Properties of hyperons in covariant chiral perturbation theory

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in collaboration with

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Introduction to χPT: Summary

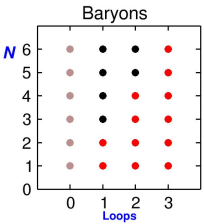
- QCD-the theory of the strong interaction:
  - very successful at high energy (PQCD);
  - problematic at the region below 1 GeV (instead of LQCD).

- **ChPT**: Effective field theory of the QCD in the low-energy limit:
  (Weinberg, Gasser and Leutwyler, ...)
  - Exploits the symmetries of the QCD Lagrangian and ground state;
  - Develops the Green functions according to the perturbative QFT techniques;
  - Perturbative parameters: external momenta or masses that are small compared with the chiral symmetry breaking scale ($\Lambda_{ChSB} \simeq 1$ GeV).

- **Power Counting**: Order in the perturbative expansion:
  - Establishes a hierarchy among the infinite Feynman diagrams contributing to a particular Green function.

- **ChPT** has been extremely successful in the mesonic sector and calculations up to $\mathcal{O}(p^6)$ have become standard.

Baryon χPT and power counting: Problems & Solutions



- Baryon mass M_0: New large scale
- Diagrams with arbitrarily large number of loops contribute to lower orders
  $->$**Power Counting is lost!**
  (Gasser et al.)

- Heavy Baryon χPT (Jenkins & Manohar):
  - Non-relativistic expansion: Considers $M_0 \simeq \Lambda_{ChSB}$;
  - Recovers the power counting pattern of meson χPT.
- Relativistic Baryon χPT:
  - Power counting breaking pieces: Analytical structure!;
  - Two remarkable schemes:
    - Infrared baryon χPT (Becher and Leutwyler);
    - EOMS-scheme (Gegelia, Japaridze and Scherer).

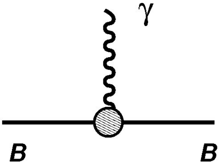
Inclusion of the Decuplet-resonances

- **Motivation**: We perform perturbations on $m_K/\Lambda_{\chi SB}$ ∼0.5 that is over the scale for the onset of Decuplet resonances $\frac{M_D - M_B}{\Lambda_{\chi SB}}$ ∼0.3.
- Problem of consistency
  - Rarita-Schwinger (RS) representation of relativistic 3/2-fermions: $\psi^\mu(x)$. RS is a field with 16 components of which only 8 (4 massless) are "physical".
  - How to introduce couplings to the RS spinor that don't activate 1/2 modes? Field-redefinition formalism: Consistent couplings "equivalent" to phenomenological ones. V. Pascalutsa.
- Problem of higher-order divergencies

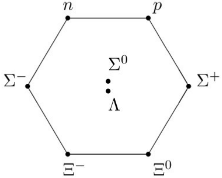
$$s^{\alpha\beta}(p) = \frac{\not{p}+m}{m^2-p^2}\left[g^{\alpha\beta} - \frac{1}{D-1}\gamma^\alpha\gamma^\beta - \frac{1}{(D-1)m}(\gamma^\alpha p^\beta - \gamma^\beta p^\alpha) - \frac{D-2}{(D-1)m^2}p^\alpha p^\beta\right];$$

  - RS propagator has a problematic high-energy behavior.
  - Higher-order ∞'s regularized in $\overline{MS}$ →Regularization-scale (μ) dependence.
- Problem of power-counting breaking
  - We use the EOMS-scheme and also obtain the HB limit (ϵ-expansion)

- General γBB vertex



- SU(3)-flavor symmetry

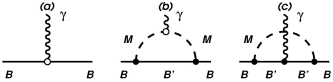


- The general vertex can be parameterized by two form factors:
$\langle\psi(p')|J^\mu|\psi(p)\rangle = |e|\bar{u}(p')\left\{\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2(t)\right\}u(p).$
- At $q^2 = 0$: $F_1(0) = Q$ (charge), $F_2(0) = \kappa$ (anomalous magnetic moment).
- The fermion (baryon) MM: $\mu \equiv (Q+\kappa)\frac{|e|}{2M}$
- SU(3)-flavor symmetry⇒Coleman-Glashow relations:

$$\mu_{\Sigma^+} = \mu_p, \qquad \mu_\Lambda = \frac{1}{2}\mu_n, \qquad \mu_{\Xi^0} = \mu_n,$$
$$\mu_{\Sigma^-} = -(\mu_n + \mu_p), \qquad \mu_{\Xi^-} = \mu_{\Sigma^-}, \qquad \mu_{\Lambda\Sigma^0} = -\frac{\sqrt{3}}{2}\mu_n,$$

- The octet-baryons MM are very well measured quantities!

Baryon-Octet MM: Numerical results at NLO



L.S.Geng, JMC, L. Alvarez-Ruso, M.J. Vicente Vacas, PRL **101**,222002 (2008)

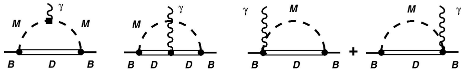
	p	n	Λ	Σ^-	Σ^0	Σ^+	Ξ^-	Ξ^0	$\Lambda\Sigma^0$	$\tilde{\chi}^2$
C-G	2.56	-1.60	-0.80	-0.97	0.80	2.56	-0.97	-1.60	1.38	0.46
HB	3.01	-2.62	-0.42	-1.35	2.18	0.42	-0.70	-0.52	1.68	1.01
IR	2.08	-2.74	-0.64	-1.13	0.64	2.41	-1.17	-1.45	1.89	1.83
EOMS	2.60	-2.16	-0.64	-1.12	0.64	2.41	-0.93	-1.23	1.58	0.18
Exp.	2.79	-1.91	-0.61	-1.16	—	2.46	-0.65	-1.25	1.61	—

- Study of the convergence of the chiral series (LO and NLO):

$$\mu_p = 3.47\,(1-0.257)\,,\quad \mu_n = -2.55\,(1-0.175)\,,\quad \mu_\Lambda = -1.27\,(1-0.482)\,,$$
$$\mu_{\Sigma^-} = -0.93\,(1+0.187)\,,\quad \mu_{\Sigma^+} = 3.47\,(1-0.300)\,,\quad \mu_{\Sigma^0} = 1.27\,(1-0.482)\,,$$
$$\mu_{\Xi^-} = -0.93\,(1+0.025)\,,\quad \mu_{\Xi^0} = -2.55\,(1-0.501)\,,\quad \mu_{\Lambda\Sigma^0} = 2.21\,(1-0.284)\,.$$

- The Covariant EOMS ($\overline{MS}$) NLO-calculation improves the C-G relations!

Baryon octet MM:Inclusion of the Decuplet-resonances

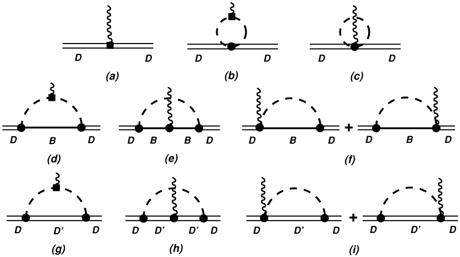


	p	n	Λ	Σ^-	Σ^0	Σ^+	Ξ^-	Ξ^0	$\Lambda\Sigma^0$	$\tilde{\chi}^2$
C-G	2.56	-1.60	-0.80	-0.97	0.80	2.56	-0.97	-1.60	1.38	0.46
HB-O	3.01	-2.62	-0.42	-1.35	2.18	0.42	-0.70	-0.52	1.68	1.01
HB-OD	3.47	-2.84	-0.17	-1.42	1.77	0.17	-0.41	-0.56	1.86	2.58
C-O	2.60	-2.16	-0.64	-1.12	0.64	2.41	-0.93	-1.23	1.58	0.18
C-OD	2.61	-2.23	-0.60	-1.17	0.59	2.37	-0.92	-1.22	1.65	0.22
Exp.	2.79	-1.91	-0.61	-1.16	—	2.46	-0.65	-1.25	1.61	—

- **Par.** $\mathcal{C}$=1.0, M_B =1.151 GeV, M_D = 1.428 GeV, $F_\phi \simeq 1.17 F_\pi$, μ=1 GeV
- In the covariant framework the NLO-decuplet contributions are small!
- The problem of consistency has also been investigated:

  L.S.Geng, JMC, M.J. Vicente Vacas, PLB **676**,63 (2009)

Predictions of BχPT on the MDMs of the decuplet at NLO



	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}
EOMS	6.04	2.84	-0.36	-3.56	3.07	0	-3.07	0.36	-2.56

- The results for the Δ^{++} and Δ^{+} are compatible with PDG values:

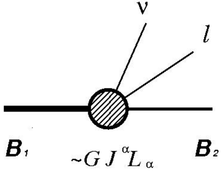
  $$\mu_{\Delta^{++}} = 5.6 \pm 1.9\mu_N, \qquad \mu_{\Delta^{+}} = 2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3\mu_N$$

  and the latest experiment $\mu_{\Delta^{++}} = 6.14 \pm 0.51\,\mu_N$ Lopez Castro et al.(2001)
- The HB result for Δ^{++} is $\mu_{\Delta^{++}} = 7.94\mu_N$
- More details and results for EQMs, MOMs and charge radii:

  L.S.Geng, JMC, M.J. Vicente Vacas **PRD80**, 034027 (2009)

Hyperon Semileptonic Decays (HSD)

- General $B_1 \longrightarrow B_2$ HSD



$$H_{HSD} = \frac{G}{\sqrt{2}} J_\alpha L^\alpha + \mathrm{h.c.}$$

- Electronic: $l = e$, $\nu = \overline{\nu}_e$

B_1	B_2
$\Lambda \longrightarrow$	p
$\Sigma^- \longrightarrow$	n
$\Xi^- \longrightarrow$	Λ
$\Xi^0 \longrightarrow$	Σ^+

$$L^\alpha = \overline{\psi}_e \gamma^\alpha (1 - \gamma_5) \psi_{\nu_e} \quad ; \quad J_\alpha = V_\alpha - A_\alpha$$

$$V_\alpha = V_{us} \bar{u} \gamma_\alpha s \quad ; \quad A_\alpha = V_{us} \bar{u} \gamma_\alpha \gamma_5 s$$

$$\langle B_2 | V_\alpha | B_1 \rangle = V_{us} \bar{u}_{B_2} \left[f_1(q^2) \gamma_\alpha + f_2(q^2) \frac{\sigma_{\alpha\beta} q^\beta}{M_1 + M_2} + f_3(q^2) \frac{q_\alpha}{M_1 + M_2} \right] u_{B_1}$$

$$\langle B_2 | A_\alpha | B_1 \rangle = V_{us} \bar{u}_{B_2} \left[g_1(q^2) \gamma_\alpha + g_2(q^2) \frac{\sigma_{\alpha\beta} q^\beta}{M_1 + M_2} + g_3(q^2) \frac{q_\alpha}{M_1 + M_2} \right] \gamma_5 u_{B_1}$$

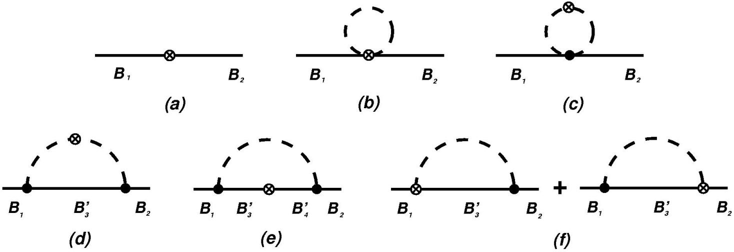
HSD observables and form factors

- To first order in $\beta = (M_1 - M_2)/M_1$:

$$R \sim V_{us}^2 \left(f_1^2 + 3g_1^2 - 4\beta g_1 g_2 \right)$$

- To know V_{us}, V_{ud} with accuracy allows to check SM (CKM unitarity) (M. Kobayashi and T. Maskawa 1/2 Nobel Prize in Physics 2008)
- **Measurements of R + Knowledge of f_1, g_1 and g_2 = V_{us} !!**
  - g_1 can be extracted from measured ratios g_1/f_1
  - f_1 protected by Ademollo-Gatto (A-G) theorem: $\delta f_1 = \mathcal{O}(\beta^2)$
  - $g_2 = 0$ in SU(3)-symmetry.
    SU(3)-breaking effects expected (e.g. Sasaki et al.(2009),...)
- Accurate determination of V_{us} requires a reliable calculation of $f_1(0)$
- In ChPT the A-G theorem forbids unknown LECs up to $\mathcal{O}(p^5)$
- **Goal:** Calculate $f_1(0)$ in relativistic ChPT and give a prediction of V_{us}
  L.S.Geng, JMC, M.J. Vicente Vacas, PRD **79**, 094022 (2009)

ChPT calculation of $f_1(0)$



- The internal baryons belong to the octet or decuplet multiplets
- At LO the contributions to $f_1(0)$ are given by g_V

$$\Sigma^- \to n : -1, \quad \Lambda \to p : -\sqrt{\frac{3}{2}}, \quad \Xi^- \to \Lambda : \sqrt{\frac{3}{2}}, \quad \Xi^0 \to \Sigma^+ : 1$$

Corrections up to NNLO $\mathcal{O}(p^4)$ are due to loops→Pure prediction

- Higher-order scale dependence gives an estimation of systematical error: $0.7\text{GeV} \leq \mu \leq 1.3\text{GeV}$

Results for V_{us} and comparison with other determinations

- Results of V_{us} for each channel

Channel	$\Lambda\ p$	$\Sigma^-\ n$	$\Xi^-\ \Lambda$	$\Xi^0\Sigma^+$
f_1/g_V	1.002±0.012	1.088±0.036	1.039±0.024	1.017±0.020
V_{us}	0.2217±0.0042	0.2090±0.0083	0.228±0.011	0.2124±0.0053

- Averaged result for V_{us} from hyperon decays in ChPT:

$$V_{us}^{Unit.} = 0.2265 \pm 0.0011$$

$$V_{us}^{SU(3)} = 0.2250 \pm 0.0027$$ Cabibbo et al. (2004)

$$V_{us}^{N_c} = 0.2199 \pm 0.0026$$ Flores-Mendieta. (2004)

$$V_{us}^{ChPT} = 0.2176 \pm 0.0029$$ Geng et al. (2009)

- These results are to be compared with other determinations ...

From $K \to \pi l \nu$ decays $V_{us} = 0.2233 \pm 0.0028$ Jamin et al. (2004)

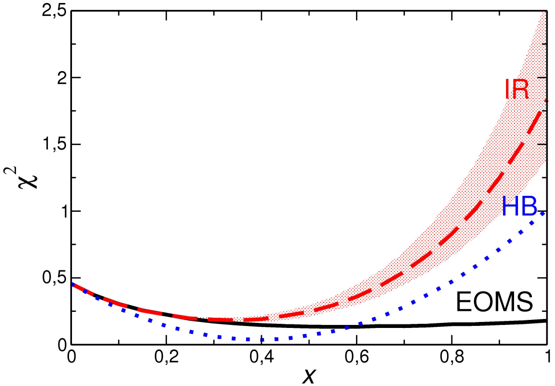
From τ decays $V_{us} = 0.2165 \pm 0.0026$ Gamiz et al. (2005)

From the f_K/f_π ratio $V_{us} = 0.2219 \pm 0.0025$ Aubin et al. (2004)

Conclusions and Outlook

- Chiral perturbation theory
  - Effective field theory of QCD at low energies
  - Exploits the spontaneous ChSB→Dynamical relevance of π,K and η's
  - Power Counting provides an organization principle in perturbations of ϵ/Λ_{ChSB}
- Relativistic BChPT (EOMS) calculations:
  - Incorporates (higher-order) **relativistic corrections**, consistent with **analyticity**
  - Decuplet resonances explicitly included
- Phenomenological applications to hyperon's properties
  - Baryon magnetic moments→Improvement over Coleman-Glashow relations
    →Predictions on $\Delta(1232)$ electromagnetic structure
  - Hyperon semileptonic decays→Allows for an accurate determination of V_{us}
- **Outlook:** Analysis of the lQCD results and experimental data
  - The octet- and decuplet-baryon masses and sigma-terms
  - The meson-baryon-baryon couplings
  - Improve the extraction of V_{us} including g_2 and other form factors

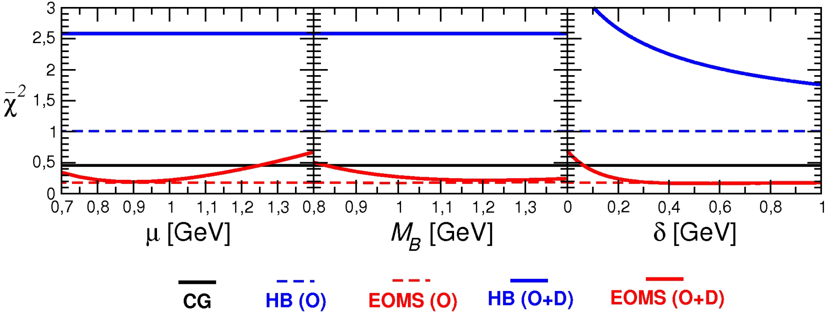
EOMS BChPT results: Graphical comparison



$x \equiv m/m_{phys}$ with m the meson masses

- The three approaches agree in the vicinity of the chiral limit.
- IR and EOMS coincide up to $x \sim 0.4$. IR description then get worse.
- Shaded area(s) represent the variation 0.8GeV$\leq M_0 \leq$ 1.1GeV.
- EOMS provides a realistic SU(3)-breaking mechanism for MM!

Baryon octet MM: Uncertainties in decuplet contributions



- Improvement over CG for 0.7 GeV$\leq \mu \leq$1.3 GeV
- Smooth dependence on the average baryon mass ($\delta = 0.231$ GeV)
- The decuplet contributions vanish at $\delta \to \infty$: Decoupling of the decuplet
- Covariant O.+D. NLO-calculation improves the C-G relations!

Numerical details of the calculation of $f_1(0)$

- Separation of results in partial contributions

	O-$\delta^{(2)}$	O-$\delta^{(3)}$	O-Total	D-$\delta^{(2)}$	D-$\delta^{(3)}$	D-Total	O+D Total
ΛN	-3.8	$0.2^{+1.2}_{-0.9}$	$-3.6^{+1.2}_{-0.9}$	0.7	$3.0^{+0.1}_{-0.1}$	$3.7^{+0.1}_{-0.1}$	$+0.1^{+1.3}_{-1.0}$
ΣN	-0.8	$4.7^{+3.8}_{-2.8}$	$3.9^{+3.8}_{-2.8}$	-1.4	$6.2^{+0.4}_{-0.3}$	$4.8^{+0.4}_{-0.3}$	$+8.7^{+4.2}_{-3.1}$
$\Xi\Lambda$	-2.9	$1.7^{+2.4}_{-1.8}$	$-1.2^{+2.4}_{-1.8}$	-0.02	$5.2^{+0.4}_{-0.3}$	$5.2^{+0.4}_{-0.3}$	$+4.0^{+2.8}_{-2.1}$
$\Xi\Sigma$	-3.7	$-1.3^{+0.3}_{-0.2}$	$-5.0^{+0.3}_{-0.2}$	0.7	$6.0^{+1.9}_{-1.4}$	$6.7^{+1.9}_{-1.4}$	$+1.7^{+2.2}_{-1.6}$

$\delta^{(i)}$ are the leading (1) and next-to-leading(2) fraction of SU(3)-breaking in %

- Comparison with other approaches

	BχPT	HBχPT*	Large N_c	QM	χQM	lQCD
ΛN	$+0.1^{+1.3}_{-1.0}$	$+5.8$	$+2\pm2$	-1.3	$+0.1$	
ΣN	$+8.7^{+4.2}_{-3.1}$	$+9.3$	$+4\pm3$	-1.3	$+0.9$	$-1.2\pm2.9\pm4$
$\Xi\Lambda$	$+4.0^{+2.8}_{-2.1}$	$+8.4$	$+4\pm3$	-1.3	$+2.2$	
$\Xi\Sigma$	$+1.7^{+2.2}_{-1.6}$	$+2.6$	$+8\pm5$	-1.3	$+4.2$	-1.3 ± 1.9