10th International Conference on Hypernuclear and Strange Particle Physics " Hyp X "
(Sept. 14- 18, 2009 "RICOTTI" in Tokai, Ibaraki, Japan)

Hyperon-nucleus folding potentials in the complex G-matrix approach

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Contents

1. Extended Soft Core (ESC) model
   - core is designed by pomeron and ω-meson
   - **ESC04** and **ESC08**

2. **hyperon-nucleus optical potential**
   - through the single-folding model approach
   - apply to **Σ-nucleus elastic scattering**
   - apply to the **quasi free Σ production**

ESC model（Σ−N interaction）

ESC core = pomeron + ω

ESC04 model → ESC08 model

Quark Pauli-forbidden state

- ESC08 model
  - **Assuming** "equal parts" of ESC and QM are similar to each other
  - adjust $^3S_1$ state by changing the pomeron strengths for the corresponding channels

$g_P \rightarrow \mathrm{sqrt}(2.5)\, g_P$

Table 1: Values of U_Σ at normal density and partial wave contributions for ESC08.

model	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Σ
ESC08	1/2	9.7	−27.6	2.6	2.0	−6.9	−1.3	−0.8	
	3/2	−9.1	53.7	−11.8	−2.0	6.8	−0.6	−0.1	+14.8
ESC04a	1/2	10.9	−31.0	2.3	2.6	−6.5	−2.2	−0.8	
	3/2	−11.7	2.2	−6.9	−2.3	5.8	−5.3	−0.2	−42.9
NSC97f	1/2	14.9	−8.3	2.1	2.5	−4.6	0.5	−0.5	
	3/2	−12.4	−4.1	−4.1	−2.1	6.0	−2.8	−0.1	−12.9

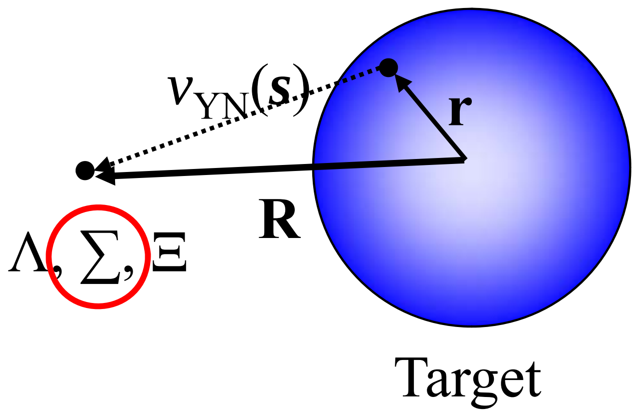
repulsive

Σ-nucleus optical potential

construct ↑ — **apply** →

- Quasi-free Σ production
- Σ-nucleus scattering

complex G-matrix folding potential



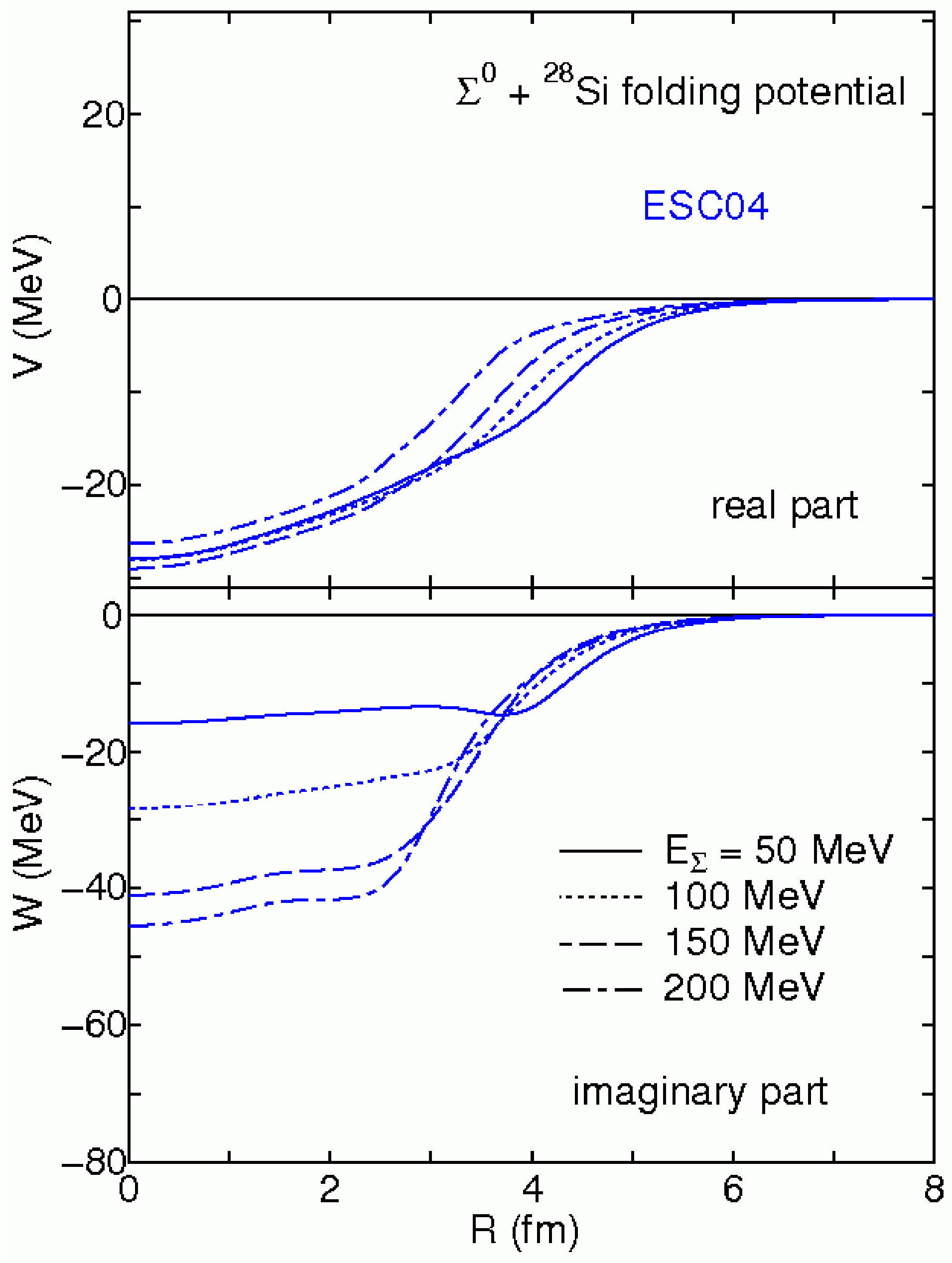
$$U_{SFM}(\mathbf{R}) = \int \rho(\mathbf{r}) v_{YN}(\mathbf{s}; \rho, E) d\mathbf{r}$$

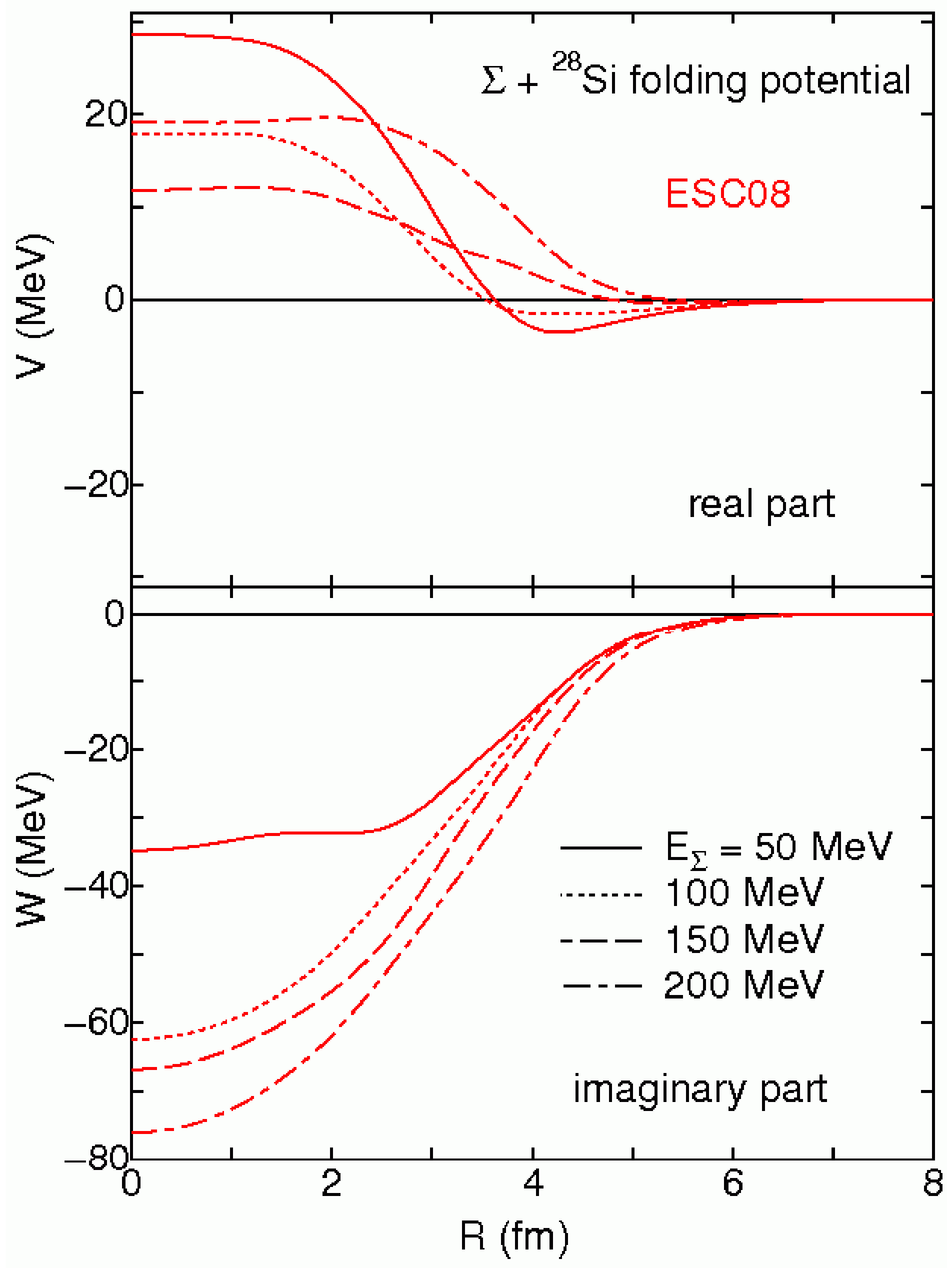
complex G-matrix interaction

Interesting problems

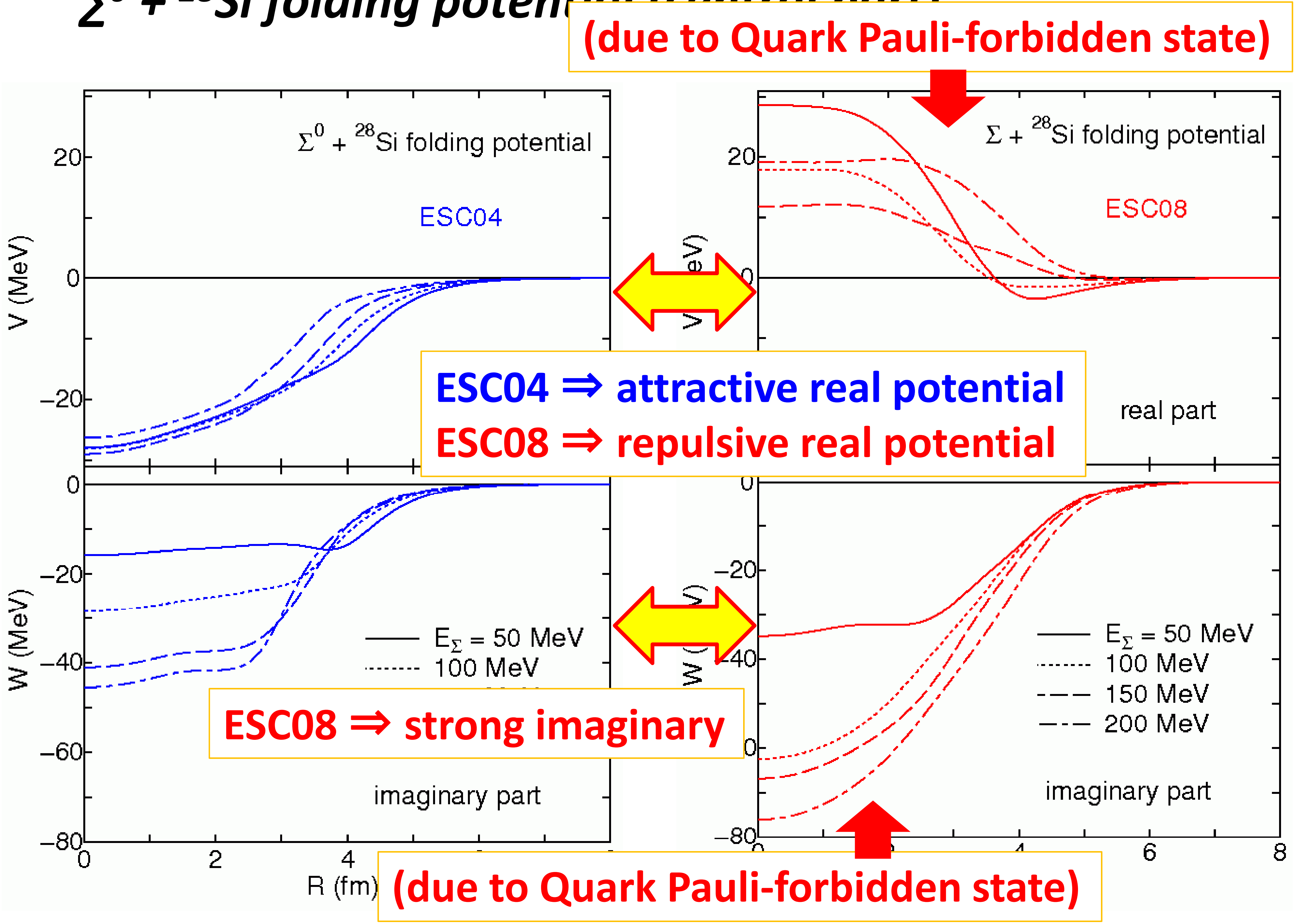
- repulsive ?
- isospin-dependence
- spin-orbit interaction
- imaginary part

Σ^0 + ^{28}Si folding potential (central part)





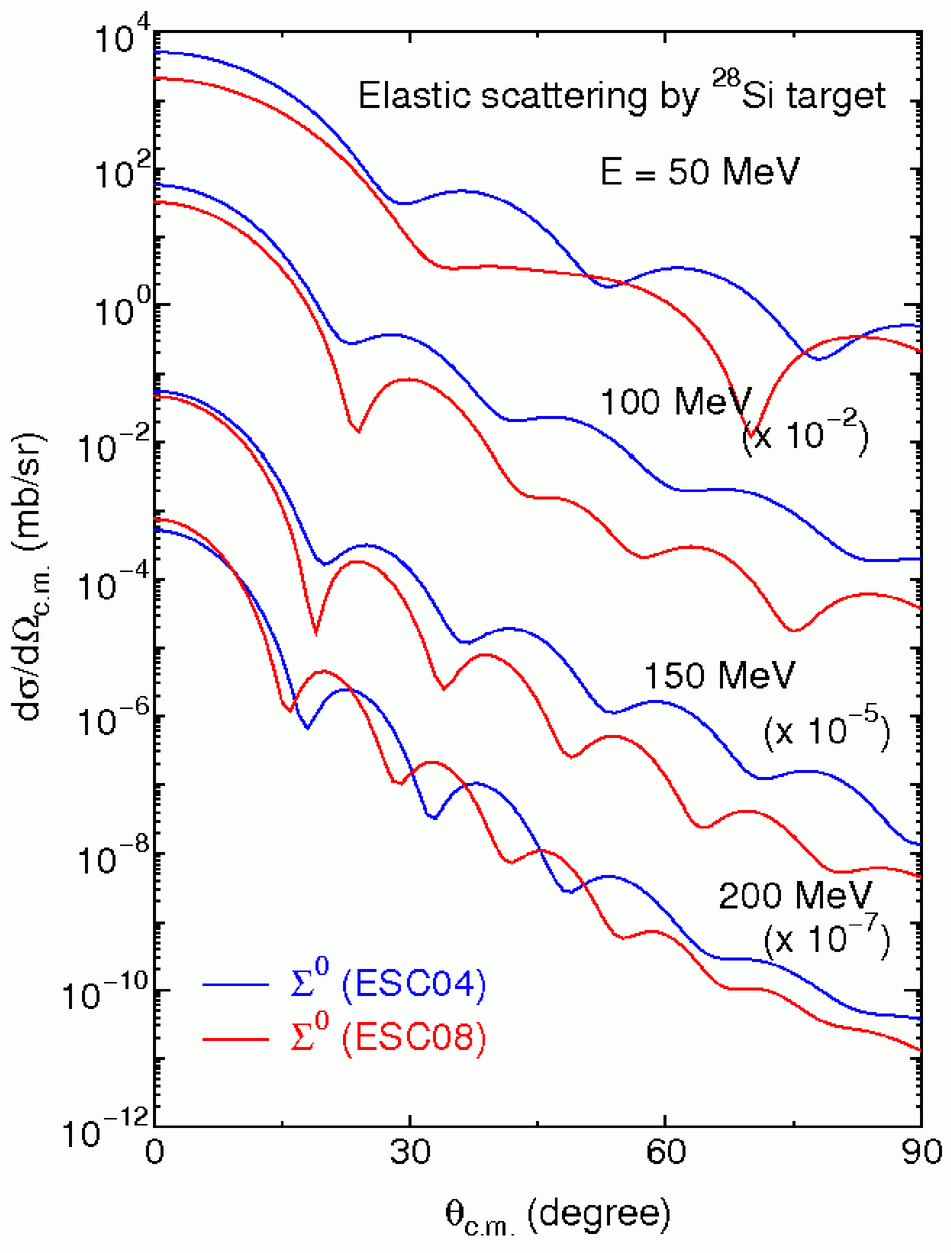
$\Sigma^0 + {}^{28}Si$ folding potential (central part)

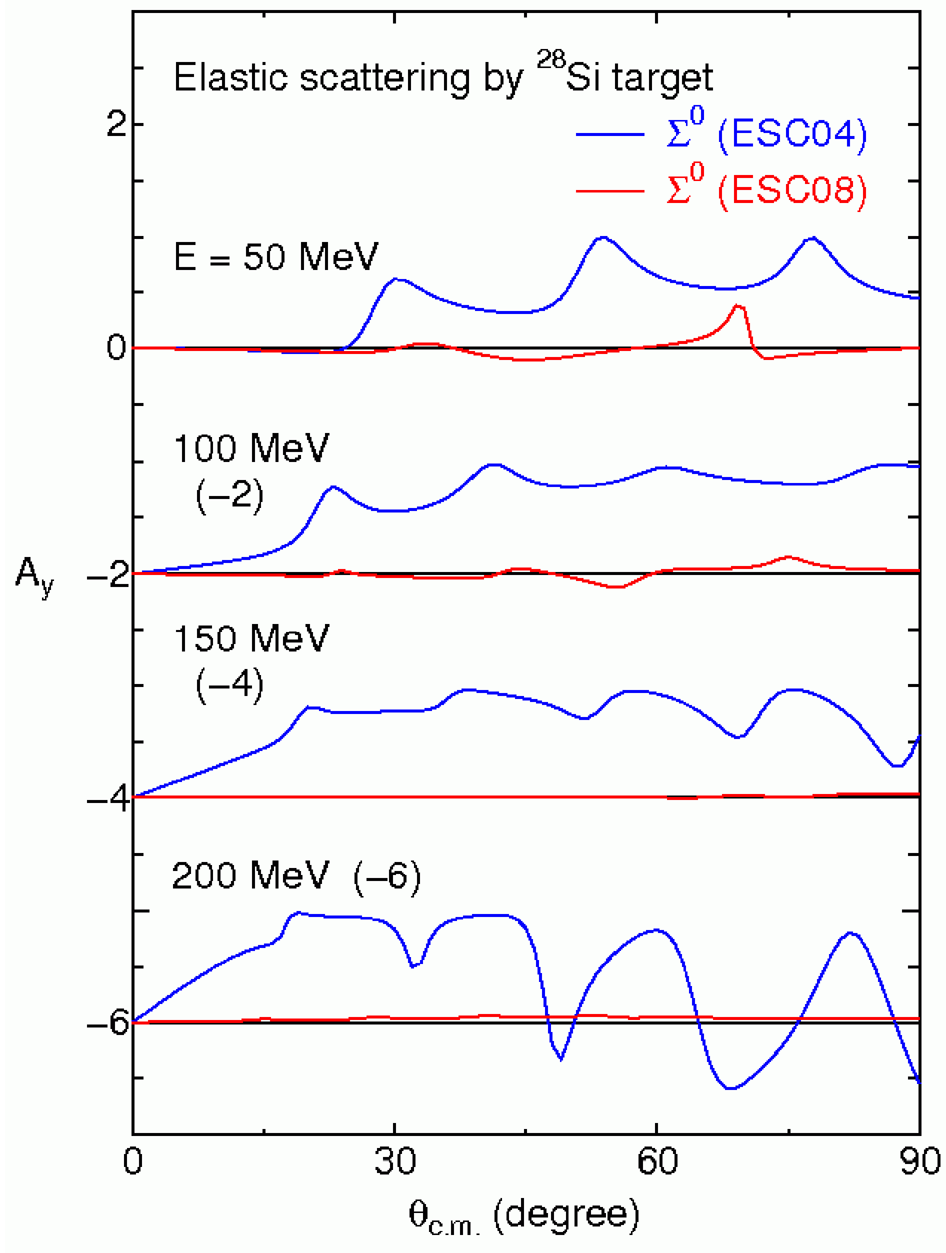


Σ^0 + ^{28}Si elastic scattering

$$U_{opt} = V + iN_W W$$

$N_W = 0.6$

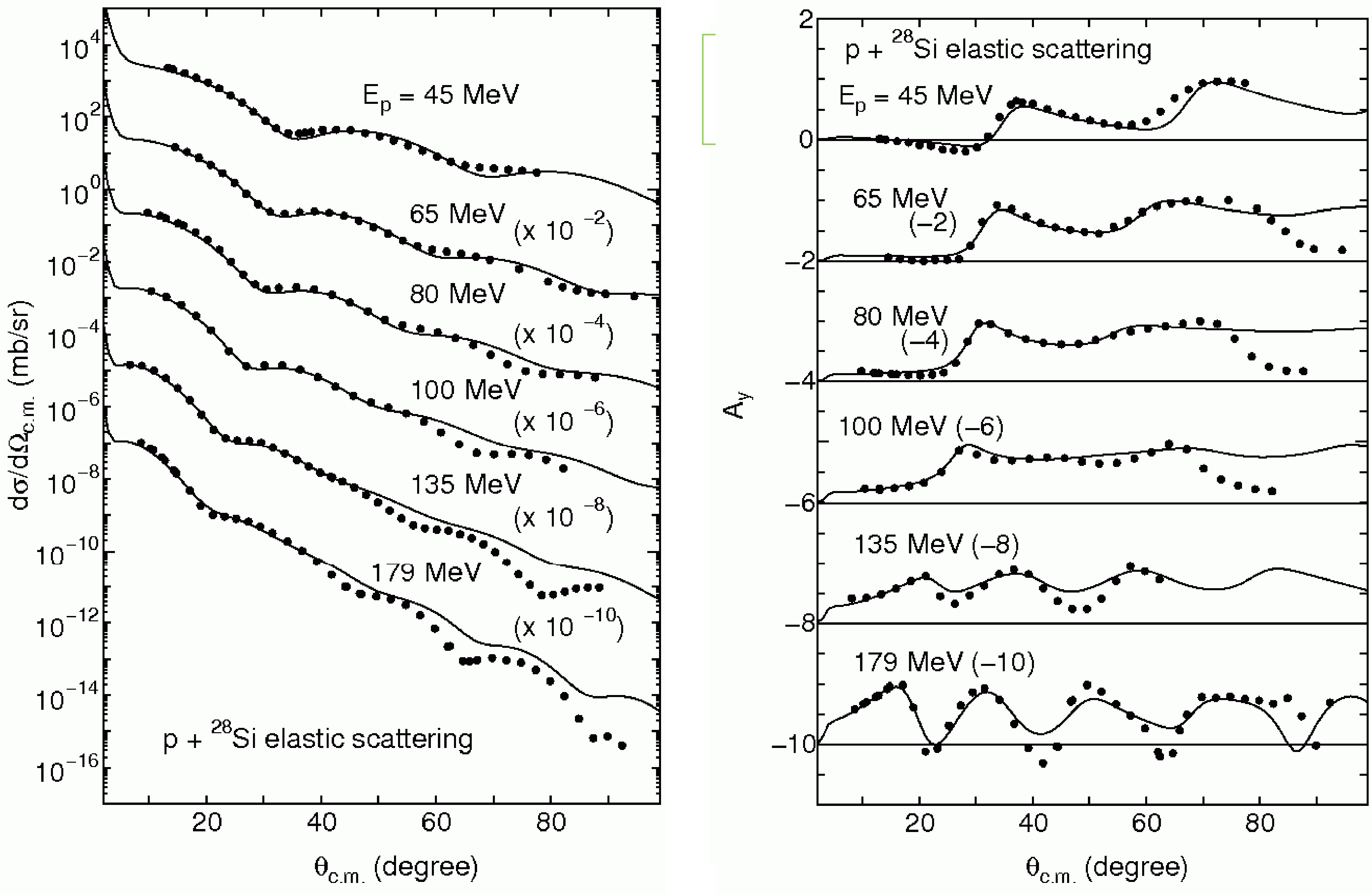




proton + ^{28}Si elastic scattering

$$U_{\rm opt} = V + iN_{\rm W}W$$

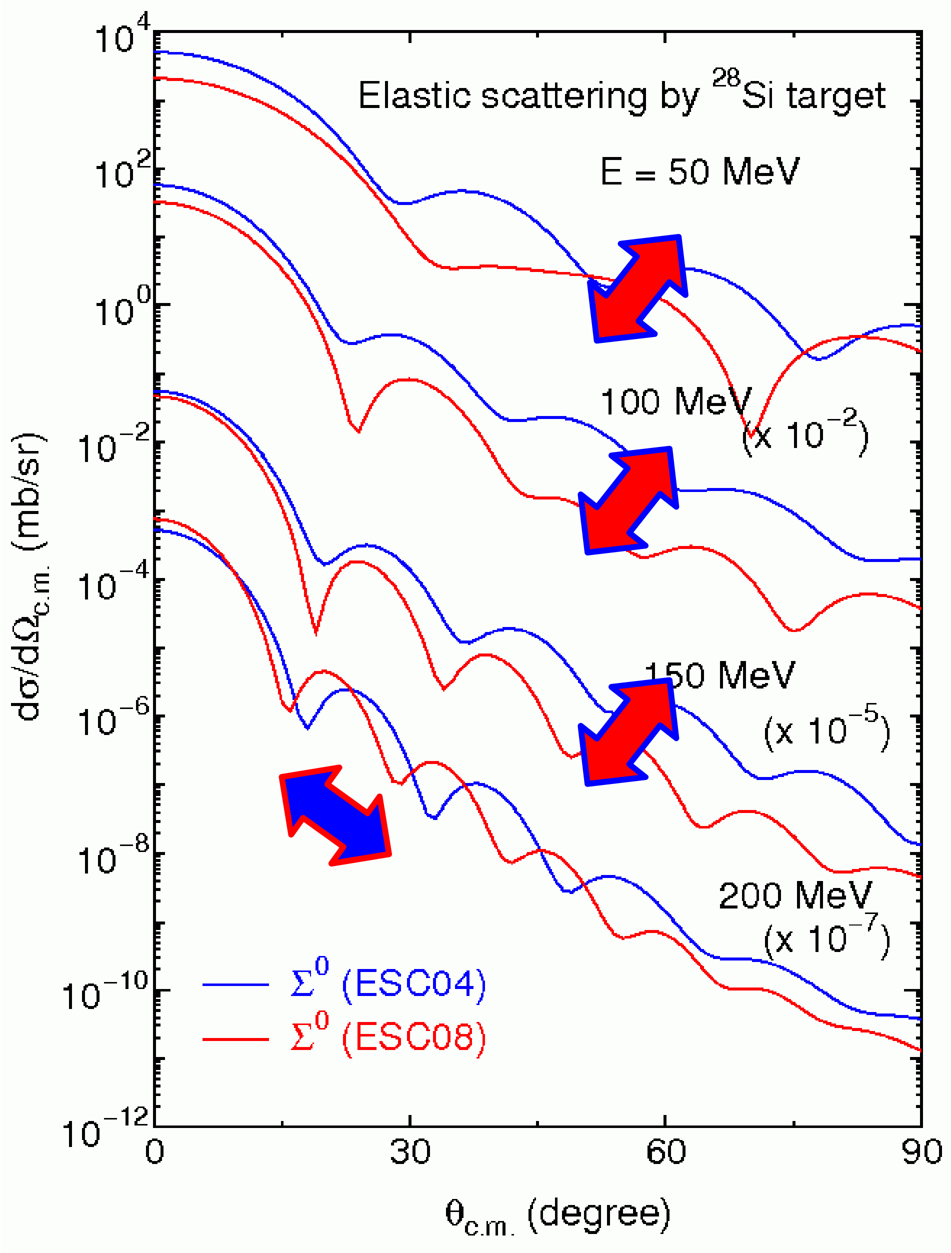
$N_{\rm W} = 0.6$

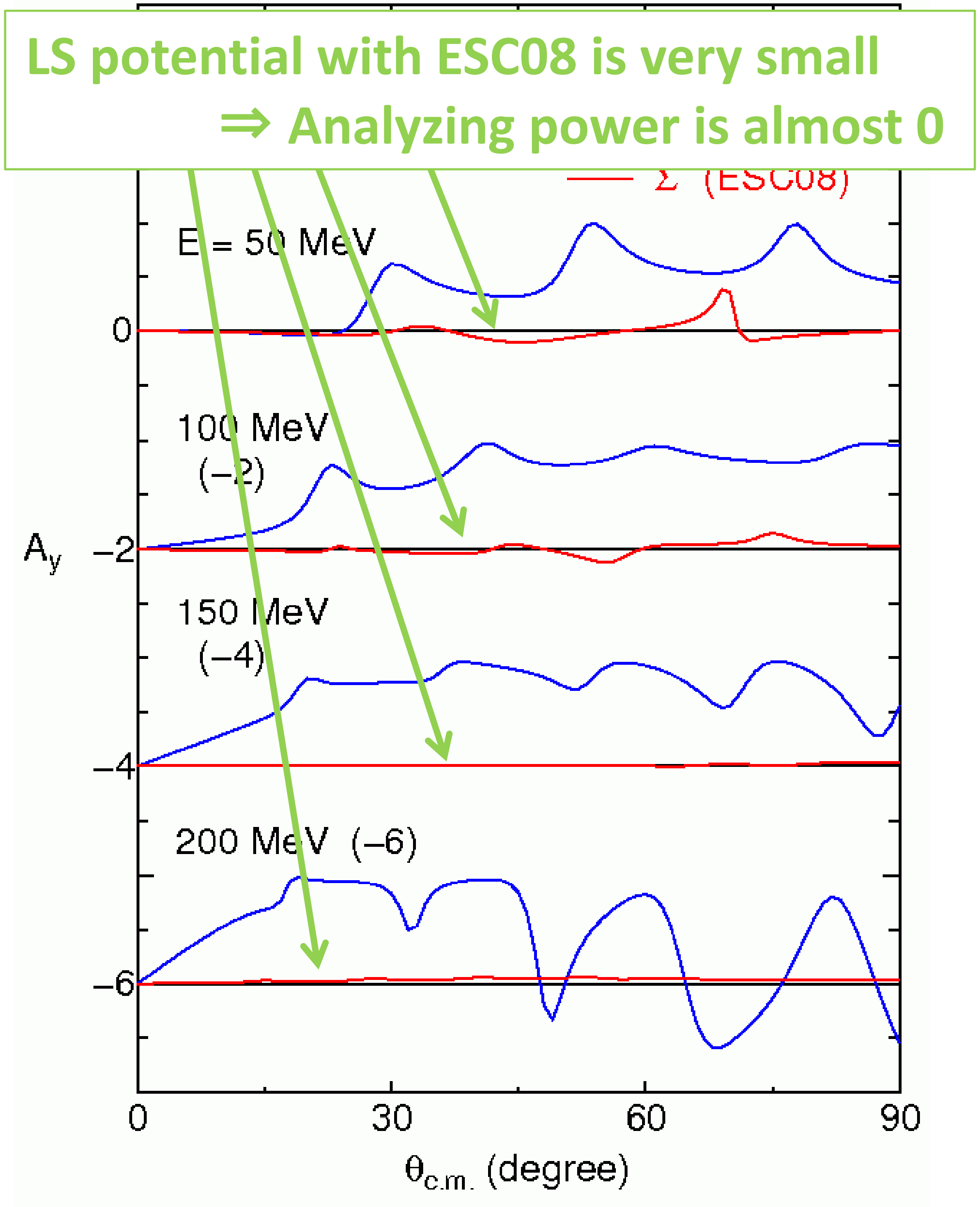


$\Sigma^0 + {}^{28}Si$ elastic scattering

$$U_{\mathrm{opt}} = V + iN_{\mathrm{W}}W$$

$N_{\mathrm{W}} = 0.6$





Σ⁻ quasi-free production spectra from (π⁻, K⁺) reaction

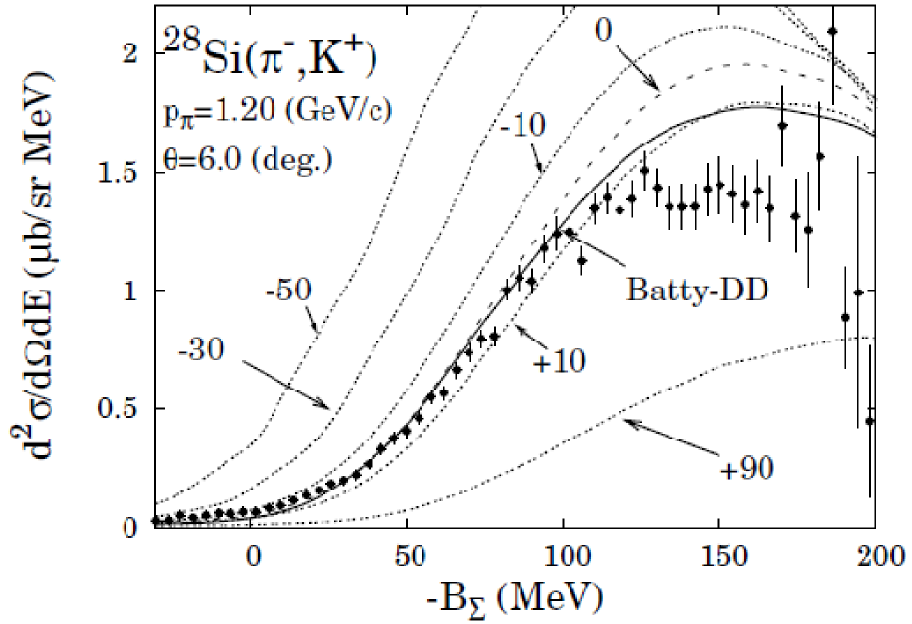


Fig. 2. Differential cross section of (π^-, K^+) reaction on ^{28}Si target at the incident momentum of p_π=1.2 GeV/c. The solid line shows result of Batty's DD potential with LOFAt + DWIA, Other line are calculated results with LOFAt + DWIA with potential depth of V_0=-50, -30, -10, 0, +10, +90 MeV(up to down), respectively. Imaginary part is fixed to be -20 MeV.

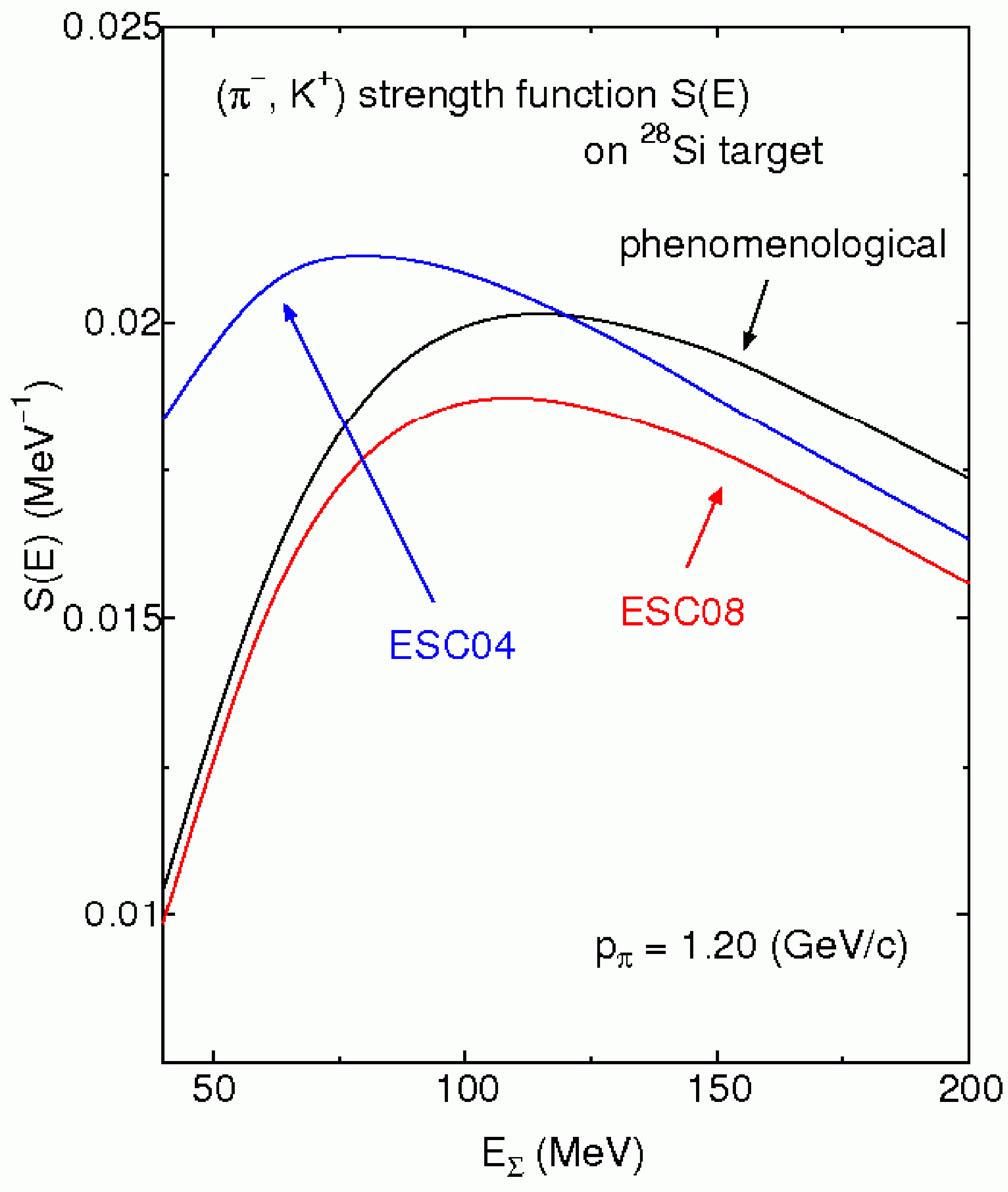
H. Maekawa, et. al., EPJ A33, 269 (2007)

$$\frac{d^2\sigma}{dE_K d\Omega_K} = \frac{1}{\pi}\left(\frac{d\sigma}{d\Omega}\right)_{\pi N \to \bar{K}\Sigma} S(E)$$

strength function
-includes the information of ∑ optical potential

apply **G-matrix folding potential** to **strength function**

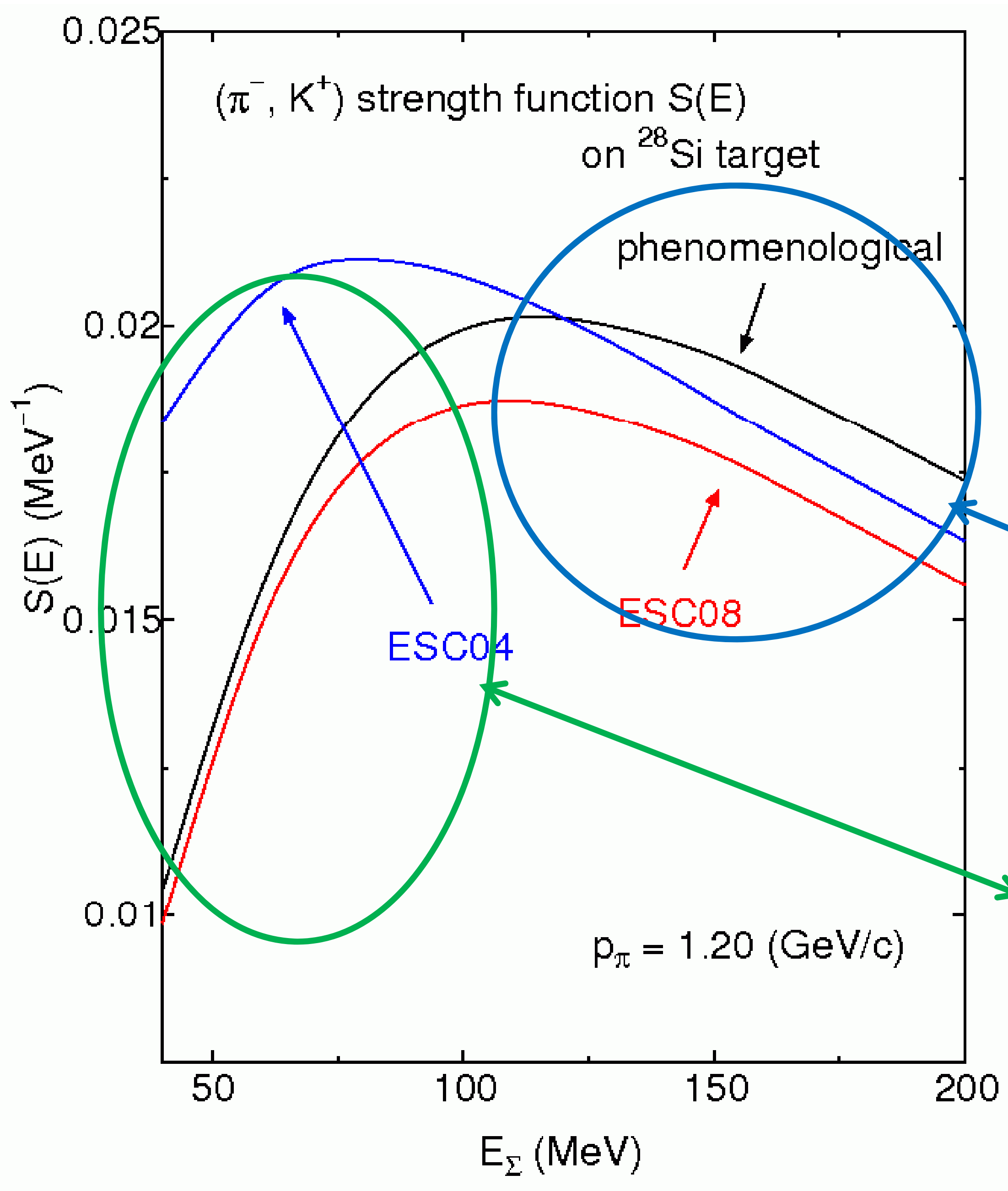
Σ^- quasi-free production spectra from (π^-, K^+) reaction



$$\frac{d^2\sigma}{dE_K d\Omega_K} = \frac{1}{\pi}\left(\frac{d\sigma}{d\Omega}\right)_{\pi N \to \bar{K}\Sigma} S(E)$$

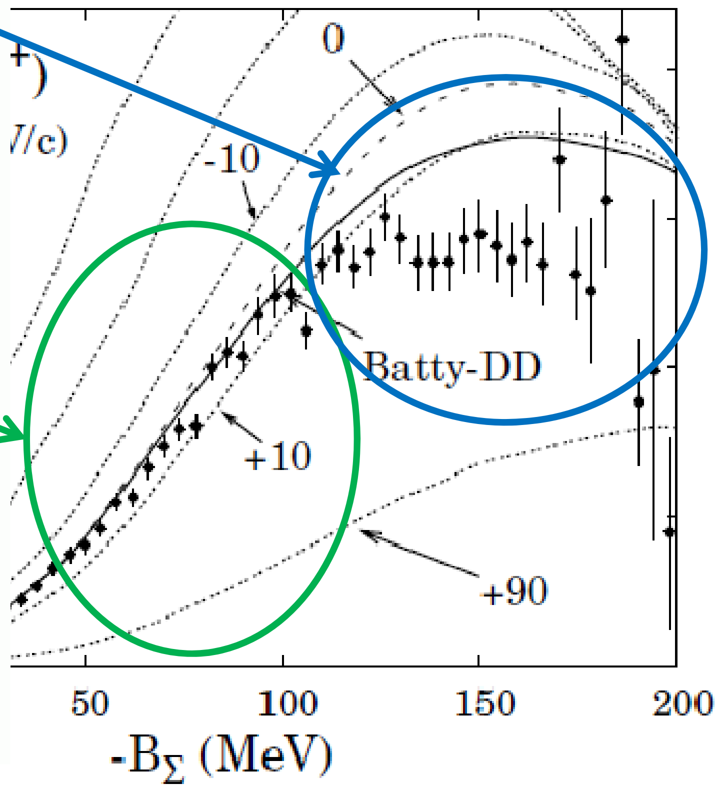
strength function
-includes the information of ∑ optical potential

Σ⁻ quasi-free production spectra from (π⁻, K⁺) reaction



$$\frac{d^2\sigma}{dE_K d\Omega_K} = \frac{1}{\pi}\left(\frac{d\sigma}{d\Omega}\right)_{\pi N \to \bar{K}\Sigma} S(E)$$

strength function



Summary

We have constructed **ΣA optical potential**
though complex G-matrix folding model approach

Elastic scattering (cross section & analyzing power)
- is demonstrated by folding potential with **ESC04** & **ESC08**

Strength function S(E)
- is calculated by folding potentials with **ESC04** & **ESC08**
- compare folding potentials with phenomenological potential
⇒ **ESC08** is apparently better than **ESC04**

Future

1. complete the quasi-free (π^-, K^+) calculation
2. the problem of imaginary part (overestimation)

$\Sigma^0 + {}^{28}Si$ folding potential (LS part)

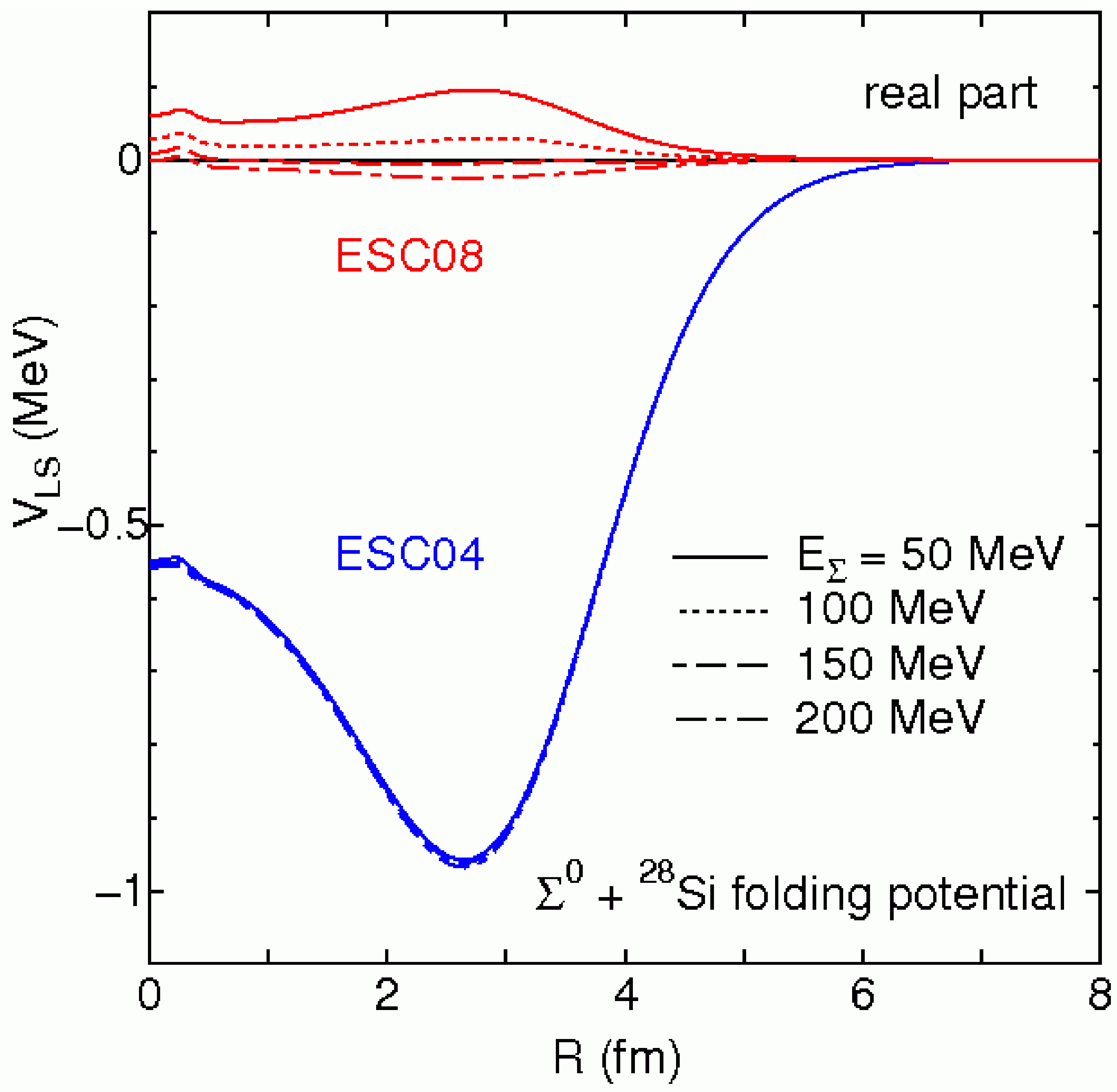


TABLE XXII: Values of U_Σ at normal density and partial wave contributions for ESC04a-d and NSC97f (in MeV).

	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Σ
ESC04a	1/2	11.6	–26.9	2.4	2.7	–6.4	–2.0	–0.8	
	3/2	–11.3	2.6	–6.8	–2.3	5.9	–5.1	–0.2	–36.5
ESC04b	1/2	9.6	–25.3	1.8	1.6	–5.4	–2.1	–0.7	
	3/2	–9.6	9.9	–5.5	–1.9	5.4	–4.6	–0.2	–27.1
ESC04c	1/2	6.4	–20.6	2.4	2.9	–6.7	–1.6	–0.9	
	3/2	–10.7	6.9	–8.8	–2.6	6.0	–5.8	–0.2	–33.2
ESC04d	1/2	6.5	–21.0	2.6	2.4	–6.7	–1.7	–0.9	
	3/2	–10.1	14.0	–8.5	–2.6	5.9	–5.7	–0.2	–26.0
NSC97f	1/2	14.9	–8.3	2.1	2.5	–4.6	0.5	–0.5	
	3/2	–12.4	–4.1	–4.1	–2.1	6.0	–2.8	–0.1	–12.9

various Nijmegen Models

attractive

repulsive

QM-based models

	$^{21}S_0$	$^{23}S_1$	$^{41}S_0$	$^{43}S_1$	sum
Fss	6.1	-20.2	-8.8	48.2	+9.8
fss2	6.7	-23.9	-9.2	41.2	+7.5

ESC04 model → ESC08 model

ESC core = pomeron + ω

- Quark model
  - quark Pauli-forbidden state have an effect on $^3S_1$

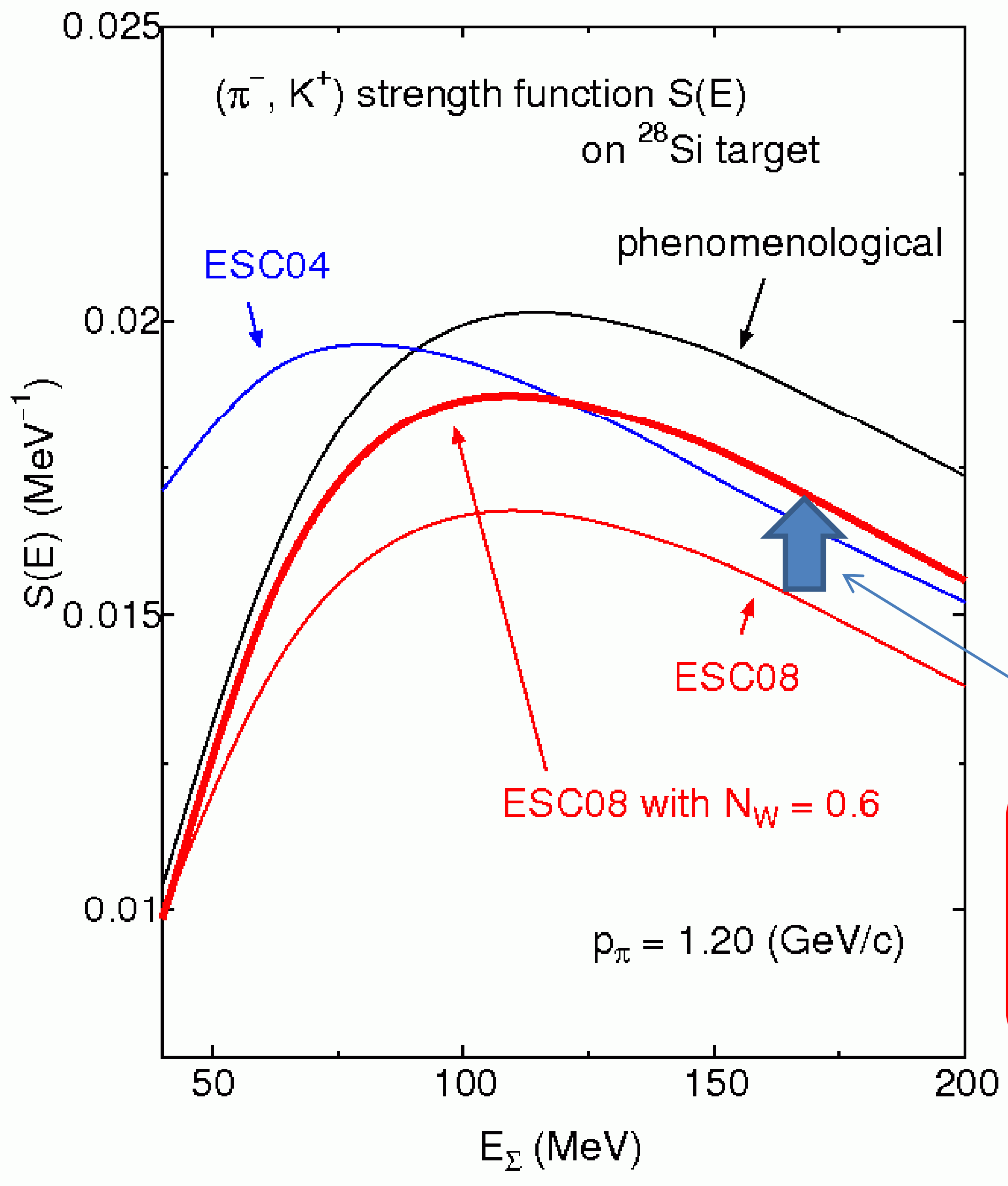
- ESC08 model
  - **<u>Assuming</u>** "equal parts" of ESC and QM are similar to each other
  - adjust $^3S_1$ state by changing the pomeron strengths for the corresponding channels

$g_P \longrightarrow \mathrm{sqrt}(2.5)\, g_P$

model	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Σ	Γ_Σ
NSC97f	1/2	14.9	−9.6	1.9	2.3	−4.0	0.4	−0.4		
	3/2	−12.2	−4.2	−3.8	−1.8	5.5	−2.7	−0.2	−13.9	16.0
ESC04a	1/2	11.6	−29.6	2.4	2.7	−6.3	−2.1	−0.8		
	3/2	−11.3	2.6	−6.8	−2.3	5.9	−5.1	−0.2	−39.2	9.8
ESC04d	1/2	6.7	−22.7	2.6	2.5	−6.7	−1.9	−0.9		
	3/2	−10.1	14.1	−8.5	−2.6	5.8	−5.7	−0.2	−27.5	11.1
ESC07-1	1/2	12.1	−13.6	3.0	0.5	−1.8	−0.2	0.0		
	3/2	−16.6	43.9	−11.7	0.6	4.4	0.1	0.2	+21.0	5.7
ESC07-2	1/2	9.7	−15.1	2.8	0.5	−1.8	−0.3	0.0		
	3/2	−16.3	35.8	−12.9	0.5	4.2	−0.7	0.2	+6.8	6.3

repulsive

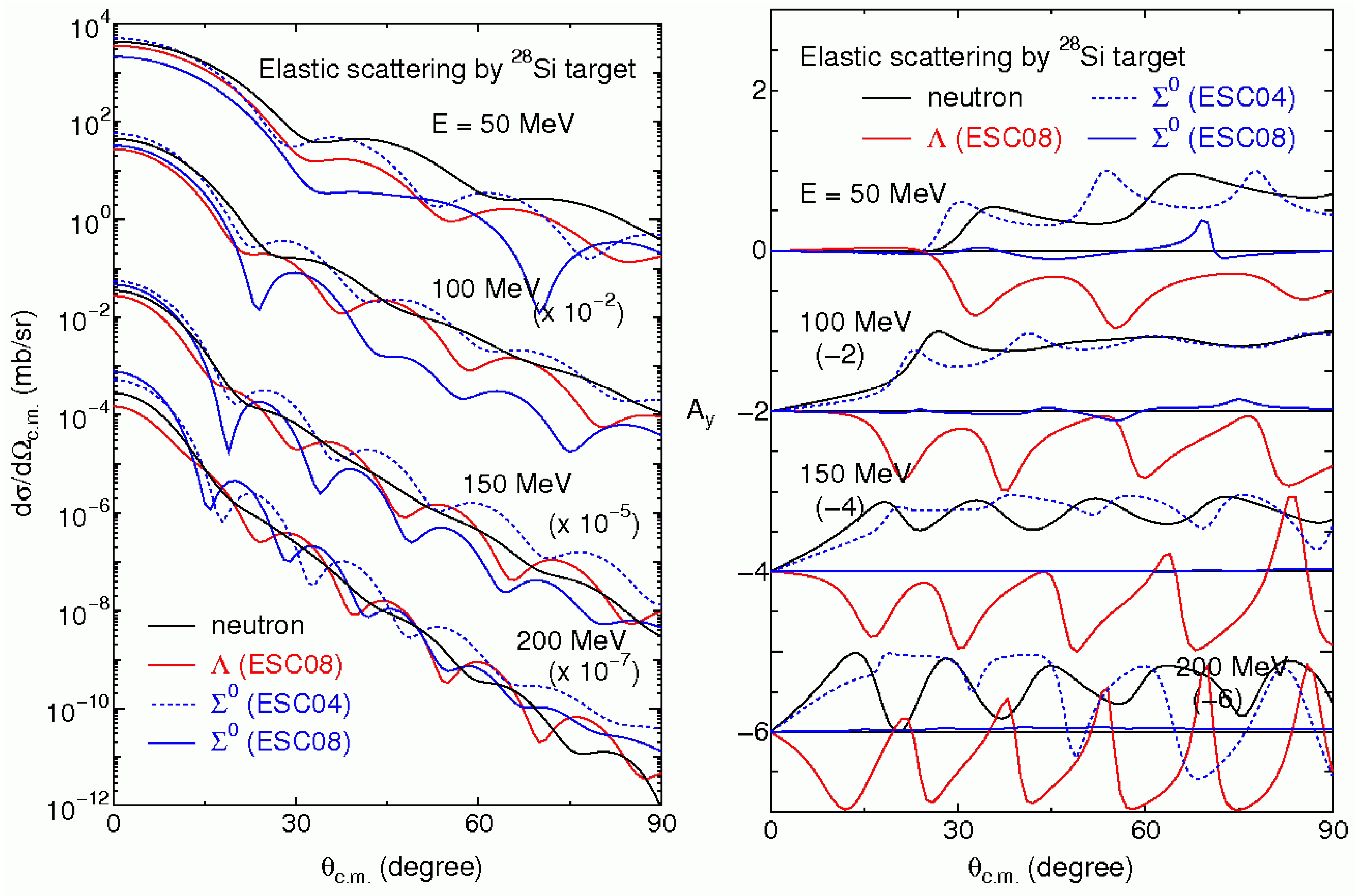
Σ^- quasi-free production spectra from (π^-, K^+) reaction



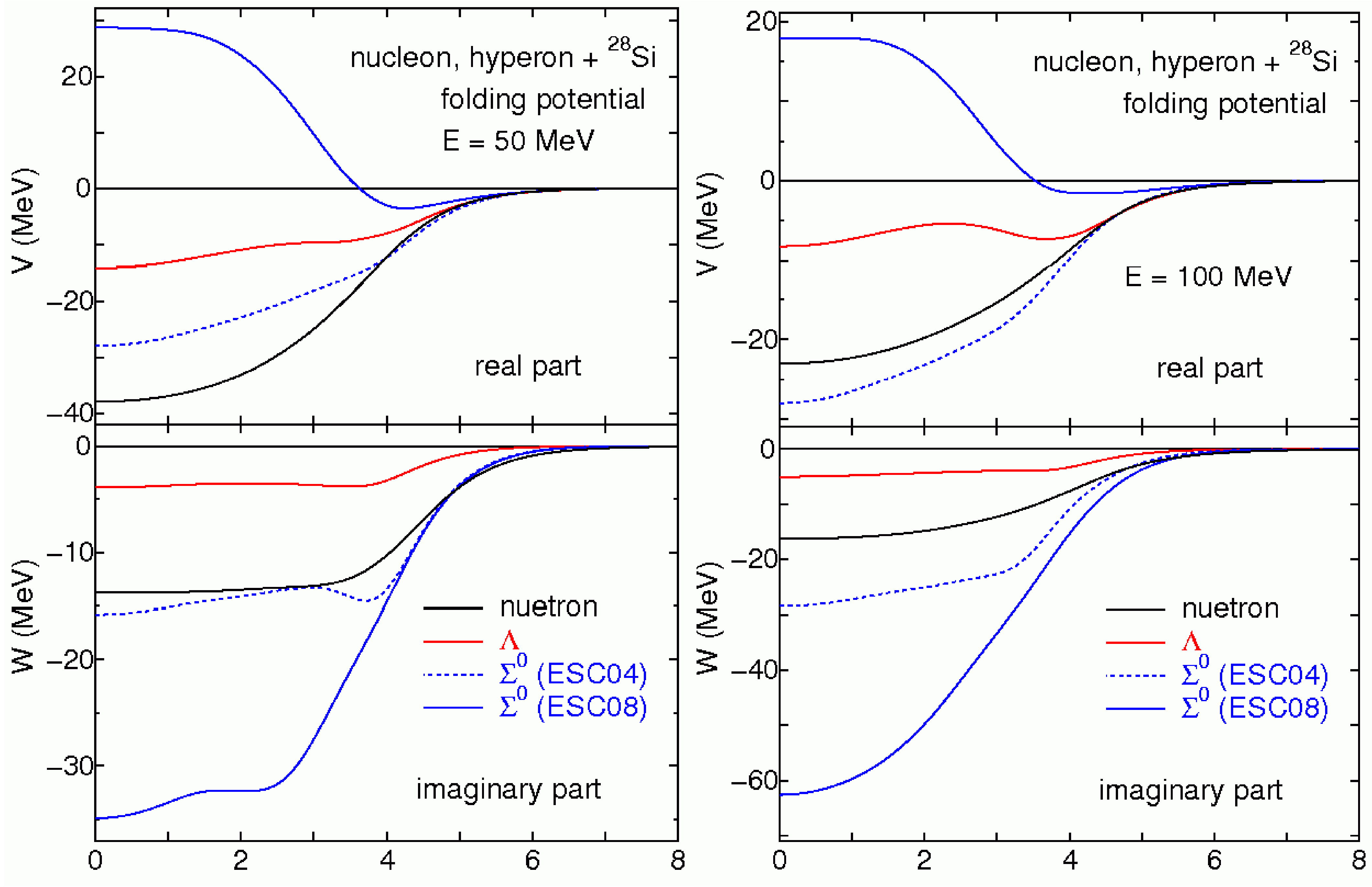
$$\frac{d^2\sigma}{dE_K d\Omega_K} = \frac{1}{\pi}\left(\frac{d\sigma}{d\Omega}\right)_{\bar{K}N\to\pi\Sigma} S(E)$$

<u>strength function</u>
-includes the information of Σ optical potential

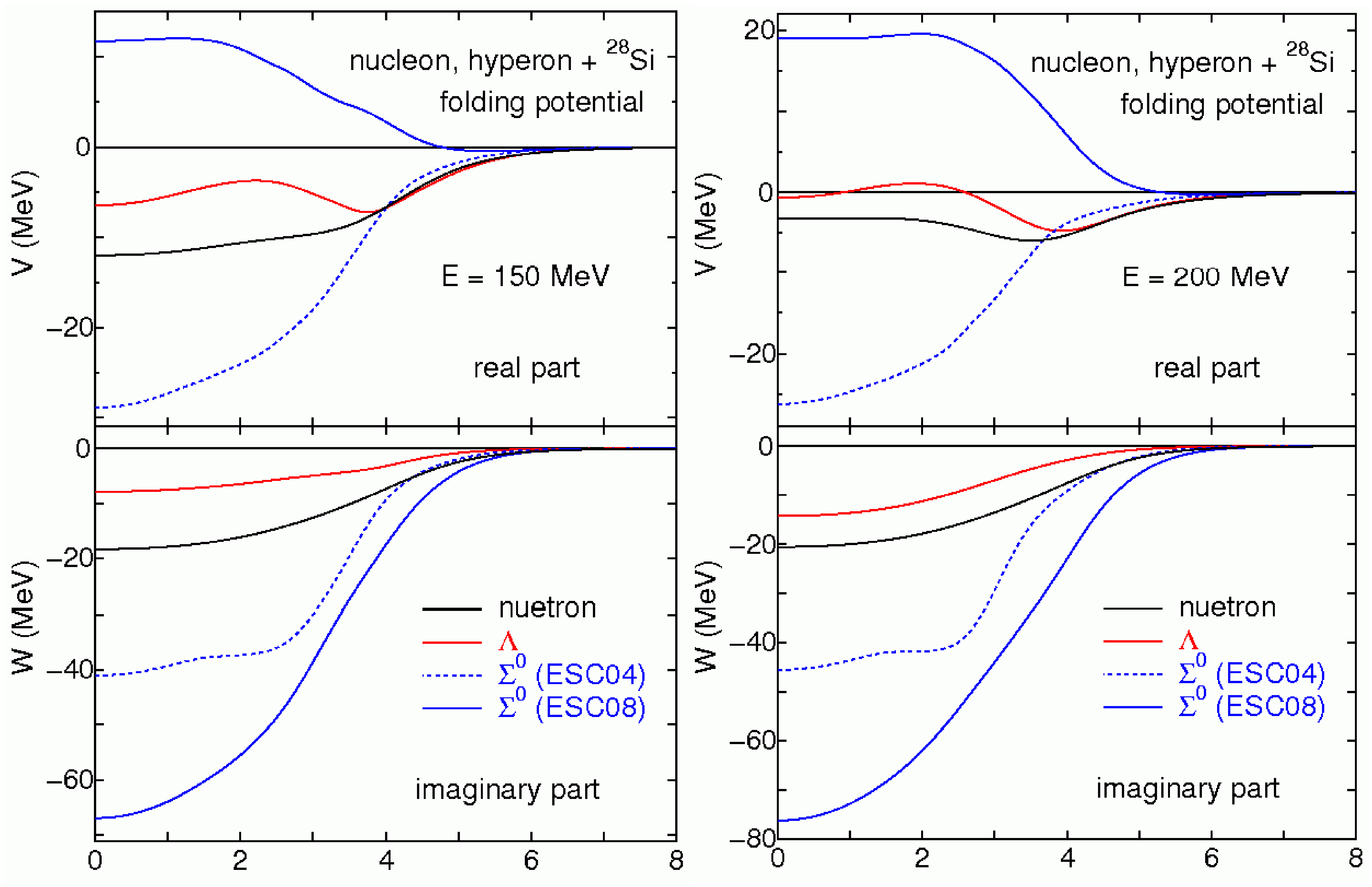
In general, G-matrix overestimates the imaginary part.
- apply N_W fixed by *NA* system



Elastic scattering by ^{28}Si target
E = 50 MeV
100 MeV
(x 10^{-2})
150 MeV
(x 10^{-5})
200 MeV
(x 10^{-7})
neutron
Λ (ESC08)
Σ^0 (ESC04)
Σ^0 (ESC08)
$d\sigma/d\Omega_{c.m.}$ (mb/sr)
$\theta_{c.m.}$ (degree)
Elastic scattering by ^{28}Si target
neutron
Σ^0 (ESC04)
Λ (ESC08)
Σ^0 (ESC08)
E = 50 MeV
100 MeV
(−2)
150 MeV
(−4)
200 MeV
(−6)
A_y
$\theta_{c.m.}$ (degree)



nucleon, hyperon + ^{28}Si
folding potential
E = 50 MeV
real part
V (MeV)
W (MeV)
imaginary part
nuetron
Λ
Σ^0 (ESC04)
Σ^0 (ESC08)
nucleon, hyperon + ^{28}Si
folding potential
E = 100 MeV
real part
V (MeV)
W (MeV)
imaginary part
nuetron
Λ
Σ^0 (ESC04)
Σ^0 (ESC08)


nucleon, hyperon + ^{28}Si
folding potential
E = 150 MeV
real part
V (MeV)
W (MeV)
imaginary part
nuetron
Λ
Σ^0 (ESC04)
Σ^0 (ESC08)
nucleon, hyperon + ^{28}Si
folding potential
E = 200 MeV
real part
imaginary part

Nijmegen soft-core models
(NSC89/97, ESC04/07)

Origin of cores

pomeron

ω meson

Repulsive cores are similar to each other in all channels

Different from Quark-model core

Tamagaki's Quark Pauli-forbidden states ?

ハイパー核で領域Ⅲを見れるか？

原子核現象を通じて核力の領域IIIの異なるmodelingを区別することはできなかった

Σ-Nucleus potentials U_Σ

Intermediate states in (π,K) reactions

Σ-nucleus scattering

·····

Interesting problems
- repulsive ?
- isospin-dependence
- spin-orbit interaction
- imaginary parts (scattering & conversion)

Are repulsive ∑-potentials obtained from Nijmegen models?

NHC-F ok
but…

No (maybe) ← standard NSC/ESC modeling

in spite of elaborate works by Rijken

Import the feature of quark model !

TABLE XXII: Values of U_Σ at normal density and partial wave contributions for ESC04a-d and NSC97f (in MeV).

	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Σ
ESC04a	1/2	11.6	−26.9	2.4	2.7	−6.4	−2.0	−0.8	
	3/2	−11.3	2.6	−6.8	−2.3	5.9	−5.1	−0.2	−36.5
ESC04b	1/2	9.6	−25.3	1.8	1.6	−5.4	−2.1	−0.7	
	3/2	−9.6	9.9	−5.5	−1.9	5.4	−4.6	−0.2	−27.1
ESC04c	1/2	6.4	−20.6	2.4	2.9	−6.7	−1.6	−0.9	
	3/2	−10.7	6.9	−8.8	−2.6	6.0	−5.8	−0.2	−33.2
ESC04d	1/2	6.5	−21.0	2.6	2.4	−6.7	−1.7	−0.9	
	3/2	−10.1	14.0	−8.5	−2.6	5.9	−5.7	−0.2	−26.0
NSC97f	1/2	14.9	−8.3	2.1	2.5	−4.6	0.5	−0.5	
	3/2	−12.4	−4.1	−4.1	−2.1	6.0	−2.8	−0.1	−12.9

various Nijmegen Models

QM-based models

	$^{21}S_0$	$^{23}S_1$	$^{41}S_0$	$^{43}S_1$	sum
Fss	6.1	-20.2	-8.8	48.2	+9.8
fss2	6.7	-23.9	-9.2	41.2	+7.5

Feature of QM core

K. Shimizu, S. Takeuchi and A.J. Buchmann, PTP, Suppl. 137 (2000)

$BB(TS)$	N	$V_{mag}(R=0)$ [MeV]	r_c [fm]
$NN(01)$	$\frac{10}{9}$	340.7	0.38
$NN(10)$	$\frac{10}{9}$	438.0	0.52
$N\Lambda(\frac{1}{2}0)$	1	379.9	0.57
$N\Sigma(\frac{1}{2}0)$	$\frac{1}{9}$	302.4	0.91
$N\Lambda(\frac{1}{2}1)$	1	262.7	0.46
$N\Sigma(\frac{1}{2}1)$	1	213.9	0.37
$N\Sigma(\frac{3}{2}0)$	$\frac{10}{9}$	389.6	0.50
$N\Sigma(\frac{3}{2}1)$	$\frac{2}{9}$	343.9	0.94

Almost Pauli-forbidden states

$SU(6)_{fs}$-contents of the various potentials on the isospin,spin basis.

	(S,I)	$V = aV_{[51]} + bV_{[33]}$
$NN \to NN$	$(0,1)$	$V_{NN}(I=1) = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$NN \to NN$	$(1,0)$	$V_{NN} = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$\Lambda N \to \Lambda N$	$(0,1/2)$	$V_{\Lambda\Lambda} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$\Lambda N \to \Lambda N$	$(1,1/2)$	$V_{\Lambda\Lambda} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$\Sigma N \to \Sigma N$	$(0,1/2)$	$V_{\Sigma\Sigma} = \frac{17}{18}V_{[51]} + \frac{1}{18}V_{[33]}$
$\Sigma N \to \Sigma N$	$(1,1/2)$	$V_{\Sigma\Sigma} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$\Sigma N \to \Sigma N$	$(0,3/2)$	$V_{\Sigma\Sigma} = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$\Sigma N \to \Sigma N$	$(1,3/2)$	$V_{\Sigma\Sigma} = \frac{8}{9}V_{[51]} + \frac{1}{9}V_{[33]}$

Adjust $V_{[51}$

Pauli-forbidden state exist in $V_{[51]}$

Recent Nijmegen approach

ESC core = pomeron + ω

Assuming
"equal parts" of ESC and QM are similar to each other

Almost Pauli-forbidden states in [51] are taken into account by changing the pomeron strengths for the corresponding channels

$g_P \rightarrow \mathrm{sqrt}(2.5)\, g_P$

ESC07 models

Quark Pauli-forbidden states の存在をいかにして実証するか？

ΣN phase-shift analysis (可能?)

Σ-nucleus potentialを通じて ☆

↓

Quasi-free Σ production

Σ-nucleus scattering

model	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Σ	Γ_Σ
NSC97f	1/2	14.9	–9.6	1.9	2.3	–4.0	0.4	–0.4		
	3/2	–12.2	–4.2	–3.8	–1.8	5.5	–2.7	–0.2	–13.9	16.0
ESC04a	1/2	11.6	–29.6	2.4	2.7	–6.3	–2.1	–0.8		
	3/2	–11.3	2.6	–6.8	–2.3	5.9	–5.1	–0.2	–39.2	9.8
ESC04d	1/2	6.7	–22.7	2.6	2.5	–6.7	–1.9	–0.9		
	3/2	–10.1	14.1	–8.5	–2.6	5.8	–5.7	–0.2	–27.5	11.1
ESC07-1	1/2	12.1	–13.6	3.0	0.5	–1.8	–0.2	0.0		
	3/2	–16.6	43.9	–11.7	0.6	4.4	0.1	0.2	+21.0	5.7
ESC07-2	1/2	9.7	–15.1	2.8	0.5	–1.8	–0.3	0.0		
	3/2	–16.3	35.8	–12.9	0.5	4.2	–0.7	0.2	+6.8	6.3

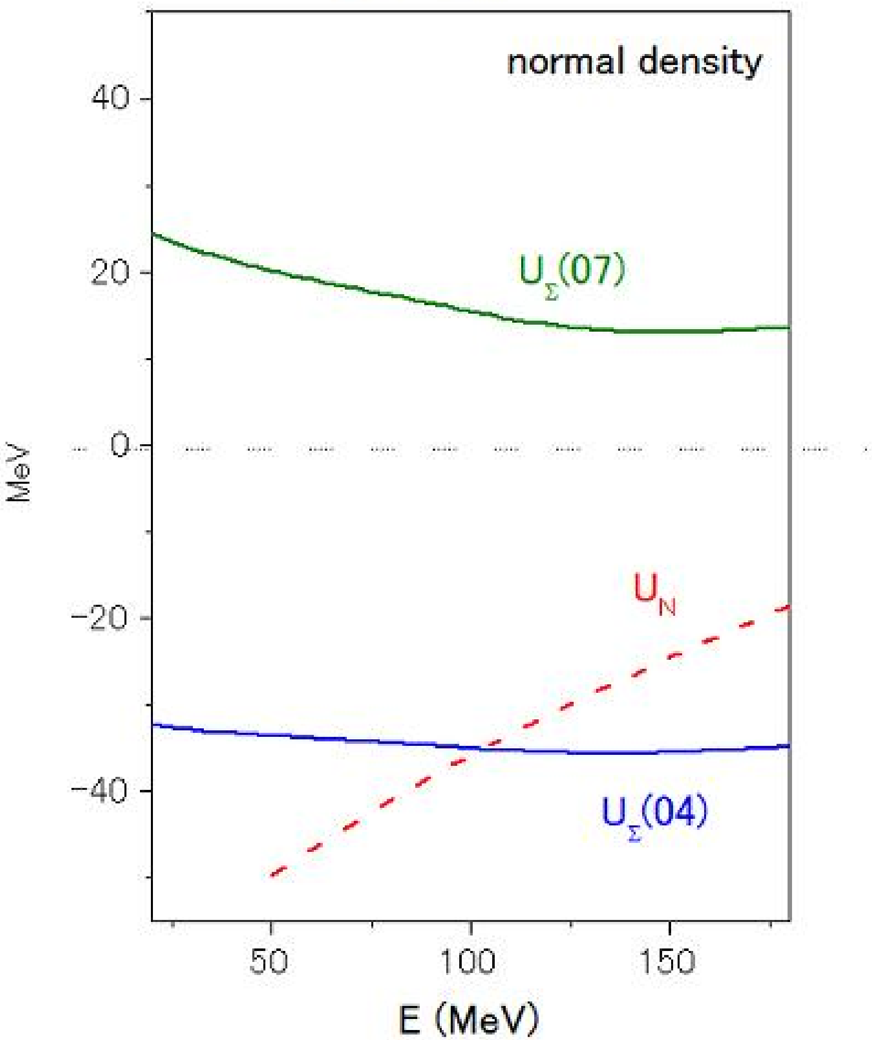
Optical potential

Σ-nucleus folding potential derived from complex G-matrix

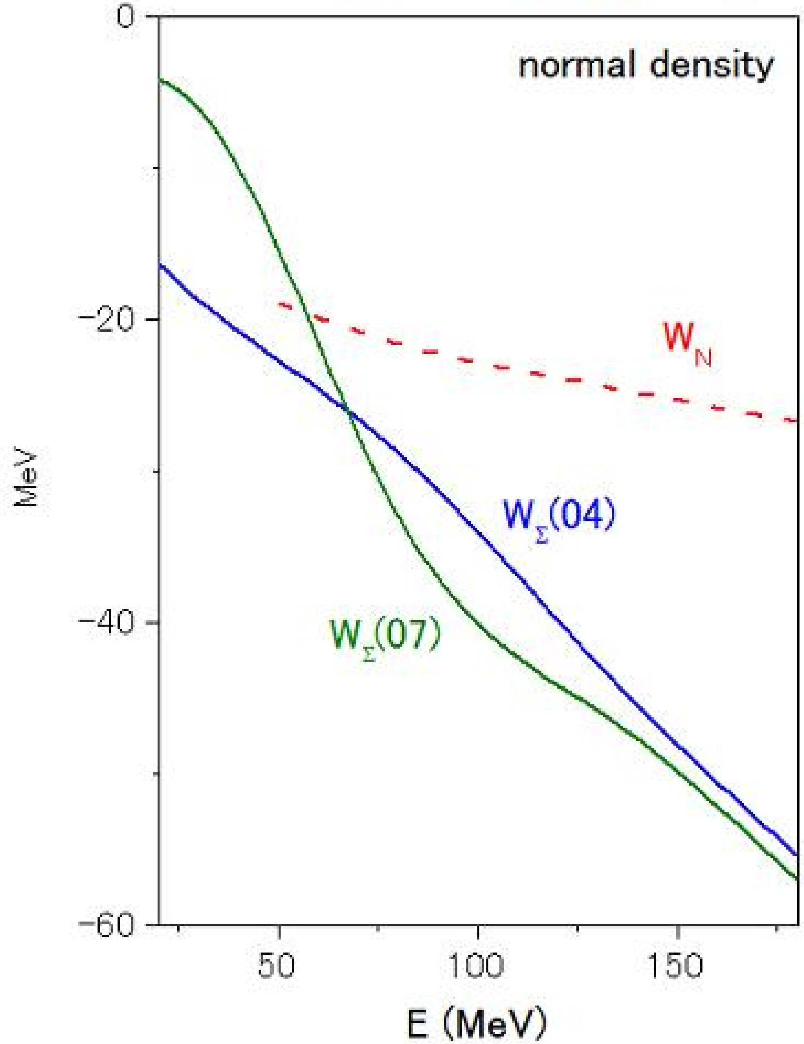
$$G_{\Sigma N}(r; E, k_F)$$

In N-nucleus scattering problem
physical observables can be reproduced
with "no free parameter"

Real part



Imaginary part



Effective Mass and E-dependence of U_Σ relation

$$m^*_\Sigma = \frac{M^*_\Sigma}{M_\Sigma}$$

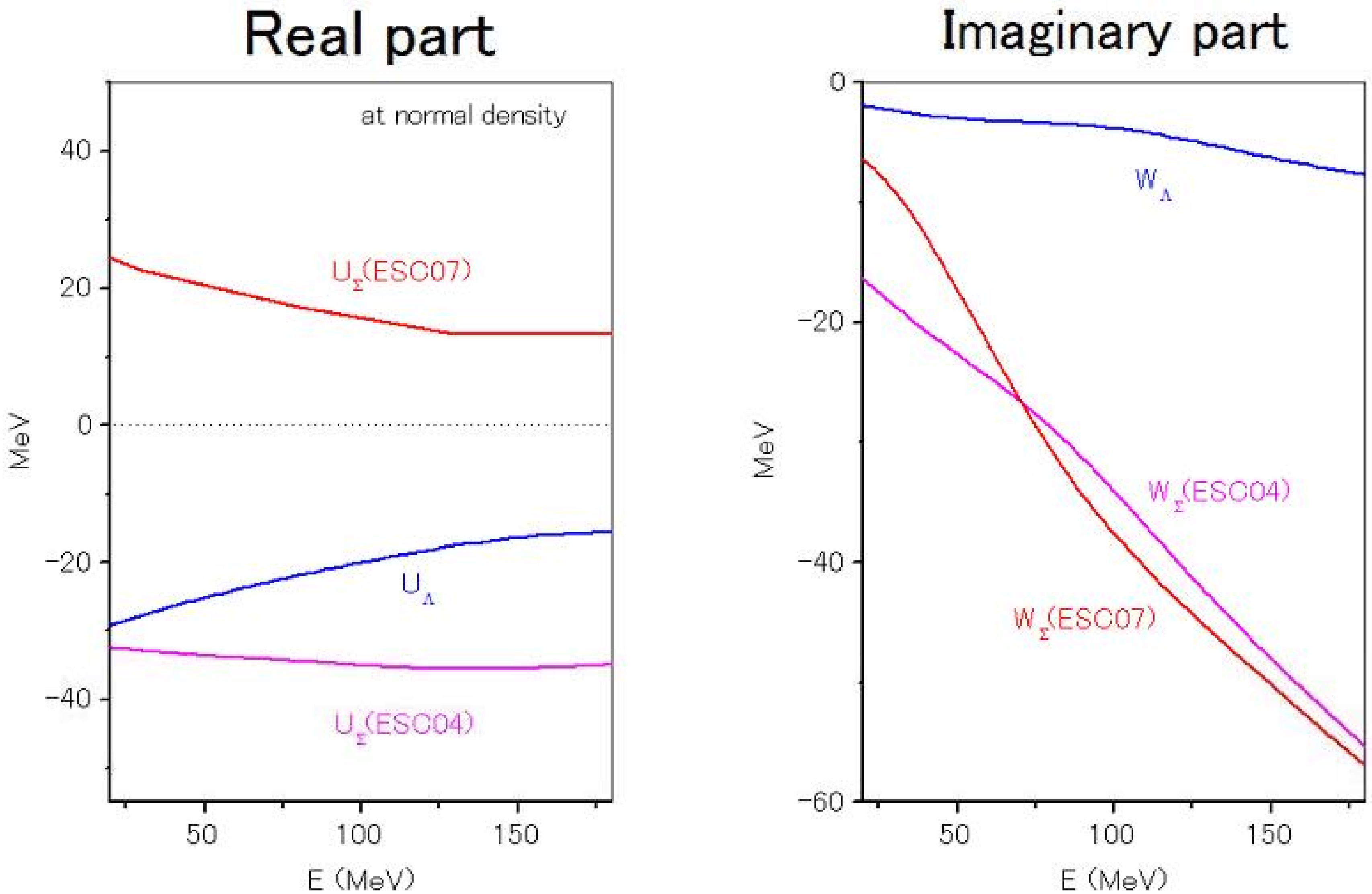
with ESC07

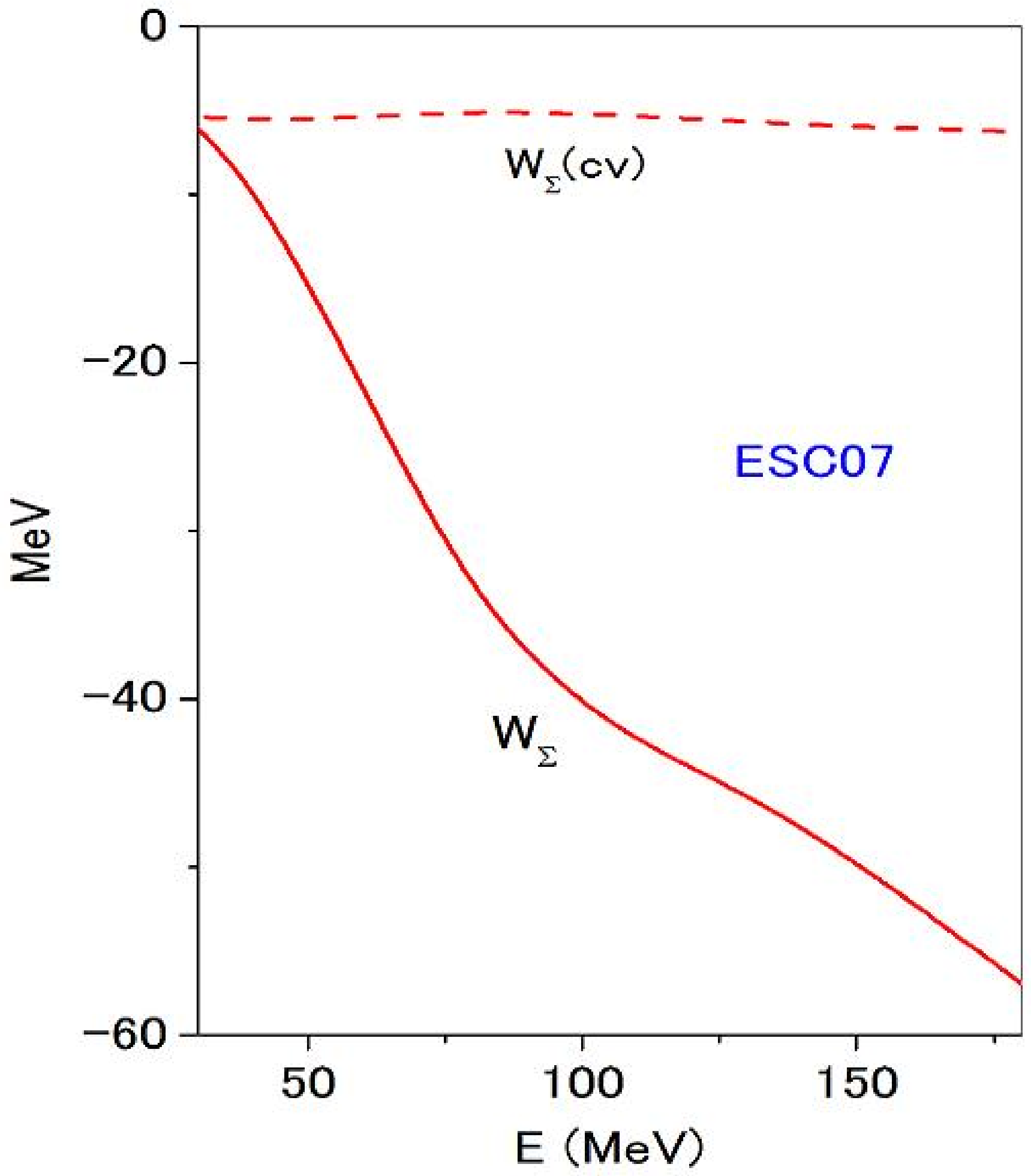
T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	dU/dT	m^*_Σ
1/2	-0.09	-0.01	0.03	0.00	-0.13	-0.03	-0.05		
3/2	0.13	-0.23	-0.07	0.01	0.10	0.05	-0.01	-0.31	1.46

$$m^*_\Sigma = \frac{1}{1 + \frac{dU_\Sigma}{dT}}$$

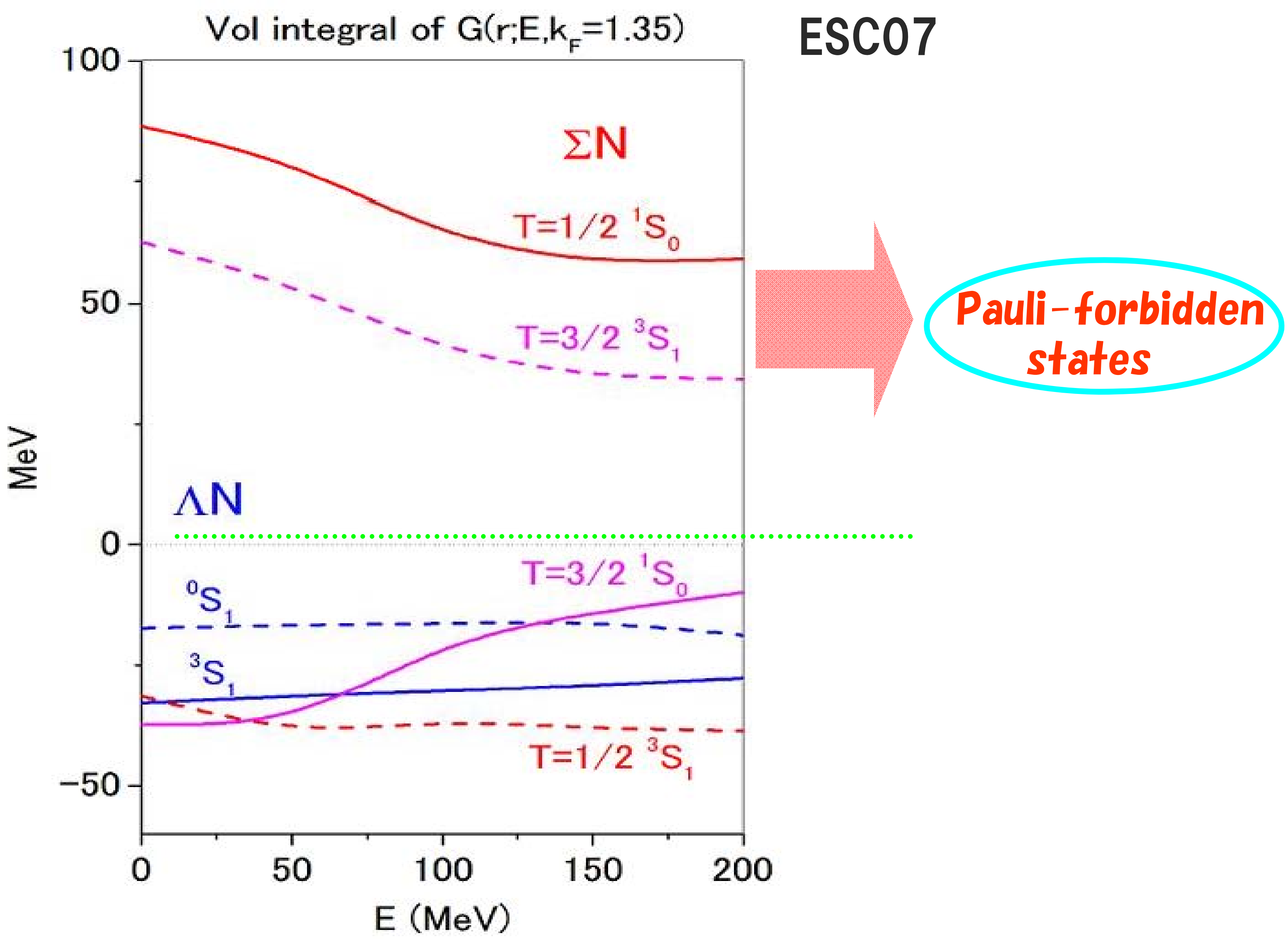
$$\frac{dU_\Sigma}{dE} = 1 - m^*_\Sigma$$

If $m_\Sigma{}^* > 1$ then $U'_\Sigma < 0$


Real part
at normal density
U_Σ(ESC07)
U_Λ
U_Σ(ESC04)
MeV
E (MeV)
Imaginary part
W_Λ
W_Σ(ESC04)
W_Σ(ESC07)
MeV
E (MeV)



0
−20
−40
−60
MeV
50
100
150
E (MeV)
W_Σ(cv)
W_Σ
ESC07



W_{scatt}が大きい理由

U_Σ(real)　cancelingが効く

W_Σには"2乗和"で効く

Improved LDA by JLM

Phys. Rev. C10 (1974) 1391

$$U(\rho, E) = \sum_{ij} a_{ij}\, \rho^i\, E^{j-1}$$

$$U(r; E) = (t\sqrt{\pi})^{-3} \int U(\rho(r'), E)\, \exp(-|\mathbf{r} - \mathbf{r}'|^2/t^2) d\mathbf{r}'$$

simple LDA： U(ρ (r),E)

by Maekawa, at al.

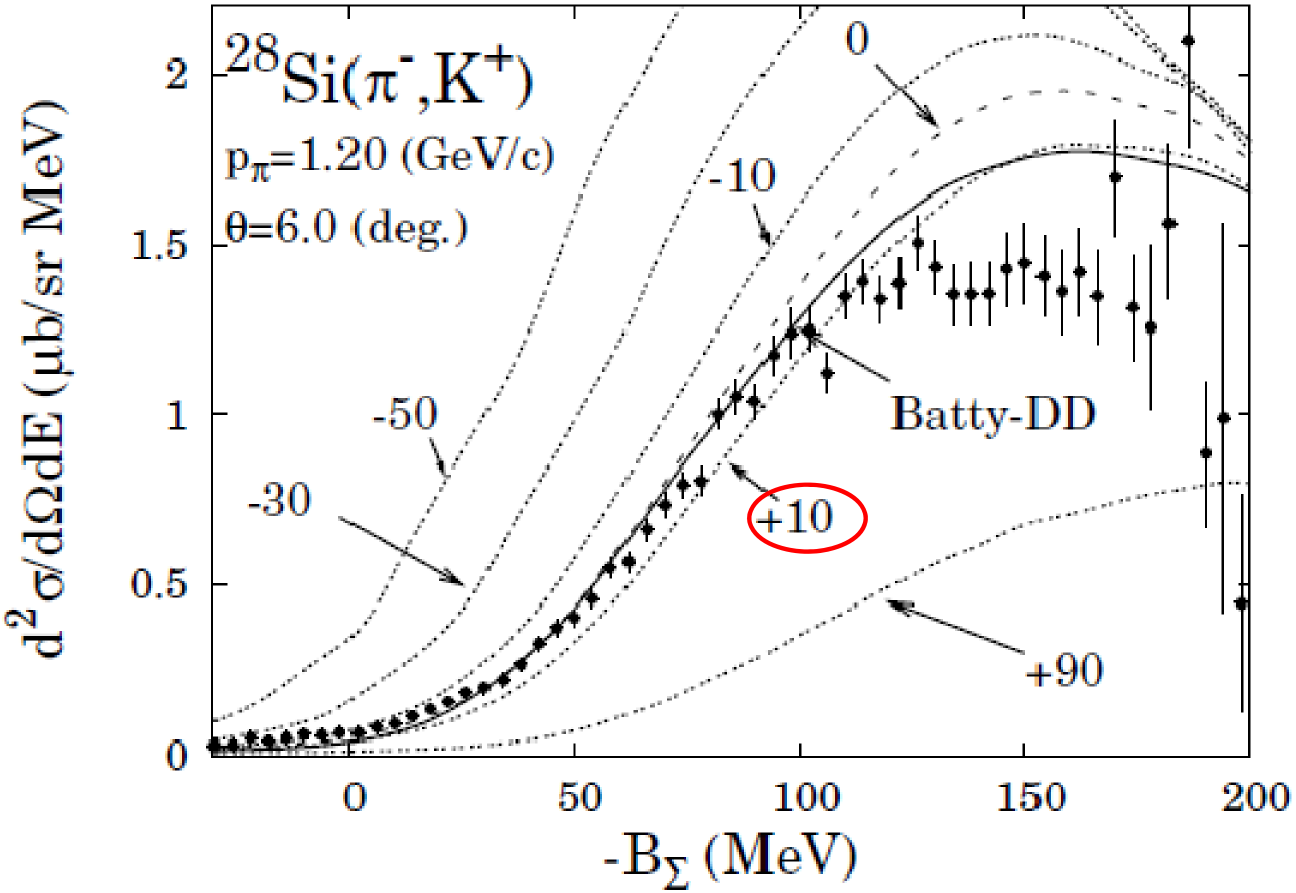
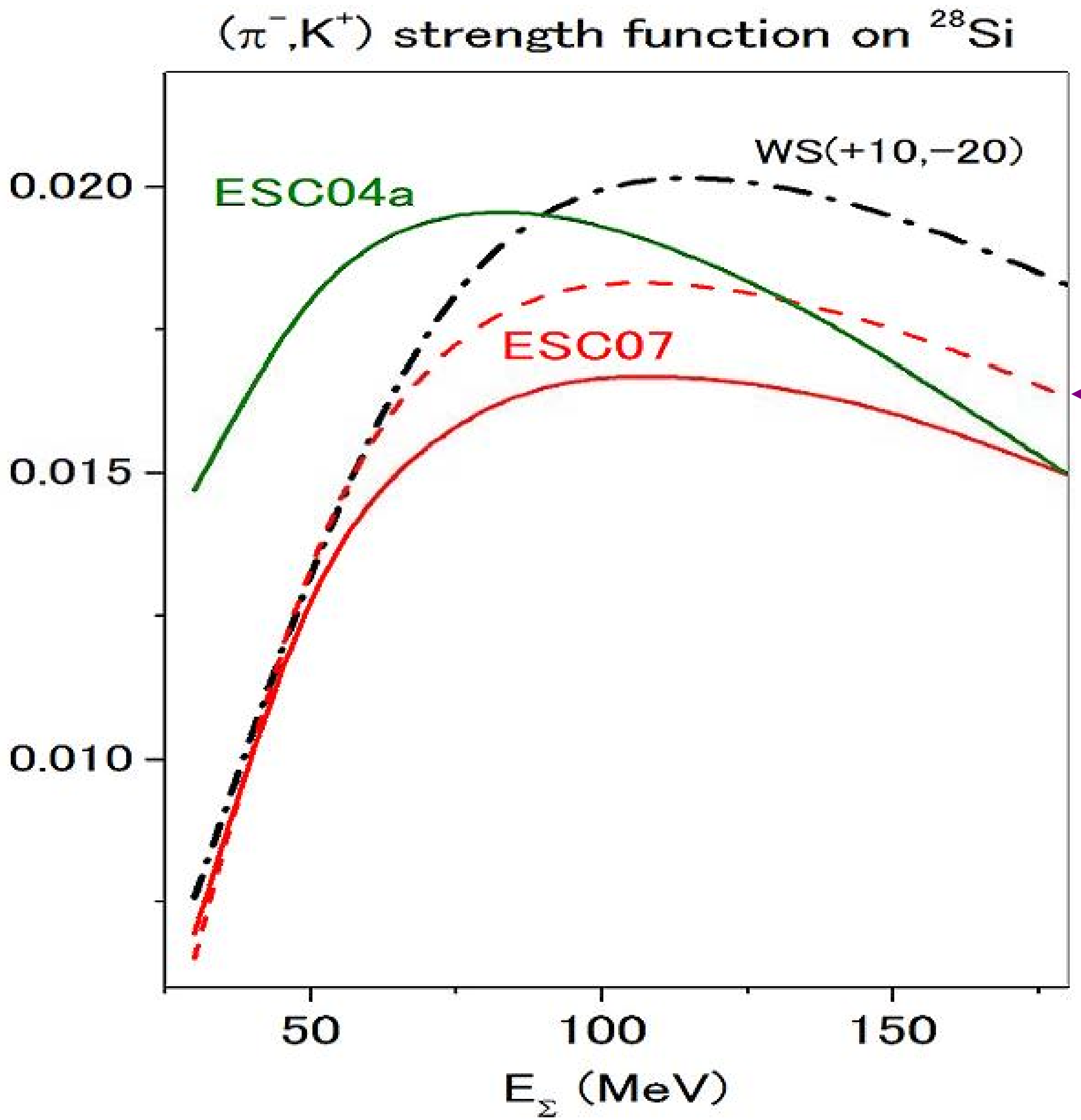


Fig. 2. Differential cross section of (π^-, K^+) reaction on ^{28}Si target at the incident momentum of p_π=1.2 GeV/c. The solid line shows result of Batty's DD potential with LOFAt + DWIA, Other line are calculated results with LOFAt + DWIA with potential depth of V_0=-50, -30, -10, 0, +10, +90 MeV(up to down), respectively. Imaginary part is fixed to be -20 MeV.



$N_W * U_{imag}$

N_W=0.65
Same as NA case

In general, G-matrix overestimates U_{imag} as seen in N-nucleus systems