S=+1 Pentaquarks in QCD Sum Rules

Phys. Rev. D 79, 114011 (2009)

Philipp Gubler (Tokyo Tech)
15.9.2009
@ J-PARC

Collaborators: M. Oka (Tokyo Tech)
T. Nishikawa (Ryotokuji University)
D. Jido (YITP)
T. Kojo (Brookhaven National Laboratory)

Contents

- The Pentaquark Θ^+ State
- The Method: QCD Sum Rules
  - A very brief introduction
- Results of our Calculation
  - $I,J^P = 0,3/2^\pm$
  - Other quantum numbers
- Conclusion and Outlook

Pentaquark Θ^+

Basic properties

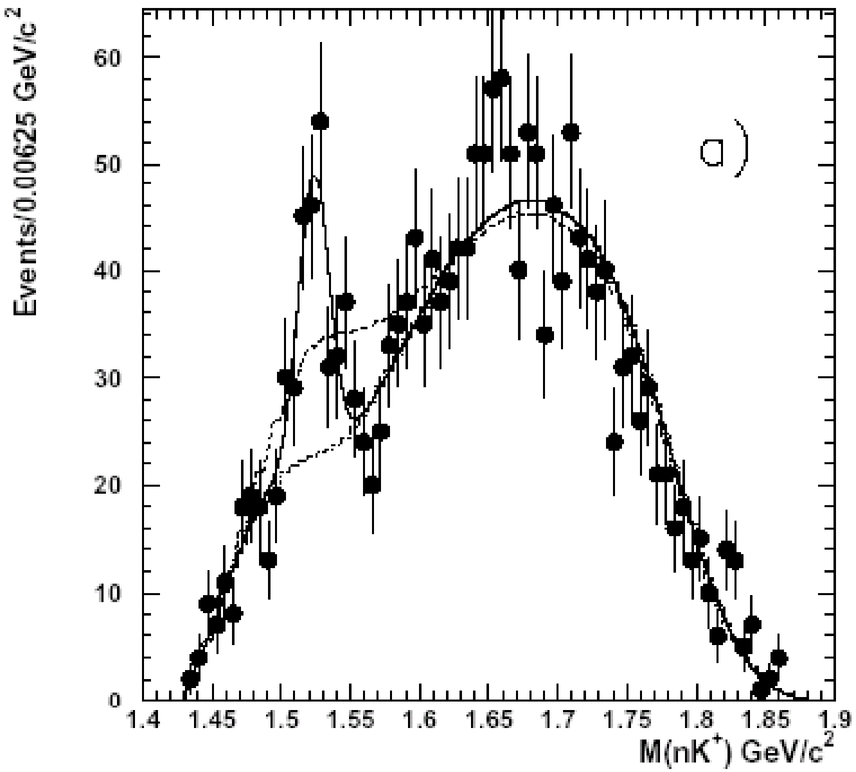
- B=1, S=1 → minimal quark content: 5 quarks $(uudd\bar{s})$
- No Isospin-partners Θ^0, Θ^{++} → I=0 (?)
- Mass: ~1540 MeV
- Narrow width: less than ~1 MeV

Why is it interesting?

- It is exotic.
- Why is it seen in some experiments, while it not seen in others?
- Why is it so narrow?

→ New dynamics in QCD ?

The SPring-8 experiment has reconfirmed a peak, so the question of the existence of Θ^+ is not settled yet.



T. Nakano *et al.*

Phys. Rev. C **79**, 025210 (2009).

For the latest experimental developments:

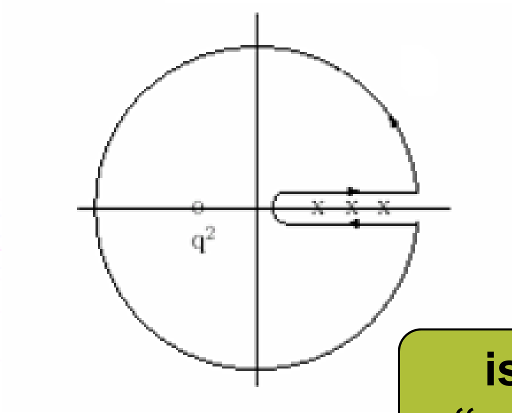
→ T. Nakano`s talk on Friday

There are still many theoretical questions that remain to be answered. (Quantum numbers, narrow width, etc.)

QCD sum rules

In this method the properties of the two point correlation function is fully exploited:

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0 | T\{\chi(x)\overline{\chi}(0)\} | 0 \rangle$$



$$\rightarrow \Pi(q^2) = \frac{1}{\pi} \int_{s_{min}}^{\infty} ds \frac{\text{Im}\Pi(s)}{s - q^2 - i\epsilon}$$

is calculated "perturbatively"

spectral function of the operator χ

$$\frac{1}{\pi}\text{Im}\Pi(s) = \lambda^2 \delta(s - m^2) + \theta(s - s_{th}) \frac{1}{\pi} \text{Im}\Pi^{OPE}(s)$$

Borel transformation → Introduction of an unphysical parameter, the Borel mass

The concrete calculation (for I,J^P=0,3/2$^\pm$)

We use the following interpolating fields:

$$\eta^1_\mu(x) = \epsilon_{cfg}[\epsilon_{abc} u^T_a(x) C\gamma_5 d_b(x)][\epsilon_{def} u^T_d(x) C\gamma_\mu\gamma_5 d_e(x)] C\bar{s}^T_g(x),$$

$$\eta^2_\mu(x) = \epsilon_{cfg}[\epsilon_{abc} u^T_a(x) C d_b(x)][\epsilon_{def} u^T_d(x) C\gamma_\mu\gamma_5 d_e(x)]\gamma_5 C\bar{s}^T_g(x)$$

$$\longrightarrow \quad \eta_\mu(x) = \cos\theta\,\eta^1_\mu(x) + \sin\theta\,\eta^2_\mu(x)$$

Using these currents, the 2-point function is calculated:

$$\Pi^{ij}_{\mu\nu}(q) = i\int d^4x e^{iqx}\langle 0|T[\eta^i_\mu(x)\bar{\eta}^j_\nu(0)]|0\rangle$$

$$= g_{\mu\nu}\left[\hat{q}\Pi^{ij}_0(q^2) + \Pi^{ij}_1(q^2)\right] + \ldots \qquad (\hat{q} \equiv q^\mu\gamma_\mu)$$

chiral even part (Π^{ij}_0)

chiral odd part (Π^{ij}_1)

Importance of the Borel window

1. The OPE Convergence

$$\left|\frac{Dimension\ \ N\ \ terms}{OPE\ \ summed\ \ up\ \ to\ \ Dimension\ \ N}\right| \leq 0.1$$

2. The Pole Contribution

$$\frac{\int_0^{s_{th}} ds e^{-\frac{s}{M^2}} Im\Pi^{OPE}(s)}{\int_0^{\infty} ds e^{-\frac{s}{M^2}} Im\Pi^{OPE}(s)} \geq 0.5$$

It is very important that these two conditions are satisfied simultaneously to obtain reliable results from QCDSR calculations!

How to obtain a high Pole Contribution (1)

We use an approach similar to the old idea of the Weinberg spectral function sum rule:

$$\langle V_\mu(x)\overline{V}_\nu(0)\rangle - \langle A_\mu(x)\overline{A}_\nu(0)\rangle \simeq 0$$
$$(x \to 0)$$

→ **leading orders in the OPE expansion are suppressed!**

T. Kojo, A. Hayashigaki, D.Jido, Phys. Rev. C **74**, 045206 (2006)

In our case we calculate the difference of two (independent) correlators with different mixing angles to obtain a good suppression of the leading OPE orders:

$$\Pi_D(q^2,\phi) \equiv \Pi_i(q^2,\theta_1) - \Pi_i(q^2,\theta_2)$$
$$(\phi \equiv \theta_1 + \theta_2)$$

How to obtain a high Pole Contribution (2)

The sum and the difference of the used interpolating fields belong to specific chiral multiplets:

$$\begin{aligned}\xi_{1,\mu} &\equiv \eta_{1,\mu} + \eta_{2,\mu} \\ &= 2(u_R^T C d_R)[(u_L^T C\gamma_\mu d_R) - (u_R^T C\gamma_\mu d_L)]C\overline{s}_R^T \\ &\quad - 2(u_L^T C d_L)[(u_L^T C\gamma_\mu d_R) - (u_R^T C\gamma_\mu d_L)]C\overline{s}_L^T,\end{aligned} \qquad (\mathbf{3},\overline{\mathbf{15}}) \oplus (\overline{\mathbf{15}},\mathbf{3})$$

$$\begin{aligned}\xi_{2,\mu} &\equiv \eta_{1,\mu} - \eta_{2,\mu} \\ &= 2(u_R^T C d_R)[(u_L^T C\gamma_\mu d_R) - (u_R^T C\gamma_\mu d_L)]C\overline{s}_L^T \\ &\quad - 2(u_L^T C d_L)[(u_L^T C\gamma_\mu d_R) - (u_R^T C\gamma_\mu d_L)]C\overline{s}_R^T.\end{aligned} \qquad (\mathbf{8},\mathbf{8})$$

$$\begin{aligned}\Pi_D(q^2,\phi) = &\frac{1}{2}\Big\{\cos\phi[\langle\xi_1\overline{\xi_1}\rangle - \langle\xi_2\overline{\xi_2}\rangle] \\ &- \sin\phi[\langle\xi_1\overline{\xi_2}\rangle + \langle\xi_2\overline{\xi_1}\rangle]\Big\}\end{aligned}$$

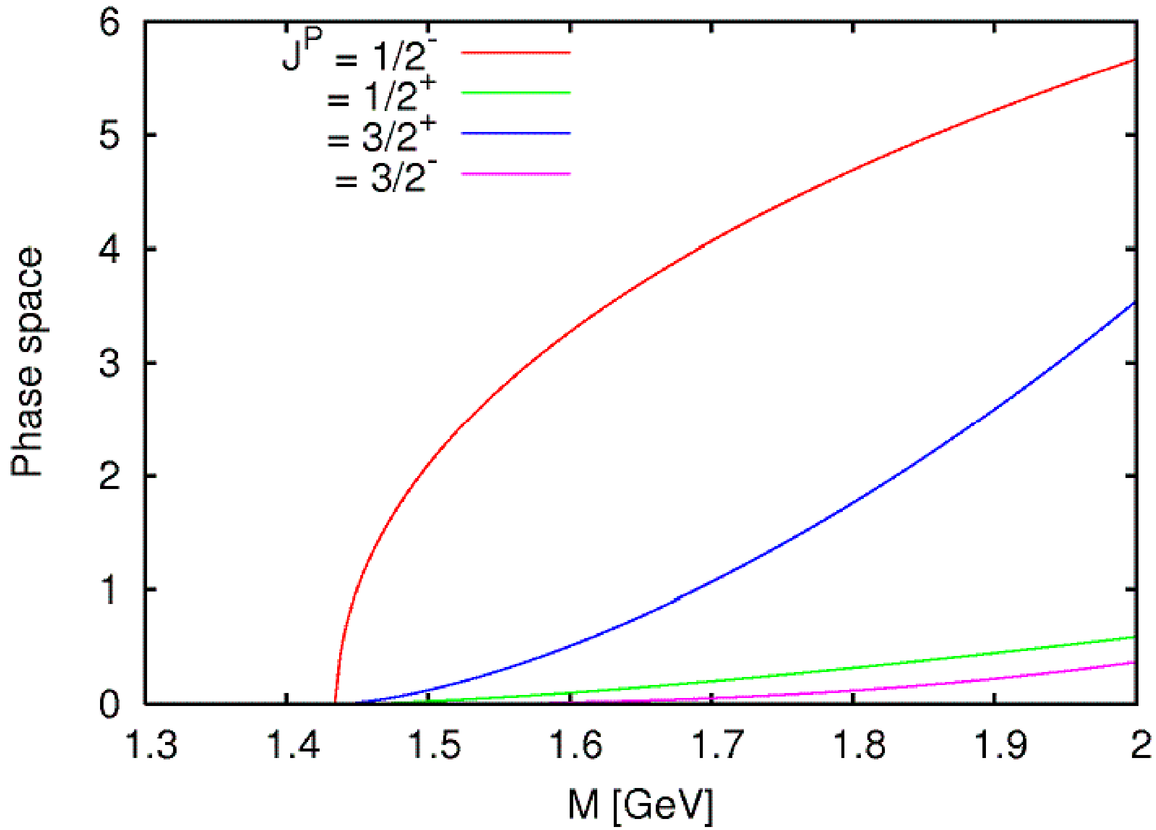
Determination of the mixing angle Φ $\rightarrow$

1) A sufficiently wide Borel window exists.

2) The calculated pentaquark mass should only weakly depend on the Borel mass M and the threshold parameter s_{th}.

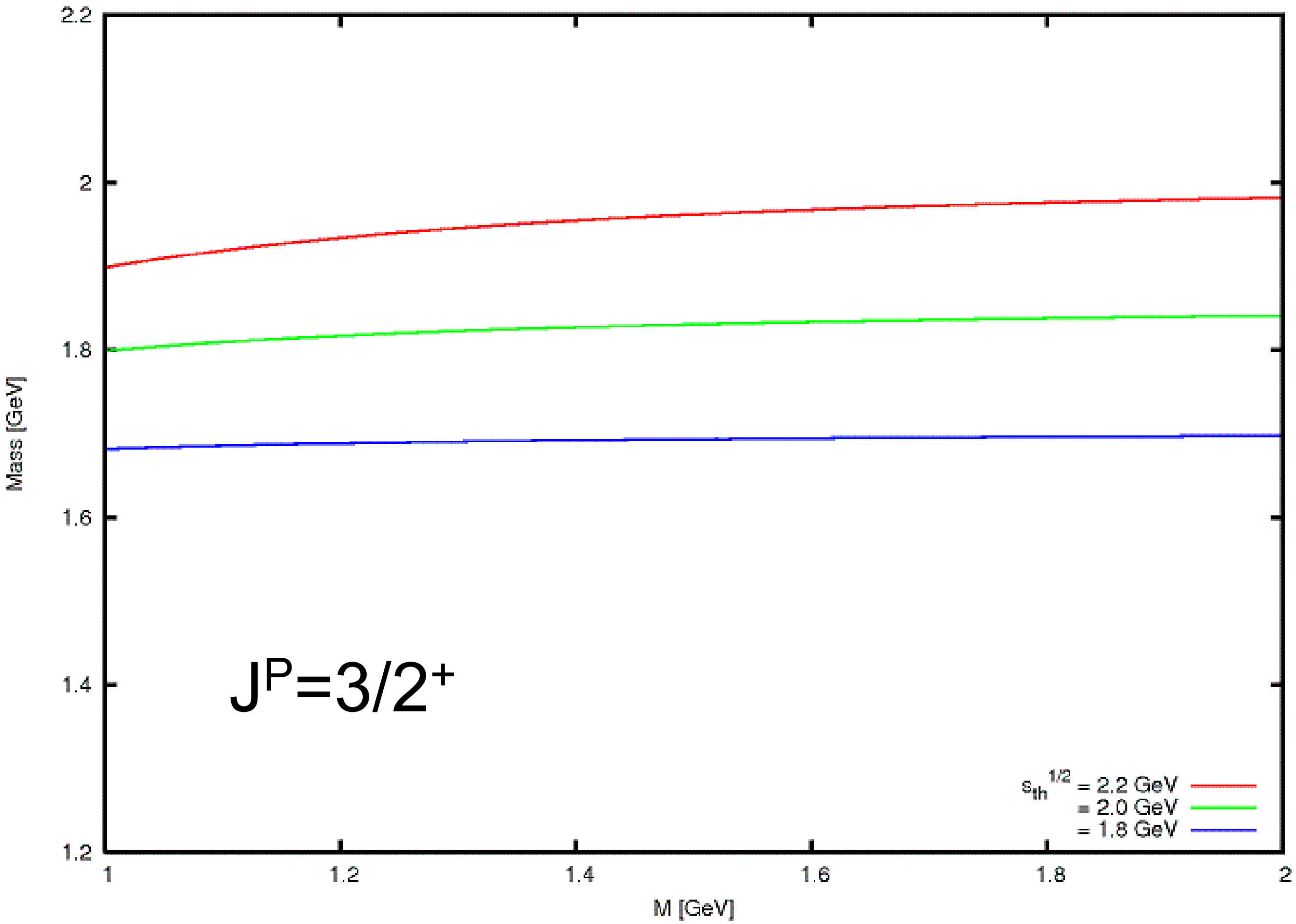
What about the KN scattering states?

Parametrizing the spectral function by the KN phase space, the dependence of the results on the Borel mass and the threshold parameter is investigated.

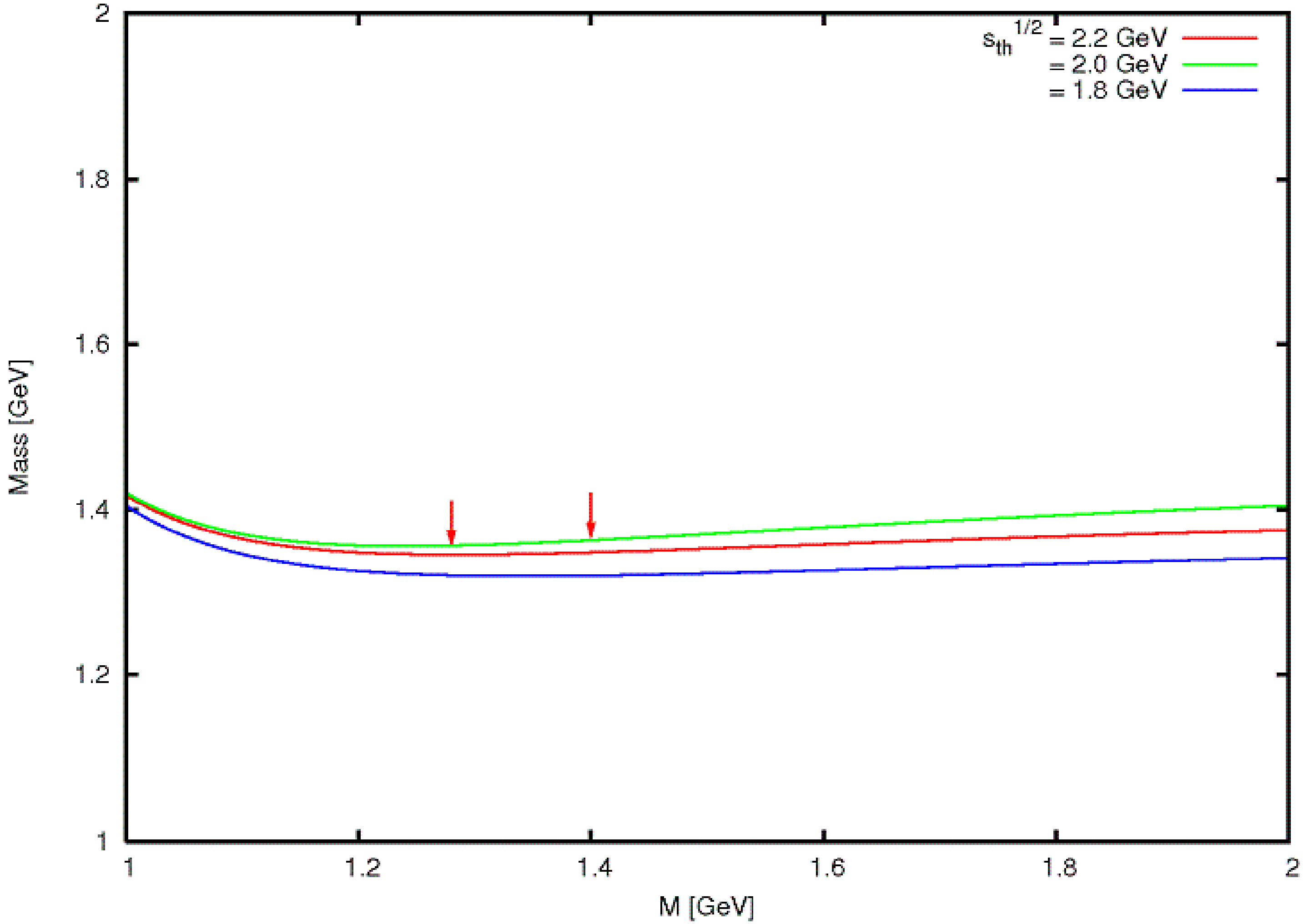


What about the KN scattering states?

Parametrizing the spectral function by the KN phase space, the dependence of the results on the Borel mass and the threshold parameter is investigated.

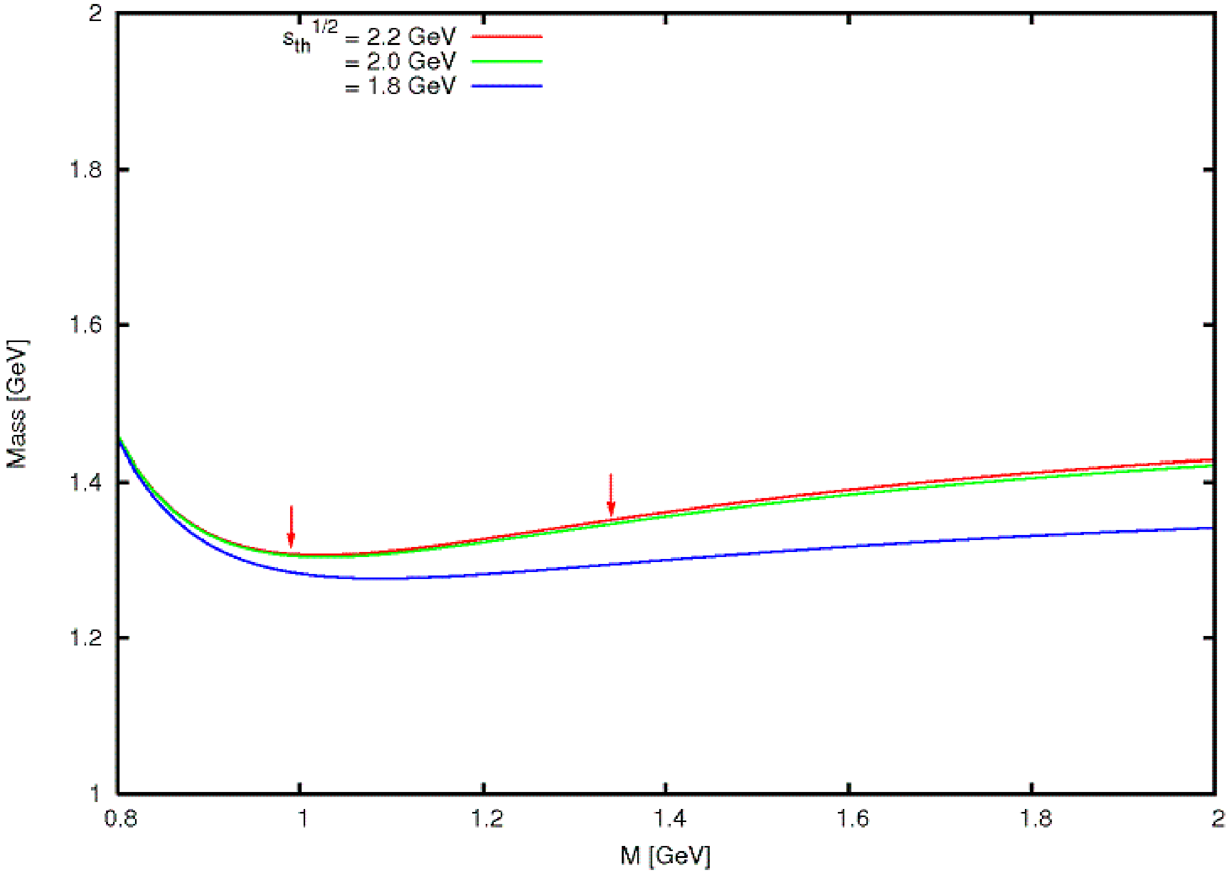


Results ($I,J^P=0,3/2^{\pm}$, Chiral even part)



$$\rightarrow m_{\Theta^+} = 1.4 \pm 0.2 \text{ GeV}$$

Results ($I,J^P=0,3/2^+$)



In the negative parity channel no valid Borel window with a flat Borel mass curve is obtained.

$$\rightarrow \ IJ^P = 0\tfrac{3}{2}^+$$

The other quantum numbers $(1,3/2^{\pm}\ 0,1/2^{\pm}\ 1,1/2^{\pm})$

The following interpolating fields are used:

$$\eta'^{1}_{\mu}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)C\gamma_5 d_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu d_e(x)]C\overline{s}_g^T(x),$$
$$\eta'^{2}_{\mu}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)Cd_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu d_e(x)]\gamma_5 C\overline{s}_g^T(x).$$

(IJ^{π}= 1,3/2$^{\pm}$)

$$\eta^{1}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)C\gamma_5 d_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu\gamma_5 d_e(x)]\gamma^\mu\gamma_5 C\overline{s}_g^T(x),$$
$$\eta^{2}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)Cd_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu\gamma_5 d_e(x)]\gamma^\mu C\overline{s}_g^T(x).$$

(IJ^{π}= 0,1/2$^{\pm}$)

$$\eta'^{1}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)C\gamma_5 d_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu d_e(x)]\gamma^\mu C\overline{s}_g^T(x),$$
$$\eta'^{2}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)Cd_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu d_e(x)]\gamma^\mu\gamma_5 C\overline{s}_g^T(x).$$

(IJ^{π}= 1,1/2$^{\pm}$)

The rest of the calculations follows the same lines as in the I,J^P = 0,3/2$^{\pm}$ case.

Summary of all obtained Results

		Parity	
		+	-
$J=\frac{3}{2}$	$I=0$	1.4 ± 0.2 GeV (KN P-wave)	no state found below 2.0 GeV (KN D-wave)
	$I=1$	1.6 ± 0.3 GeV (KN P-wave)	no state found below 2.0 GeV (KN D-wave)
$J=\frac{1}{2}$	$I=0$	no state found below 2.0 GeV (KN P-wave)	1.5 ± 0.3 GeV (?) (KN S-wave)
	$I=1$	no state found below 2.0 GeV (KN P-wave)	1.6 ± 0.4 GeV (KN S-wave)

Why can`t we observe this state ?

The width is expected to be quite large.
Most probably these are not the experimentally observed states

Conclusion and Outlook

- Our results suggest that the $I,J^{\pi}=0,3/2^{+}$ seems to be the most probable candidate for the experimentally observed $\Theta^{+}(1540)$.
- Other states have been found for $I,J^{\pi}=1,3/2^{+}$ $0,1/2^{-}$ and $1,1/2^{-}$ at somewhat higher mass values. Some of these states may be found in future experiments.
- As we have obtained a spin 3/2 state with positive parity, the problem of the narrow width will need further consideration.

  Calculation of the width using the QCD sum rule approach would be interesting.

Backup Slides

The theoretical (QCD) side

The operator product expansion (OPE) is used:

$$i\int d^4x e^{iqx}\langle 0|T\{\chi(x)\overline{\chi}(0)\}|0\rangle = C_I(q^2)I+\sum_n C_n(q^2)\langle 0|O_n|0\rangle$$

$$\begin{aligned}\langle 0|O_n|0\rangle = \; & \langle 0|\overline{q}q|0\rangle, \\ & \langle 0|G^a_{\mu\nu}G^{a\mu\nu}|0\rangle, \\ & \langle 0|\overline{q}\sigma_{\mu\nu}\tfrac{\lambda^a}{2}G^{a\mu\nu}q|0\rangle, \\ & \langle 0|\overline{q}q\overline{q}q|0\rangle, \ldots\end{aligned}$$

The phenomenological (hadronic) side

Sharp resonance + continuum is assumed:

$$\tfrac{1}{\pi}Im\Pi(s) = \lambda^2\delta(s-m^2)+\theta(s-s_{th})\tfrac{1}{\pi}Im\Pi^{OPE}(s)$$

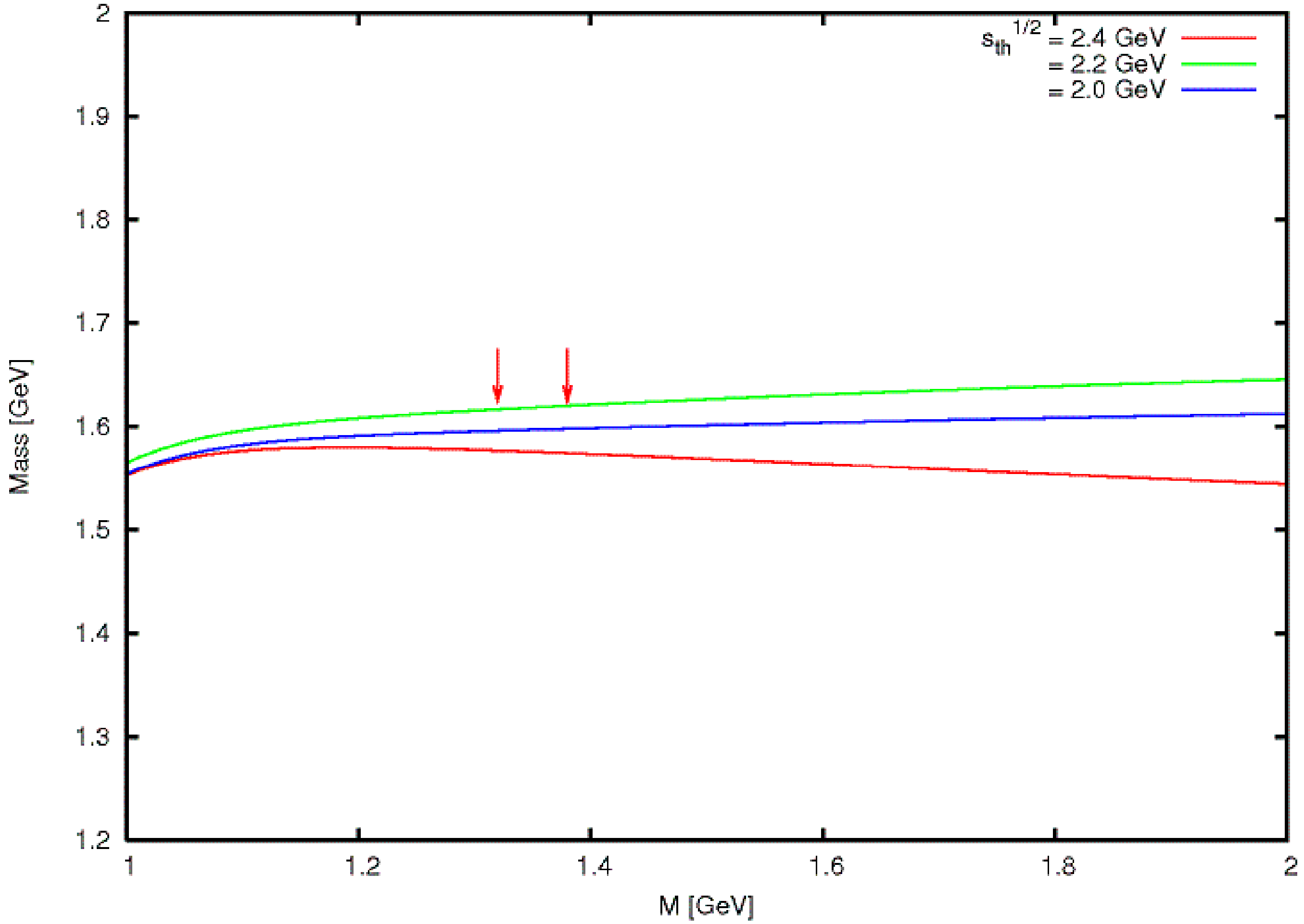
$$
\begin{aligned}
&-i\int d^4x e^{iqx}\langle 0|T[\eta_\mu^{I=0}(x)\bar{\eta}_\nu^{I=0}(0)]|0\rangle_{g_{\mu\nu}q'} \\
&= -\frac{1}{2^{19}3^2 5^2\pi^8}q^{10}\ln(-q^2)\cdot(\cos^2\theta^{I=0}+\sin^2\theta^{I=0}) \\
&- \frac{m_s\langle\bar{s}s\rangle}{2^{13}3^3\pi^6}q^6\ln(-q^2)\cdot(\cos^2\theta^{I=0}+\sin^2\theta^{I=0}) \\
&+ \frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{18}3^4 5\pi^6}q^6\ln(-q^2)\cdot(7\cos^2\theta^{I=0}+12\cos\theta^{I=0}\sin\theta^{I=0}+7\sin^2\theta^{I=0}) \\
&- \frac{\langle\bar{q}q\rangle^2}{2^9 3^2 5\pi^4}q^4\ln(-q^2)\cdot(\cos^2\theta^{I=0}-9\sin^2\theta^{I=0}) \\
&+ \frac{m_s\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{16}3^2 5\pi^6}q^4\ln(-q^2)\cdot(37\cos^2\theta^{I=0}+12\cos\theta^{I=0}\sin\theta^{I=0}+37\sin^2\theta^{I=0}) \\
&+ \frac{5m_s\langle\bar{s}s\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{15}3^3\pi^4}q^2\ln(-q^2)\cdot(\cos^2\theta^{I=0}+\sin^2\theta^{I=0}) \\
&- \frac{\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle}{2^{12}3^3\pi^4}q^2\ln(-q^2)\cdot(26\cos^2\theta^{I=0}+7\cos\theta^{I=0}\sin\theta^{I=0}+198\sin^2\theta^{I=0}) \\
&+ \frac{m_s\langle\frac{\alpha_s}{\pi}G^2\rangle\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{17}3^4\pi^4}\ln(-q^2)\cdot(211\cos^2\theta^{I=0}+156\cos\theta^{I=0}\sin\theta^{I=0}+211\sin^2\theta^{I=0}) \\
&- \frac{m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle}{2^5 3^3\pi^2}\ln(-q^2)\cdot(\cos^2\theta^{I=0}-5\sin^2\theta^{I=0}) \\
&+ \frac{\langle\bar{q}g\sigma\cdot Gq\rangle^2}{2^{15}3^4\pi^4}\ln(-q^2)\cdot(489\cos^2\theta^{I=0}+44\cos\theta^{I=0}\sin\theta^{I=0}+1959\sin^2\theta^{I=0}) \\
&+ \frac{\langle\bar{q}q\rangle^2\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{11}3^4\pi^2}\ln(-q^2)\cdot(151\cos^2\theta^{I=0}+4\cos\theta^{I=0}\sin\theta^{I=0}+133\sin^2\theta^{I=0}) \\
&+ \frac{m_s\langle\bar{q}q\rangle^2\langle\bar{s}g\sigma\cdot Gs\rangle}{2^8 3^3\pi^2 q^2}\cdot(17\cos^2\theta^{I=0}+\cos\theta^{I=0}\sin\theta^{I=0}-13\sin^2\theta^{I=0}) \\
&- \frac{5m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{q}g\sigma\cdot Gq\rangle}{2^7 3^2\pi^2 q^2}\cdot\sin^2\theta^{I=0} \\
&- \frac{\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{14}3^4\pi^2 q^2}\cdot(1365\cos^2\theta^{I=0}+65\cos\theta^{I=0}\sin\theta^{I=0}+849\sin^2\theta^{I=0}) \\
&+ \frac{\langle\bar{q}q\rangle^4}{2\cdot 3^3 q^2}\cdot(\cos^2\theta^{I=0}-\sin^2\theta^{I=0}) \\
&+ \frac{m_s\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{12}3^4\pi^2 q^4}\cdot(271\cos^2\theta^{I=0}+68\cos\theta^{I=0}\sin\theta^{I=0}-209\sin^2\theta^{I=0}) \\
&- \frac{m_s\langle\bar{s}s\rangle\langle\bar{q}g\sigma\cdot Gq\rangle^2}{2^{10}3^3\pi^2 q^4}\cdot(6\cos^2\theta^{I=0}+17\sin^2\theta^{I=0}) \\
&- \frac{m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^9 3^4 q^4}\cdot(21\cos^2\theta^{I=0}+35\sin^2\theta^{I=0}) \\
&+ \frac{97\langle\bar{q}q\rangle^3\langle\bar{q}g\sigma\cdot Gq\rangle}{2^6 3^4 q^4}\cdot(\cos^2\theta^{I=0}-\sin^2\theta^{I=0})
\end{aligned}
$$

Perturbative part, contributing mainly to the continuum

$\rightarrow$

$$
\begin{aligned}
&\Pi(\tfrac{\phi}{2}+\tfrac{\pi}{4})-\Pi(\tfrac{\phi}{2}-\tfrac{\pi}{4}) \\
&= +\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{16}3^3 5\pi^6}q^6\ln(-q^2)\cdot\cos\phi \\
&+ \frac{\langle\bar{q}q\rangle^2}{2^8 3^2\pi^4}q^4\ln(-q^2)\cdot\sin\phi \\
&+ \frac{m_s\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{14}3\cdot 5\pi^6}q^4\ln(-q^2)\cdot\cos\phi \\
&- \frac{\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle}{2^{12}3^3\pi^4}q^2\ln(-q^2)\cdot(7\cos\phi+172\sin\phi) \\
&+ \frac{13m_s\langle\frac{\alpha_s}{\pi}G^2\rangle\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{15}3^3\pi^4}\ln(-q^2)\cdot\cos\phi \\
&+ \frac{m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle}{2^4 3^2\pi^2}\ln(-q^2)\cdot\sin\phi \\
&+ \frac{\langle\bar{q}g\sigma\cdot Gq\rangle^2}{2^{14}3^4\pi^4}\ln(-q^2)\cdot(22\cos\phi+735\sin\phi) \\
&+ \frac{\langle\bar{q}q\rangle^2\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{10}3^4\pi^2}\ln(-q^2)\cdot(2\cos\phi-9\sin\phi) \\
&+ \frac{m_s\langle\bar{q}q\rangle^2\langle\bar{s}g\sigma\cdot Gs\rangle}{2^8 3^3\pi^2 q^2}\cdot(\cos\phi-30\sin\phi) \\
&- \frac{5m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{q}g\sigma\cdot Gq\rangle}{2^7 3^2\pi^2 q^2}\cdot\sin\phi \\
&- \frac{\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{14}3^4\pi^2 q^2}\cdot(65\cos\phi-516\sin\phi) \\
&- \frac{\langle\bar{q}q\rangle^4}{3^3 q^2}\cdot\sin\phi \\
&+ \frac{m_s\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{10}3^4\pi^2 q^4}\cdot(17\cos\phi-120\sin\phi) \\
&- \frac{11m_s\langle\bar{s}s\rangle\langle\bar{q}g\sigma\cdot Gq\rangle^2}{2^{10}3^3\pi^2 q^4}\cdot\sin\phi \\
&- \frac{7m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^8 3^4 q^4}\cdot\sin\phi \\
&- \frac{97\langle\bar{q}q\rangle^3\langle\bar{q}g\sigma\cdot Gq\rangle}{2^5 3^4 q^4}\cdot\sin\phi
\end{aligned}
$$

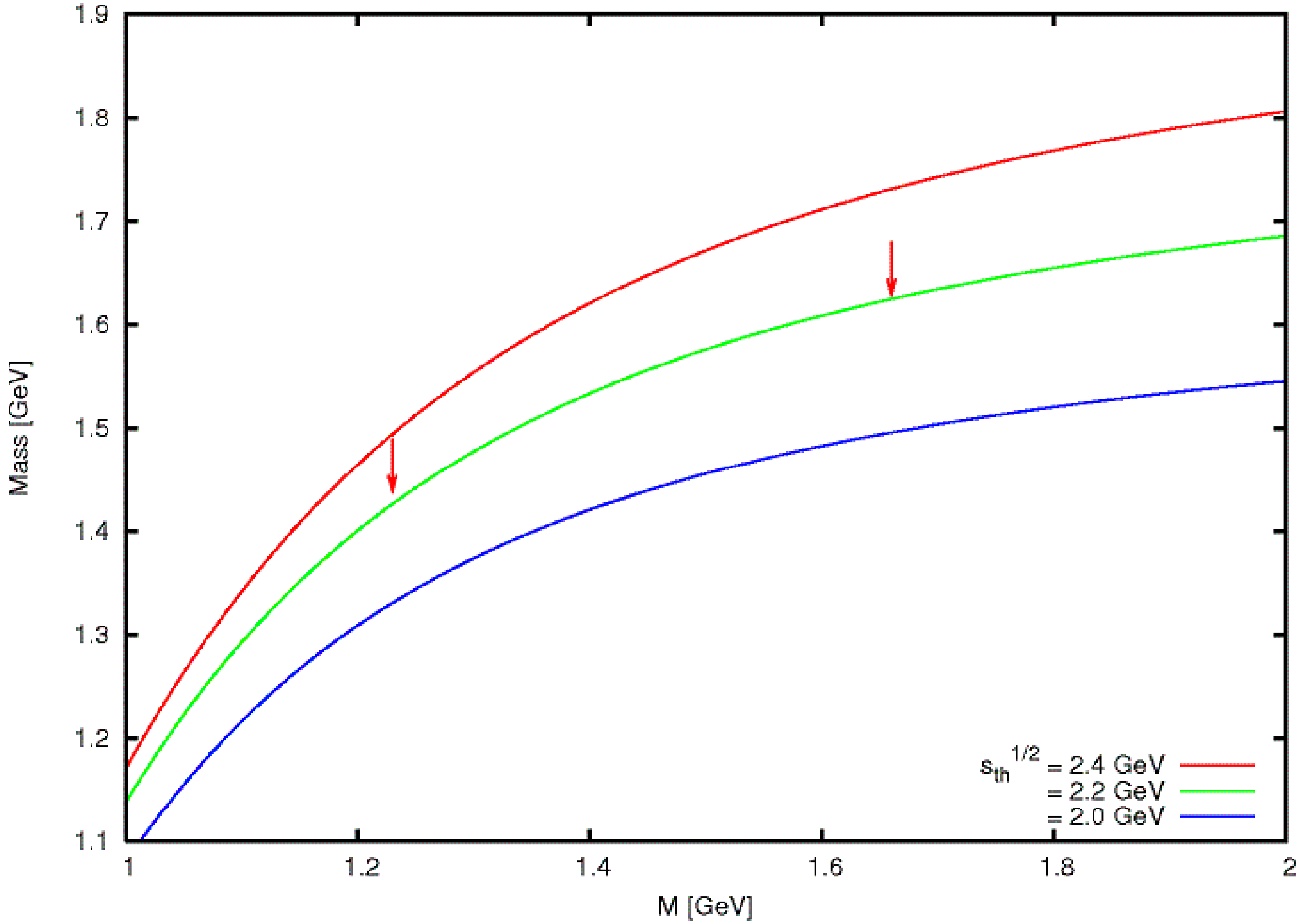
Results $(1, 3/2^{\pm})$



$\rightarrow \quad m = 1.6 \pm 0.3$ GeV

Parity projection $\rightarrow \quad IJ^P = 1\frac{3}{2}^+$

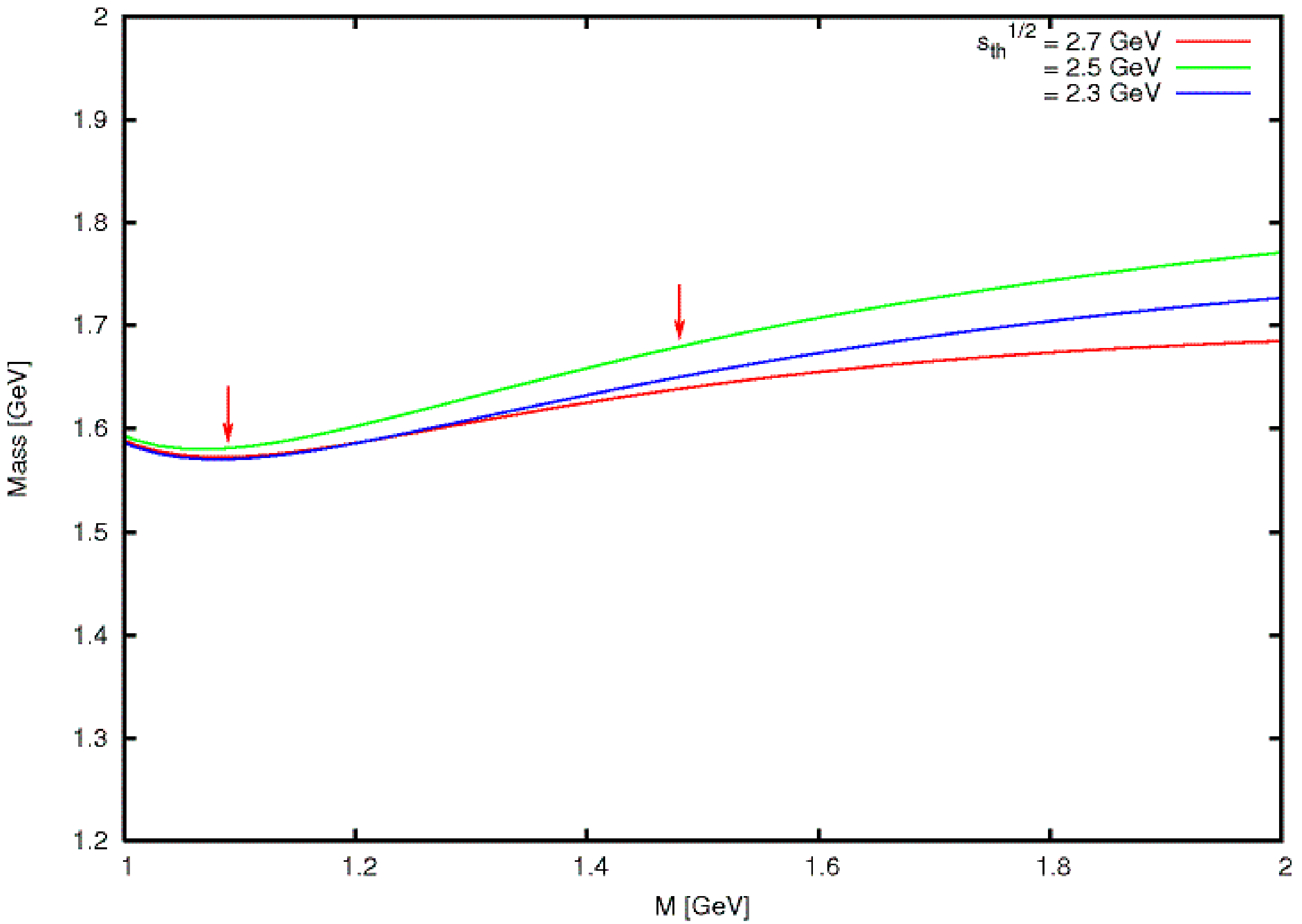
Results (0,1/2$^{\pm}$)



$\rightarrow \quad m = 1.5 \pm 0.3 \text{ GeV}$

Parity projection $\rightarrow \quad IJ^P = 0\frac{1}{2}^-$

Results $(1,1/2^{\pm})$



$$\rightarrow \quad m = 1.6 \pm 0.4 \text{ GeV}$$

Parity projection $\rightarrow \quad IJ^P = 1\frac{1}{2}^-$