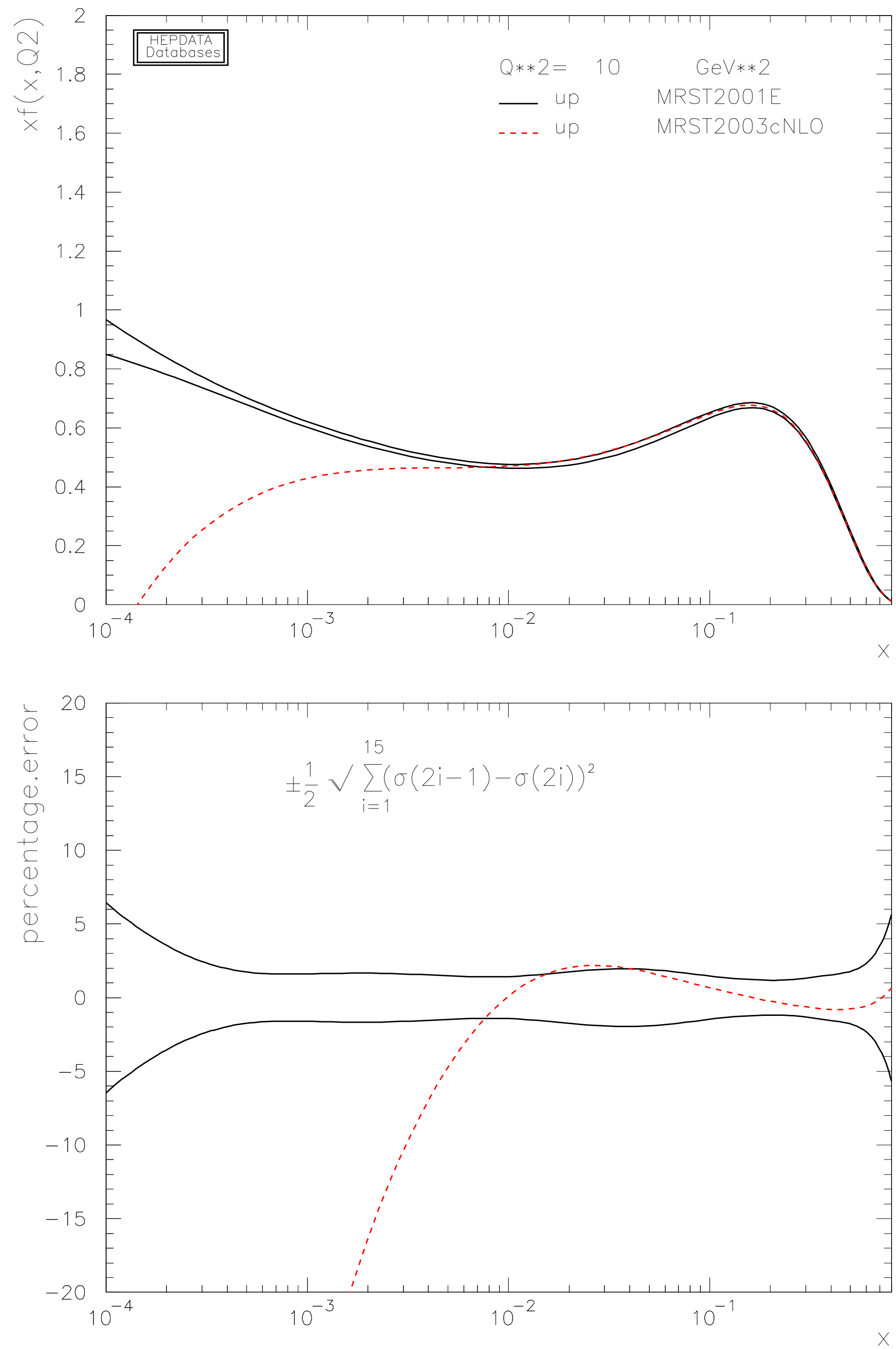
Systematic errors difficult to understand, and not usually Gaussian in nature.

Our theory is never perfect – not simply a matter of tying down unknown constants.

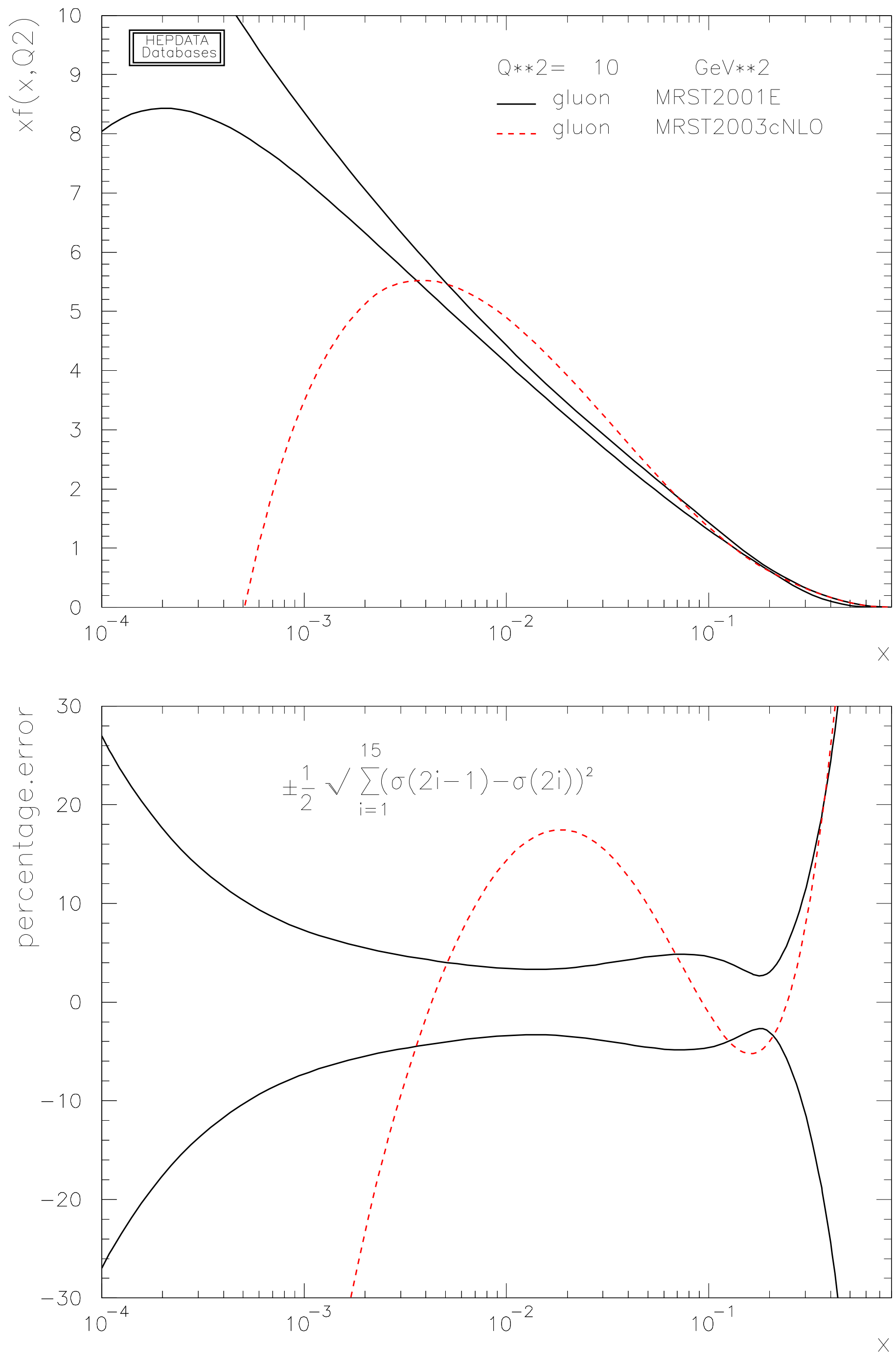
Corrections possible at low Q^2, small x and very high x.

Safer to fit in restricted kinematic range, but use all available data.

Conservative partons consistent with usual partons, for directly fit quarks) in range where data fit, and everything much more self consistent (more like $\Delta\chi^2 = 5$).



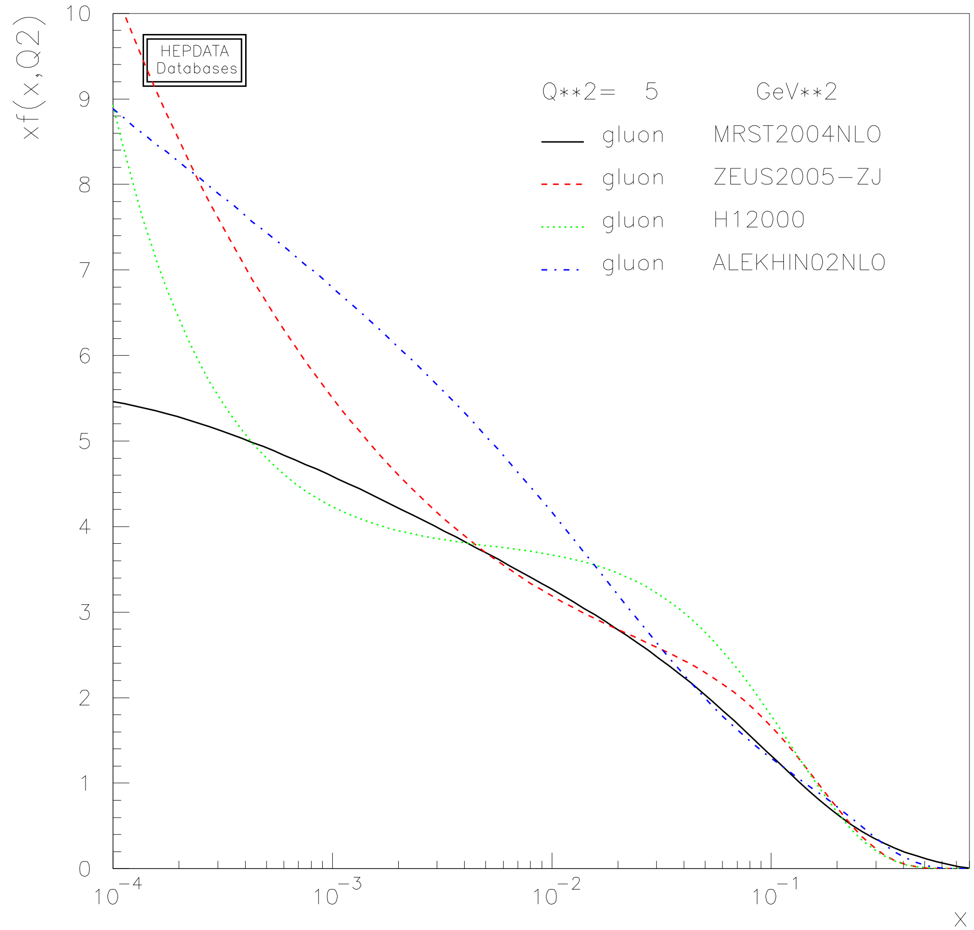
Much worse for more indirectly fit gluon.



In fact gluon still very uncertain at low x and Q^2.

All partons fit to same small-x HERA data.

The direct constraint from any reasonable direct measurement of $F_L(x, Q^2)$ would help this situation.



Large x resummations..

Terms like $\alpha_S^n \ln^{2n-1}(1-x)$ in series.

Resummations well understood. Claims of $20-30\%$ changes in partons at $x=0.8$, $Q^2 = 30\mathrm{GeV}^2$.

However, compared to NLO. Much of correction already in NNLO.

Although resummation well-understood for structure functions, not known for differential Drell-Yan. Difficult to include in global fit.

Also resummations and higher twist begin to mix.

Can perform fits with the known large $\ln(1-x)$ terms included explicitly up to NNNLO.

Also parameterize higher twist contributions by

$$F_i^{\mathrm{HT}}(x,Q^2) = F_i^{\mathrm{LT}}(x,Q^2)\left(1+\frac{D_i(x)}{Q^2}\right)$$

x	LO	NLO	NNLO	NNNLO
0–0.0005	-0.07	-0.02	-0.02	-0.03
0.0005–0.005	-0.03	-0.01	0.03	0.03
0.005–0.01	-0.13	-0.09	-0.04	-0.03
0.01–0.06	-0.09	-0.08	-0.04	-0.03
0.06–0.1	-0.02	0.02	0.03	0.04
0.1–0.2	-0.07	-0.03	-0.00	0.01
0.2–0.3	-0.11	-0.09	-0.04	0.00
0.3–0.4	-0.06	-0.13	-0.06	-0.01
0.4–0.5	0.22	0.01	0.07	0.11
0.5–0.6	0.85	0.40	0.41	0.39
0.6–0.7	2.6	1.7	1.6	1.4
0.7–0.8	7.3	5.5	5.1	4.4
0.8–0.9	20.2	16.7	16.1	13.4

In this type of expansion $\ln(1-x)$-corrections become indistinguishable from $1/W^2$ corrections at low W^2.

Small-x

NLO and NNLO gluons small at small x and Q^2.

Simple absorptive corrections/higher twist do not have much effect. Misleading results by considering LO partons.

Empirical higher twist absent (previous table). Absorptive corrections, i.e.

$$\frac{\partial(xg(x,Q^2))}{\partial \ln Q^2} = \ldots - 3\frac{\alpha_S^2(Q^2)}{R^2Q^2}\int_x^1 \frac{dx'}{x'}\,[x'g(x',Q^2)]^2$$

can help a little (Martin, Ryskin, Watt).

Empirical resummation corrections improve global fit, e.g.

$$P_{gg} \to .... + \frac{1}{x}\left[A\bar{\alpha}_S^4\left(\frac{\ln^3(1/x)}{6} - \frac{\ln^2(1/x)}{2}\right) + B\bar{\alpha}_S^5\left(\frac{\ln^4(1/x)}{24} - \frac{\ln^3(1/x)}{6}\right)\right],$$

$$P_{qg} \to .... + \alpha_S\frac{N_f}{3\pi x}\left[C\bar{\alpha}_S^3\left(\frac{\ln^2(1/x)}{2} - \ln(1/x)\right) + D\bar{\alpha}_S^4\left(\frac{\ln^3(1/x)}{6} - \frac{\ln^2(1/x)}{2}\right)\right].$$

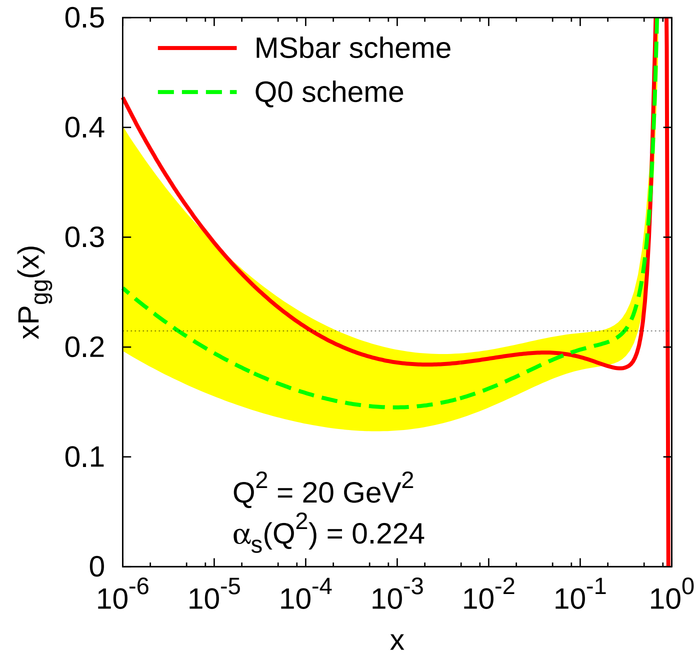
improve global fit significantly at NLO and NNLO.

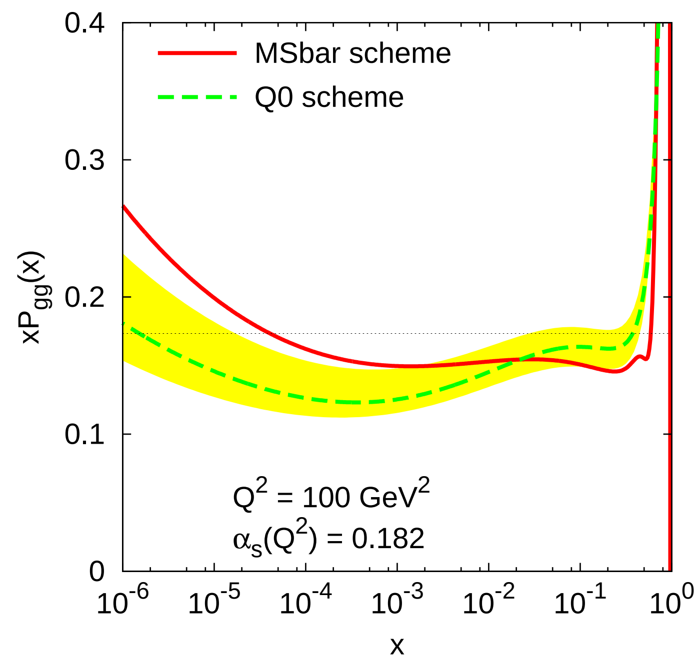
Lot of recent activity in small-resummations again recently.

Papers in 2006 by Altarelli, Ball and Forte, Ciafaloni, Colferai, Salam and Stasto, White, RT.

Lots of differences in approaches. All include running coupling. All agree moderate effects compared to original resummation ideas.

All get dip, then very low x rise in P_{gg}.



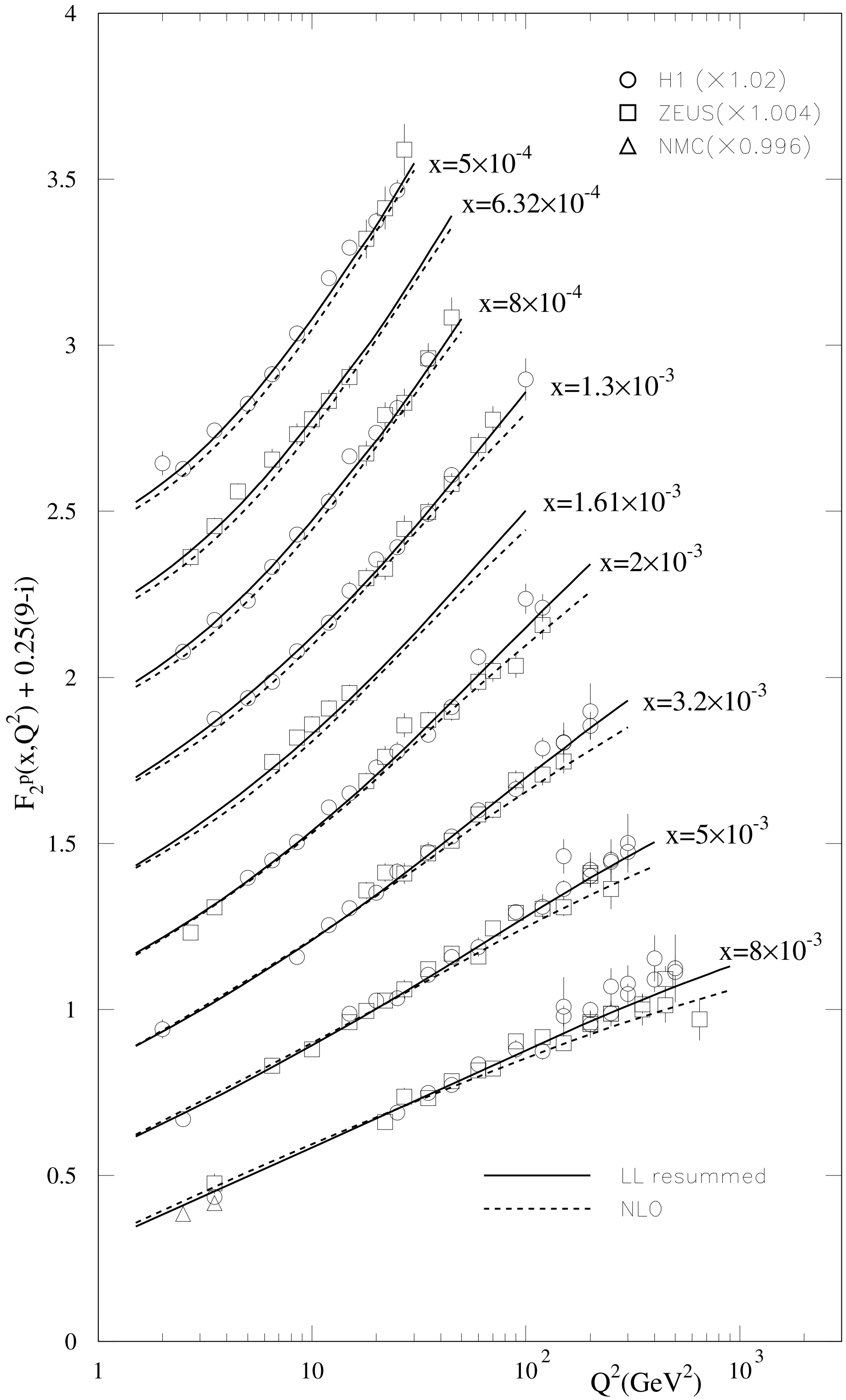


Phenomenology only at LO, but with consistent VFNS.

Better fit than NLO-in-α_S in terms of $dF_2(x,Q^2)/d\ln Q^2$, (White, RT).

Enhancement of evolution too great at small x. Gluon and $F_L(x,Q^2)$ too small at moderate x. Need the full NLO generalization.

More on this in parallel sessions (and on saturation – little removed from partons – other plenary talk).



Conclusions

Lots of different types of partons defined. NNLO in principle on a very sound footing. Current available partons all slightly approximate. Soon to change.

NNLO in principle preferable to NLO and LO – significant corrections. NNLO best for testing QCD, but latter two still needed for many applications.

A general VFNS is preferable in same sense as is NNLO. However, again FFNS sometimes needed. Don't really see the need for ZM-VFNS partons – can be misleading.

Partons for special occasions (e.g. NuTeV $\sin^2\theta_W$) all available – QED corrections, isospin violation MRST, $s(x,Q^2) \neq \bar{s}(x,Q^2)$ CTEQ, DIS-scheme, $etc.$

Many used partons out of date (or wrong).

Variations in treatment of systematic errors does not have enormous effect, unless completely dominant (Tevatron jets). Far smaller than effect of varying which data sets are actually fit.

Personally always wary of partons obtained from data sets which only provide a limited sets of direct constraints.

Analysis of joint H1/ZEUS data suggests systematic errors may lead to incorrect pull on partons. Best to minimize these by combining data if possible.

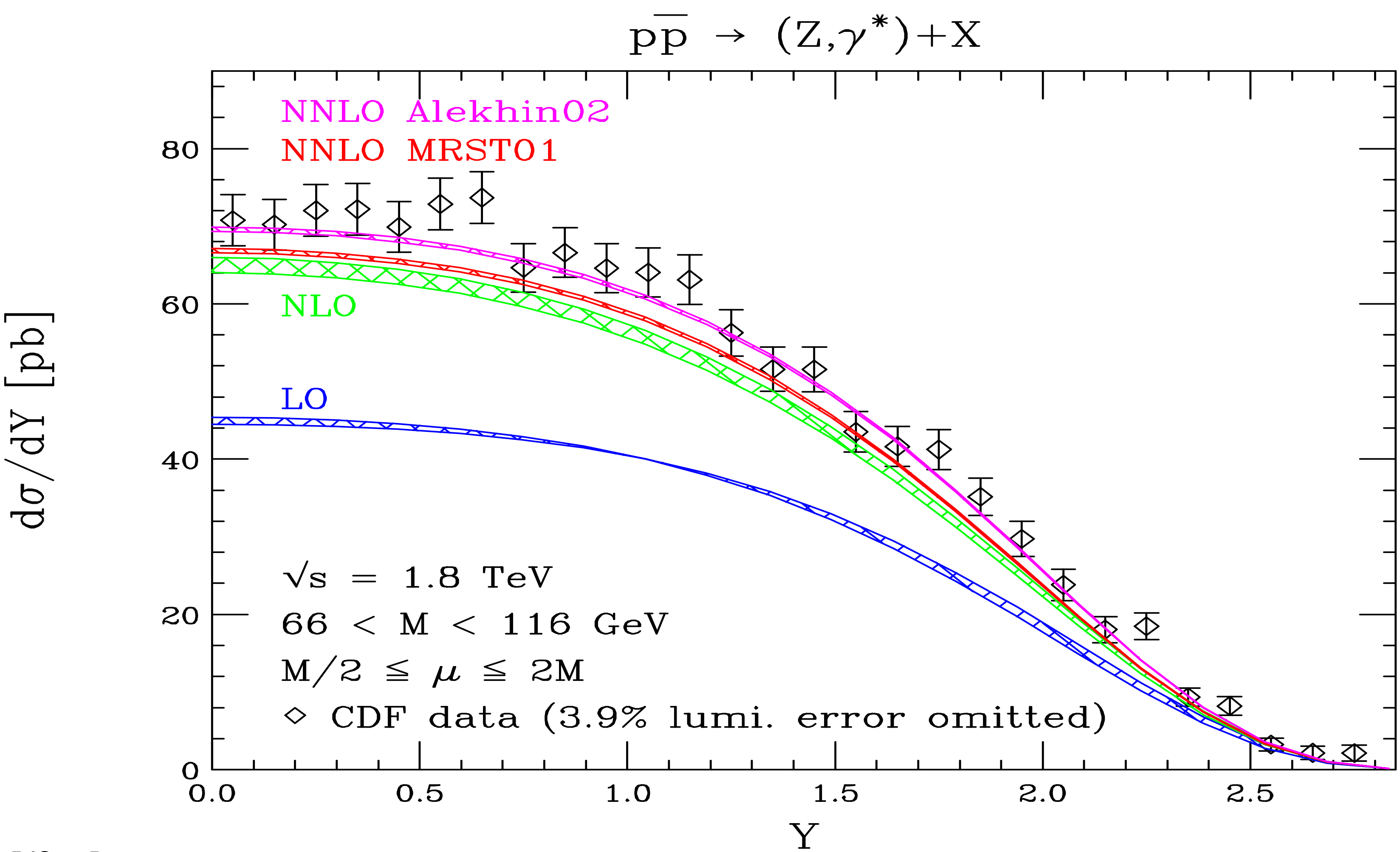
Lack of perfect theory perhaps makes simultaneous fit to all data difficult. Incorrect pull on unconstrained partons.

Improvements due to resummations, high- and small-x. Getting closer. Not there yet.

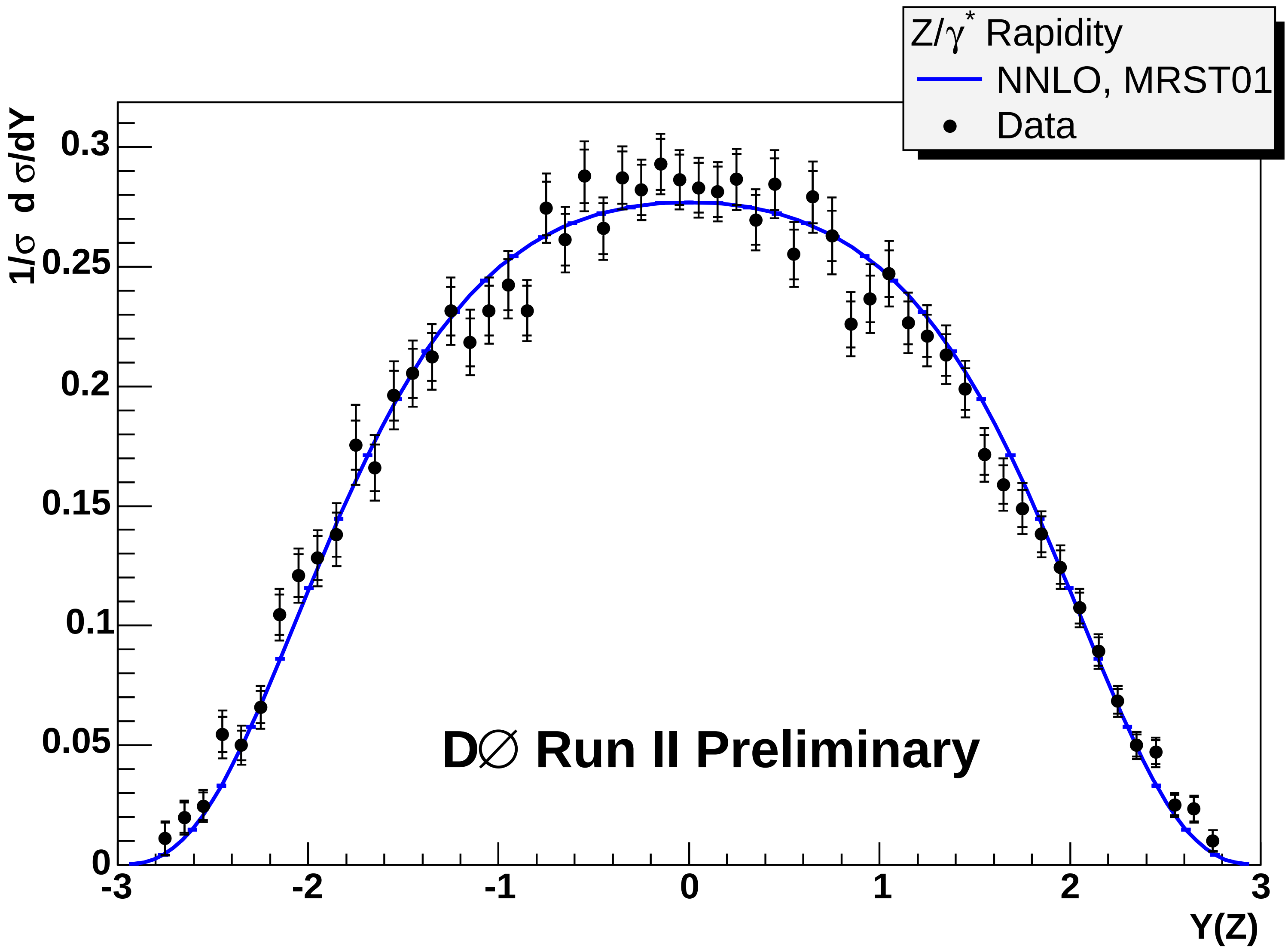
Big problem. No direct constraint on gluon in most of range. Big variations. Not sure of best theoretical approach. Helped by as good a measurement of $F_L(x, Q^2)$ as can be squeezed in.

Rapidity distribution of Z-boson

(Anastasiou-Dixon-Melnikov-Petriello 03)



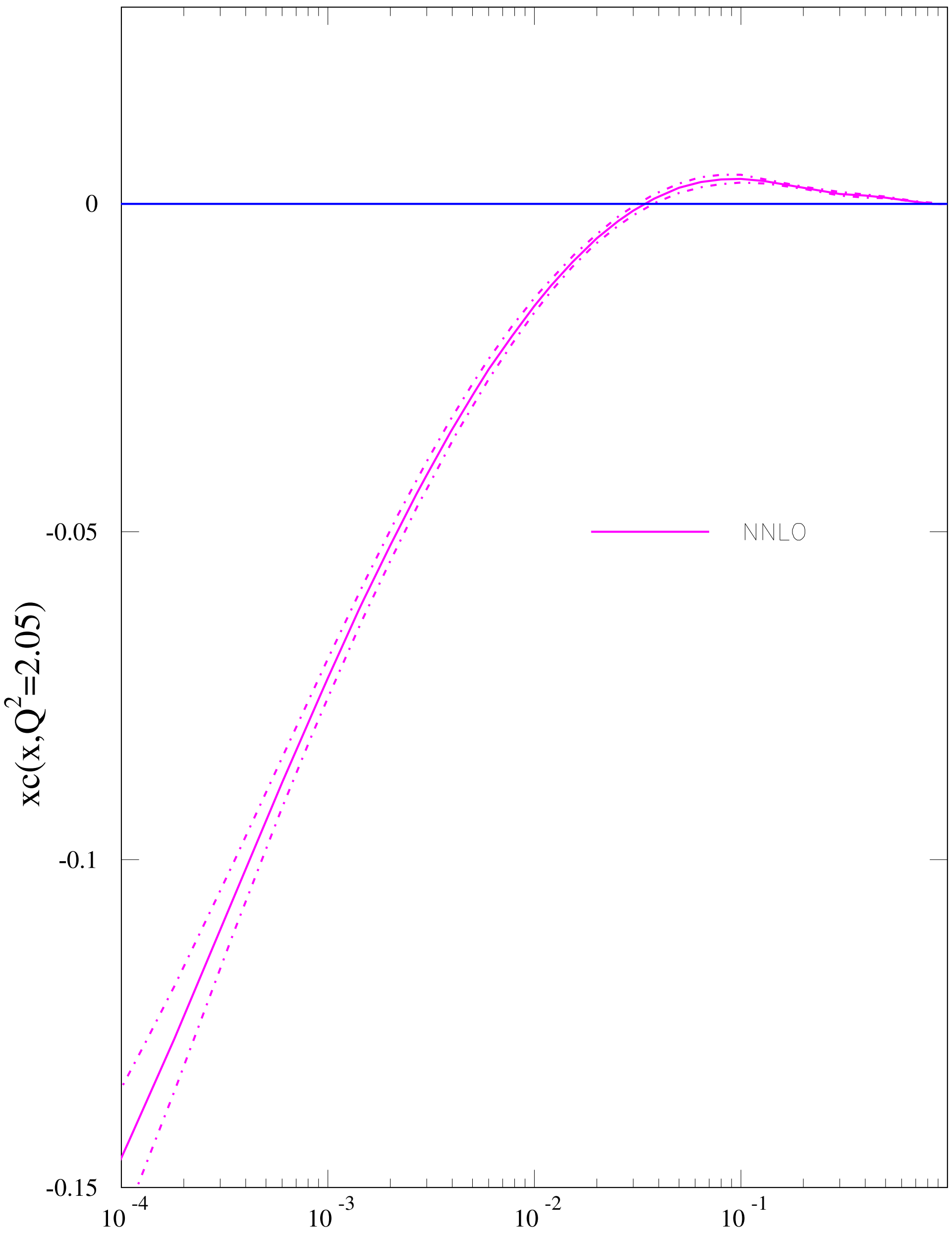
Comparison of MRST prediction for Z rapidity distribution with preliminary data.



Heavy flavour no longer turns on from zero at $\mu^2 = m_c^2$

$$(c+\bar{c})(x, m_c^2) = A^2_{Hg}(m_c^2) \otimes g(m_c^2)$$

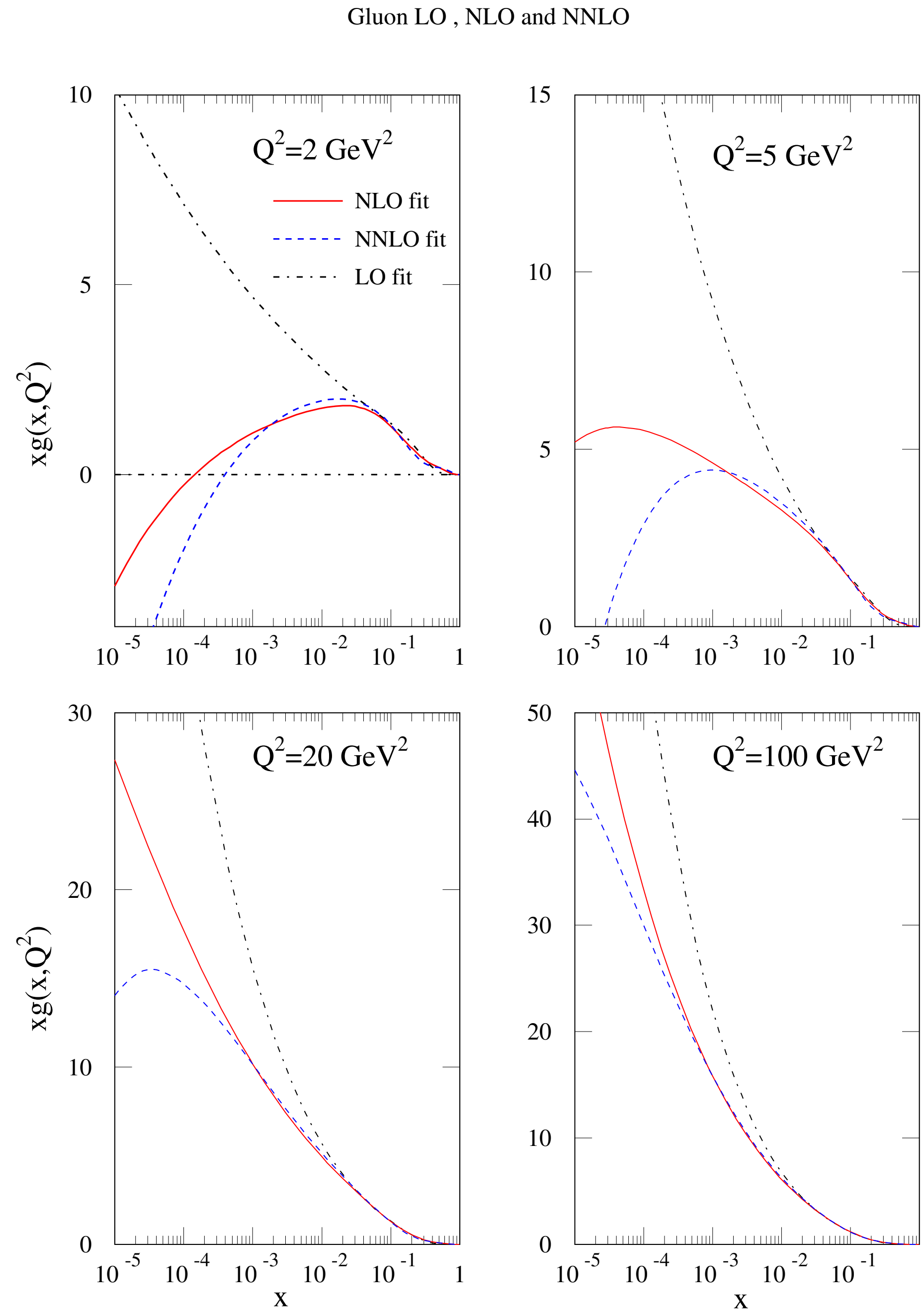
In practice turns on from negative value, (for general gluon).



The gluon extracted from the global fit at LO, NLO and NNLO.

Additional and positive small-x contributions in P_{qg} at each order lead to smaller small-x gluon at each order.

Note - this conclusion relied on correct application of flavour thresholds in a General Variable Flavour Number Scheme at NLO not present in earlier approximate NNLO MRST fits. Correct treatment of flavour particularly important at NNLO because discontinuities in unphysical quantities appear at this order.

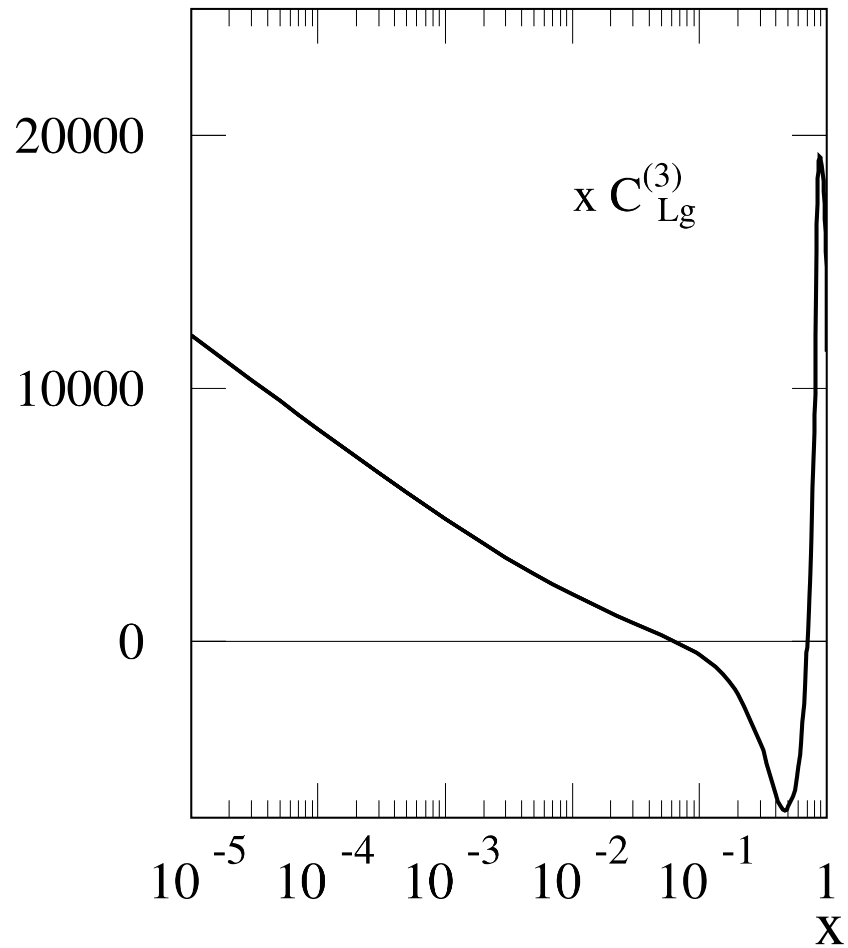


The NNLO $\mathcal{O}(\alpha_s^3)$ longitudinal coefficient function $C^3_{Lg}(x)$ given by

$$C^3_{Lg}(x) = n_f\left(\frac{\alpha_S}{4\pi}\right)^3\left(\frac{409.5\ln(1/x)}{x} - \frac{2044.7}{x} - \dots\right.$$

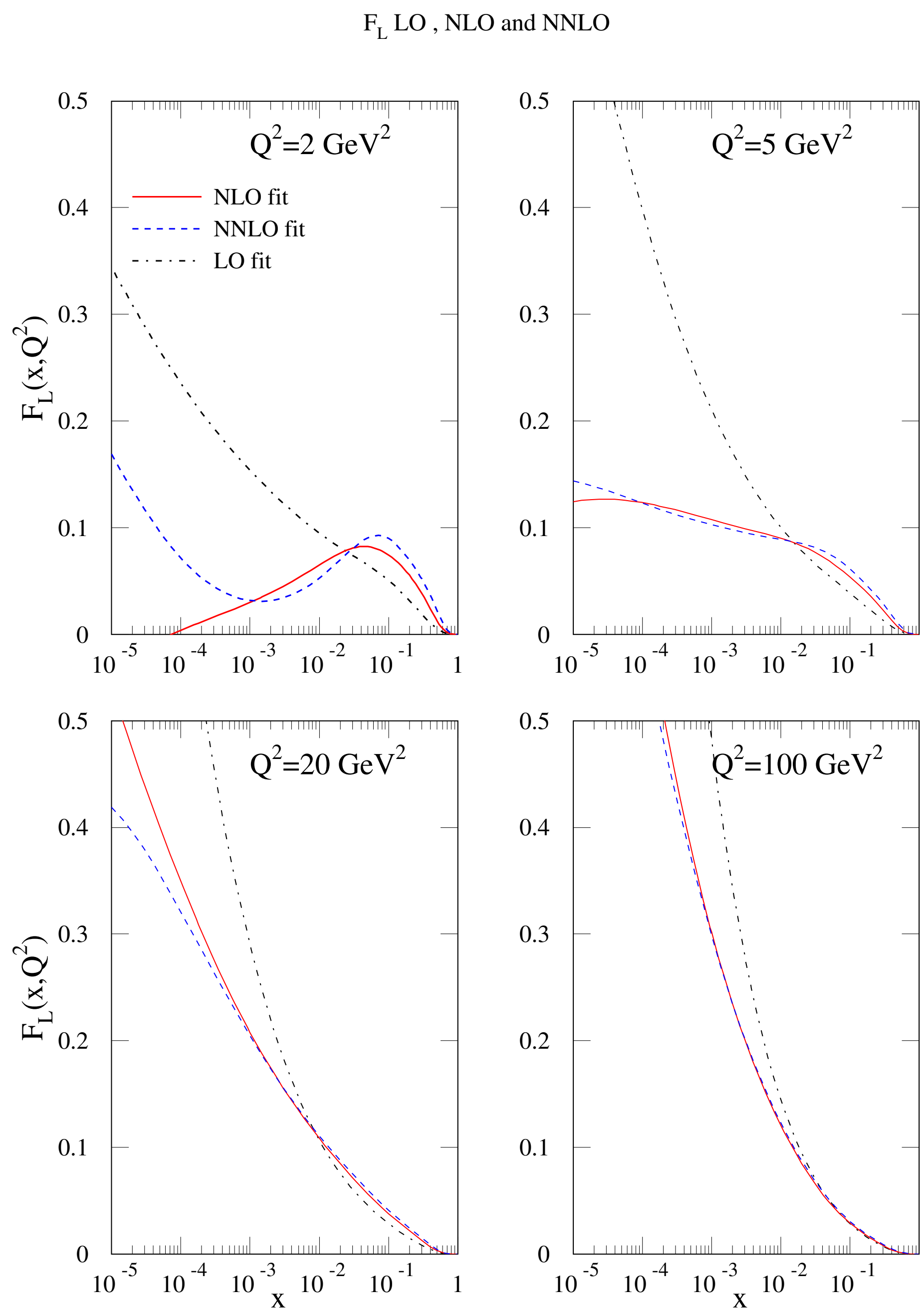
Clearly a significant positive contribution at small x.

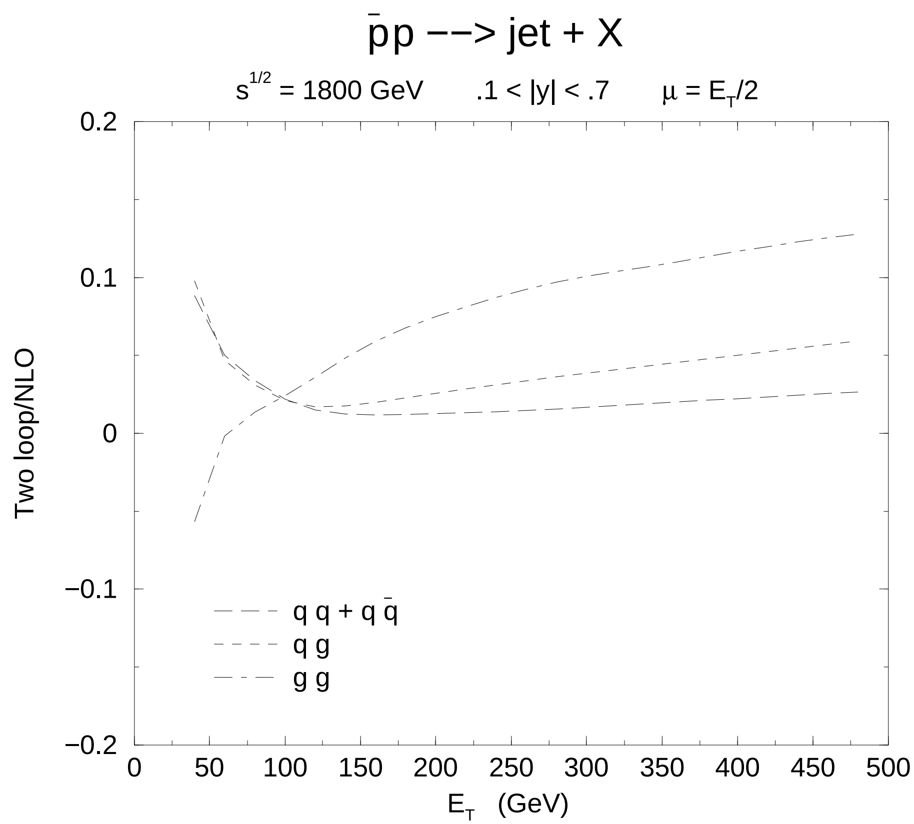
Counters decrease in small-x gluon.

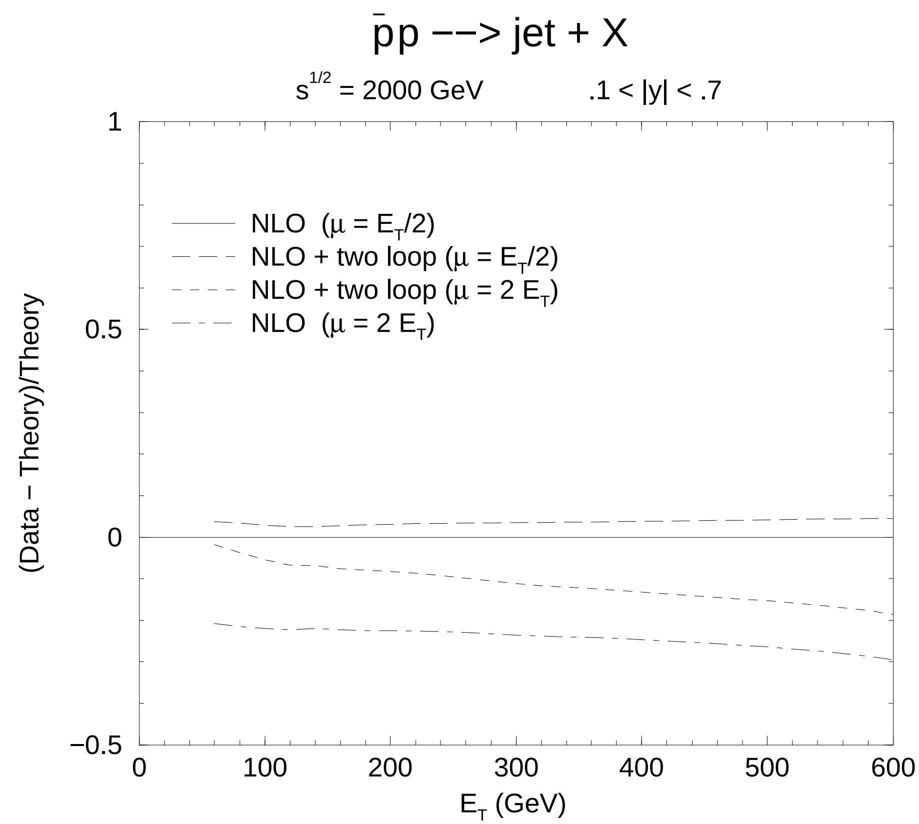


$F_L(x,Q^2)$ predicted from the global fit at LO, NLO and NNLO.

NNLO coefficient function more than compensates decrease in NNLO gluon.







Also good NNLO estimates Kidonakis, Owens. Calculated threshold correction logarithms. Expected to be significant component of total NNLO correction.

Issue concerning application within given jet definition – non-global logarithms.

$\rightarrow$ Flat $3 - 4\%$ correction. Consistent with what is known from NLO. Smaller than systematics on data.

Comparison for $\bar{u}$ quark.