Shell-model structure of light hypernuclei

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Source	# γ-rays	# doublets
Ge Hyperball	~ 22	9
NaI ${}^{13}_{\Lambda}$C	3	1
NaI ${}^{4}_{\Lambda}$H/${}^{4}_{\Lambda}$He	2	2

Parameters in MeV

	Δ	S_Λ	S_N	T
$A = 7-\ ?$	0.430	-0.015	-0.390	0.030
$A = 11-16$	0.330	-0.015	-0.350	0.024

1. G-Matrix elements from $N\Lambda$-$N\Sigma$ calculation fitted with sums of Gaussians, Yukawas, OBEP forms, ...

2. Hypernuclear two-body matrix elements calculated using Woods-Saxon wave functions.

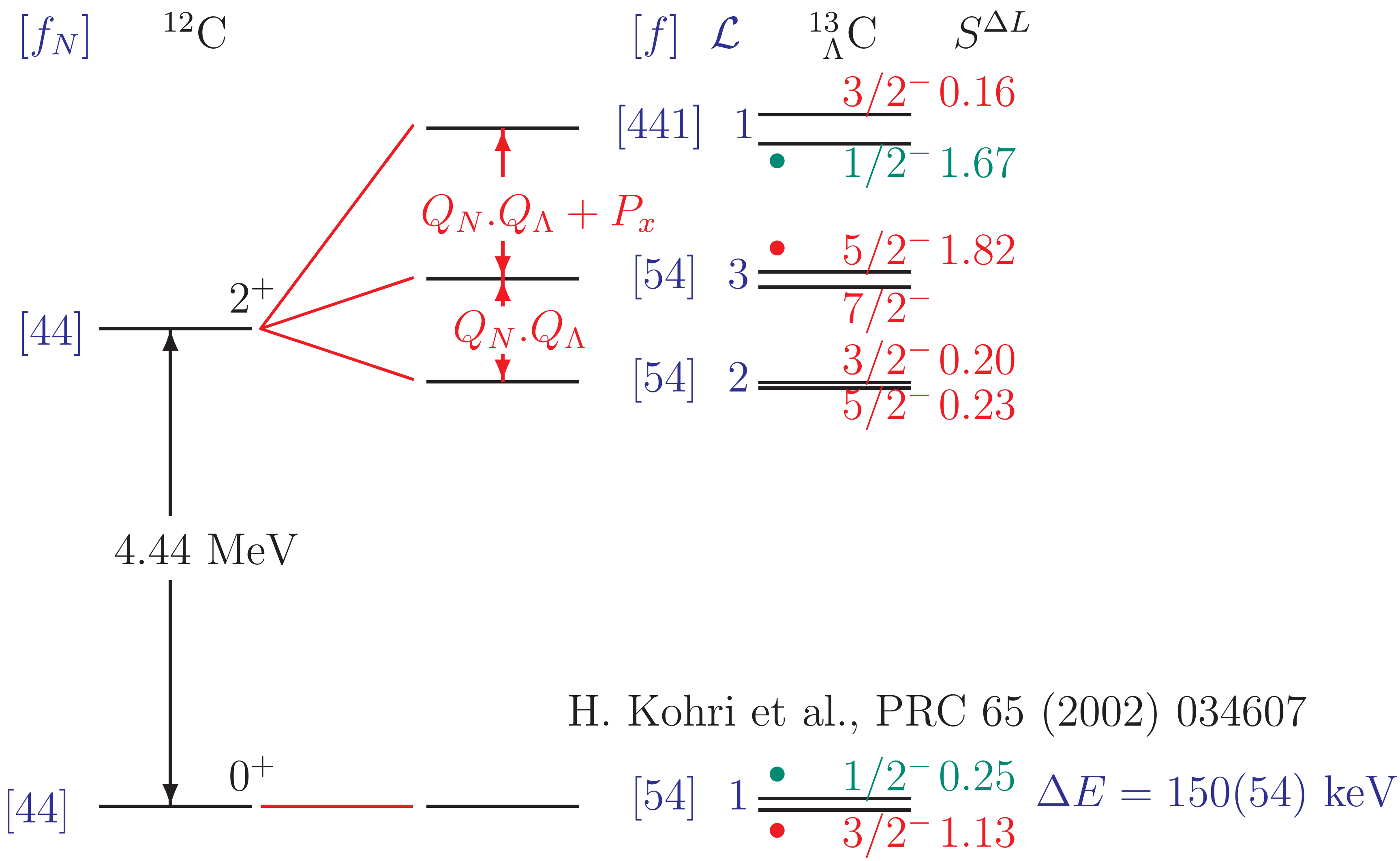
		p-shell					s-shell	
		$\overline{V}$	Δ	S_Λ	S_N	T	$\overline{V}_s$	Δ_s
fit-djm	${}^{7}_{\Lambda}$Li	-1.142	0.438	-0.008	-0.414	0.031	-1.387	0.497
	${}^{16}_{\Lambda}$O	-1.161	0.441	-0.007	-0.401	0.030		
nsc97f	${}^{7}_{\Lambda}$Li	-1.086	0.421	-0.149	-0.238	0.055	-1.725	0.775
esc04a	${}^{7}_{\Lambda}$Li	-1.287	0.381	-0.108	-0.236	0.013	-1.577	0.850

- First two lines show that matrix elements are roughly constant with A - same YNG interaction, WS wells have $R = r_0 A^{1/3}$, but rms radii of p-shell nuclei are roughly constant

		singlet even	triplet even	singlet odd	triplet odd	even	odd	in odd states
nsc97f	$\overline{V}$	−0.421	−0.834	0.070	0.102	−1.255	0.172	repulsive
	Δ					0.571	−0.148	
esc04a	$\overline{V}$	−0.421	−0.791	0.029	−0.100	−1.212	−0.072	strong spin
	Δ					0.632	−0.248	dependence
fit-djm	$\overline{V}$	−0.331	−0.701	−0.036	−0.069	−1.032	−0.105	attractive
	Δ					0.387	0.051	

- Most new YN models use some constraint to ensure a more attractive s-wave interaction than triplet to bind hypertriton, fit A=4 $0^+/1^+$ doublet, and Δ for p-shell hypernuclei.
- The p-wave central interaction is not constrained, but may be important - along with Λ-Σ coupling - to simultaneously fit data on s-shell and p-shell hypernuclei.
- Next two slides show an old example that puts some constraint on the exchange character of the central interaction.

$^{12}C(0^+, 2^+) \times p_\Lambda$ states of $^{13}_{\Lambda}C$



$B_\Lambda = 11.69 \pm 0.12$ MeV $\qquad 3/2^-/1/2^- \qquad E_x = 10.83/10.98$ MeV

$^{13}C(K^-,\pi^-)^{13}_{\Lambda}C$ M. May et al., PRL 47, 1106 (1981)

Theory: E.H. Auerbach et al., PRL 47, 1110 (1981); Ann. Phys. 148, 381 (1983)

Basic data: Separation between 10.4 and 16.4 MeV peaks at 0° and shift in position of upper peak at 15°.

Woods-Saxon: p_Λ bound at 0.8 MeV

$^{13}_{\Lambda}C$	nsc97f	esc04a	djm	Experiment
$1/2^-_2 - 1/2^-_1$	6.94	6.05	6.18	6.0 ± 0.4
$1/2^-_2 - 5/2^-_1$	2.18	1.23	1.37	1.7 ± 0.4

Odd-state tensor, even-state tensor, and Λ-Σ mixing work against one-body and two-body spin-orbit interactions in the "single-particle" p_Λ splitting. Mixing of $2^+ \times p_\Lambda$ into $0^+ \times p_\Lambda$ (typically 5%) also contributes to the spacing.

Λ-Σ and spin-dependent contributions to ground-state binding energies

	${}^{7}_{\Lambda}$Li	${}^{8}_{\Lambda}$Li	${}^{9}_{\Lambda}$Li	${}^{9}_{\Lambda}$Be	${}^{10}_{\Lambda}$B	${}^{11}_{\Lambda}$B	${}^{12}_{\Lambda}$B	${}^{13}_{\Lambda}$C	${}^{15}_{\Lambda}$N	${}^{16}_{\Lambda}$N
	$1/2^+$	1^-	$3/2^+$	$1/2^+$	1^-	$5/2^+$	1^-	$1/2^+$	$3/2^+$	1^-
Λ-Σ	78	160	183	4	35	66	103	28	59	62
Δ	419	288	350	0	125	203	108	−4	40	94
S_Λ	0	−6	−10	0	−13	−20	−14	0	12	6
S_N	94	192	434	207	386	652	704	841	630	349
T	−2	−9	−6	0	−15	−43	−29	−1	−69	−45
Sum	589	625	952	211	518	858	869	864	726	412
Expt	5.58	6.80	8.50	6.71	8.89	10.24	11.37	11.69		13.76
		6.84	8.29		9.11					*
$\overline{V}$	−0.94	−1.02	−1.06		−1.05	−1.04	−1.05	−0.96		−0.93

* $B_\Lambda = 13.76(16)$ MeV: F. Cusanno (Plenary 5)

To get a rough $\overline{V}$, take $B_\Lambda({}^{5}_{\Lambda}\mathrm{He}) = 3.12$ MeV as s_Λ single-particle energy, and subtract sum from experimental B_Λ value.

Double one-pion exchange ΛNN interaction

Gal, Soper, and Dalitz: Ann. Phys. (N.Y.) 63, 53 (1971)

Independent of Λ spin. Averaged over s_Λ wave function gives

$$V_{NN}^{eff} = \sum_{klm} Q_{lm}^{k}(r_1, r_2)\,[\sigma_1, \sigma_2]^k \cdot [C_l(\hat{r_1}), C_m(\hat{r_2})]^k \;\; \tau_1 \cdot \tau_2$$

Parameters in MeV

Q_{00}^0	Q_{22}^0	Q_{22}^1	$Q_{02}^2 = Q_{20}^2$	Q_{22}^2
0.026	1.037	−0.531	−0.049	0.245

- Q_{00}^0 and Q_{22}^0 give repulsive contributions to B_Λ that depend quadratically on the number of p-shell nucleons in the core.
- Q_{22}^1 represents an anti-symmetric spin-orbit interaction that behaves rather like S_N

Shell-model calculations

- Both $|p^n \alpha_c J_c T \times s_\Lambda\rangle$ and $|p^n \alpha_c J_c T_c \times s_\Sigma\rangle$ configurations included. In general, T_c can take three values. E.g. for ${}^{10}_{\Lambda}$Li, $T_c = 1/2, 3/2, 5/2$.
- Supermultiplet basis $|p^n [f_c] \beta_c (L_c S_c) J_c T_c\rangle$ is very good for p shell $\Rightarrow$ states with different $[f_c]$ (often T_c) well separated. E.g. $\sim$ 15 MeV for lowest $T_c\!=\!1/2 \times \Sigma$ and $T_c\!=\!3/2 \times \Sigma$ in ${}^{10}_{\Lambda}$Li example.
- Easy, but not really necessary, to use all possible states in the diagonaliztion.
- Need $N\Lambda$-$N\Lambda$ (parametrized, Δ,..), $N\Lambda$-$N\Sigma$ (see following slides), and $N\Sigma$-$N\Sigma$ (for T=1/2 and T=3/2; from YNG-type interaction) two-body matrix elements. All can be represented in the same way.
- Diagonal energies of Λ and Σ states differ by $\sim$ 80MeV, plus core energy differences, plus contributions from YN interactions.

Ground-state doublet spacings of ${}^{10}_{\Lambda}$B and ${}^{12}_{\Lambda}$C

KEK E566 Hyperball2 Y. Ma (parallel 2-B)

Λ-Σ coupling

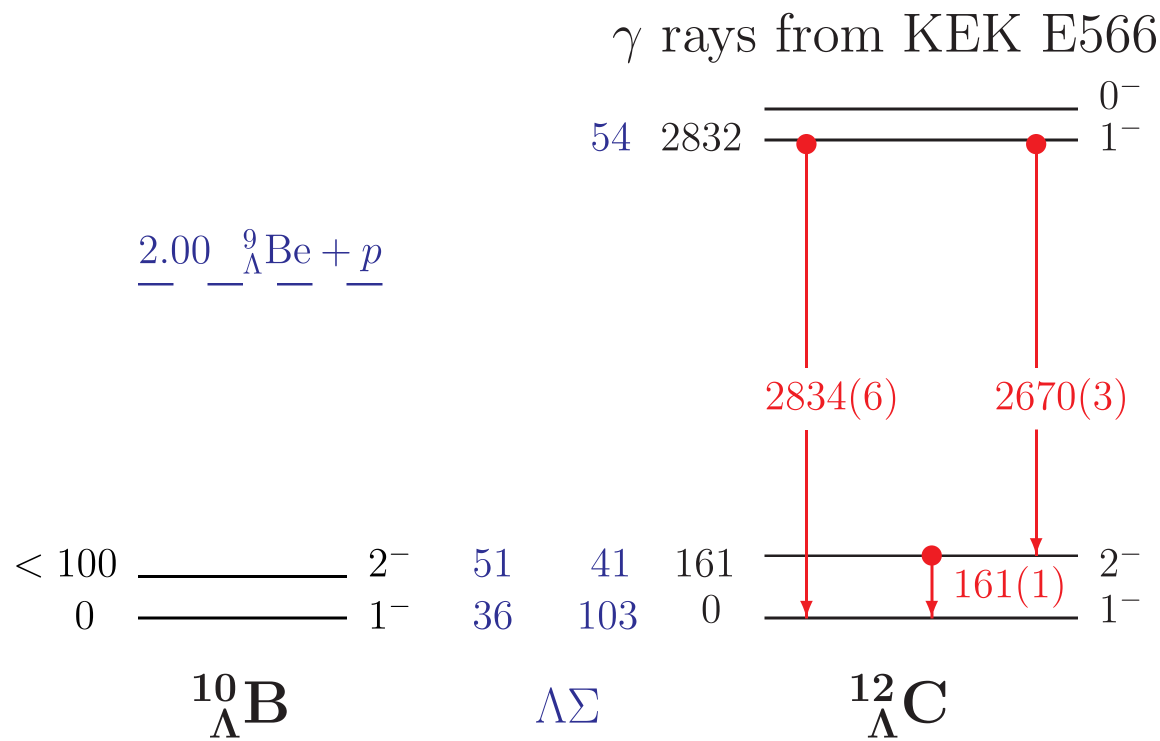
BNL ${}^{10}\mathrm{B}(3^+)(K^-,\pi^-\gamma){}^{10}_{\Lambda}\mathrm{B}(2^-)$

KEK ${}^{12}\mathrm{C}(0^+)(\pi^+,K^+\gamma){}^{12}_{\Lambda}\mathrm{C}(1^-_2)$

Core nuclei ^{9}B, ^{11}C similar

Particle-hole conjugates in p shell

Spacings of $2^-/1^-$ $(3/2^- \times s_\Lambda)$ doublets should be similar; mainly due to ΛN spin-spin interaction



Ground-state wave functions of ^{9}B and ^{11}C for a variety of p-shell interactions. The columns labelled %L=1 and %L=2 give the total percentages of the given L with S=1/2.

	Interaction	L=1 – L=2	% [41]	%L=1	%L=2
^{9}B	fitd	0.919 – 0.343	96.2	84.7	13.0
	fit4	0.898 – 0.375	94.7	80.6	16.1
	CK616	0.925 – 0.317	95.7	85.9	12.3
	Otsuka	0.868 – 0.400	91.4	76.5	19.8
			% [43]		
^{11}C	fitd	0.778 – 0.415	77.7	68.0	20.3
	fit4	0.734 – 0.520	81.0	60.8	29.3
	CK616	0.762 – 0.480	81.1	65.5	26.2
	Otsuka	0.737 – 0.463	75.7	63.7	25.4
SU3 K=3/2		$\sqrt{21/26} - \sqrt{5/26}$			
Coef. Δ				$2/3$	$-2/5$

$p_N s_\Lambda$ Λ-Σ coupling parameters from several of the Nijmegen baryon-baryon potentials.

Source	Interaction	$\bar{V}'$	Δ'	S'_Λ	S'_N	T'
Akaishi (s-shell)	NSC97e/f	1.45	3.04	−0.09	−0.09	0.16
Yamamoto	NSC97f	0.96	3.62	−0.07	−0.07	0.31
Halderson ∗	NSC97e	0.75	3.51	−0.45	−0.24	0.31
Halderson	NSC97f	1.10	3.73	−0.45	−0.23	0.30
Halderson	ESC04a	−2.30	−2.59	−0.17	−0.17	0.23

∗ D. Halderson, Phys. Rev. 77, 034304 (2008).

- ${}^4_\Lambda$H/${}^4_\Lambda$He 0^+ $\bar{V}'_s + 3/4\ \Delta'_s$
- ${}^4_\Lambda$H/${}^4_\Lambda$He 1^+ $\bar{V}'_s - 1/4\ \Delta'_s$
- Effective central interaction from second-order tensor; ESC04 interactions have a peculiar radial behavior (see Halderson) but the overall strength is not so different from the other interacions (when resticted to the p shell).

fitd p-shell interaction - nsc97 Λ-Σ coupling

	ΛΣ	Δ	S_Λ	S_N	T	ΔE
${}^{12}_{\Lambda}$C		0.529	1.446	0.038	−1.773	
	61	175	−22	−13	−42	153 keV
${}^{10}_{\Lambda}$B		0.570	1.426	0.008	−1.100	
	−15	188	−21	−3	−26	120 keV

CK616 p-shell interaction - esc04a Λ-Σ coupling

	ΛΣ	Δ	S_Λ	S_N	T	ΔE
${}^{12}_{\Lambda}$C		0.443	1.542	0.027	−2.145	
	111	146	−23	−9	−51	167 keV
${}^{10}_{\Lambda}$B		0.575	1.422	0.015	−1.645	
	−88	190	−21	−5	−39	34 keV

Can write the central Λ-Σ coupling interaction as

$$\sqrt{4/3}\; t_N \cdot t_Y \; \bar{V}' + \sqrt{4/3}\; s_N \cdot s_Y \; t_N \cdot t_Y \; \Delta'$$

where the factor $\sqrt{4/3}$ arises from defining t_Y as an operator that changes a Λ into a Σ [$^{10}_{\Lambda}$Li - A. Umeya (parallel 2-B)].

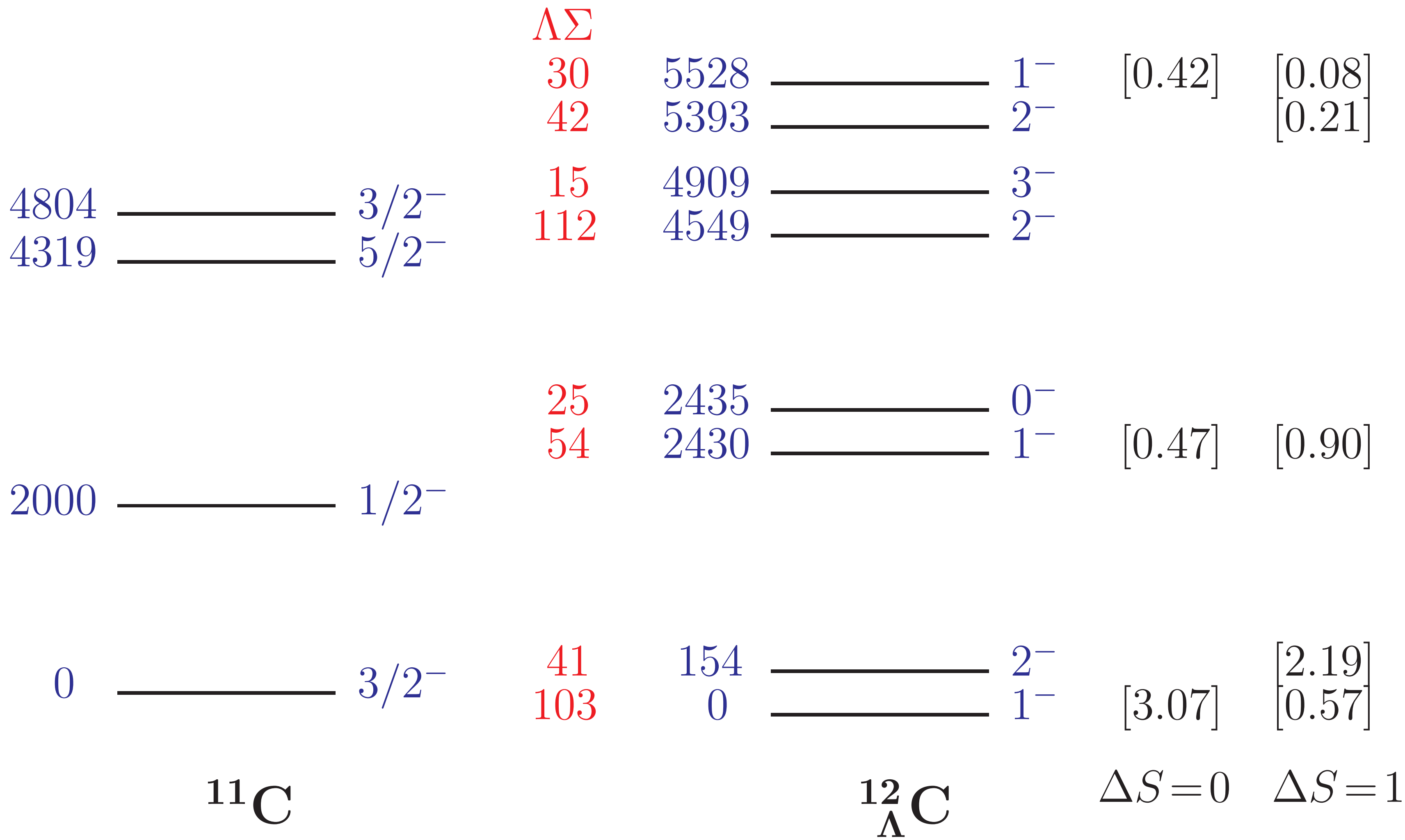
The important Λ-Σ coupling matrix elements involve a Σ coupled to the ground and $1/2^-$ (L=1) states of the core.

Diagonal matrix element $\sqrt{4/3}\sqrt{T(T+1)}\overline{V} + a(J)\langle 3/2|| \sum_i s_i t_i||3/2\rangle$

Off-diagonal matrix element $b(J)\langle 1/2|| \sum_i s_i t_i||3/2\rangle$

		$^{10}_{\Lambda}$B	$^{12}_{\Lambda}$C
$J=1$	$\langle 3/2^- \times \Sigma \|\|V\|\| 3/2^- \times \Lambda\rangle$	$\overline{V} - 5/13\ \Delta$	$\overline{V} + 5/13\ \Delta$
	$\langle 1/2^- \times \Sigma \|\|V\|\| 3/2^- \times \Lambda\rangle$	$7/13\sqrt{2}\ \Delta$	$-7/13\sqrt{2}\ \Delta$
$J=2$	$\langle 3/2^- \times \Sigma \|\|V\|\| 3/2^- \times \Lambda\rangle$	$\overline{V} + 3/13\ \Delta$	$\overline{V} - 3/13\ \Delta$

Actual shifts are not so large (previous slide) because of the contribution from the $1/2^- \times \Sigma$ configuration; the $^{12}_{\Lambda}$C doublet spacing is substantially increased; at this level, the non-central Λ-Σ coupling components are also important.

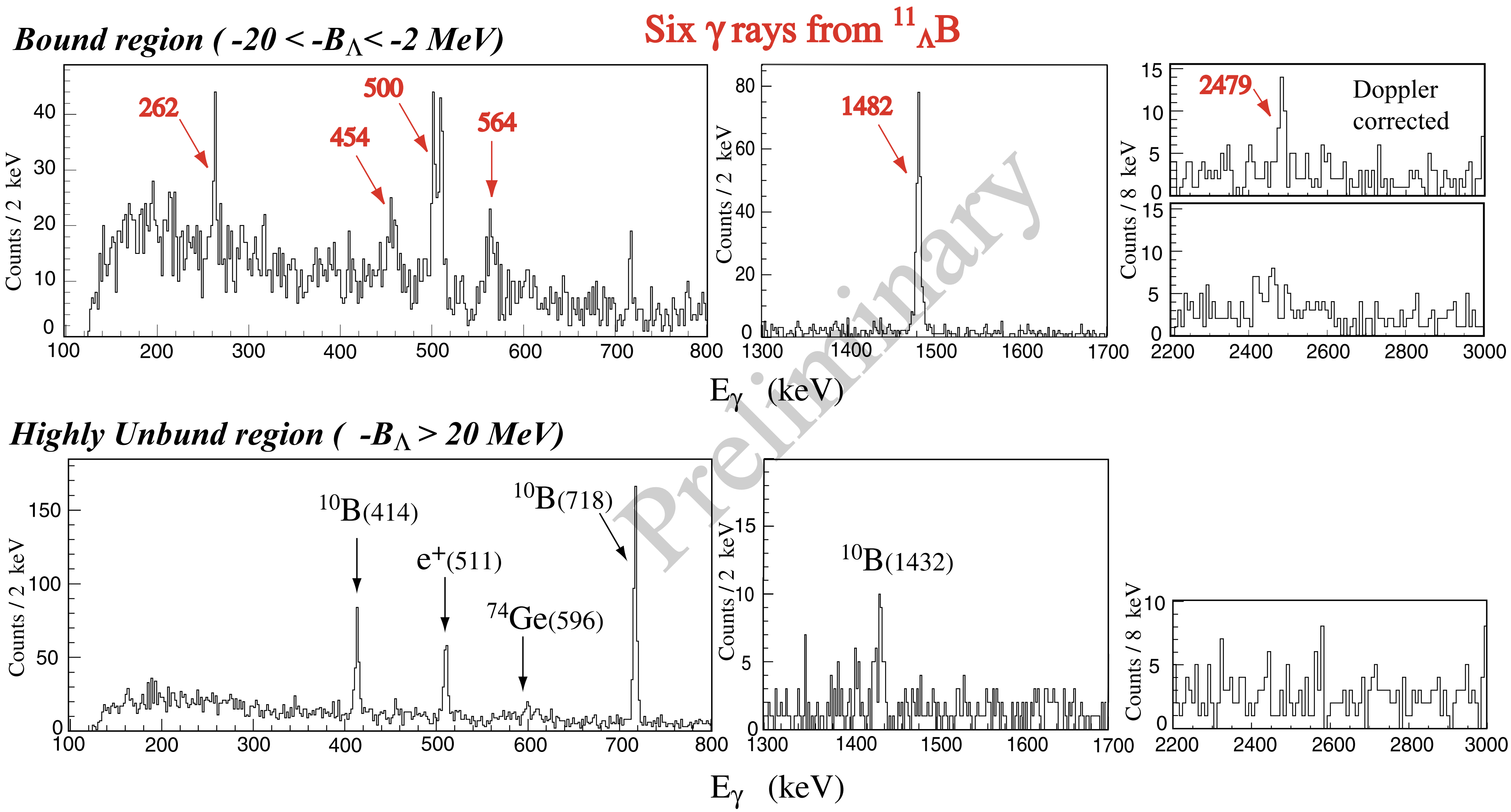


$^{12}C(e,e'K^+)^{12}_{\Lambda}\mathrm{B}$ - can translate energies of 3 peaks into $^{12}_{\Lambda}\mathrm{C}$ energies

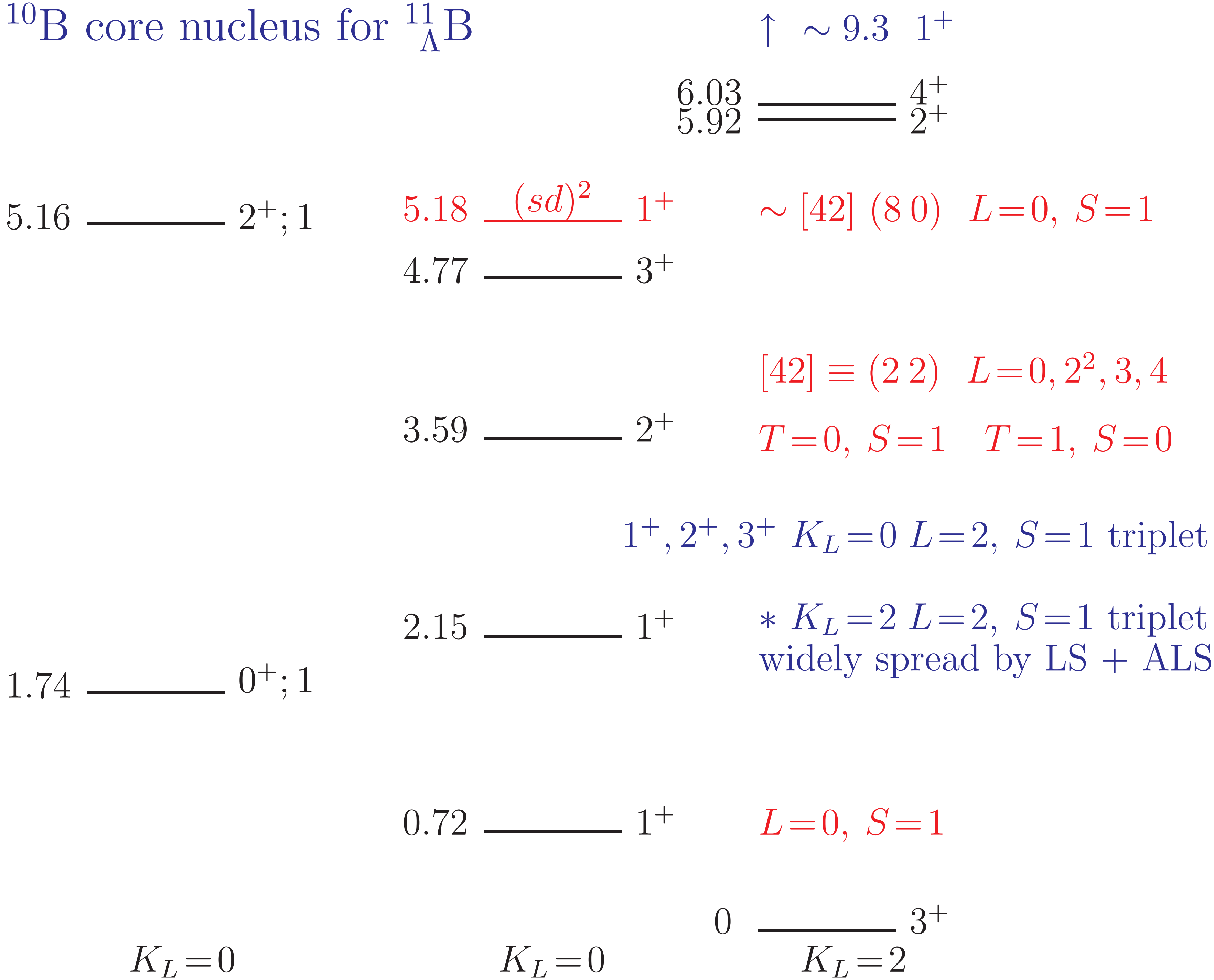
[Hall A/Hall C] $E_x(1^-_2) = [2.65 - 2.80]$ $E_x(1^-_3) = [6.05 - 6.23]$

The excited 1^- states are raised by S_N, but not enough to reproduce the new γ-ray and $(e,e'K^+)$ data.

KEK E518 $^{11}B(\pi^+,K^+\gamma)\,^{11}_{\Lambda}B$

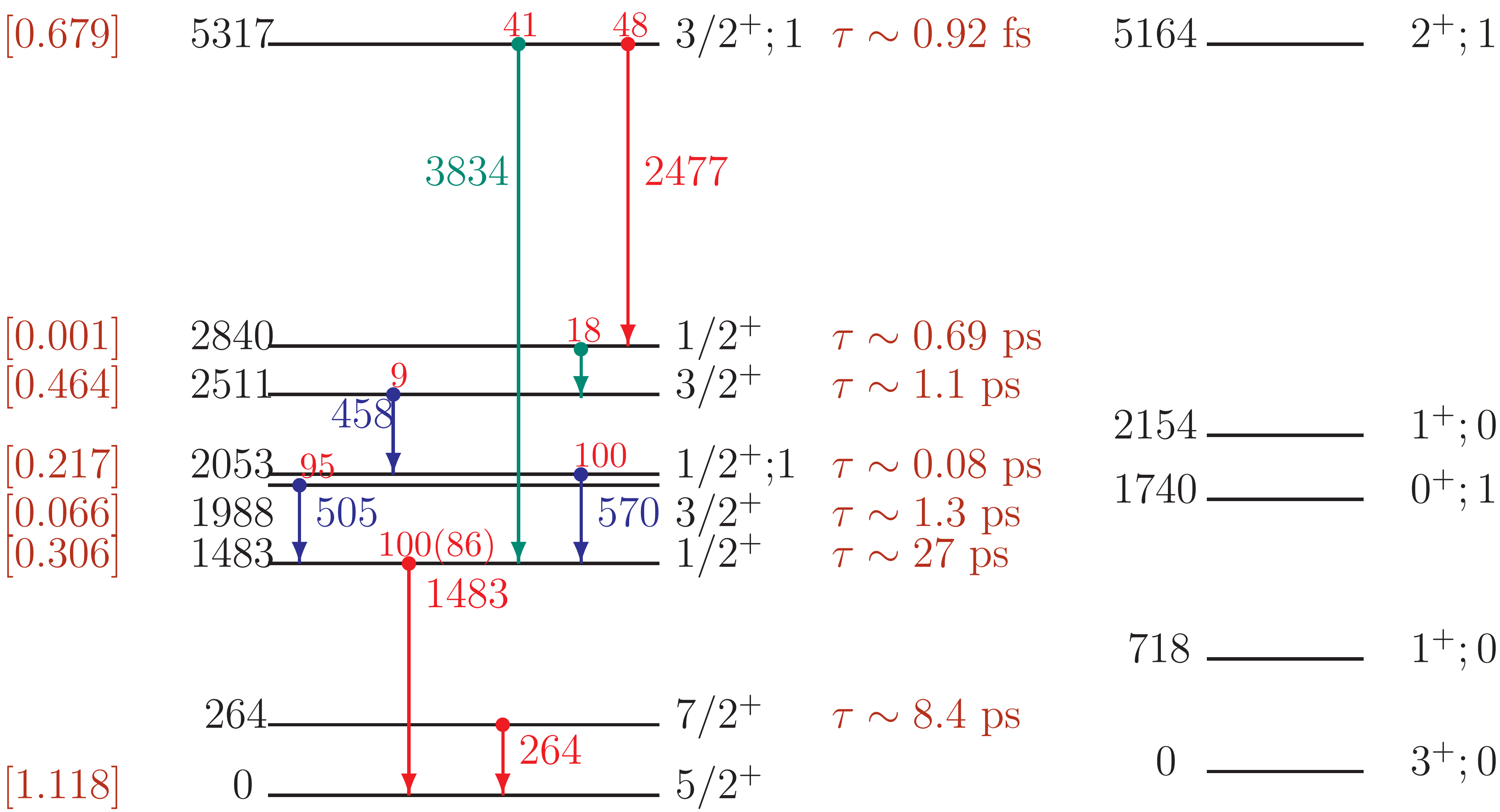


^{10}B core nucleus for $^{11}_{\Lambda}$B

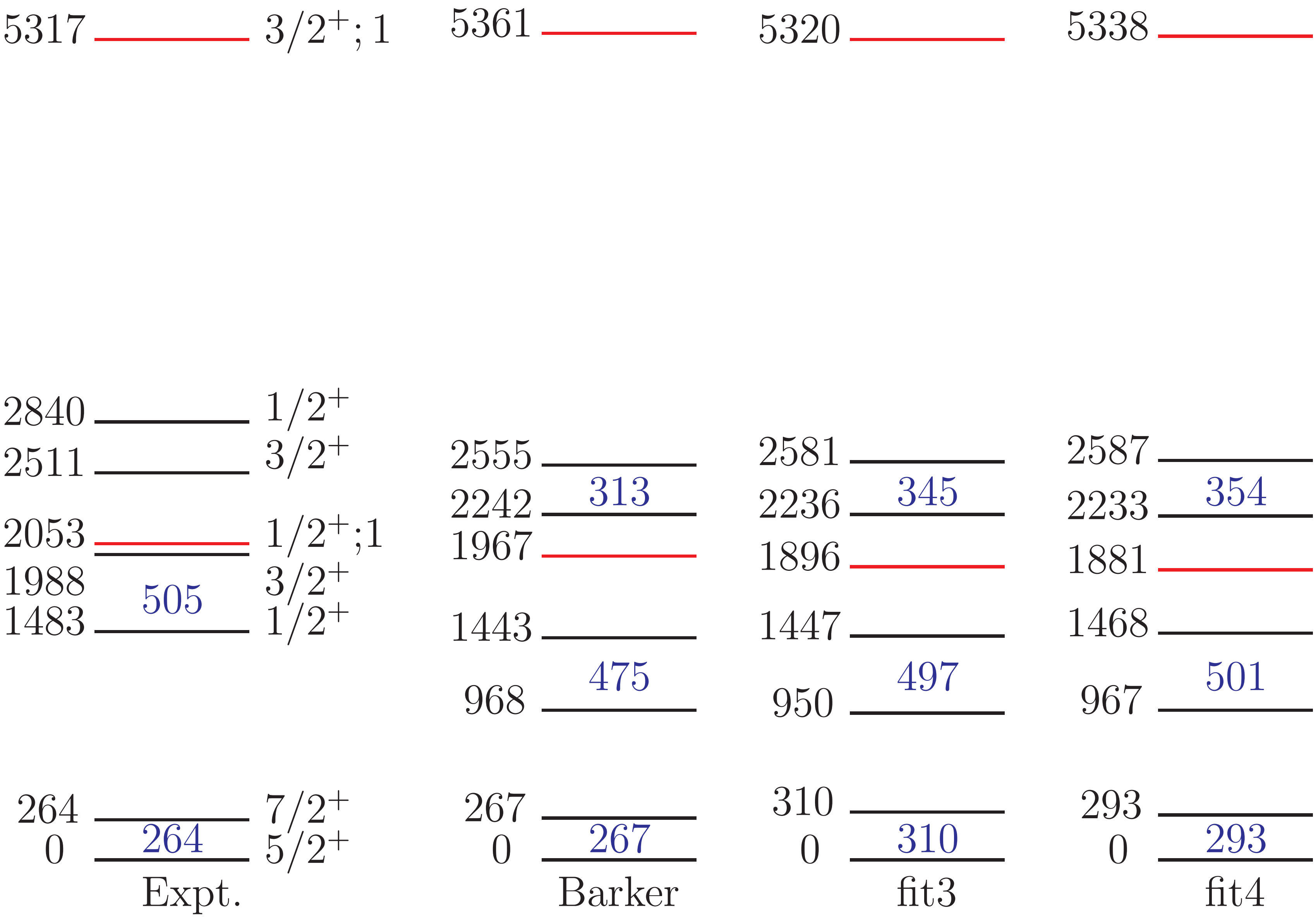


- See DJM, Nucl. Phys. A 804, 84 (2008) for more details on ^{10}B and $^{11}_{\Lambda}$B, in particular γ-ray intensities and lifetime limits in $^{11}_{\Lambda}$B from KEK E518.
- In the next slide, the red arrows indicate the most certain assignments, the blue arrows correspond to observed γ-rays, and the green arrows indicate other branches of interest. The decay of the states in the upper $1/2^+/3/2^+$ doublet are complex.
- KEK E566 shows that the 505 keV γ-ray directly feeds the 1483 keV level.
- Data on E2/M1 mixing ratios in ^{10}B implies that there is very small mixing of $L=0$ and $L=2$ in the core 1^+ wave functions. The Barker, fit3, and fit4 interactions satisfy this requirement.
- The lowest $1/2^+$ is raised substantially ($>$ 400 keV with respect to the $5/2^+$ gs by the action of S_N, but not nearly enough.

Speculations on the placement of ${}^{11}_{\Lambda}$B γ rays.



Shell-model calculations for $^{11}_{\Lambda}$B



$(sd)_N s_\Lambda$ matrix elements

	J	HO (b = 1.7 fm)	WS (BE = 3 MeV)	WS (BE = 1 MeV)
$\langle 1s_{1/2}s_\Lambda \vert V \vert 1s_{1/2}s_\Lambda\rangle$	0	-1.690	-0.977	-0.657
	1	-1.237	-0.724	-0.488
$\langle d_{3/2}s_\Lambda \vert V \vert 1s_{1/2}s_\Lambda\rangle$	1	-0.137	-0.097	-0.071
$\langle d_{3/2}s_\Lambda \vert V \vert d_{3/2}s_\Lambda\rangle$	1	-0.508	-0.487	-0.428
	2	-0.540	-0.512	-0.447
$\langle d_{5/2}s_\Lambda \vert V \vert d_{3/2}s_\Lambda\rangle$	2	0.140	0.129	0.109
$\langle d_{5/2}s_\Lambda \vert V \vert d_{5/2}s_\Lambda\rangle$	2	-1.133	-0.999	-0.866
	3	-1.065	-0.936	-0.811
BE of pair in ^{10}B		-2.16	-1.56	-1.23

Doublet spacings in p-shell hypernuclei

	J_u^π	J_l^π	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE^{th}	ΔE^{exp}
${}^{7}_{\Lambda}$Li	$3/2^+$	$1/2^+$	72	628	−1	−4	−9	693	692
${}^{7}_{\Lambda}$Li	$7/2^+$	$5/2^+$	74	557	−32	−8	−71	494	471
${}^{8}_{\Lambda}$Li	2^-	1^-	151	396	−14	−16	−24	450	(442)
${}^{9}_{\Lambda}$Li	$3/2^+_2$	$1/2^+$	−80	231	−13	−13	−93	−9	
${}^{11}_{\Lambda}$B	$7/2^+$	$5/2^+$	56	339	−37	−10	−80	267	264
${}^{11}_{\Lambda}$B	$3/2^+$	$1/2^+$	61	424	−3	−44	−10	475	505
${}^{12}_{\Lambda}$C	2^-	1^-	61	175	−12	−13	−42	153	161
${}^{15}_{\Lambda}$N	$3/2^+_2$	$1/2^+_2$	65	451	−2	−16	−10	507	481
${}^{16}_{\Lambda}$O	1^-	0^-	−33	−123	−20	1	188	23	26
${}^{16}_{\Lambda}$O	2^-	1^-_2	92	207	−21	1	−41	248	224

Remarks

- The nine observed doublets are fit remarkably well with the only caveat that the the two doublets in ${}^{7}_{\Lambda}$Li require a somewhat larger value of Δ.
- ${}^{8}_{\Lambda}$Li is shown because the 1^- ground-state wave function involves substantial mixing ($\sim 11\%$) of configurations based on the $3/2^-$ and $1/2^-$ core states and this leads to an abnormally large $\Lambda\Sigma$ contribution for a T=1/2 core.
- In most cases, Δ and $\Lambda\Sigma$ coupling work in the same direction although the ratio of contributions varies substantially. ${}^{9}_{\Lambda}$Li is shown because the contributions work in opposite directions, as in ${}^{10}_{\Lambda}$B but giving a larger effect.
- ${}^{9}_{\Lambda}$Li, and some other interesting cases, could be reached using the $(K^-, \pi^0\gamma)$ reaction, which is hopefully in J-PARC's future. Access to states reached via spin-flip amplitudes in the $(K^-, \pi^-\gamma)$ reaction is in the near future at J-PARC.