Structure Functions and low x
Summary - Theory

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Main topics in working group

Updates in (semi) global determinations of pdfs – Guffanti, Alekhin, MRST, CTEQ (Tung) and Kumano (nuclear partons distributions).

Ever increasing sophistication (complication?) of theory.

Implementation of new heavy flavour prescriptions and/or NNLO corrections, new data in fits. Difficult to disentangle issue of NNLO and heavy flavours.

Lots of recent developments in small-x resummations. Hopefully beginning of detailed understanding and phenomenology.

Things get more involved in very small x region with non-linear effects. Saturation scale moving ever smaller?

Where do we need to stop?

Importance of $F_L(x, Q^2)$ for theoretical understanding of QCD particularly at small x.

Guffanti performed NNLO fit for non-singlet parton distributions $u_v(x,Q^2)$ and $d_v(x,Q^2)$ by fitting to $F_2^{p,d}(,x,Q^2), x > 0.3$ (too low?) and $F_2^p - F_2^d, x < 0.3$.

Aim for very accurate results.Generally successful.

BBG Non-Singlet Analysis | Results

Non-Singlet Analysis

Results - α_s, Λ_{QCD} and PDF moments

α_s determination

	$\alpha_s(M_Z^2)$	expt	theory
NNLO			
MRST03	0.1153	± 0.0020	± 0.0030
A02	0.1143	± 0.0014	± 0.0009
SY01(ep)	0.1166	± 0.0013	
SY01(νN)	0.1153	± 0.0063	
BBG	**0.1134**	$^{+0.0019}_{-0.0021}$	
World Average	**0.1182**	± 0.0027	

PDF moments

f	n	**BBG**(NNLO)	**MRST04**	**A02**
u_v	2	0.2986 ± 0.0029	0.285	0.304
	3	0.0871 ± 0.0011	0.082	0.087
	4	0.0333 ± 0.0005	0.032	0.033
d_v	2	0.1239 ± 0.0026	0.115	0.120
	3	0.0315 ± 0.0008	0.028	0.028
	4	0.0105 ± 0.0004	0.009	0.010
$u_v - d_v$	2	0.1747 ± 0.0039	0.171	0.184
	3	0.0556 ± 0.0014	0.055	0.059
	4	0.0228 ± 0.0007	0.022	0.024

Comparison with lattice results

BBG	Lattice
N3LO - $\Lambda_{QCD}^{(4)}$ MeV	Alpha Collaboration - $\Lambda_{QCD}^{(2)}$ MeV
234 ± 26	$245 \pm 16 \pm 16$

[M. Della Morte, *et al.*,Nucl.Phys.B713,(2005),378]

		BBG	Lattice
f	n	NNLO	QCDSF
$u_v - d_v$	2	0.1747 ± 0.0039	0.191 ± 0.012

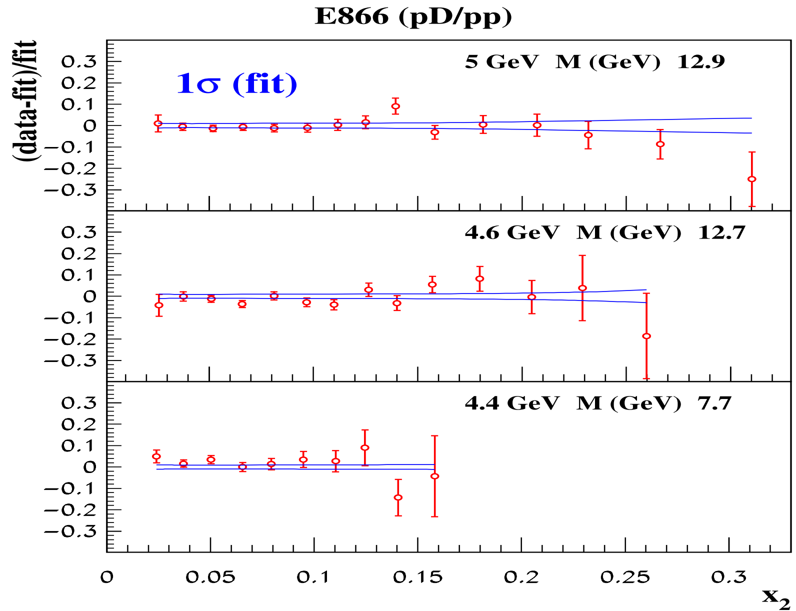
[G. Schierholz, *private communication*]

A. Guffanti (UoE) | NNLO analysis ... | DIS06 | 12 / 17

Global Fits

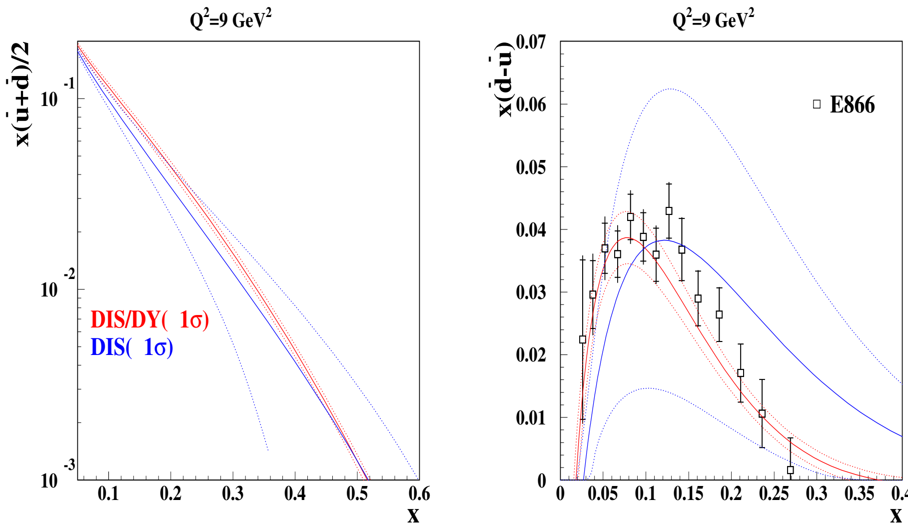
Alekhin includes E605 Drell-Yan data and E866 Drell-Yan ratio data in structure function fit.

Fit with no problems. Improves accuracy on high-x sea and gives first real constraint on $\bar{u} - \bar{d}$.



Sea in the nucleon

In fact using $\Delta\chi^2 = 1$ uncertainties now very small.



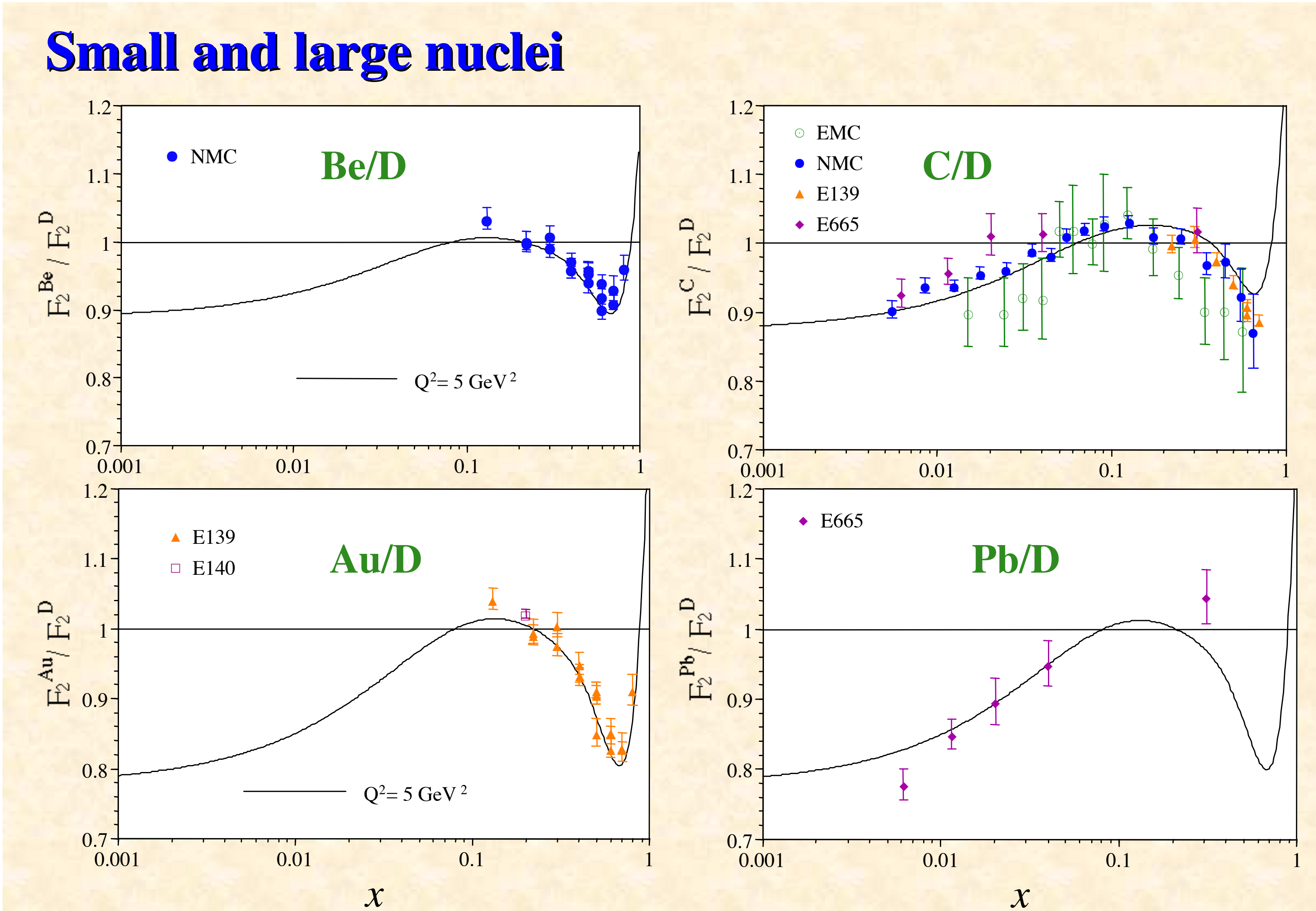
Alekhin working at NNLO and approx N^3LO.

Theoretical input to the global DIS fit

- The massless NNLO QCD corrections for the light quarks and gluons (splitting and coefficient functions.
- Account of the heavy quarks up to $\mathcal{O}(\alpha_S^2)$ by Laenen-Riemersma-Smith-van Neerven.
- Account of the target-mass corrections by Georgi-Politzer, correction for the Fermi-motion in deuterium, and the twist 4 terms.
- *The massless $\mathcal{O}(\alpha_S^3)$ corrections to the coefficient functions*

I do not agree with definition of NNLO regarding charm and bottom. Last step only part of a full N^3LO correction – not necessarily indicative except at high x. Claims reasonable stability down to $Q^2 = 0.5\mathrm{GeV}^2$ for $0.06 \leq x \leq 0.12$.

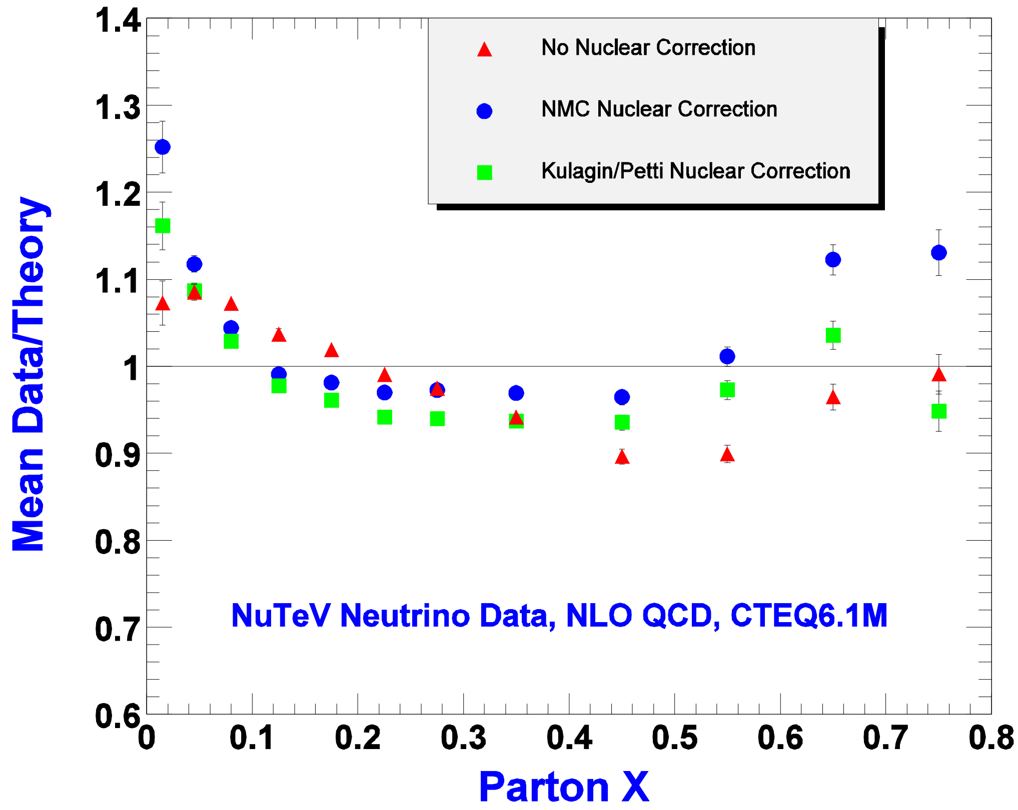
Issues with **new data**. Most interest in NuTeV structure function data. Larger than CCFR data at high x. Useful for flavour separation. Rely on nuclear correction. A determination of these was reported by Kumano.



CTEQ – Tung find NuTeV data difficult to fit.

Can Nuclear Corrections Help?

- *For each x, data are combined and errors are weighted;*
- *See a systematic x-dependent deviation that cannot be reduced substantially by nuclear correction models.*



MRST – previous nuclear correction $R(x)$ obtained from EMC effect clearly ruled out.

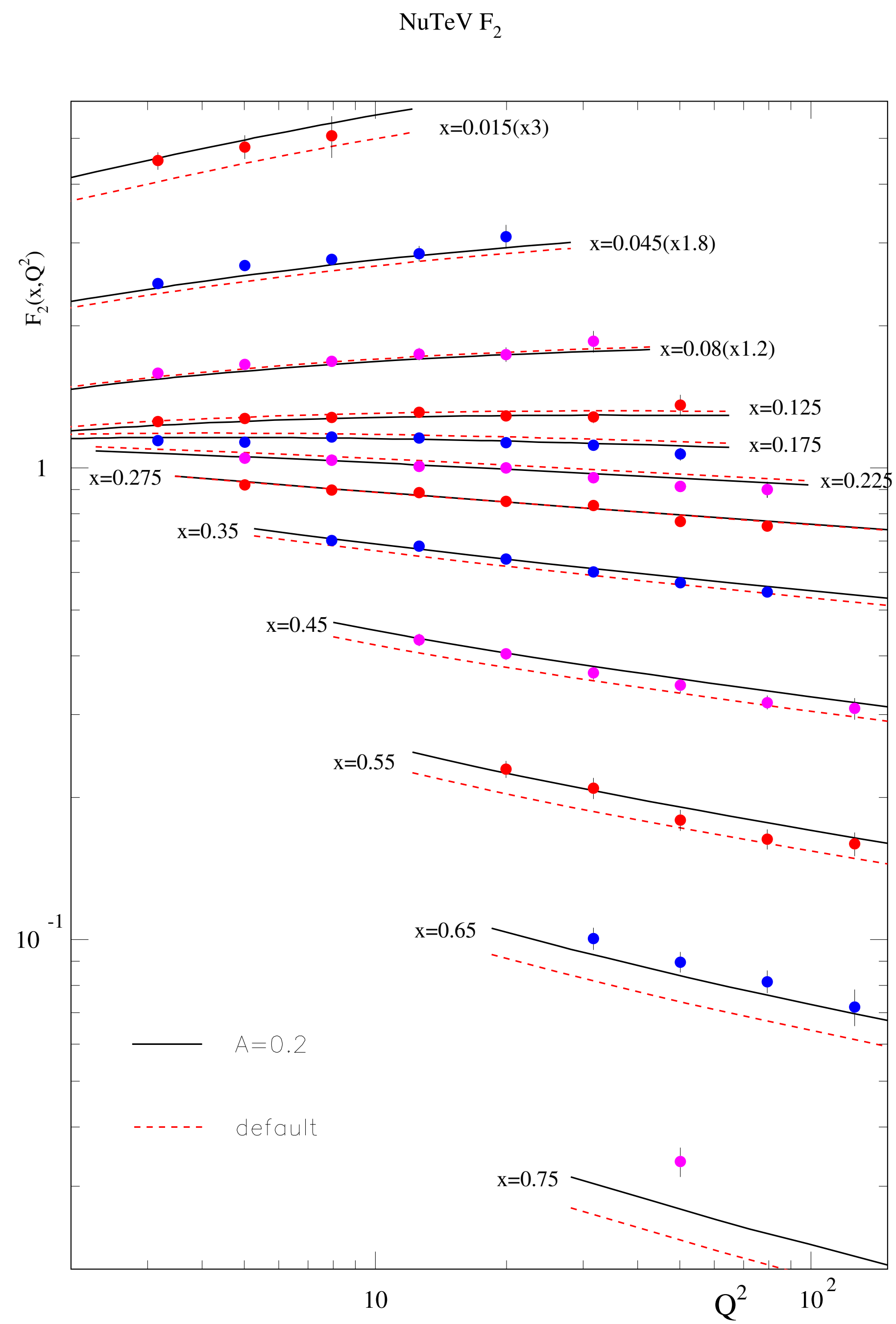
Try $R^{eff} = 1 + A*(R-1)$. $\rightarrow A = 0.2$.

Fit then good.

Partons in region of high correction already well-determined. Nuclear correction different for ν than for charged leptons?

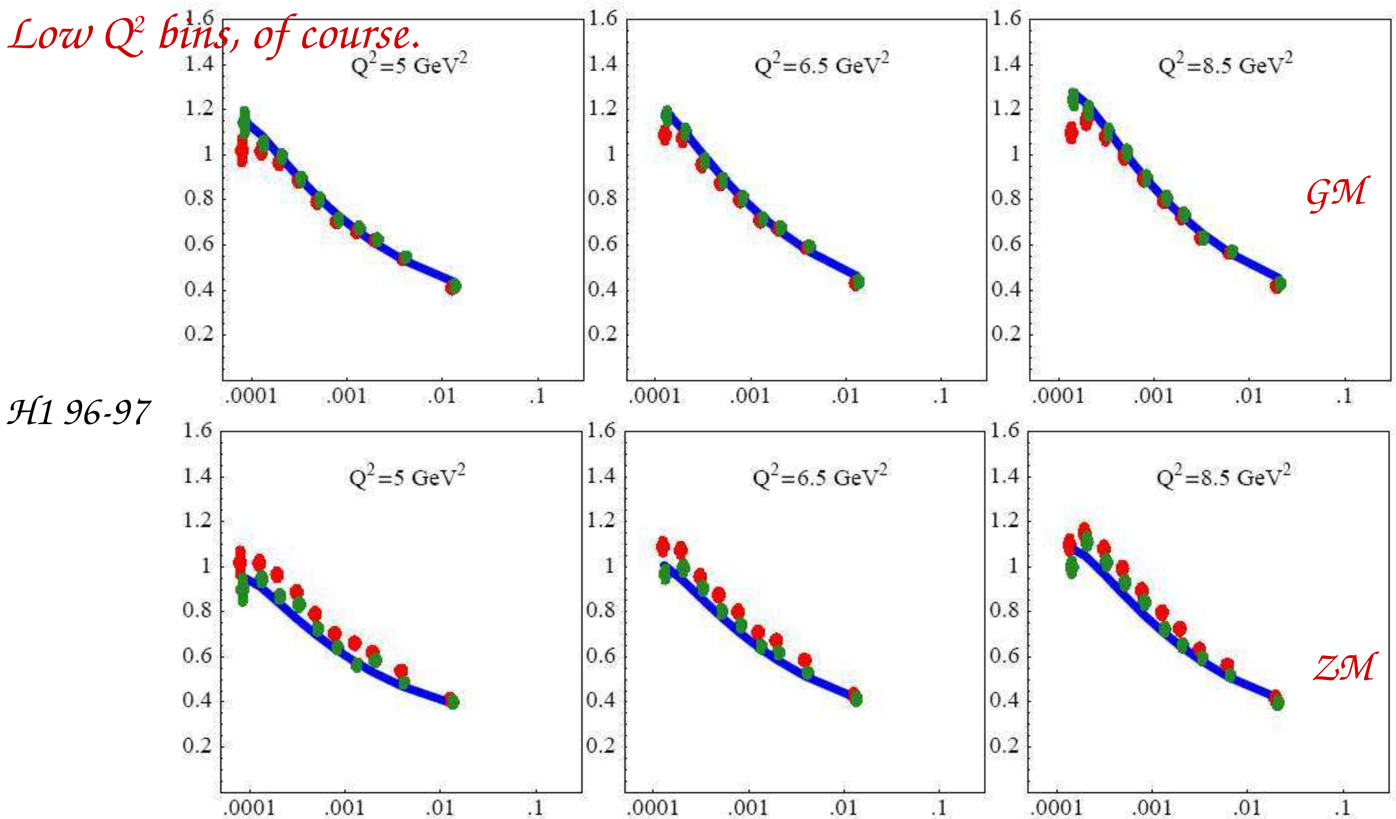
Important information in the region $x < 0.3$ - not too sensitive to corrections, but problem caused much interest. Conclusion not clear.

However CHORUS data more similar to CCFR.



CTEQ include all HERA data – fit directly to cross-sections for first time. Requires $F_L(x, Q^2)$ at high y. No NNLO?

Where does the General Mass Formalism make a difference? Compare with CTEQ6.1M (ZM)



ZEUS 96-97 data show the same effects

Also implement new heavy flavour prescription (HFS summary). Overshoot *raw* high-y data. Systematic errors remove $F_L(x, Q^2)$ turnover?

MRST have implemented full NNLO VFNS and Drell-Yan cross-sections $\longrightarrow$ (provisional) full NNLO partons with uncertainties.

Difference in charm procedure affects gluon compared to approx MRST2004 NNLO fit. Correct heavy flavour treatment vital.

Quality of full fit at

NLO $\chi^2 = 2406/2287$

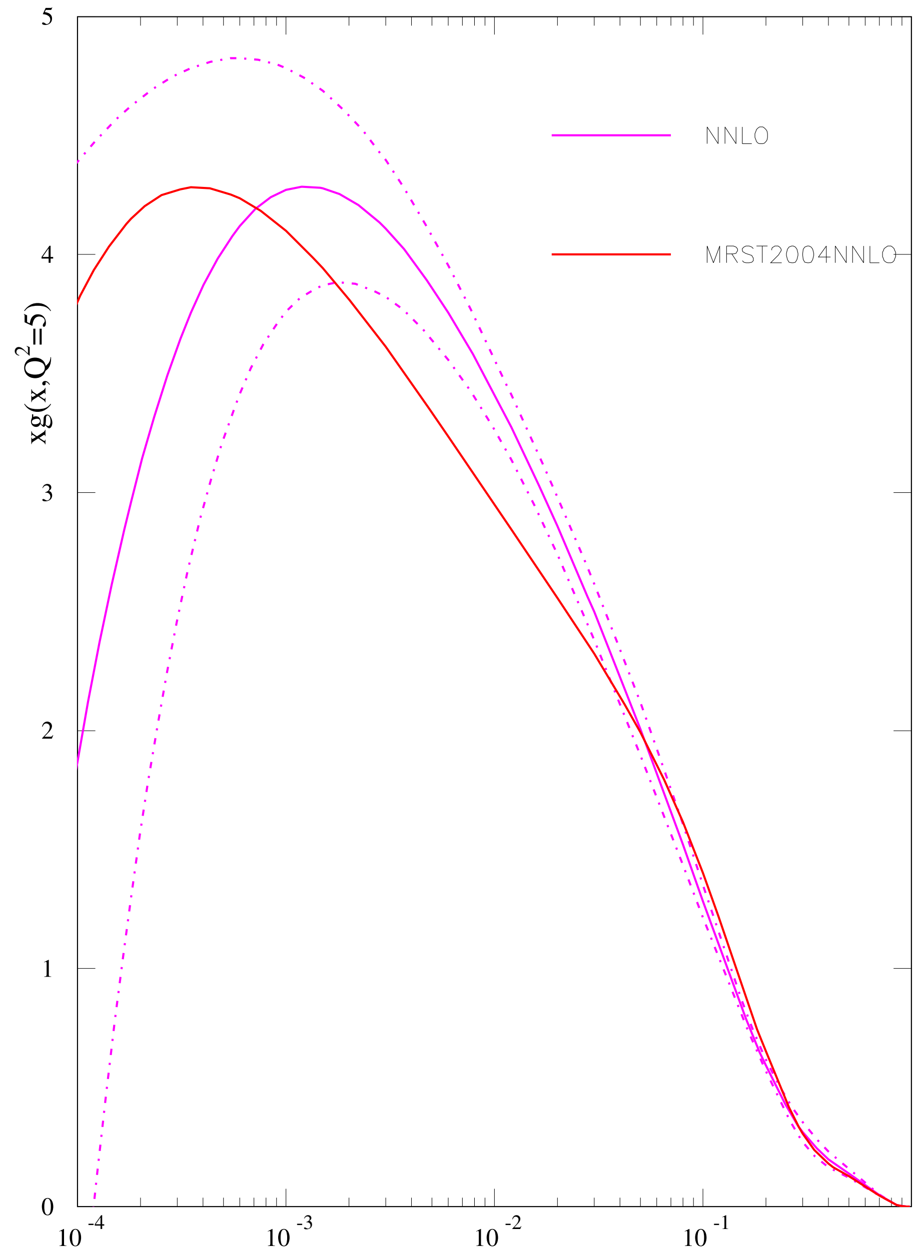
NNLO $\chi^2 = 2366/2287$.

NNLO fairly consistently better than NLO - not for Drell-Yan data.

Definite tendency for $\alpha_S(M_Z^2)$ to go up with all changes.

NLO $\alpha_S(M_Z^2) = 0.121$,

NNLO $\alpha_S(M_Z^2) = 0.119$.



Small-x resummations.

Presentation on various ways to include resummations from BFKL equation on top of fixed order expansion.

White – a resummation of $\ln(1/x)$ terms along with running coupling corrections only (RT, 1999).

$$xP_{gg} = \sum_{n} \alpha_S^n \sum_{m=0}^{n-1} \ln^{n-m-1}(1/x)\beta_0^m$$

Mainly analytic results with small numerical corrections.

Also quark-gluon splitting P_{qg} *etc.* and full implementation of heavy flavour VFNS.

Full LO resummed partons.

Colferai – approach based on Ciafaloni, Colferai, Salam and Stasto best for processes with two hard scales. Also includes running coupling (effects not as explicit) and resummation of collinear singularities.

$$\int_0^\infty dk^2 (k^2)^{-\gamma-1} K^n_{BFKL}(k^2) = \chi^n(\gamma), \qquad \chi^n(\gamma) \quad \sim \quad \frac{1}{\gamma^{2n+1}}, \quad \frac{1}{(1-\gamma)^{2n+1}}$$

where evolution variable $s/(QQ_0)$, conjugate variable N.

Consideration of changes of evolution variable to s/Q^2 (DIS) and $s/Q_0^2 \rightarrow$ resummation (Salam, 1998)

$$\chi^n_N(\gamma) \quad \sim \quad \frac{1}{(\gamma+N/2)^{n+1}}, \quad \frac{1}{(1-\gamma+N/2)^{n+1}}$$

Natural calculations in DIS scheme with Q_0 regularization i.e. incoming gluon off-shell, $k^2 = Q_0^2 \neq 0$.

Look at transformation to $\overline{MS}$ scheme – regularization in $4+2\epsilon$ dimensions – i.e. how fixed order defined.

Forte – approach of Altarelli, Ball and Forte.

1. Duality – in some limit have

$$\chi(\gamma(N,\alpha_S),\alpha_S) = N, \quad \gamma(\chi(\alpha_S,M),\alpha_S) = M$$

i.e. Q^2 evolution and x evolution dual. Not most important issue.

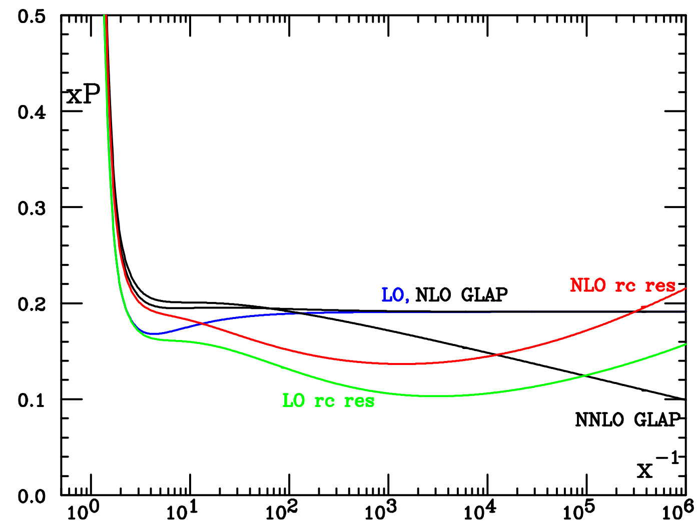
2. Explicit imposition of momentum conservation.

3. Also now include the running of the coupling.

4. Symmetrization, i.e. let $1/M^n \to 1/(M+N/2)^n$, $1/(1-M)^n \to 1/(1-M+N/2)^n$ as CCSS.

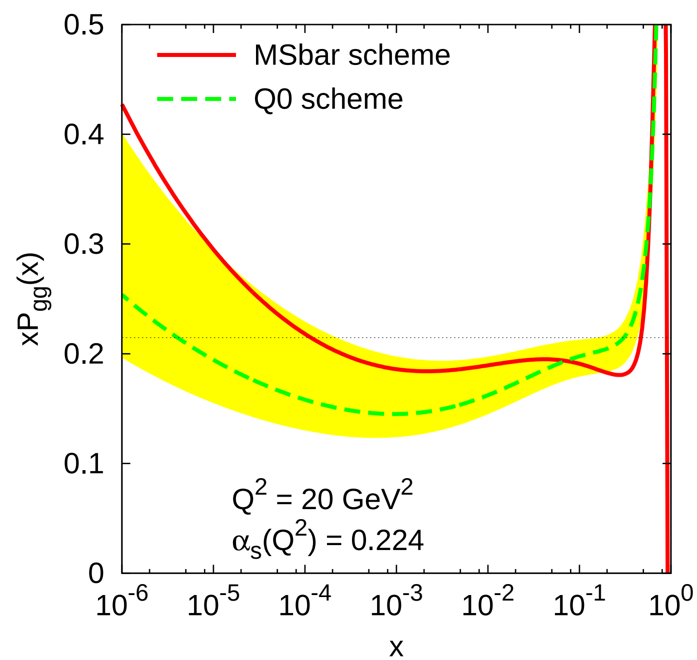
Overall leads to resummed $P_{gg}(x,Q^2)$.

Plots of P_{gg} for $\alpha_S \sim 0.2$ for Forte (top) and Colferai (bottom). White, RT at NLO similar but dip a bit lower.



Despite lots of differences in approaches. (All include running coupling.) All get dip, then very low x rise in P_{gg}. LO lower than NLO different for Forte

→ all agree moderate effects compared to original resummation ideas.

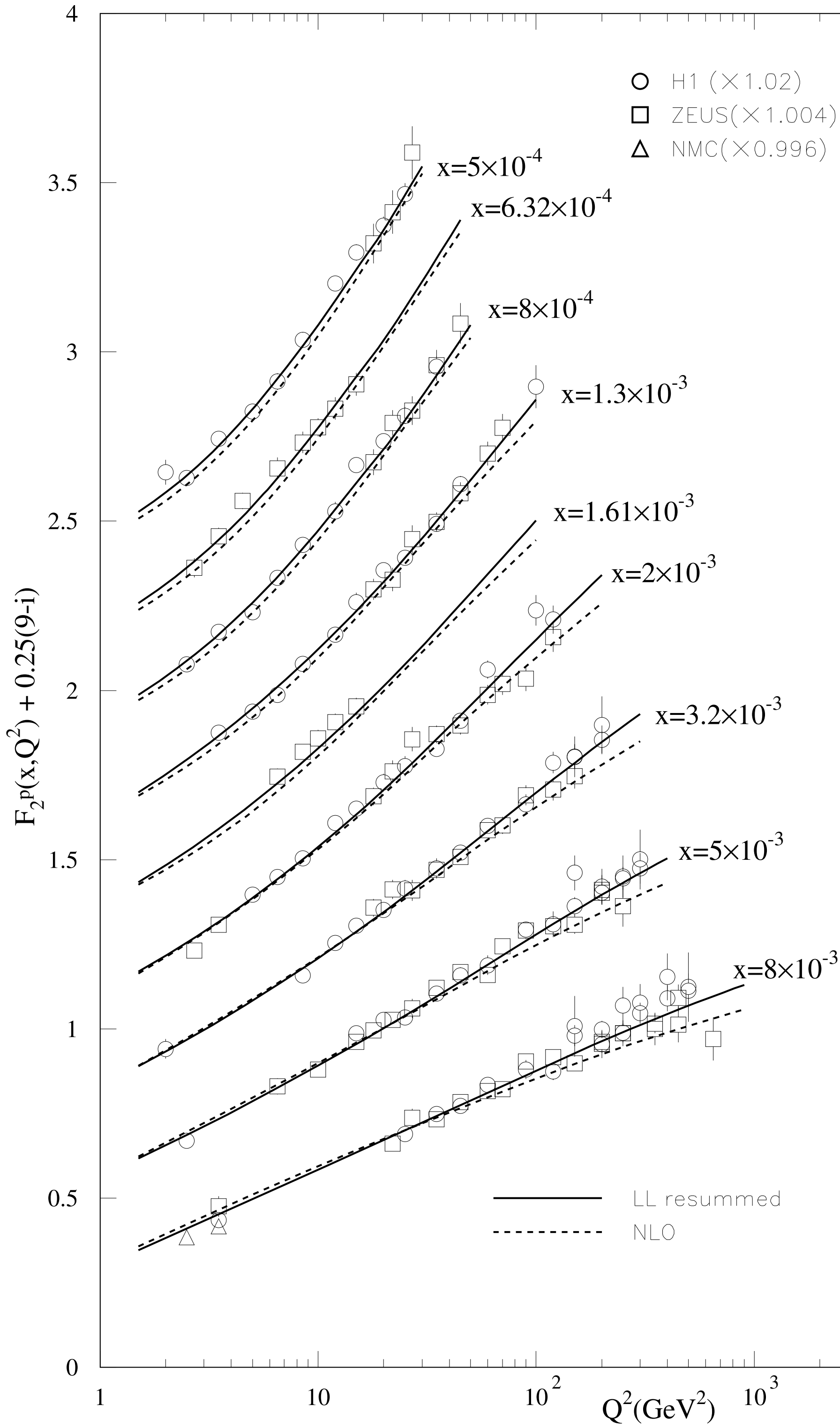


White examines phenomenology, but only at LO in resummation.

Impact factor required for P_{qg} not yet known at NLO.

Better fit than NLO-in-α_S in terms of $dF_2(x,Q^2)/d\ln Q^2$.

Enhancement of evolution too great at small x. Gluon and $F_L(x,Q^2)$ too small at moderate x. Need the full NLO generalization.



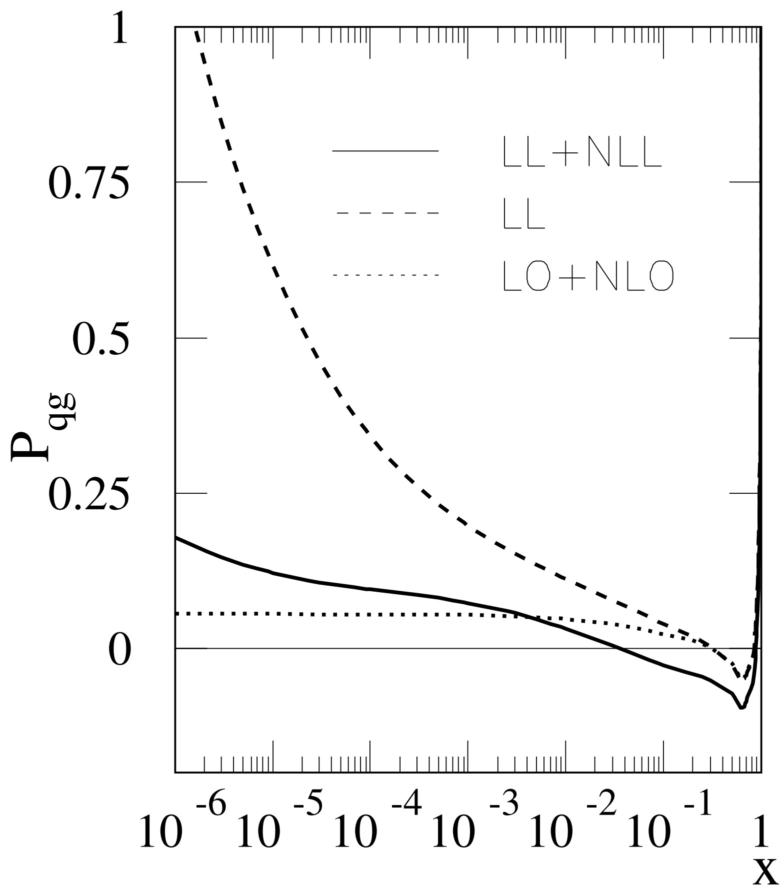
Colferai and White both examine improvement to P_{qg}.

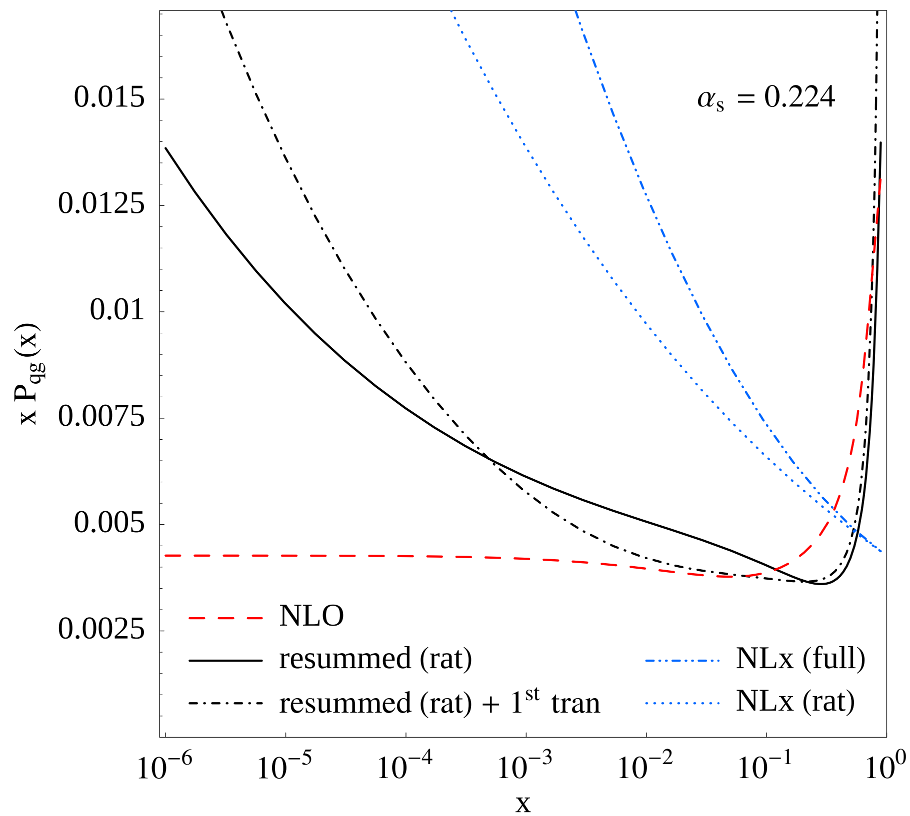
Two approaches are qualitatively similar. Contain different higher order information.

White – estimate of NLO corrections to impact factor, coupling resummation.

Colferai – resummation of P_{gg} beyond NLO via collinear resummation.

Both suggest effects of NLO resummation small but significant.





Non-Linear Corrections at very small x.

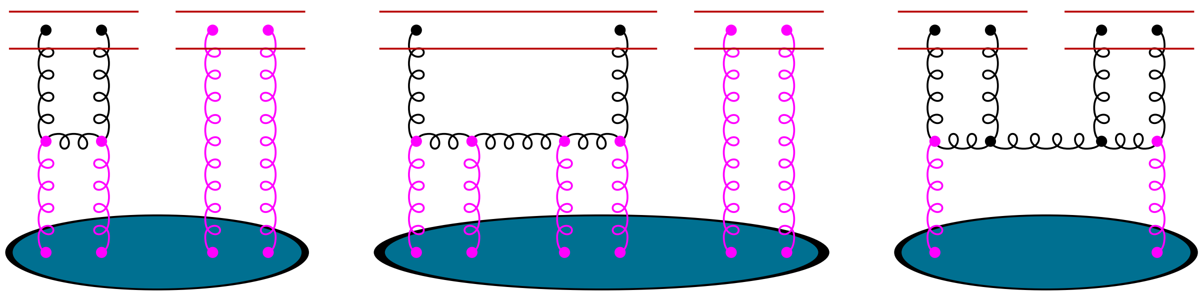
Various discussions on how to improve small-x treatments to include the saturation corrections, and more. Soyez – nonlinear evolution equation in rapidity Y extended to include fluctuations as well as recombination. Opposite sign (generally).

Fluctuations

CEA DSM-SPhT Service de Physique Théorique

Consider evolution of $\langle T^{(2)} \rangle$

[E. Iancu, D. Triantafyllopoulos]
Also A. Mueller, S. Munier, A. Shoshi, S. Wong



$$\partial_Y \left\langle T^{(2)} \right\rangle = \underbrace{\bar{\alpha}\mathcal{M} \otimes \left\langle T^{(2)} \right\rangle}_{\text{BFKL}} - \underbrace{\bar{\alpha}\mathcal{M} \otimes \left\langle T^{(3)} \right\rangle}_{\text{saturation}} + \underbrace{\bar{\alpha}\alpha_s^2 \mathcal{K} \otimes \langle T \rangle}_{\text{fluctuations}}$$

- saturation $\longrightarrow T \sim 1$ dense regime
- fluctuations $\longrightarrow T \sim \alpha_s^2$ dilute regime

G. Soyez DIS 2006, April 19-24, Tsukuba, Japan High-energy QCD and satistical physics – p. 5/19

Hatta demonstrated origin of fluctuations via a formal derivation of a Bremsstrahlung Hamiltonian which can be used to give the evolution of n-dipole densities.

Evolution equation for the dipole densities

Dipole number operator $$D(\boldsymbol{x},\boldsymbol{y}) \equiv -\frac{1}{g^2 N_c}\rho^a_\infty(\boldsymbol{x})\rho^a_\infty(\boldsymbol{y})$$

Dipole number density $$\langle D(\boldsymbol{x},\boldsymbol{y})\rangle_\tau \equiv \int D[\rho]\, D(\boldsymbol{x},\boldsymbol{y})\, Z_\tau[\rho] \approx \frac{1}{2}\Big(n_\tau(\boldsymbol{x},\boldsymbol{y}) + n_\tau(\boldsymbol{y},\boldsymbol{x})\Big)$$

$$\xrightarrow[H_{\text{BREM}}]{} \quad \frac{\partial}{\partial\tau} n_N = H_{\text{BFKL}} n_N$$

Dipole pair density $$\langle D(\boldsymbol{x}_1,\boldsymbol{y}_1) D(\boldsymbol{x}_2,\boldsymbol{y}_2)\rangle_\tau \sim n_N^{(2)}(\boldsymbol{x}_1,\boldsymbol{y}_1;\boldsymbol{x}_2,\boldsymbol{y}_2)$$

$$\xrightarrow[H_{\text{BREM}}]{} \quad \frac{\partial}{\partial\tau} n_N^{(2)} = H_{\text{BFKL}} n_N^{(2)} + n_N^{(1)}$$

fluctuation term

"seed" of $n_N^{(2)}$

Inclusion of fluctuations lead to dispersion about saturation scale Soyez. Move onset of saturation in dipole cross-section considerably downwards.

Describing F_2

DSM-SPhT Service de Physique Théorique

Following fits to the F_2^p data:

Saturation fit: [Iancu, Itakura, Munier]

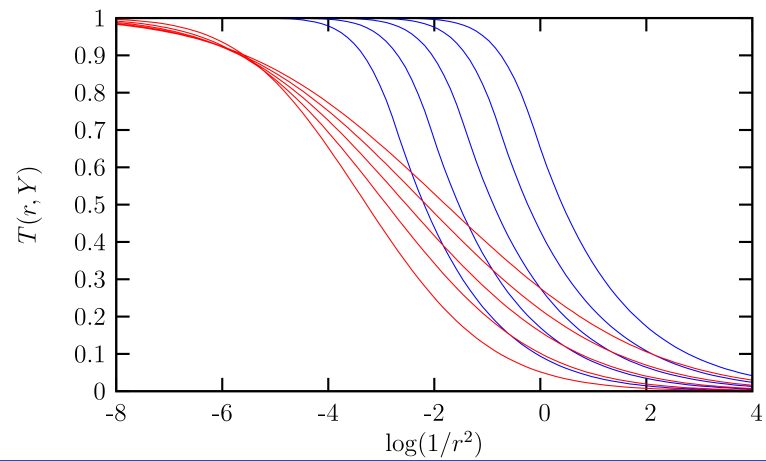
$$\langle T(r,Y)\rangle = \begin{cases} (r^2Q_s^2)^{\gamma_c} e^{-\frac{2\log^2(rQ_s)}{CY}} & r < 1/Q_s \\ 1 - e^{-a-b\log^2(rQ_s)} & r > 1/Q_s \end{cases}$$

$$Q_s^2(Y) = \lambda Y,\ \rho_s = \log(Q_s^2)$$

Saturation+fluctuations fit: [in preparation]

$$\langle T(r,Y)\rangle = \int d\rho_s\, T(r,\rho_s) \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(\rho_s-\bar\rho_s)^2}{\sigma^2}}$$

$$T(r,\rho_s) = \begin{cases} r^2Q_s^2 & r < 1/Q_s \\ 1 & r > 1/Q_s \end{cases}$$



G. Soyez DIS 2006, April 19-24, Tsukuba, Japan High-energy QCD and satistical physics – p. 16/19

Kutak modifies LO BFKL equation for non-linear recombination term.

Also inclusion of impact parameter dependence.

Inclusion of high-x effects in gluon evolution (though not in gluon quark impact factor, i.e. effectively P_{qg}).

3. Extended BK-equation for gluon density with impact parameter dependence.

(Kutak, Motyka)

$$f_g(x,k^2,b) = \tilde{f}_g^{(0)}(x,k^2,b) + K_1 \otimes f_g - K_2 \otimes f_g^2 \qquad (4)$$

where

$$\tilde{f}_g^{(0)}(x,k^2,b) = S(b)\frac{\alpha_s(k^2)}{2\pi}\int_x^1 dz P_{gg}(z,b)\frac{x}{z}g(\frac{x}{z},k_0^2)$$

$$K_1 \otimes f_g = 2N_c\frac{\alpha_s(k^2)}{2\pi}k^2\int_x^1\frac{dz}{z}\int_{k_0^2}\frac{dk'^2}{k'^2}$$

$$\left\{\frac{f_g\left(\frac{x}{z},k'^2,b\right)\Theta\left(\frac{k^2}{z}-k'^2\right) - f_g\left(\frac{x}{z},k^2,b\right)}{|k'^2-k^2|} + \frac{f_g\left(\frac{x}{z},k^2\right)}{[4k'^4+k^4]^{\frac{1}{2}}}\right\}$$

$$+\frac{\alpha_s(k^2)}{2\pi}\int_x^1\frac{dz}{z}\left[(zP_{gg}(z,b)-2N_c)\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}f_g\left(\frac{x}{z},k'^2,b\right)\right.$$

$$S(b) = \frac{1}{\pi R^2}e^{-b^2/R^2}$$

Where $R = 2.8GeV^{-1}$ **and conventional unintegrated gluon density is obtained:**

$$f(x,k^2) = \int d^2b f_g(x,k^2,b) \qquad (5)$$

5

Inclusion of impact parameter dependence reduces the effect of the nonlinear term.

Lowers the saturation scale in Q^2 for a given x.

Rough phenomenology – "hardly see effect of saturation" for HERA.

Further NLO corrections likely to reduce this further.

The b dependent saturation scale can be defined as follows:

$$\frac{\partial h(x,k^2,b)}{\partial log(1/x)} = 0. \quad (5)$$

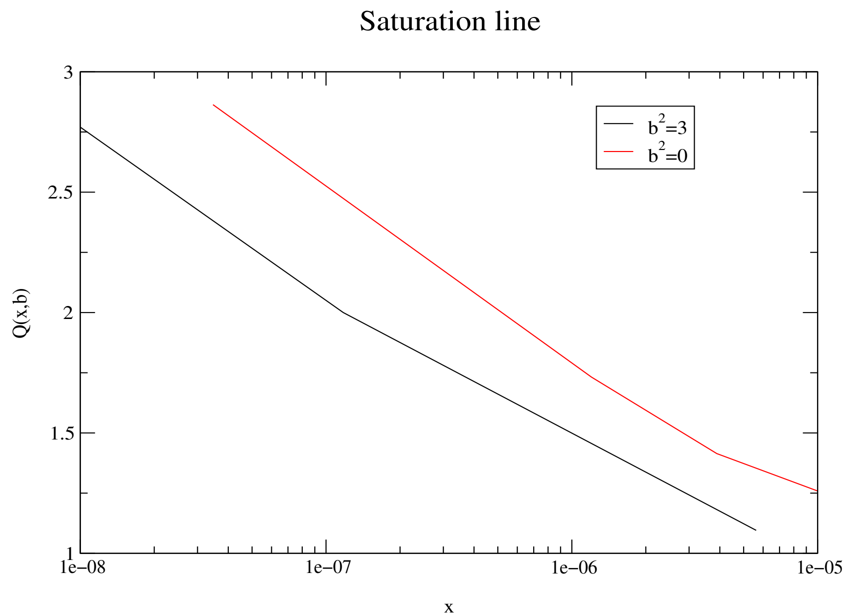
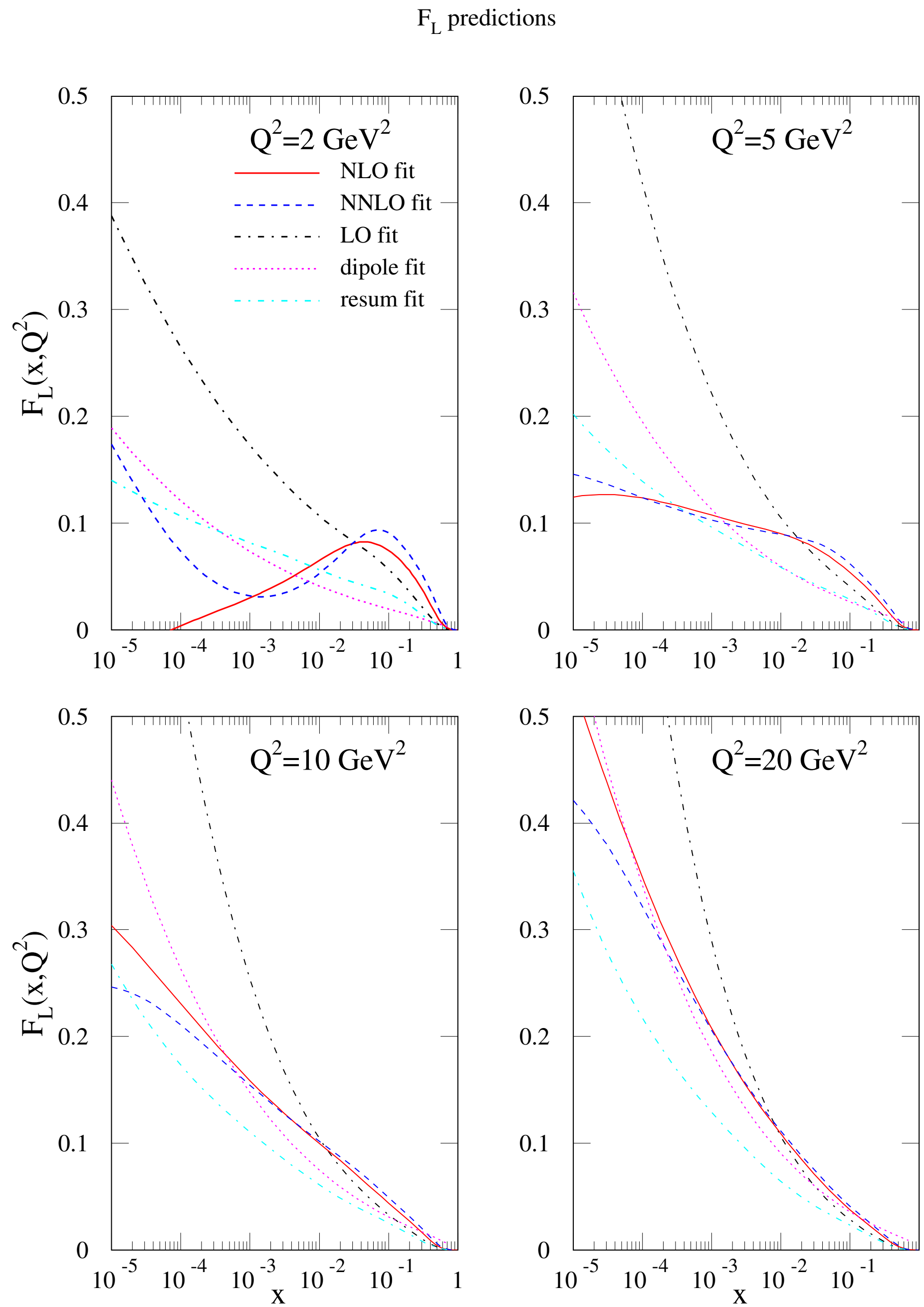


Figure 5:

$F_L(x, Q^2)$ can be calculated using some variation of all of the above methods $\rightarrow$ much variation.

$F_L(x, Q^2)$ predicted from the global fit at LO, NLO and NNLO, from a fit which performs a resummation of small-x terms, and from a dipole model type fit.

Implies a measurement of $F_L(x, Q^2)$ over as wide a range of x and Q^2 as possible would be very useful.



Conclusions

Little agreement in *global* fit analyses. Not everyone wants to go to NNLO. Not everyone agrees how to do it in detail. I believe we are now finally at stage where NNLO parton analyses are complete and reliable. Should be done. Work a little better than NLO in general.

Rather similar results coming from groups working on small-x resummations to be used on top of fixed order calculations. Will probably still argue about how results obtained. Effect of resummations moderate until very small x. Empirically can improve fit a little even over NNLO. Resummations NLO at best. Fixed order NLO + NLO resummations (large and small x) better than fixed order NNLO (Tung)?

Progress in nonlinear small-x equations, e.g. fluctuations. To me always seem to be pushing saturation scale lower. Nice if this could match on to higher x better. Usually confined to unknown small x region, missing higher x corrections. (Please do not show data plot where main difference between two competing models is at $x = 0.01, Q^2 = 1000\mathrm{GeV}^2$. DGLAP must be appropriate here.)

Lots of improvement in how to calculate using different techniques. Not enough idea yet where each approach is applicable/needed. Need better (real) phenomenology and, of course, more useful data.