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Neutron-rich Hypernuclei

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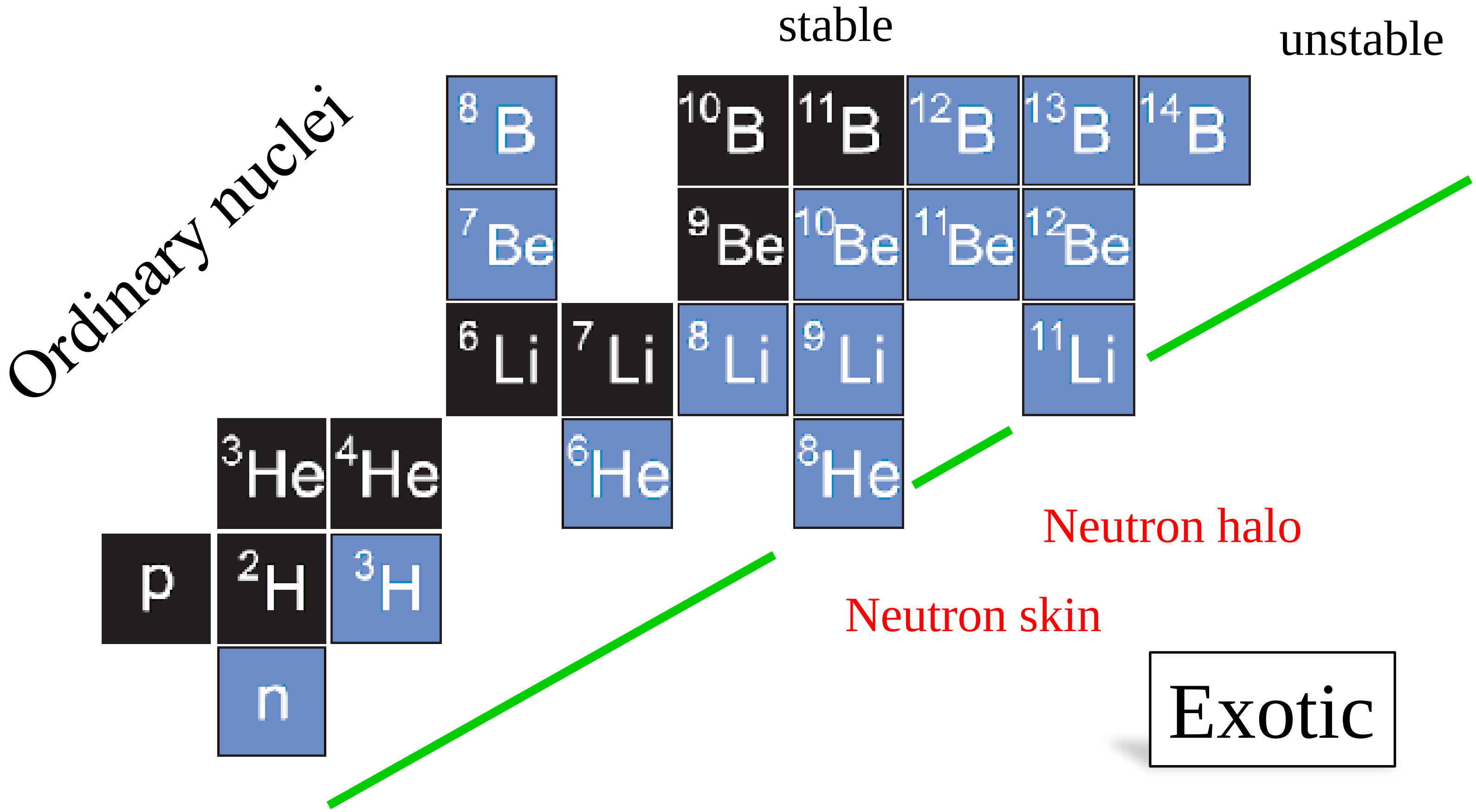
Contents

1. Introduction
2. The Λ-Σ coupling in nuclei
3. Production of neutron-rich Λ hypernuclei
   3.1. Calculation of the DWIA **--- (π^-,K^+) reaction**
   3.2. Numerical results **--- $^{10}_{\Lambda}$Li**
4. Discussion
5. Summary and conclusion

1. Introduction

Neutron-rich nuclei



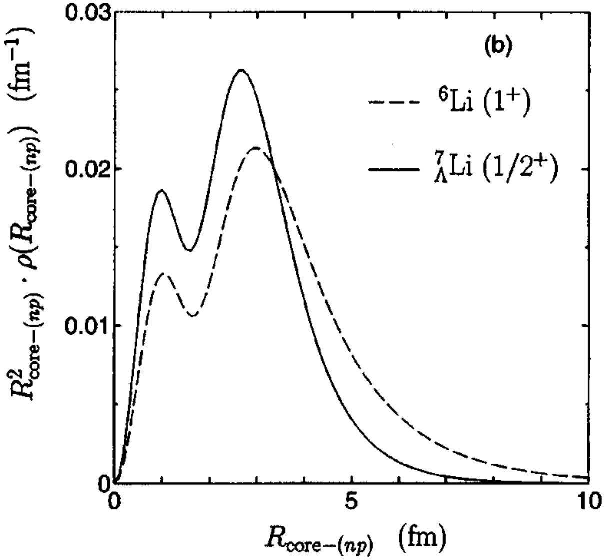
Neutron-rich nuclei have interesting phenomena…
→The neutron-excess environment gives us new information on the structure of exotic nuclei and also NN interactions.

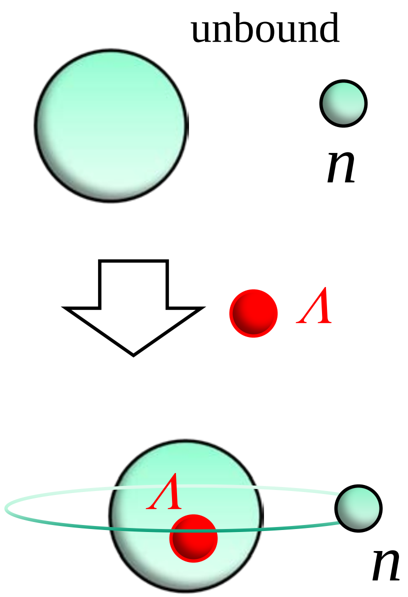
Neutron-rich Λ hypernuclei

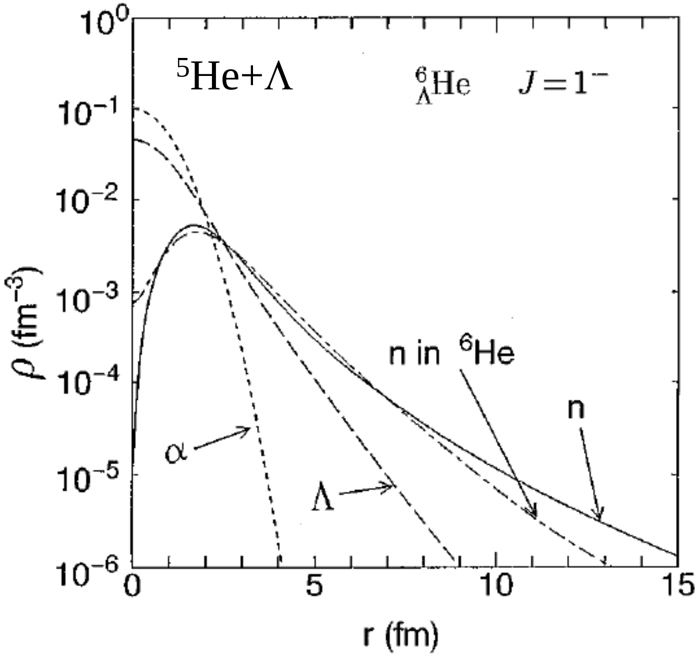
Λ-hypernuclei may be even better candidates than ordinary nuclei to exhibit unusually large neutron-excess N/Z and halo phenomena.

L. Majling, NPA585 (1995) 211c.

-- Stabilizing (gule-like) role of the Λ in nuclei







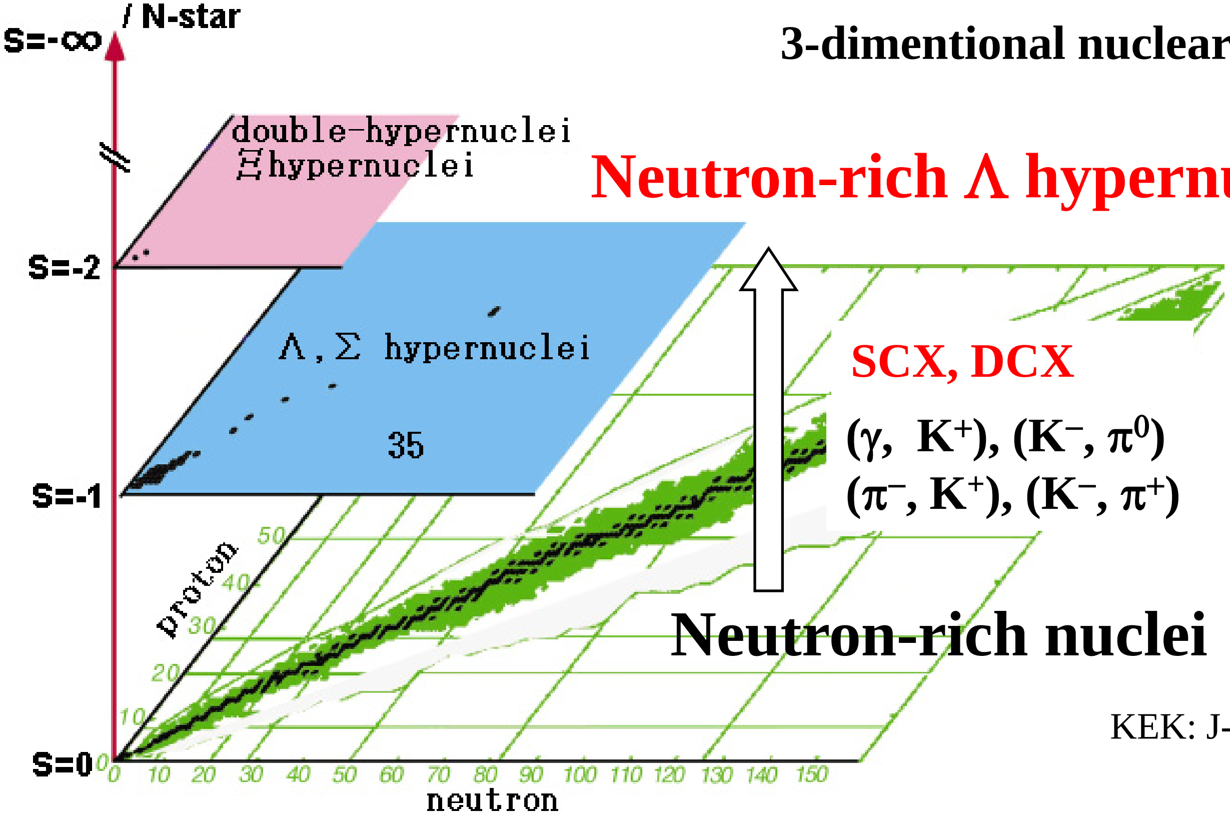
Shrinkage effects (19% for the ^{6}Li core)

T. Motoba, et al.,PTP70(1983)189
E. Hiyama, et al.,PRC59(1999)2351

Hypernuclei with neutron-skin or neutron halo

E. Hiyama, et al.,PRC59(1999)2351
Tretyakova, Lanskoy, EPJ.A5(1999) 391.

Nuclear chart with strangeness



3-dimentional nuclear chart

Neutron-rich Λ hypernuclei

SCX, DCX

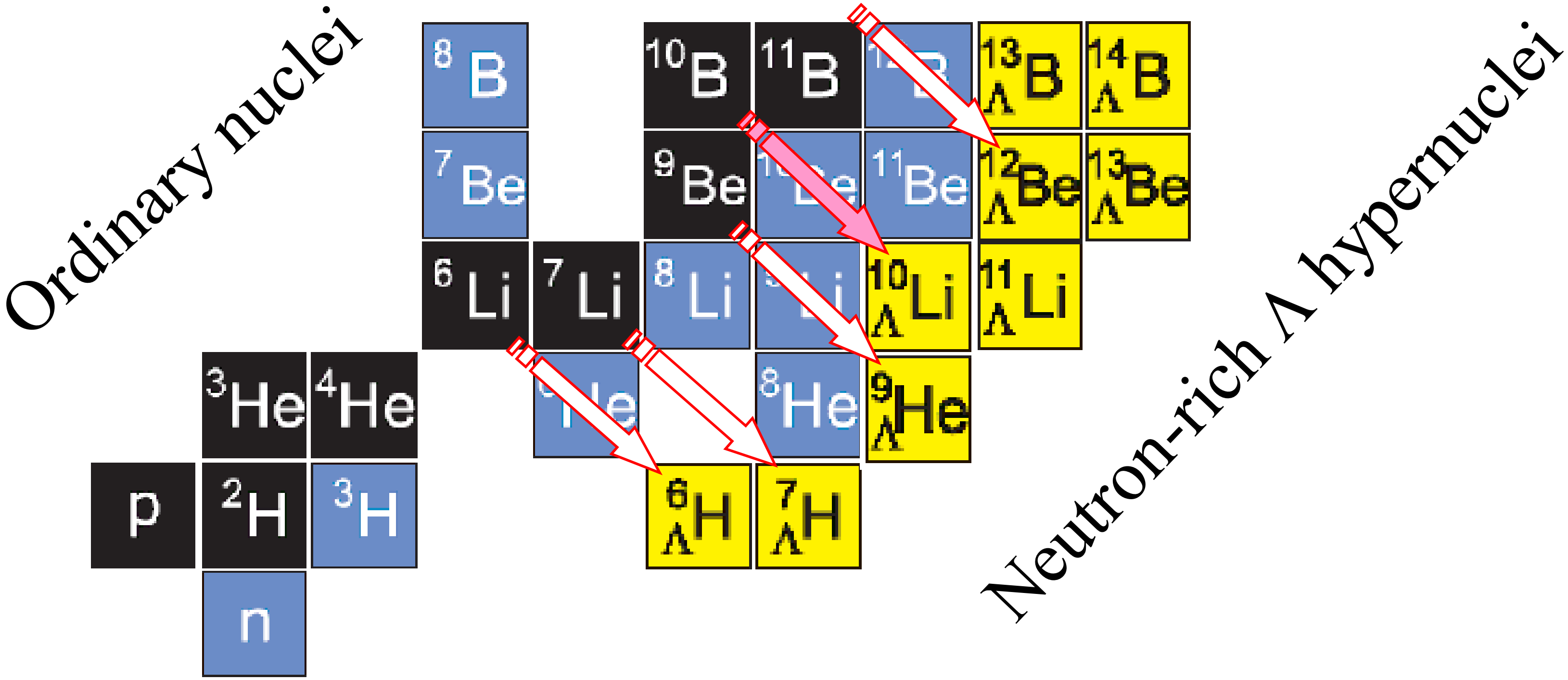
(γ, K^+), (K^-, π^0)
(π^-, K^+), (K^-, π^+)

Neutron-rich nuclei

KEK: J-PARC

To produce the neutron-rich Λ hypernuclei, we need the charge-exchange reactions on the (stable) nuclear targets.

Neutron-rich Λ hypernuclei



Several experiments attempted to measure the yields of such hypernuclei.

Double Charge Exchange (DCX) reactions

(K^{-}, π^{+}), (π^{-}, K^{+})

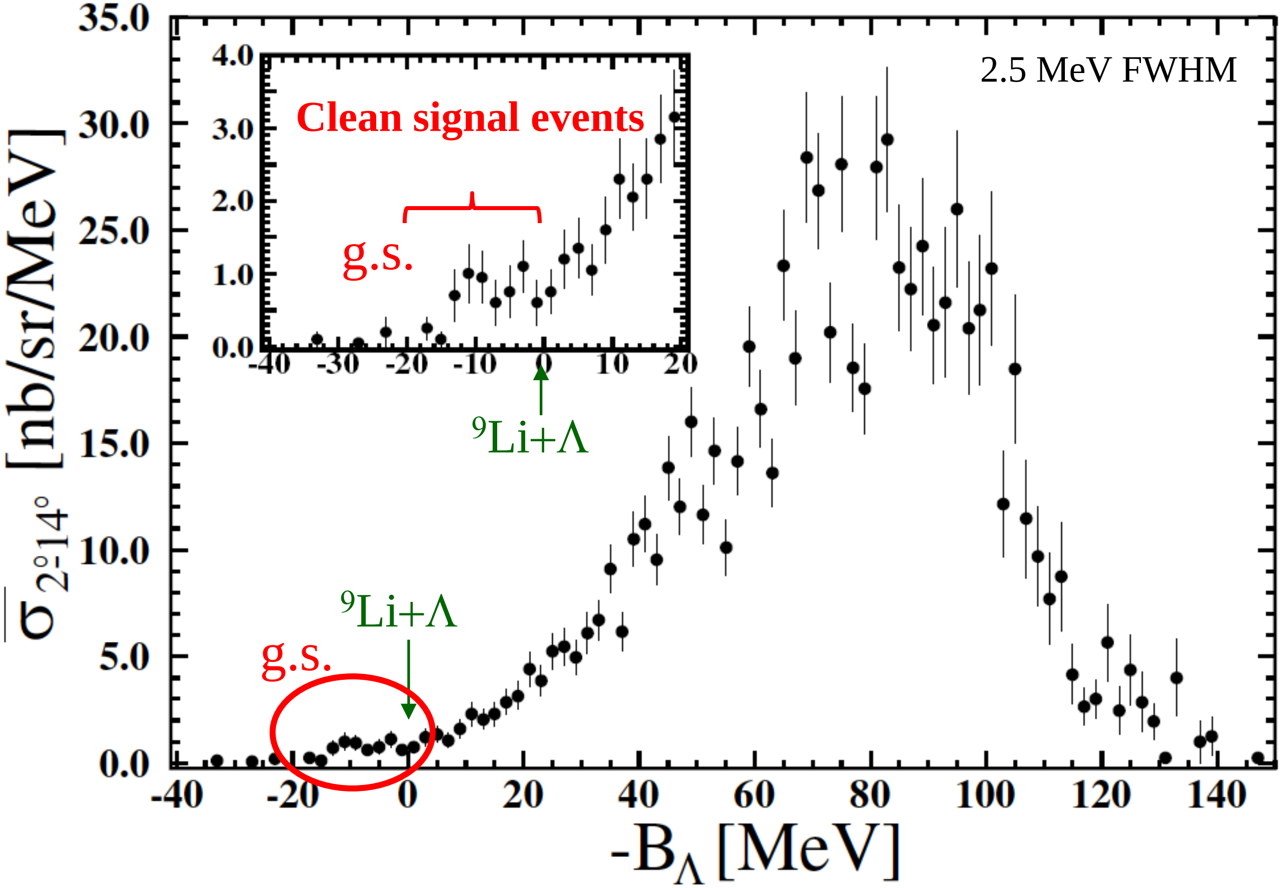
KEK	^{9}Be(K^-,π^+)$^{9}_{\Lambda}$He, ^{12}C(K^-,π^+)$^{12}_{\Lambda}$Be , ^{16}O(K^-,π^+) $^{16}_{\Lambda}$C	K.Kubota et al., NPA 602 (1996).
	^{10}B(π^-,K^+) $^{10}_{\Lambda}$Li at p_π=1.05, 1.20 GeV/c.	P. K. Saha, et al., PRL94(2005)052502.
DAΦNE	^{6}Li(K^-,π^+)$^{6}_{\Lambda}$H, ^{7}Li(K^-,π^+)$^{7}_{\Lambda}$H	M. Agnello, et al., PLB640 (2006) 145.

First measurements of neutron-rich Λ hypernuclei

$^{10}B(\pi^-, K^+)^{10}_{\Lambda}Li$

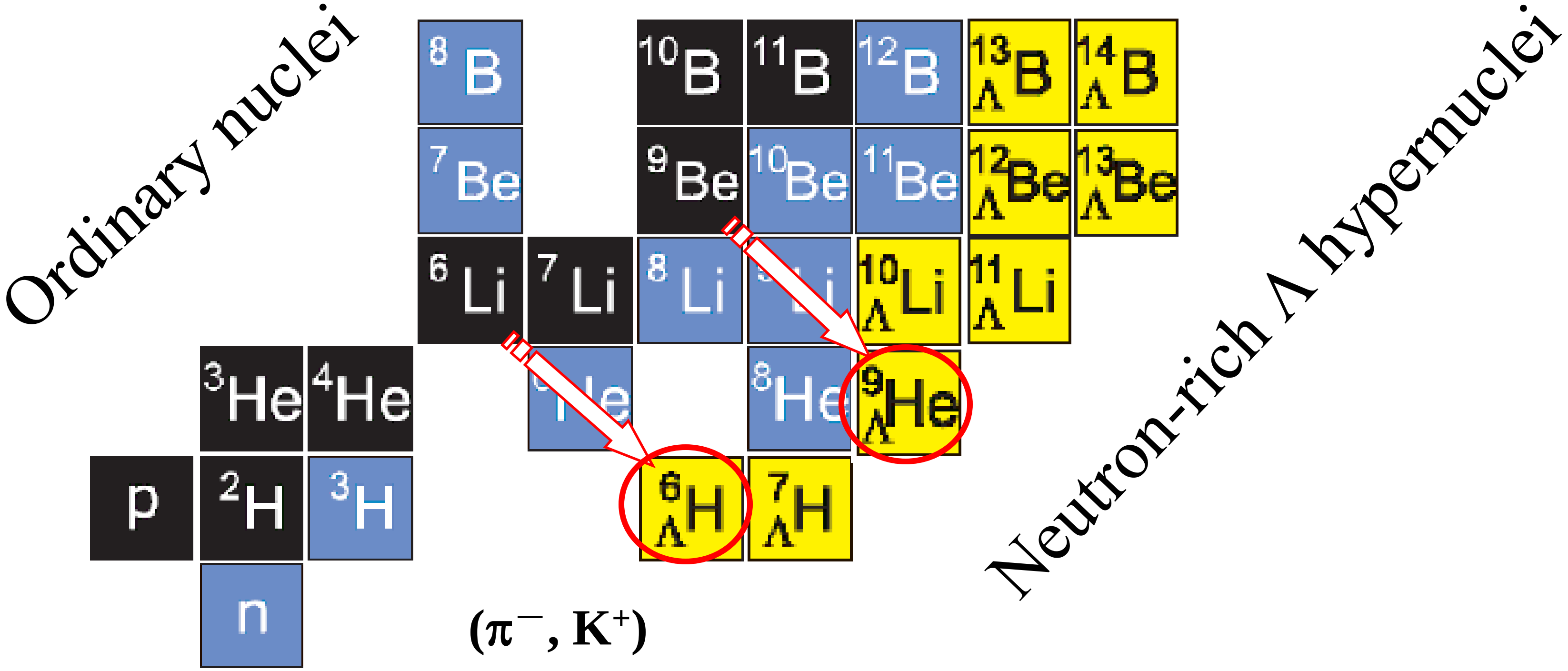
DCX (π^-,K^+) reaction at 1.2GeV/c

KEK-PS-E521 P. K. Saha, et al., PRL94(2005)052502



The production with the (π^-,K^+) *reaction may open to new fields of neutron-rich hypernuclear studies.*

Neutron-rich Λ hypernuclei



Double Charge Exchange (DCX) reactions

To investigate more exotic structure of ...

^{6}Li(π^-,K$^+$)$^{6}_{\Lambda}$H, ^{9}Be(π^-,K$^+$)$^{9}_{\Lambda}$He at p_{π}=1.20 GeV/c

J-PARC E10: Production of Neutron-Rich Λ-Hypernuclei with the Double Charge-Exchange Reactions

A.Sakaguchi et al, http://j-parc.jp/NuclPart/Proposal_e.html

In this talk,

I will discuss production and spectroscopy of the neutron-rich Λ hypernuclei by the DCX (π^-, K^+) reaction on nuclear targets, in a theoretical viewpoint.

$$^{10}\mathrm{B}(\pi^-, K^+)\,^{10}_{\Lambda}\mathrm{Li}$$

I will focus on

i. the Λ-Σ coupling which might be enhanced by the neutron-excess environment

ii. an important role of the Σ admixture in neutron-rich hypernuclei.

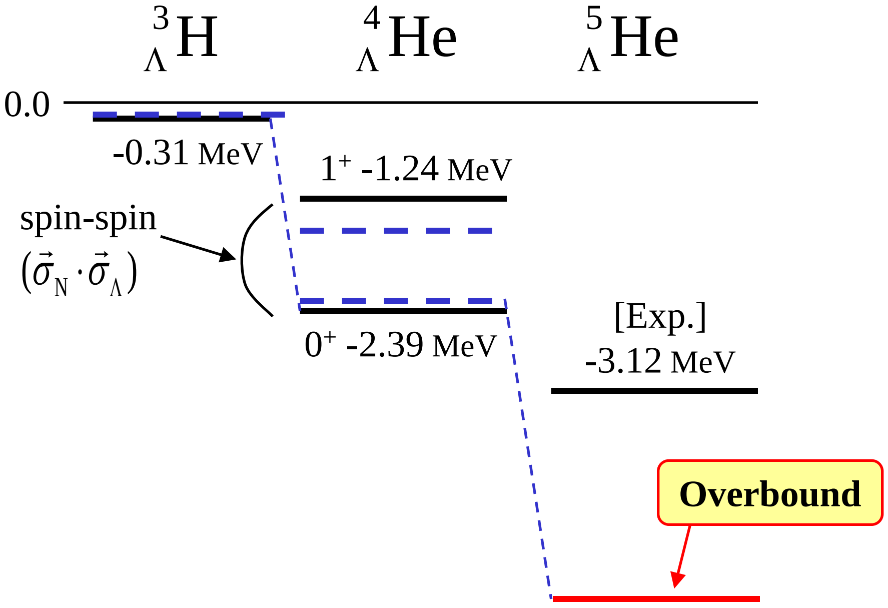
2. The Λ-Σ coupling in nuclei

“coherent” and “incoherent”

Y.Akaishi, T.H, S. Shinmura, Khin Swe Myint,
PRL84 (2000)3539 .

Overbinding Problem on s-Shell Hypernuclei

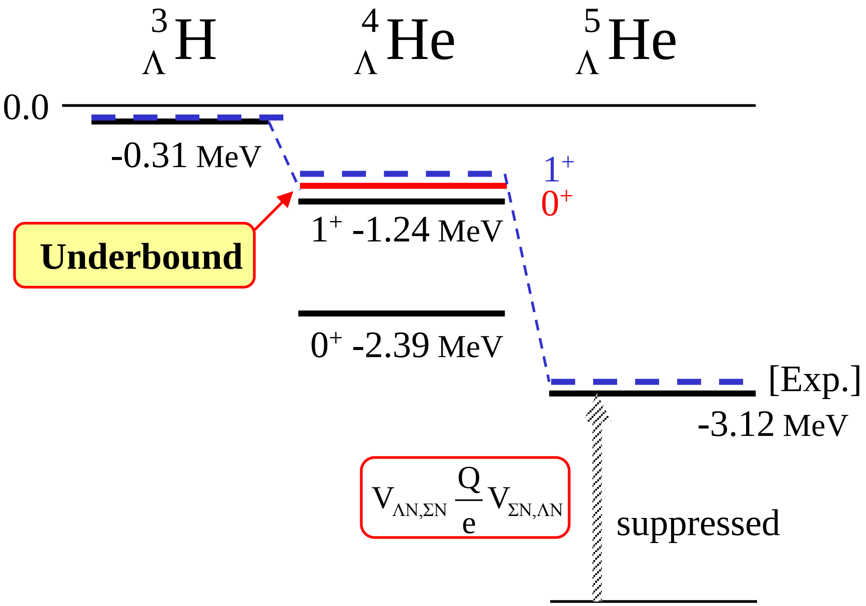
The Overbinding Problem



ΛN single-channel calc.

Dalitz et al., NP **B47** (1972) 109.

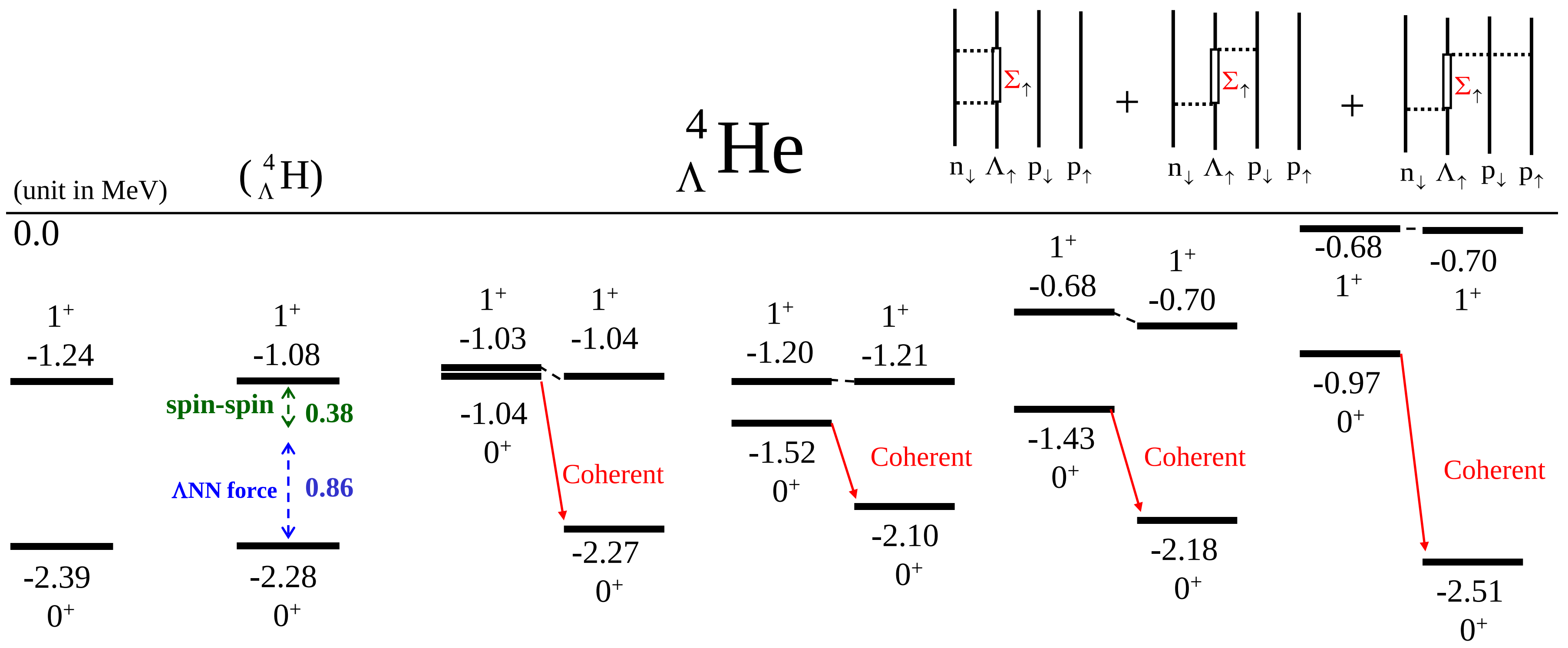
The Underbinding Problem



g-matrix calc. with ΛN-ΣN(D2)

Akaishi et al., PRL **84** (2000) 3539.

Coherent Λ-Σ coupling in s-shell Λ hypernuclei



$n_\downarrow$ $\Lambda_\uparrow$ $p_\downarrow$ $p_\uparrow$ $\Sigma_\uparrow$ + $n_\downarrow$ $\Lambda_\uparrow$ $p_\downarrow$ $p_\uparrow$ $\Sigma_\uparrow$ + $n_\downarrow$ $\Lambda_\uparrow$ $p_\downarrow$ $p_\uparrow$ $\Sigma_\uparrow$

$^{4}_{\Lambda}$He

(unit in MeV) ($^{4}_{\Lambda}$H)

0.0

1^+ -1.24

-2.39 0^+

1^+ -1.08

spin-spin 0.38

ΛNN force 0.86

-2.28 0^+

1^+ -1.03 1^+ -1.04

-1.04 0^+

Coherent

-2.27 0^+

1^+ -1.20 1^+ -1.21

-1.52 0^+

Coherent

-2.10 0^+

1^+ -0.68 1^+ -0.70

-1.43 0^+

Coherent

-2.18 0^+

-0.68 1^+ -0.70 1^+

-0.97 0^+

Coherent

-2.51 0^+

$P_{\mathrm{coh.}\Sigma} = 1.9\%$ $P_{\mathrm{coh.}\Sigma} = 0.7\%$ $P_{\mathrm{coh.}\Sigma} = 0.9\%$ $P_{\mathrm{coh.}\Sigma} = 2.0\%$

Exp.

phenomenological
$V_{\Lambda N} + V_{\Lambda NN}$
$\bar{V} = 6.20$

D2 **SC97e(S)** **SC97f(S)** **SC89(S)**

VMC

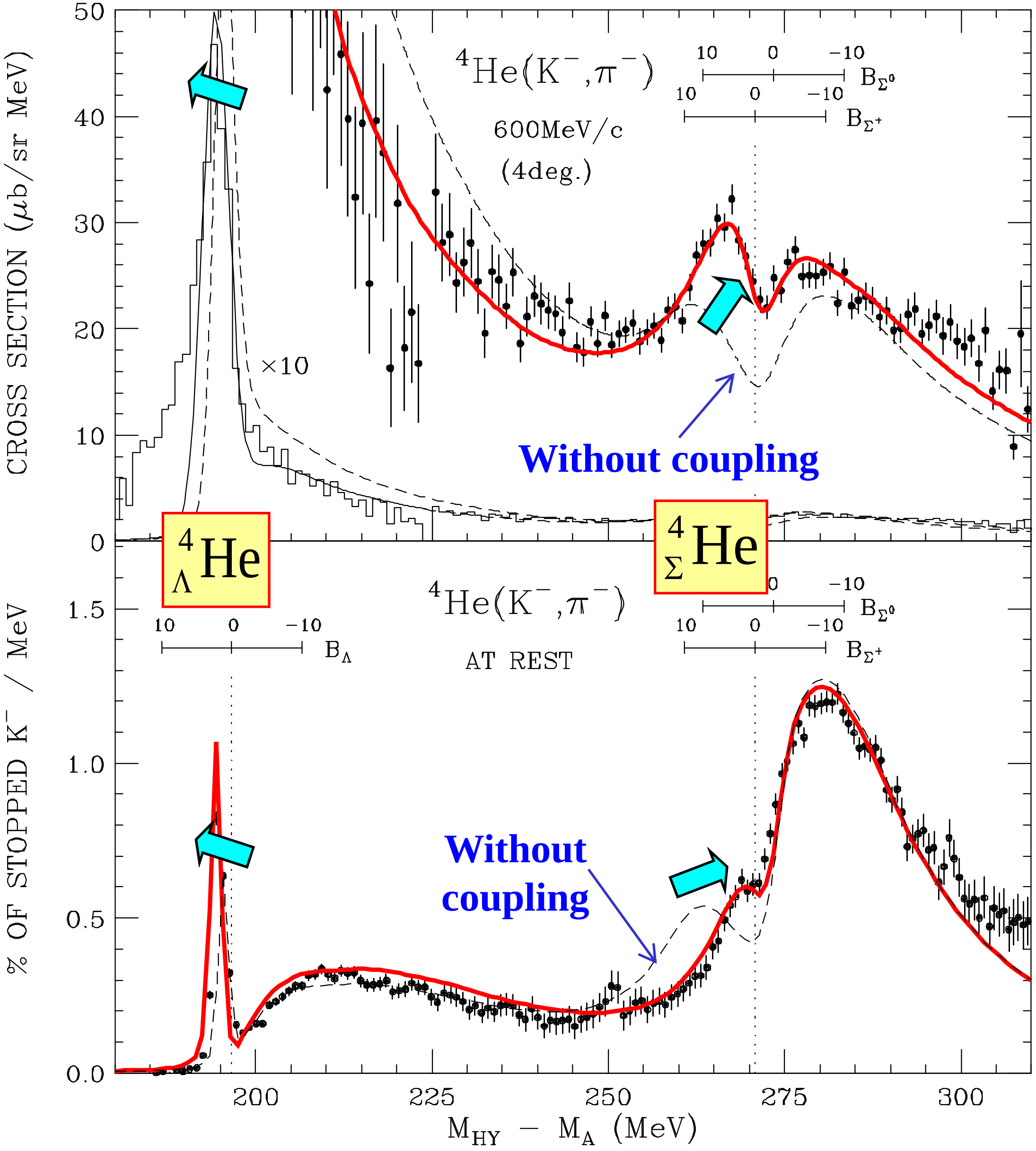
R. Sinha, Q.N.Usmani,
NPA684(2001)586c

Breuckner-Hartree-Fock

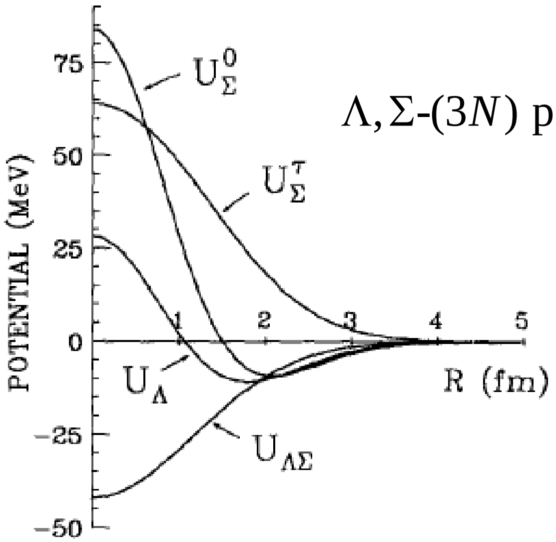
Y. Akaishi, et al., PRL84(2000)3539.

The coherent Λ-Σ coupling overcomes the underbinding problem, together with ΛN spin-spin interaction.

Λ-Σ coupling effects on the (K^-, π^-) spectrum



$^4_\Lambda$He-$^4_\Sigma$He

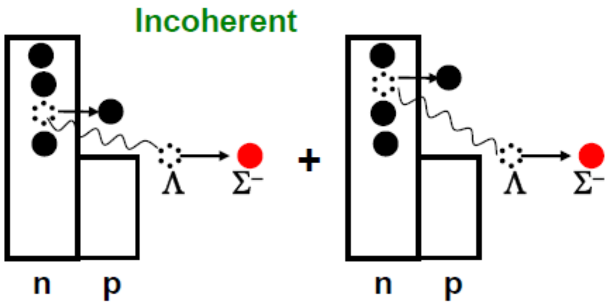


T.Harada, PRL81(1998)5287;
NPA672(2000)181;
NPA601(2001)68c.

The Λ-Σ coupling effects play an important role in reproducing the spectrum.

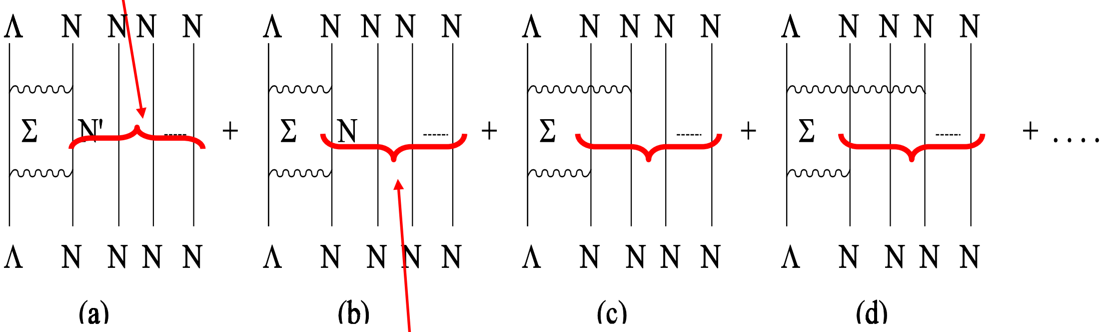
The Λ-Σ coupling effects in neutron matter

S.Shinmura, Khin Swe Myint, T.H., Y.Akaishi, J.Phys.G28(2002)L1



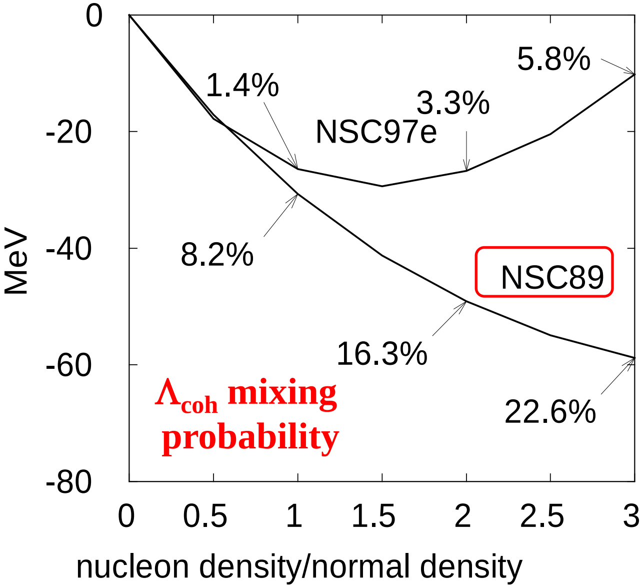
usually included in g-matrix

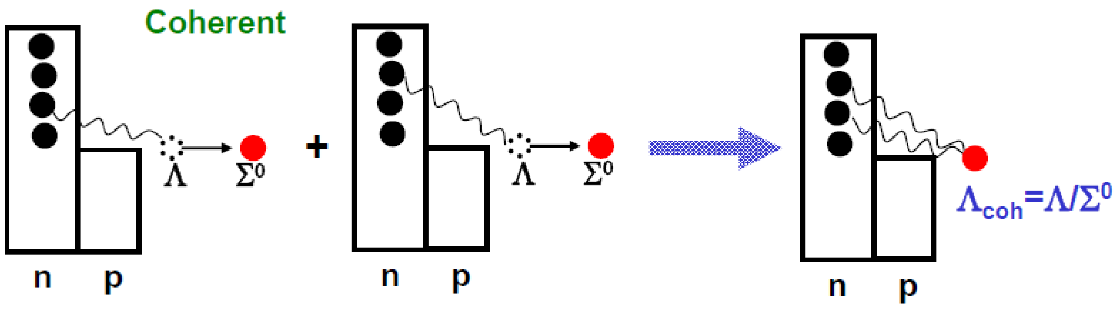
Excited (1p1h) states



Ground states

Single particle potential for Λ_{coh}.



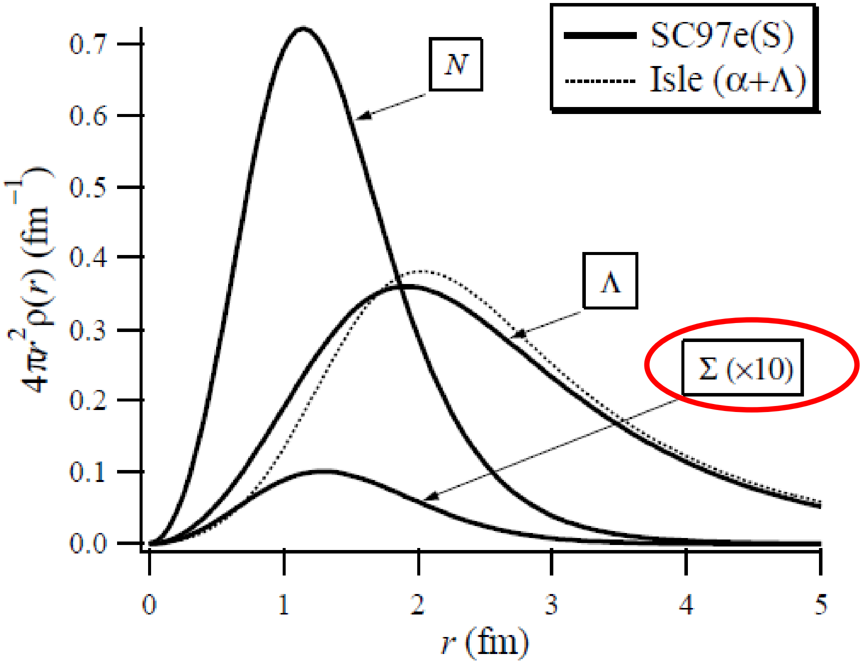


coherent Λ-Σ coupling

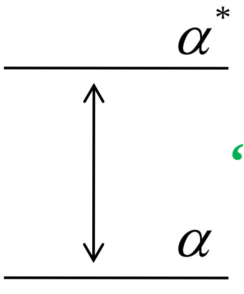
The Λ_{coh} mixing is enhanced in the neutron-excess environment.

Ab initio calculation of $^{5}_{\Lambda}$He with full realistic interactions

H.Nemura et al., PRL89(2002)142504



Better understanding of
the Λ-Σ coupling and Tensor force



$T_C = 1 \otimes \Sigma$ **~ 1.5 % admixture**

“Incoherent Λ-Σ coupling”

$T_C = 0 \otimes \Lambda$

	$L=0$		$L=2$		
	$S=\frac{1}{2}$		$S=\frac{3}{2}$		$S=\frac{5}{2}$
	$S_c=0$	$S_c=1$	$S_c=1$	$S_c=2$	$S_c=2$
$^{5}_{\Lambda}$He					
$(T_c=0)\otimes\Lambda$	89.14	0.03	0.19	3.74	5.36
$(T_c=1)\otimes\Sigma$	0.10	0.09	1.34	~ 0	0.01
^{4}He	89.56		10.44		

-The Σ admixture of
~1.5 % appears in $^{5}_{\Lambda}$He.

-The α-particle is not
a rigid core.

⇩

The incoherent Σ admixture is also important.

Remarks

Overbinding problem in s-shell hypernuclei has been solved.

Coherent Λ-Σ coupling is essential dynamics in the $^{4}_{\Lambda}$He-$^{4}_{\Sigma}$He system.

Neutron-rich Λ hypernuclei can provide additional evidences for coherent Λ-Σ coupling, together with incoherent Λ-Σ coupling.

How to produce such neutron-rich Λ hypernuclei ?

3. Production of neutron-rich Λ hypernuclei

“Feasibility of extracting a Σ^- admixture probability in the neutron-rich hypernucleus”

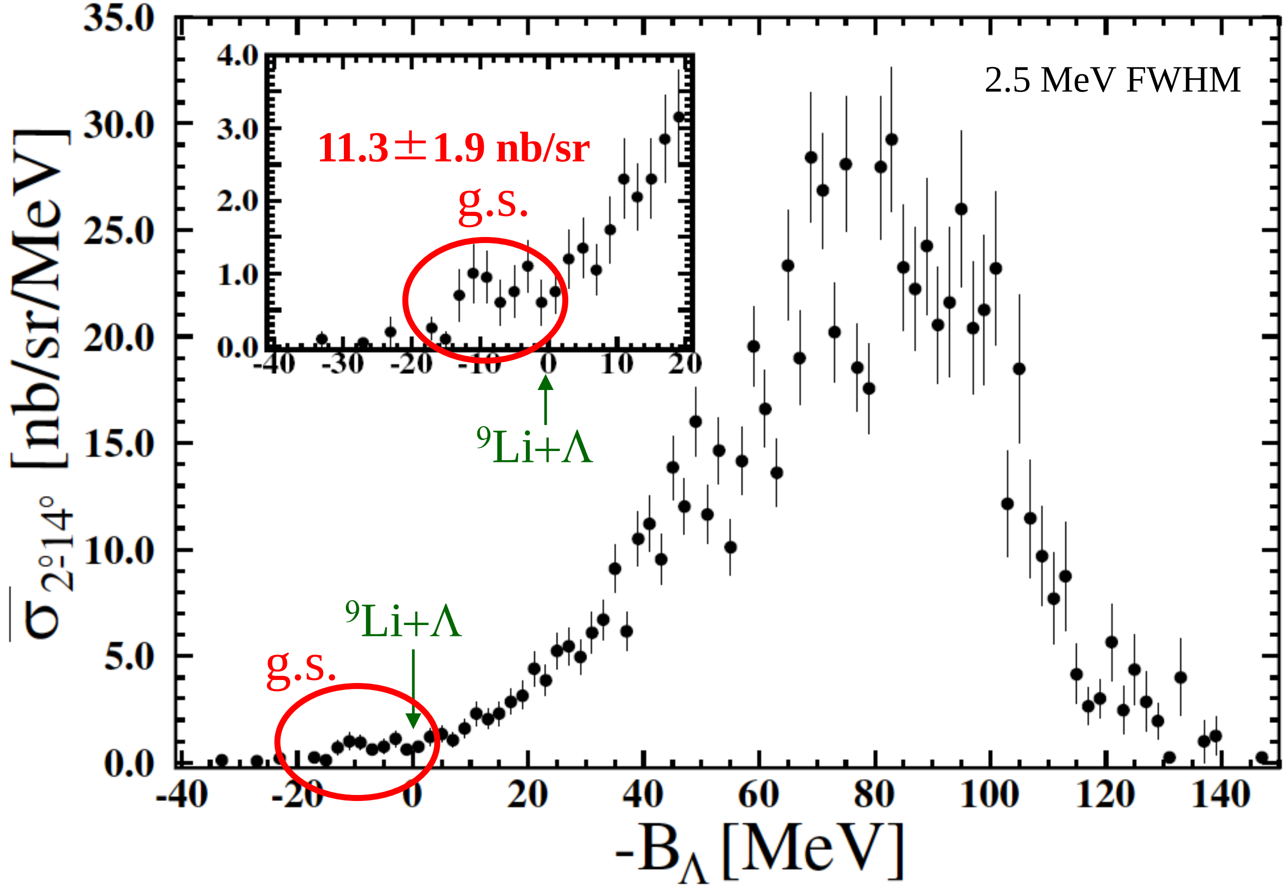
$${}^{10}_{\Lambda}\mathrm{Li}$$

First production of neutron-rich Λ hypernuclei

$^{10}\mathrm{B}(\pi^-,K^+)^{10}_{\Lambda}\mathrm{Li}$

Λ spectrum by DCX (π^-,K^+) reaction at 1.2GeV/c

KEK-PS-E521 P. K. Saha, et al., PRL94(2005)052502



Cross sections

- p_π=1.20 GeV/c

$$\frac{d\sigma}{d\Omega_L} \approx 11.3 \pm 1.9 \text{ nb/sr}$$

- p_π=1.05 GeV/c

$$\frac{d\sigma}{d\Omega_L} \approx 5.8 \pm 2.2 \text{ nb/sr}$$

～1/1000

$^{12}\mathrm{C}(\pi^+,K^+)^{12}_{\Lambda}\mathrm{C}$ (1.2 GeV/c)

17.5±0.6 μb/sr

The DCX (π^-,K^+) *reaction at 1.2GeV/c can produce the neutron-rich Λ hypernucler states, whereas the cross section is as small as 1/1000 of the* (π^+,K^+) *reaction.*

(π^-, K^+) — Double Charge Exchange (DCX) Reaction

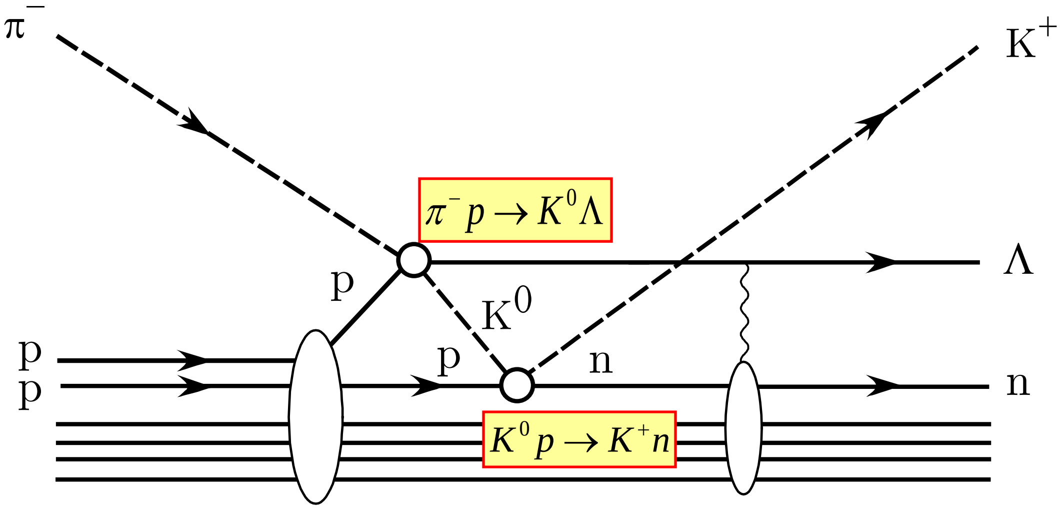
Two-step process:

$$\pi^- p \to K^0 \Lambda$$

$$K^0 p \to K^+ n$$

$$\pi^- p \to \pi^0 n$$

$$\pi^0 p \to K^+ \Lambda$$

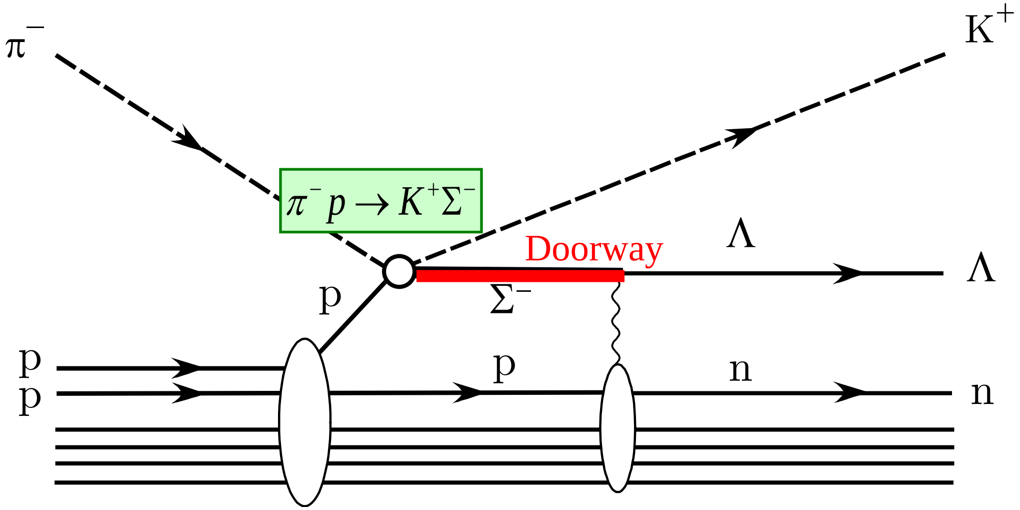


One-step process:

$$\pi^- p \to K^+ \Sigma^-$$

$$\Sigma^- p \leftrightarrow \Lambda n$$

via Σ^- doorways caused
by ΛN-ΣN coupling



(π^-, K^+) －Double Charge Exchange (DCX) Reaction

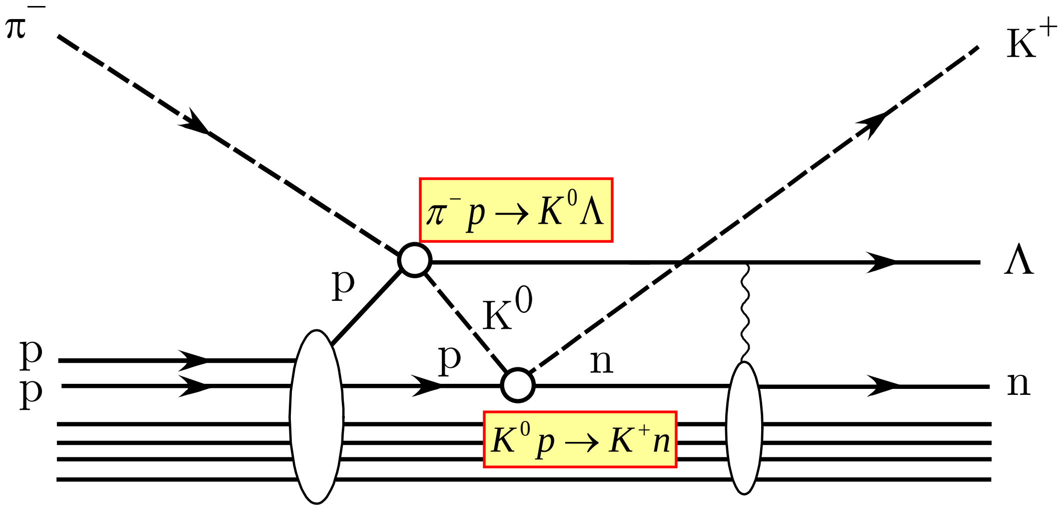
***Two-step process*:**

$$\pi^- p \to K^0 \Lambda$$

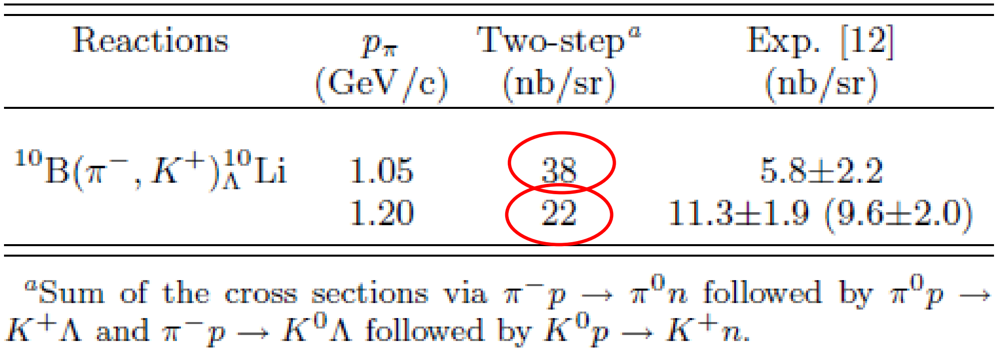
$$K^0 p \to K^+ n$$

$$\pi^- p \to \pi^0 n$$

$$\pi^0 p \to K^+ \Lambda$$

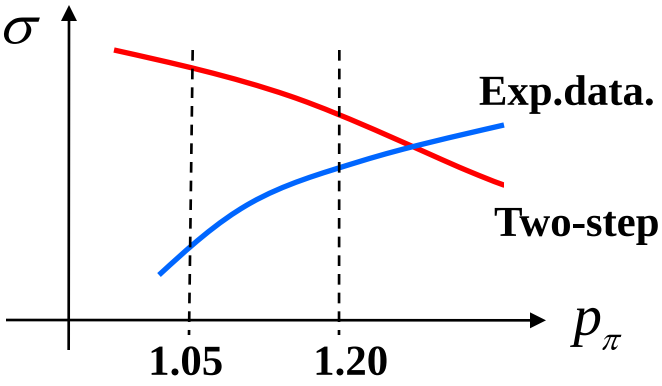


Pioneer theoretical works by Tretyakova and Lanskoy. Phys.At.Nucl.66(2003)1651
arXiv:nucl-th/0411004v1



Reactions	p_π (GeV/c)	Two-step[a] (nb/sr)	Exp. [12] (nb/sr)
$^{10}\mathrm{B}(\pi^-, K^+)^{10}_{\Lambda}\mathrm{Li}$	1.05	38	5.8±2.2
	1.20	22	11.3±1.9 (9.6±2.0)

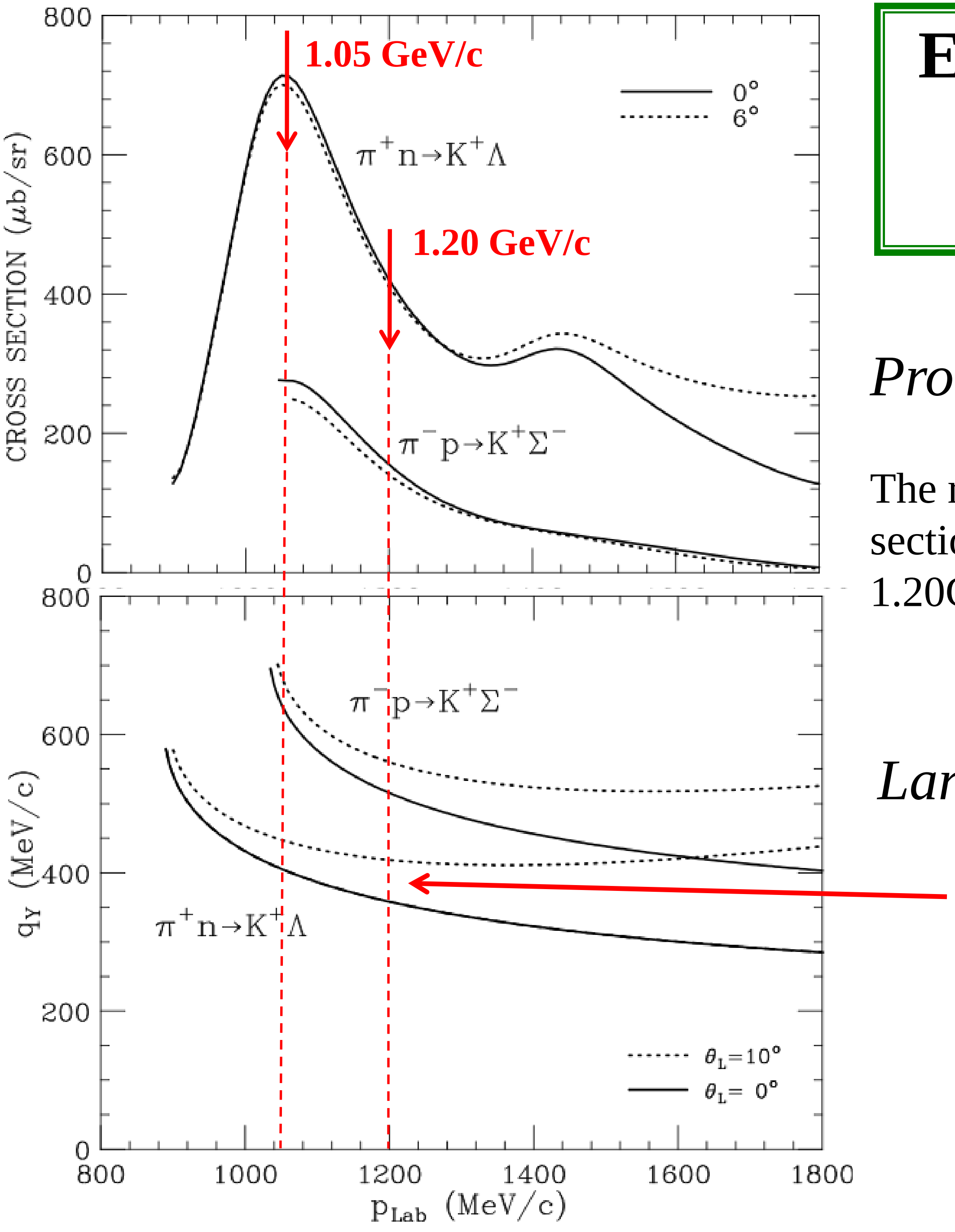
[a] Sum of the cross sections via $\pi^- p \to \pi^0 n$ followed by $\pi^0 p \to K^+ \Lambda$ and $\pi^- p \to K^0 \Lambda$ followed by $K^0 p \to K^+ n$.



-- *The two-step mechanism is dominant.*

-- *The momentum dependence of the cross section is different from that of the data.*

Elementary processes of the $\pi^+p \rightarrow K^+\Lambda$ and $\pi^-p \rightarrow K^+\Sigma^-$ reactions



Production cross sections

The reason comes from the fact that the cross section at 1.05GeV/c is larger than that at 1.20GeV/c.

Large momentum transfer

$$q_\Lambda \approx 400 \text{ MeV/c}$$

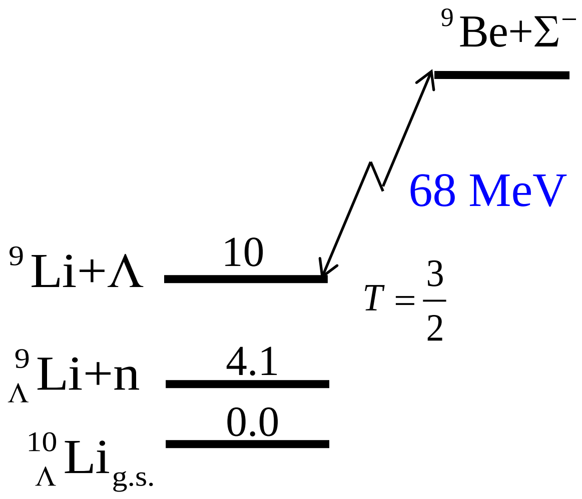
3.1. Calculation of the DWIA

in one-step mechanism

$$^{10}\mathrm{B}(\pi^-,K^+)^{10}_{\Lambda}\mathrm{Li}$$

Model for final states of the hypernucleus

Simple single-particle shell model wf.



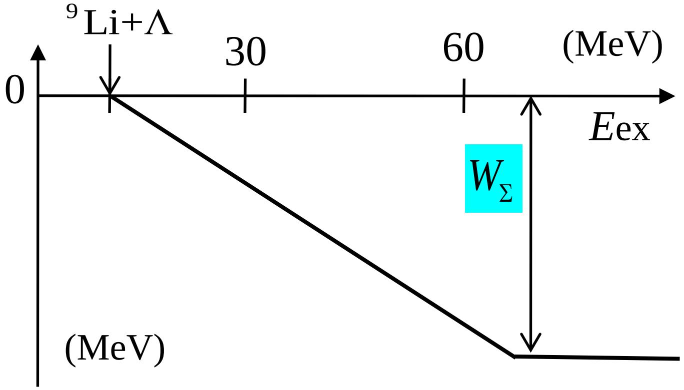
$$\left|{}^{10}_{\Lambda}\mathrm{Li}\right\rangle = \varphi_{\Lambda}(r)\left|{}^{9}\mathrm{Li}\otimes\Lambda\right\rangle + \varphi_{\Sigma^-}(r)\left|{}^{9}\mathrm{Be}^{*}\otimes\Sigma^-\right\rangle$$

Σ^- admixture probability $\quad P_{\Sigma^-} = \langle \varphi_{\Sigma^-} | \varphi_{\Sigma^-} \rangle$

Hyperon-nucleus potentials **Woods-Saxon form**

$$U_{Y=\Lambda,\Sigma} = -(V_Y + iW_Y \times g(E_{ex}))/(1+\exp[(r-R)/a])$$

30 MeV for Λ spreading potential : energy-dependent = excited states

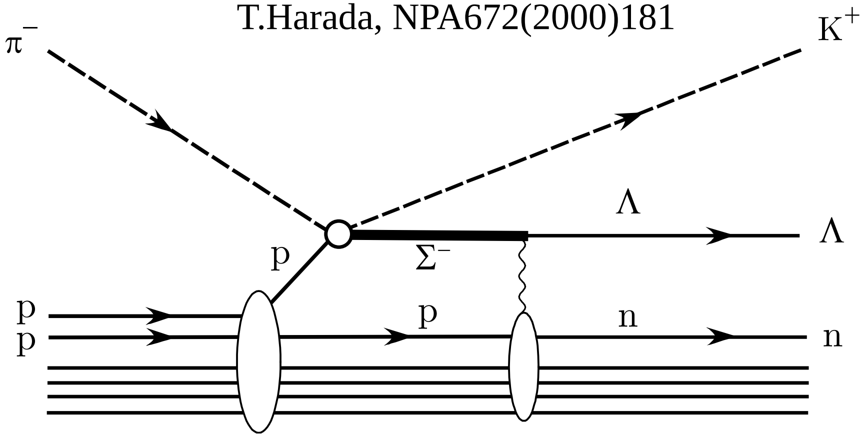


$$U_X = \left\langle {}^{9}\mathrm{Li}\text{-}\Lambda \left| \sum_j \frac{1}{\sqrt{3}} v_{\Lambda\Sigma} \vec{\tau}_j \cdot \vec{\phi} \right| {}^{9}\mathrm{Be}^{*}\text{-}\Sigma^- \right\rangle$$
$$= V_X/(1+\exp[(r-R)/a])$$

coupling Λ-Σ potential ← coherent + incoherent

Coupled-channel calculations in one-step process

Coupled-channel Green's function


T.Harada, NPA672(2000)181

$$\hat{\mathbf{G}}(\omega)=\hat{\mathbf{G}}^{(0)}(\omega)+\hat{\mathbf{G}}^{(0)}(\omega)\hat{\mathbf{U}}\hat{\mathbf{G}}(\omega)$$

$$\hat{\mathbf{G}}^{(0)}(\omega)=\begin{bmatrix} G_{\Lambda}^{(0)} & \\ & G_{\Sigma^-}^{(0)} \end{bmatrix} \quad \hat{\mathbf{U}}=\begin{bmatrix} U_{\Lambda} & U_X \\ U_X & U_{\Sigma} \end{bmatrix}$$

$$\mathrm{Im}\,\hat{G}=\underbrace{\hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\,\hat{G}_{\Lambda}^{(0)}\}\hat{\Omega}^{(-)}}_{\Lambda\text{ escape}}+\underbrace{\hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\,\hat{G}_{\Sigma^-}^{(0)}\}\hat{\Omega}^{(-)}}_{\Sigma^-\text{ escape}}+\underbrace{\hat{G}^{\dagger}\{W_{Y,T}\}\hat{G}}_{\substack{\text{Spreading (nuclear-core breakup)}\\ \text{= Complicated excited states}}}$$

Strength function *Green's function method* **Morimatsu, Yazaki, NPA483(1988)493**

$$S(\omega)=\sum_f |<f|\hat{O}|i>|^2\,\delta(\omega+E_K-E_\pi)=-\frac{1}{\pi}\mathrm{Im}\int d\mathbf{r}d\mathbf{r}'F^{\dagger}(\mathbf{r})\boxed{G(\omega+i\varepsilon;\mathbf{r},\mathbf{r}')}F(\mathbf{r}')$$

Green's function

Distorted waves for mesons

Eikonal distortion: $\bar{\sigma}=(\sigma_\pi+\sigma_K)/2=20\ \text{mb},\qquad \alpha_\pi=\alpha_K=0$

Elementary cross section $\pi^- p\rightarrow K^+\Sigma^-$

$\beta\,[d\sigma/d\Omega]$ Optimal Fermi-averaging ～10-20 μb/sr (p_π=1.2GeV/c)

3.2. Numerical results

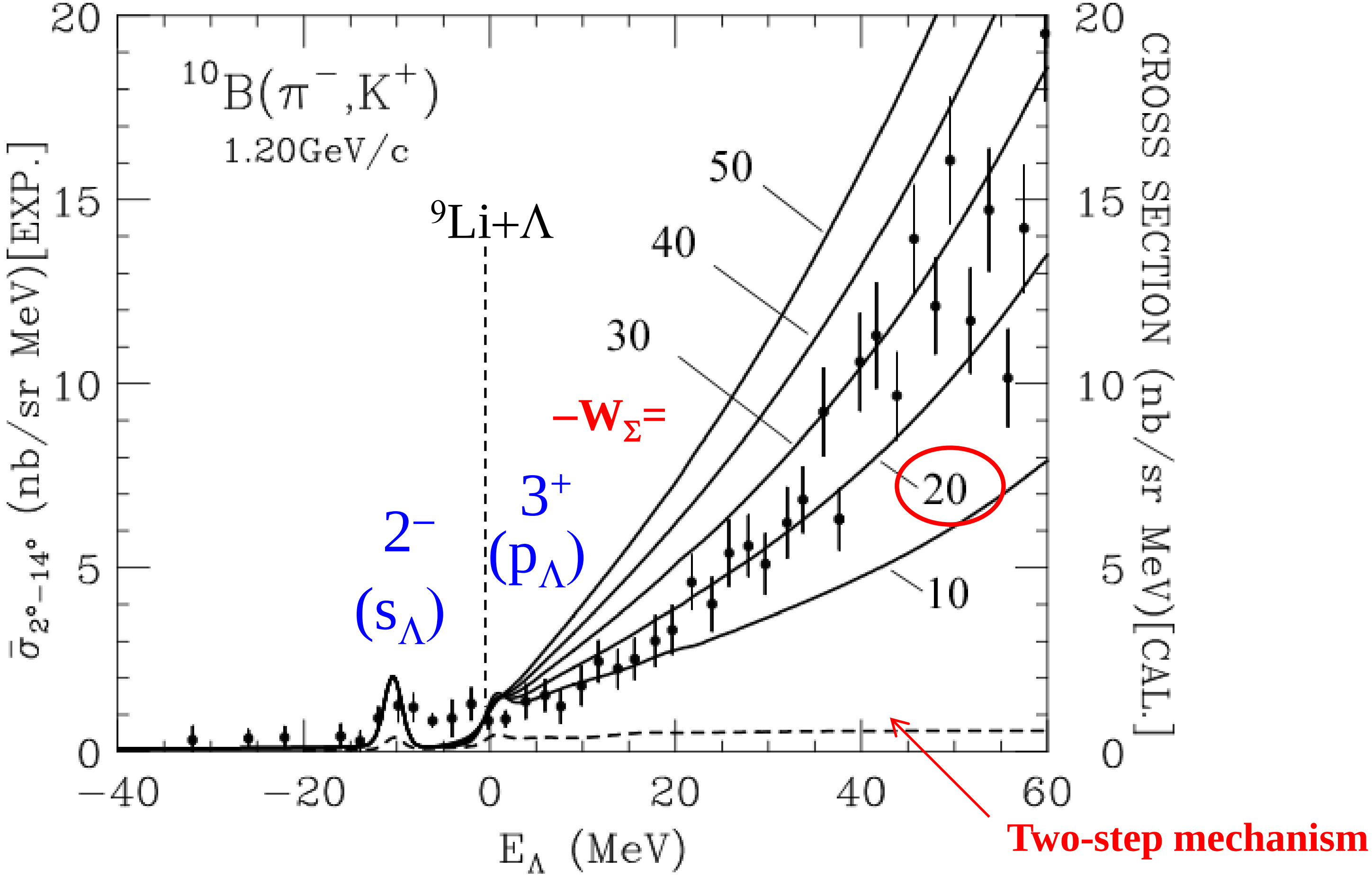
Results (1)

Λ spectrum by DCX ((π^-,K^+)) reaction at 1.2GeV/c

Spreading potential dep. W_Σ

$U_X = 11$ MeV is fixed. $P_{\Sigma^-} = 0.57\%$

^{10}B



Harada, Umeya, Hirabayashi, PRC79(2009)014603

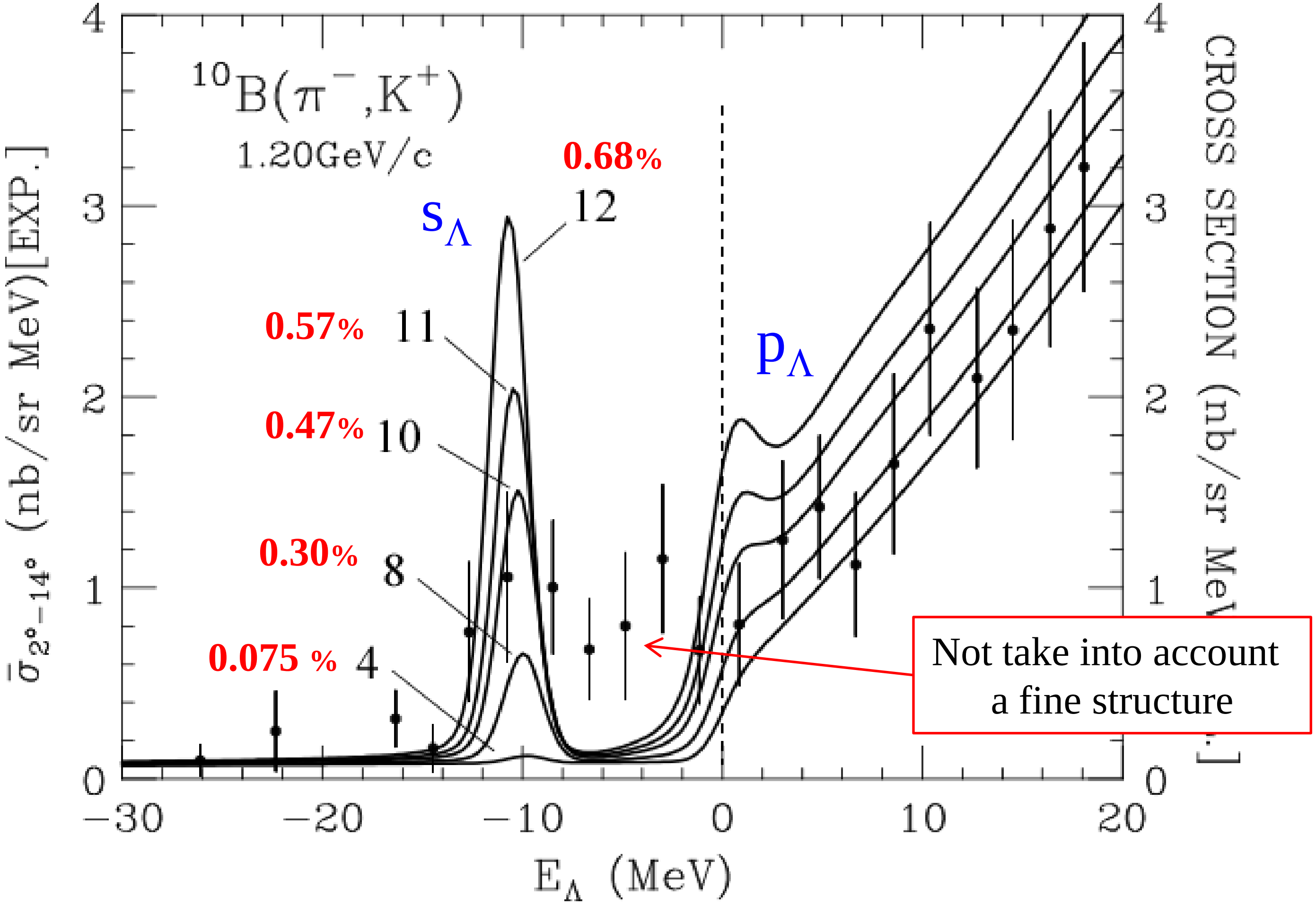
The calculated spectrum with $-W_\Sigma$= 20-30 MeV can reproduce the shape of the data in the continuum region, and these values of $-W_\Sigma$ are consistent with the analysis of Σ^- QF production by the (π^-, K^+) reactions.

Results (2)

Λ spectrum by DCX (π^-,K^+) reaction at 1.2GeV/c

Coupling Λ-Σ potential dep. V_X $-W_\Sigma = 20$ MeV is fixed.

^{10}B



Not take into account a fine structure

Harada, Umeya, Hirabayashi, PRC79(2009)014603

For the order of V_X = 10-12 MeV ($P_{\Sigma-}$~ 0.5 %), the calculated spectra can fairly reproduce the data.

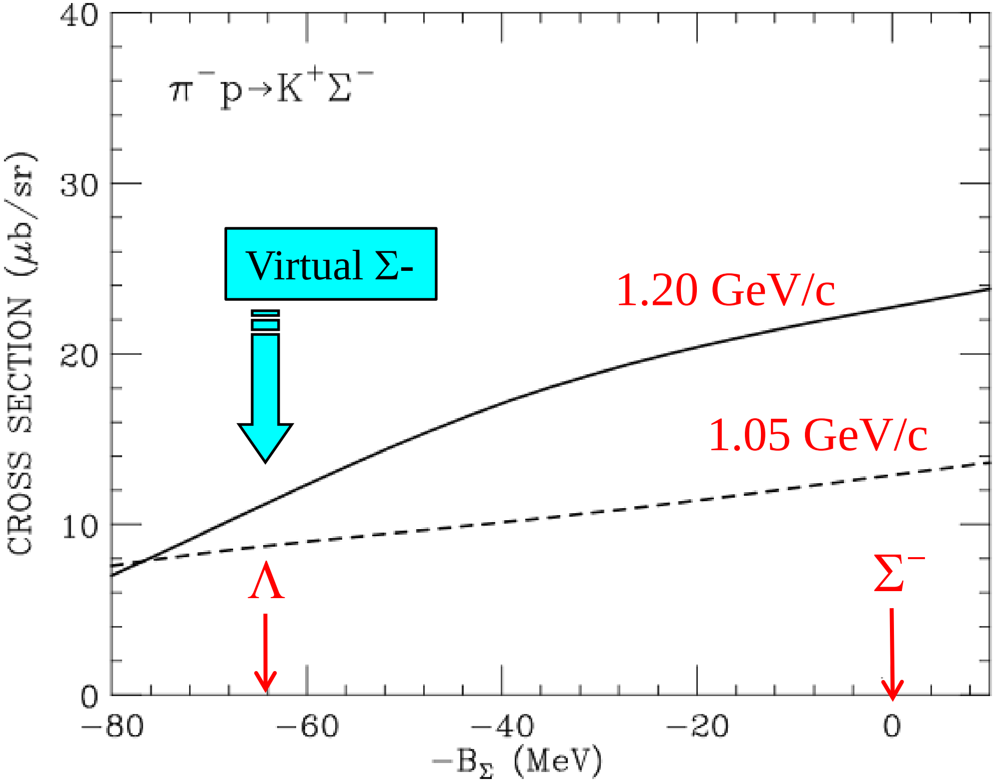
Momentum dependence of the cross sections in one-step mechanism

$$\beta\left\langle \frac{d\sigma}{d\Omega} \right\rangle^{\mathrm{opt}}$$

Optimal Fermi-averaging
for the $\pi^- p \rightarrow K^+ \Sigma^-$ T-matrices

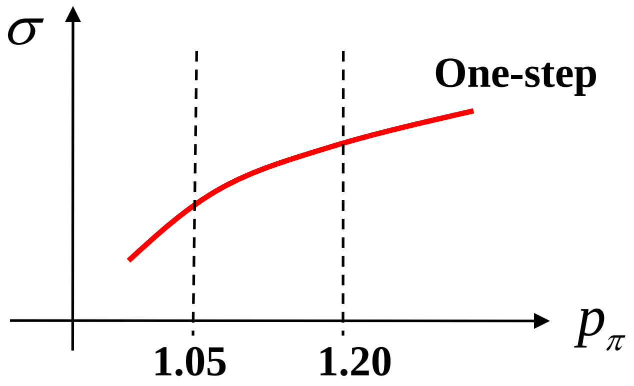
+ Threshold effects

T.Harada, Y.Hirabayashi, NPA744(2004)323.



$\pi^- p \rightarrow K^+ \Sigma^-$ threshold (1045 GeV/c)

This simple estimation seems to reproduce the momentum dependence of the data.



But more theoretical studies are needed !!

Remarks

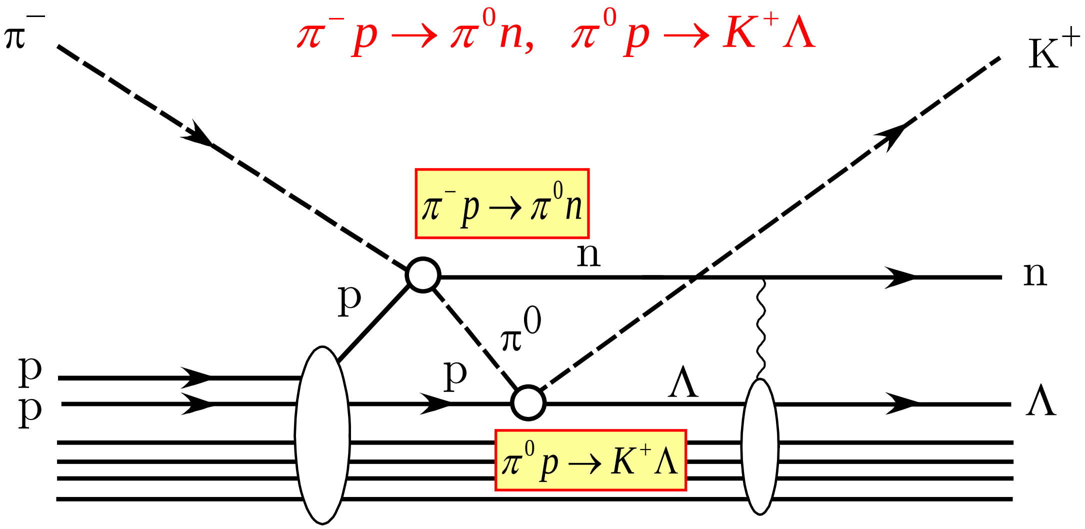
The calculated spectrum by the one-step mechanism fully explains the ^{10}B(π^-, K$^+$) data.

The one-step mechanism dominates in the (π^-, K$^+$) reaction.

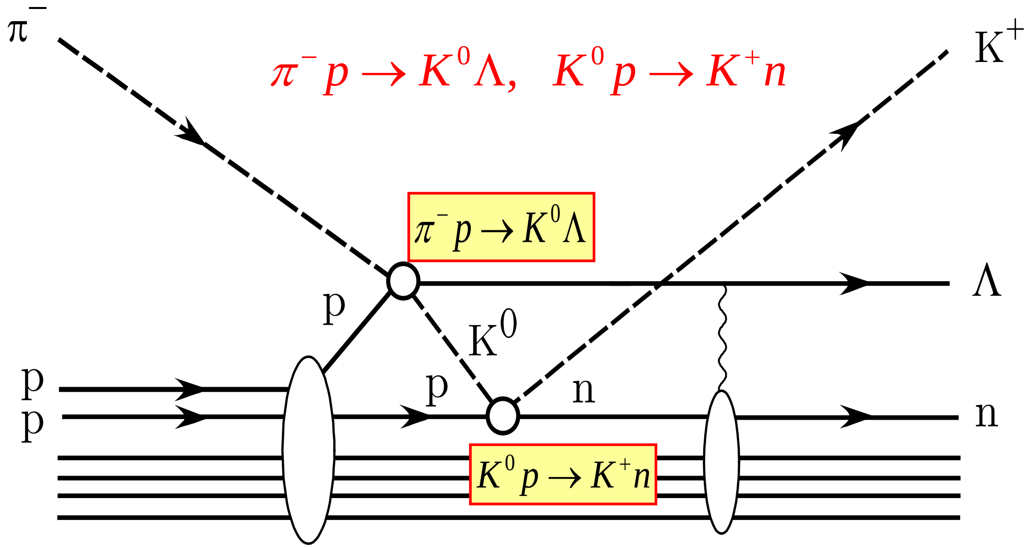
Our phenomenological calculation provides the ability to extract a production mechanism from the data of this reaction.

4. Discussion

Two-step processes in (π^-, K^+) reactions



$\pi^- p \to \pi^0 n, \quad \pi^0 p \to K^+ \Lambda$



$\pi^- p \to K^0 \Lambda, \quad K^0 p \to K^+ n$

Fermi-averaged cross sections

$[d\sigma/d\Omega]_{0^\circ}$

1.05 MeV/c	1.20 MeV/c
3.37 mb/sr	0.67 mb/sr
0.33 mb/sr	0.21 mb/sr
0.50 mb/sr	0.35 mb/sr
1.98 mb/sr	1.96 mb/sr

↑

Main components

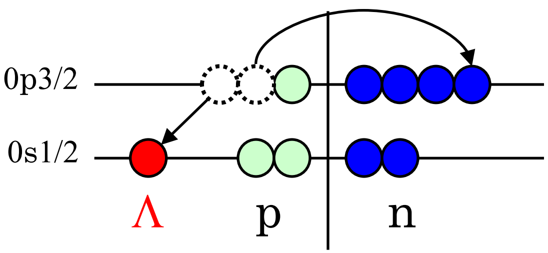
by Tretyakova-Lanskoy,
Phys. At. Nucl. 66(2003)1651

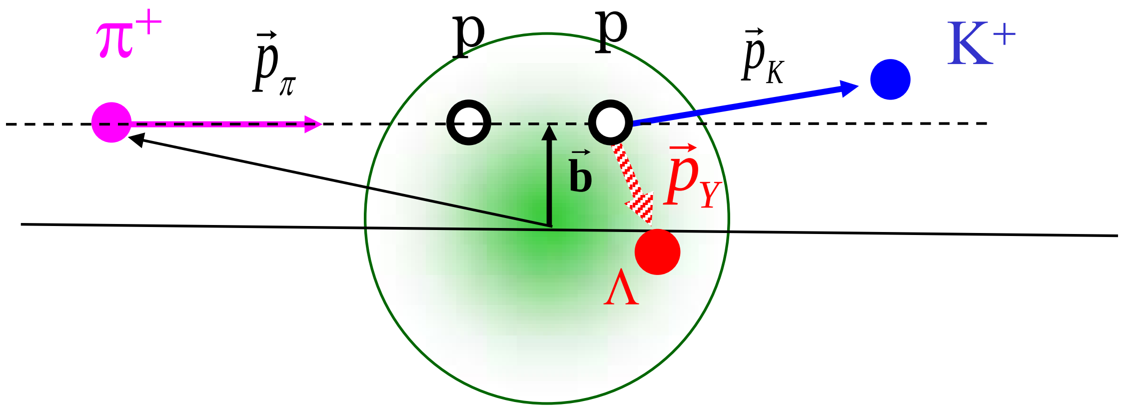
Two-step DWIA calculations

C.B. Dover, Nukleonica 25 (1980) 521

Harmonic oscillator model in the Eikonal approximation

$^{10}_{\Lambda}$Li





T-matrix elements

$$T_{fi} = -\frac{i}{v_\pi}\int d^{(2)}\mathbf{b}\int dz'\int dz\,\theta(z'-z)\, e^{-i\mathbf{p}_{\pi\perp}\cdot\mathbf{b}}e^{-i(p_{\pi^-}-p_\pi)z'}e^{-i(p_\pi-p_{K^+z})z}$$

$$\times \boxed{e^{-i\frac{1}{v_{K^+}}\int_z^\infty U_{\pi^-}(\mathbf{b},z)dz}\; e^{-i\frac{1}{v_\pi}\int_{z'}^{z} U_\pi(\mathbf{b},z)dz}\; e^{-i\frac{1}{v_{\pi^-}}\int_{-\infty}^{z'} U_{\pi^-}(\mathbf{b},z)dz}}$$

Eikonal distortion

$$\times\, t_1(\mathbf{b})t_2(\mathbf{b})\left\langle f\left|\sum_{l\neq j}V_-(l)\delta^{(3)}(\mathbf{r}-\mathbf{r}_l)\tau_-(j)\delta^{(3)}(\mathbf{r}-\mathbf{r}_j)\right|i\right\rangle$$

HO wf.

HO wf.

Production cross sections

$$\left[\frac{d\sigma_{fi}}{d\Omega}\right]_{0^\circ} = \frac{8\pi^2\alpha_1\alpha_2}{p_{\pi^0}^2}\left[\frac{d\sigma}{d\Omega}\right]_{0^\circ}^{\pi^-p\to\pi^0n}\left[\frac{d\sigma}{d\Omega}\right]_{0^\circ}^{\pi^-p\to K^+\Lambda} N_{\mathrm{eff}}^{\mathrm{pp}}$$

Effective number (Nuclear structure)

Integrated lab cross sections $^{10}_{\Lambda}$Li (2^-)

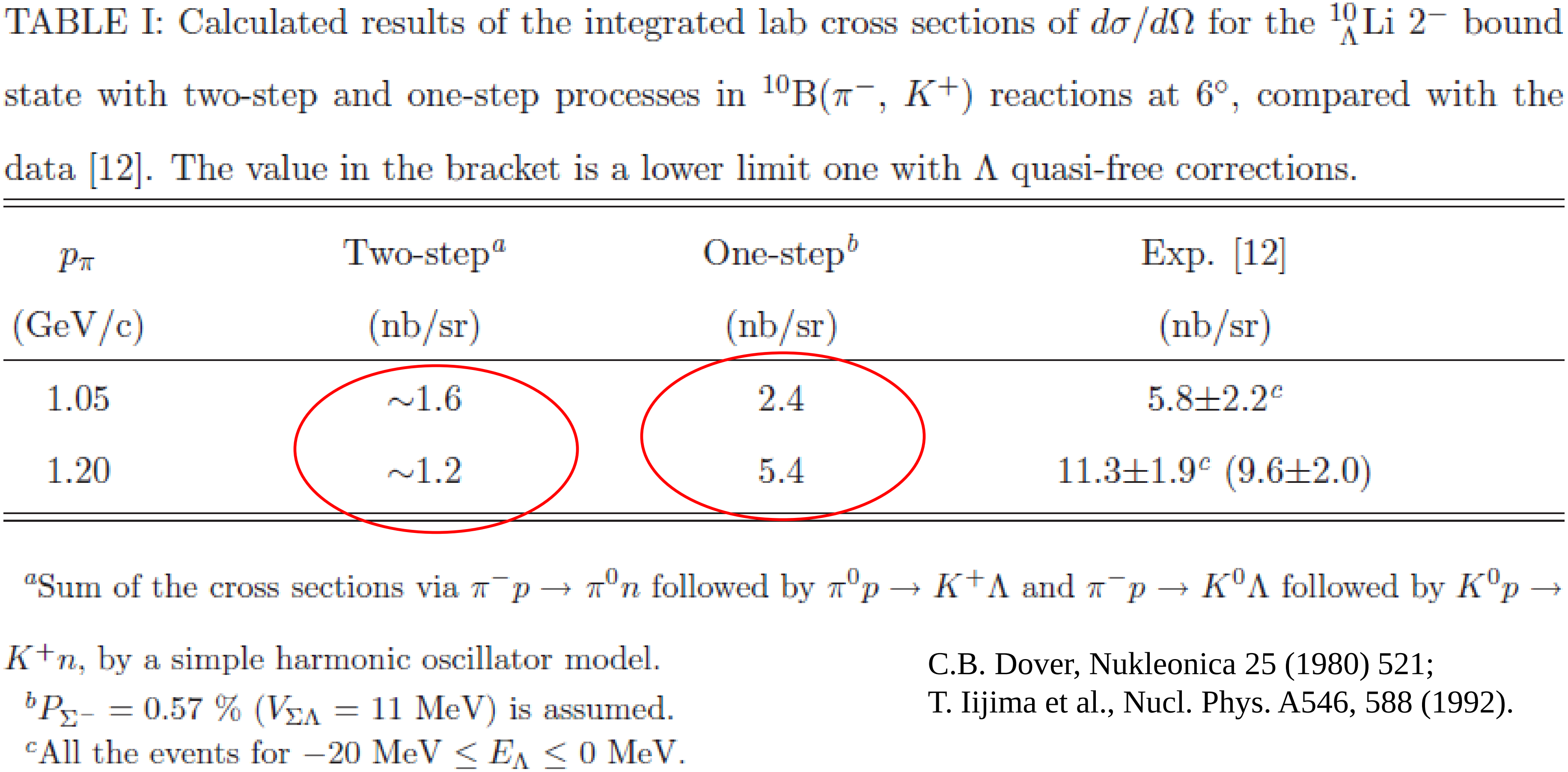


TABLE I: Calculated results of the integrated lab cross sections of $d\sigma/d\Omega$ for the $^{10}_{\Lambda}$Li 2^- bound state with two-step and one-step processes in ^{10}B(π^-, K^+) reactions at 6°, compared with the data [12]. The value in the bracket is a lower limit one with Λ quasi-free corrections.

p_π (GeV/c)	Two-step[a] (nb/sr)	One-step[b] (nb/sr)	Exp. [12] (nb/sr)
1.05	~1.6	2.4	5.8±2.2[c]
1.20	~1.2	5.4	11.3±1.9[c] (9.6±2.0)

[a] Sum of the cross sections via $\pi^- p \to \pi^0 n$ followed by $\pi^0 p \to K^+ \Lambda$ and $\pi^- p \to K^0 \Lambda$ followed by $K^0 p \to K^+ n$, by a simple harmonic oscillator model.

[b] $P_{\Sigma^-} = 0.57$ % ($V_{\Sigma\Lambda} = 11$ MeV) is assumed.

[c] All the events for -20 MeV $\leq E_\Lambda \leq 0$ MeV.

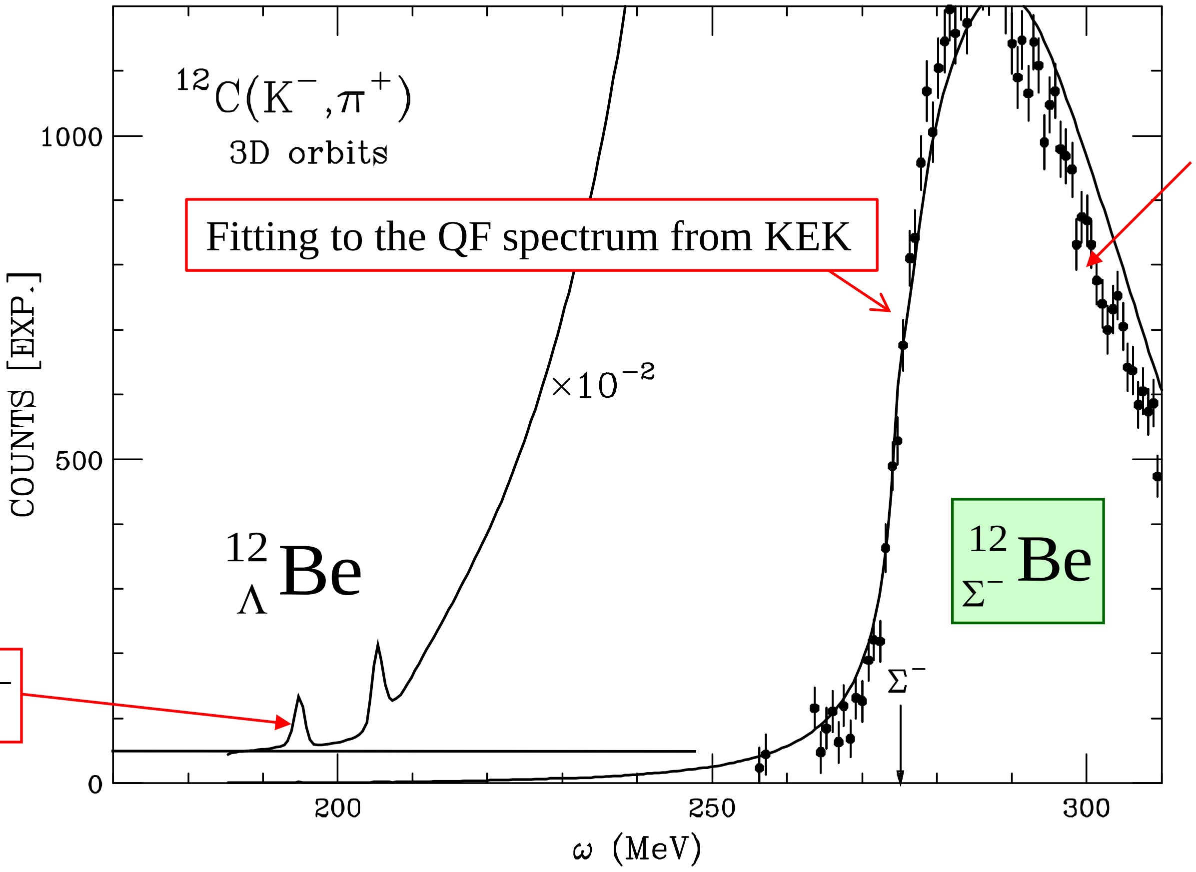
C.B. Dover, Nukleonica 25 (1980) 521;
T. Iijima et al., Nucl. Phys. A546, 588 (1992).

The calculated cross section at 1.2GeV/c by two-step mechanism is rather small (1-2nb/sr) *compared to the cross section by the one-step mechanism.*

Calculation for DCX (stopped K^-, π^+) reactions

If the Σ^- admixture probability of ~0.6 % is assumed in ${}^{12}_{\Lambda}\mathrm{Be}$, we demonstrate the (stopped K^-,π^+) spectrum on a ^{12}C target.

^{12}C



Fitting to the QF spectrum from KEK

${}^{12}_{\Sigma^-}\mathrm{Be}$

Early KEK data

Integrated production rate

$\sim 4\times10^{-6}/K^-$

This result is consistent with recent data form DAΦNE.

The DAΦNE data: U.L. $\sim (2.0\pm0.4)\times10^{-5}/K^-$

M.Agnello, et al.,PLB640(2006)145.

Summary

We have discussed production and spectroscopy of the neutron-rich Λ hypernuclei and feasibility of extracting a Σ^- admixture probability in the neutron-rich ${}^{10}_{\Lambda}$Li hypernucleus.

- The calculated spectrum of the ${}^{10}_{\Lambda}$Li hypernucleus by the one-step mechanism via Σ^- doorways fully explains the data of the DCX ^{10}B(π^-, K$^+$) reaction at 1.20GeV/c, rather than by the two-step mechanism.

Σ^- admixture probability ~the order of 10^{-1} % for ${}^{10}_{\Lambda}$Li

due to coherent and incoherent Λ-Σ couplings

- The sensitivity to the potential parameters implies that the nuclear (π^-, K$^+$) reactions provide the high ability for the theoretical analysis of precise wave functions in the neutron-rich hypernuclei.

→ Systematic studies may well separate these couplings.

Conclusion

Studies of the neutron-rich hypernuclei
are
very interesting and exciting at J-PARC.

Future subjects:

The detailed analysis based on microscopic calculations
is needed to understand the structure and production of the
neutron-rich nuclei.

→ A.Umeya, Parallel session 2-B on 17 (Thu) 15:00- Li-isotope
→ T. Harada, Poster session II (T12) on 15 (Tue) 18:00- (K–,K+)