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Ξ- binding in a Generalized Mass Formula and the Quark-Meson Coupling Model

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Hypernucleus: Hyperons Bound in Nuclei

Protons and neutrons (Baryons) have **up (u)** and **down (d) quarks** inside. Hyperons are also Baryons with **strange** (s) quark(s) in addition to **up/down** quarks.

Baryons:

- p=(**uud**), n=(**udd**) ⇨ **Nucleons**

- Λ^0=(**uds**), Σ^+(**uus**), Ξ^-=(**dss**), Ξ^0=(**uss**) ⇨ **Hyperons (Y)**

Hypernucleus: consists of nucleons (n, p) + hyperon (Y)

Found so far:

Hypernuclei with:

Λ^0	(S= -1)	~ Fifty	$\Lambda\Lambda$-hypernuclei (Three)
Σ^+	(S= -1)	One	
Ξ^-	(S= -2)	Six (with very large error bars)	

Binding energies of hypernuclei have been extensively studied in Relativistic Mean Field (RMF) model.

Generalized Bethe-Weizsäcker mass formula

As a properly constructed mass formula provides a **quick check** on the more complex **RMF** calculation, Dover and Gal [**NPA560 (1993) 559**] developed a generalized Bethe-Weizsäcker mass formula from a {p,n} Fermi gas model.

Table I. Parameter sets for use in the GBW formula.[a]

	a_V	$b_V^{(\Lambda)}$	$b_V^{(\Xi)}$	a_Σ	a_Ξ	$a_{\Sigma\Xi}$	a_Y	a_{YY}
set I	10.7	−35.5	−16.75	43	23.7	57.1	45	7.7
set II	28.7	− 5.5	− 4.75	43	23.7	57.1	45	7.7

[a] The parameter values are given in MeV. Independently of which set is used, the Coulomb and surface parameters are given by $a_c = a_c^{(Y)} = 0.72$ MeV, $a_s^{(Y)} = a_s = 15$ MeV.

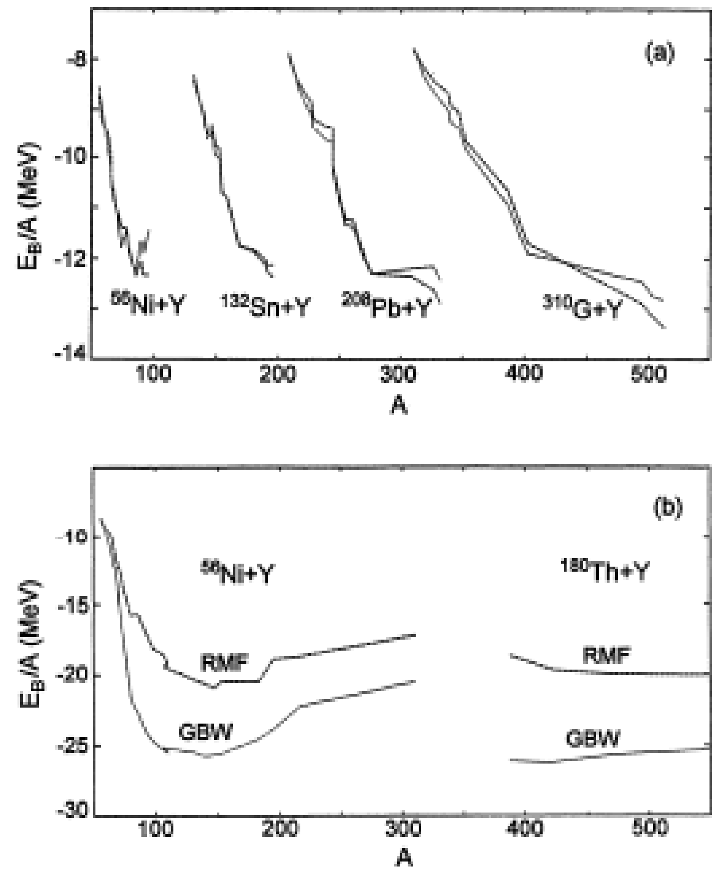


Fig. 3. E_B/A as function of A for multi-strange nuclei based on nuclear cores as marked. The upper and lower parts, (a) and (b) respectively, correspond to Model 1 and Model 2 respectively of the RMF calculations[11] (dashed lines), and to parameter sets I and II respectively of the GBW formula[10] (dotted lines), as given in Table I.

Later on, Balberg et al **[PPTP, 117 (1994)]** fitted the **RMF** results of Schaffner et al [**Ann. Phys. 235 (1994)35]** by re-adjusting the parameters of the of Dover and Gal.

Set I	without YY interactions
Set II	with YY interactions

Dover and Gal's Generalized BW (GBW) mass formula for Strange Hadronic Matter

C. B. Dover and A. Gal, Nucl. Phys. A560(1993)559

Formula for multi Λ :

A = N+Zc+Λ , x= (N-Zc)/A and y=[(N+Zc)/2 –Λ]/A

$$BE(N,Z,\Lambda) = a_v A - b_v yA - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_x x^2 A - a_y y^2 A$$

Formula for multi strange hypernuclei (including cascades) :

A=N+Zc+Λ+Ξ⁻+Ξ⁰ , x= (N-Zc)/A, y= N+Zc+Λ+Ξ⁰+Ξ⁻, u = (Ξ⁰–Ξ⁻)/A

$$BE(p,n,\Lambda,\Xi^o,\Xi^-) = a_v A - b_v^y yA - b_v^w wA - a_s A^{2/3}$$
$$- a_c \left(w + \frac{u}{2} - \frac{x}{2} \right)^2 A^{5/3} - a_x x^2 A - a_u u^2 A - a_w w^2 A - a_y y^2 A - a_{wy} wyA$$

w= {N+Zc–(Ξ0+Ξ–)}/A charge Z = Zc –Ξ- = (w+u/2-x/2)A

Liquid Drop Mass Formula for Binding Energy

Bethe-Weiszäcker (BW) mass formula for medium-heavy nuclei:

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A} + \delta$$

a_v=15.777 MeV, **a_s =18.34 MeV, a_c=0.71 MeV, a_{sym}=23.21 MeV**,

δ = 12 $A^{-1/2}$ for even N and even Z

= -12 $A^{-1/2}$ for odd N and odd Z

= 0 for odd A

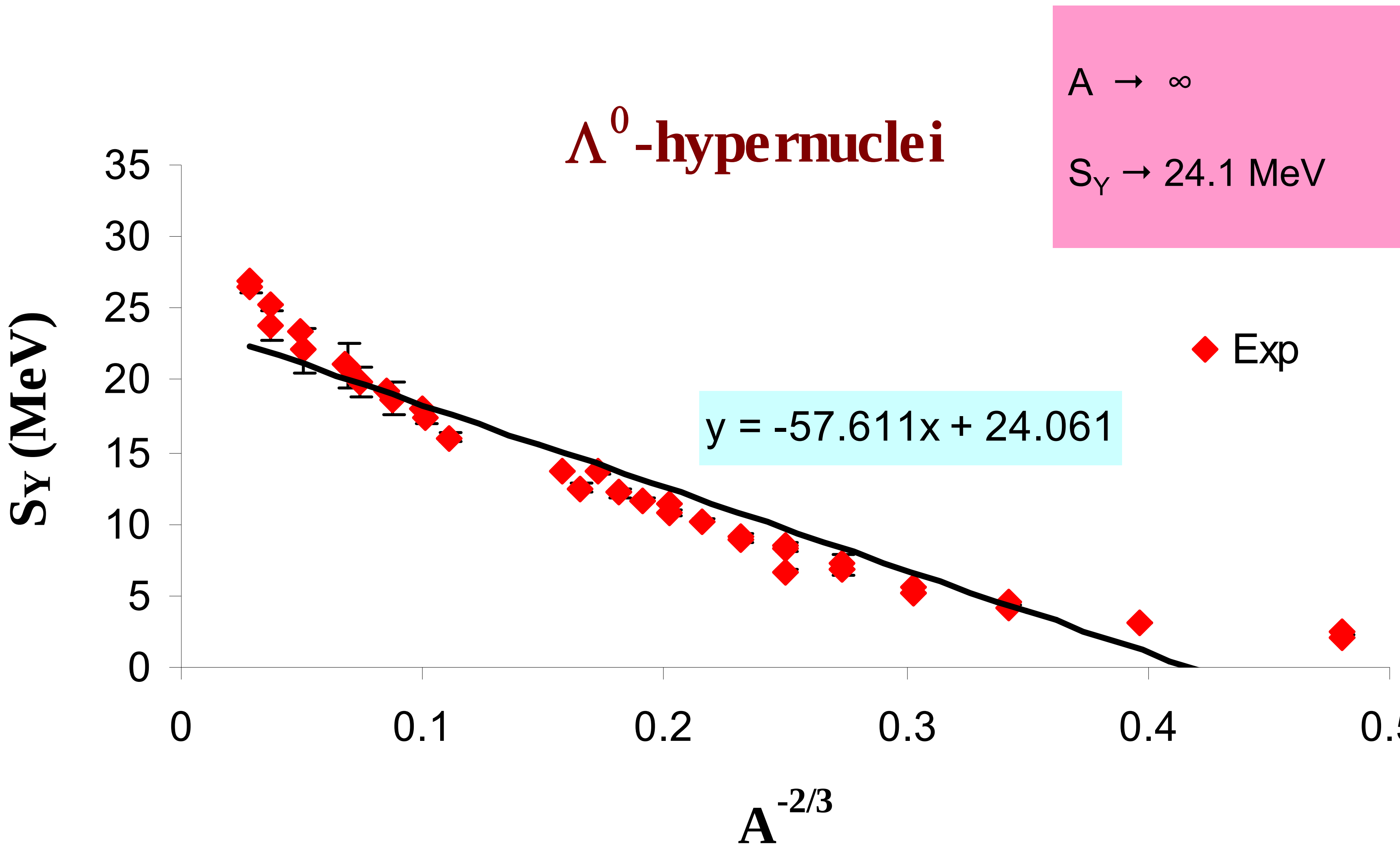
Modified BW formula for light nuclei (BWM):

C. Samanta & S. Adhikari, Phys. Rev. C 65 (2002) 037301

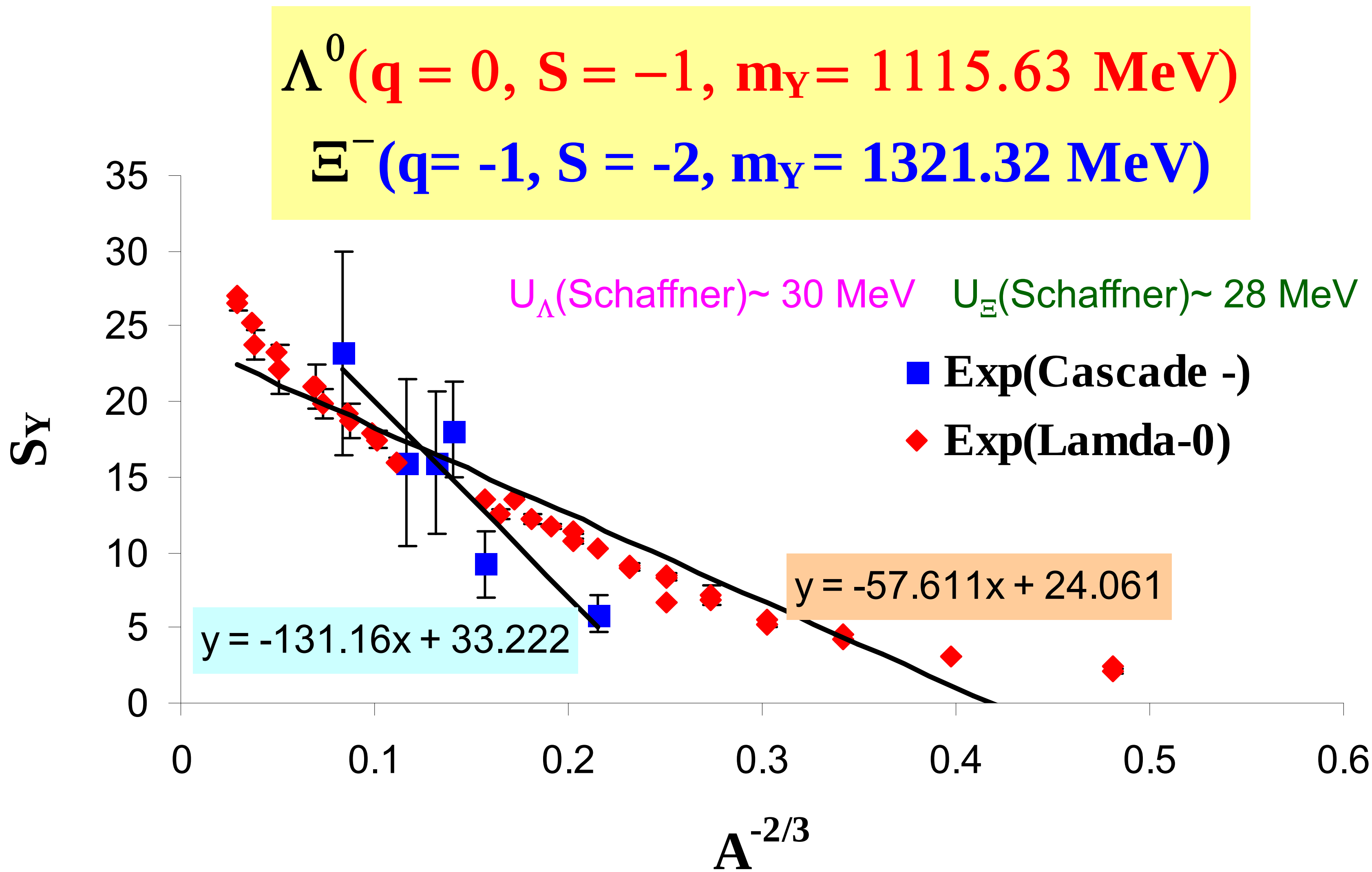
$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A\left(1+\exp^{-A/17}\right)} + \delta_{new}$$

$$\delta_{new} = \delta\left(1-\exp^{-A/30}\right)$$

Λ^0-separation Energy versus Mass Number Plot



Λ^0, Ξ^- Separation Energy versus Mass Number Plot



Generalized Mass Formula for All Nuclei (BWMH)

A systematic search using experimental hyperon-separation energy **(S_Y)** data for Λ^o, $\Lambda\Lambda$, Σ^+, Ξ^--hypernuclei leads to a generalized mass formula (called **BWMH)** which is valid for normal nuclei (S=0) as well as Hypernuclei (S≠0).

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(N-Z_c)^2}{A\left(1+\exp^{-A/17}\right)} + \delta_{new} - n_Y\left[48.7\frac{|S|}{A^{2/3}} + 0.0035 m_Y - 26.7\right]$$

Explicitly depends on the strangeness (S_Y) and mass (m_Y) of the hyperon

Hyperon	S_Y	m_Y (MeV)
Λ	-1	1115.63
Ξ^0	-2	1315
Ξ^-	-2	1321.32

n_Y = no. of hyperons in a nucleus
m_Y = mass of hyperon in MeV
S = strangeness no.of the hyperon,
A = N + Z_c+ n_Y = total no. of baryons
Z_c = no. of protons,
Z = Z_c + n_Y q = Z_Q = net charge no.
q = charge no. of Hyperon with proper sign (viz., q= -1, 0, 1)

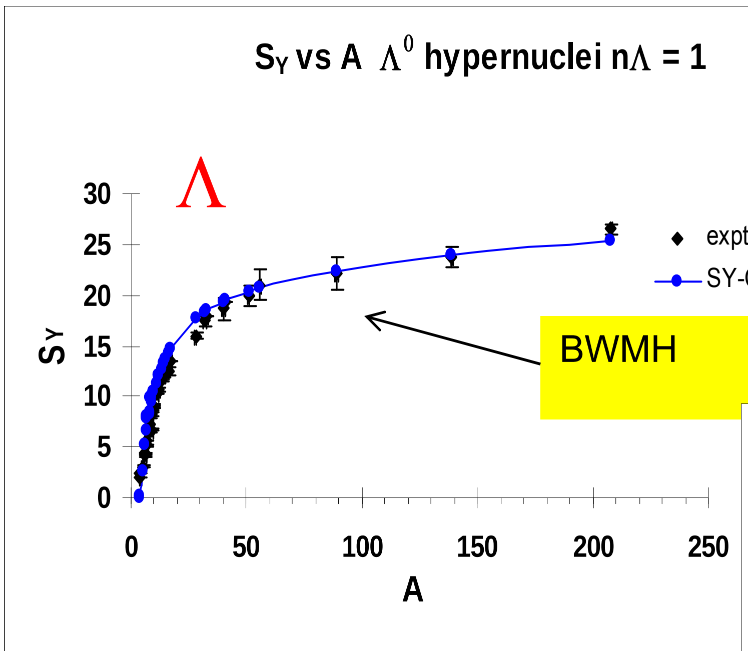
C. Samanta, P. Roy Chowdhury ,D. N. Basu , J.Phys.G:Nucl.Part.Phys.32(2006) 363-373

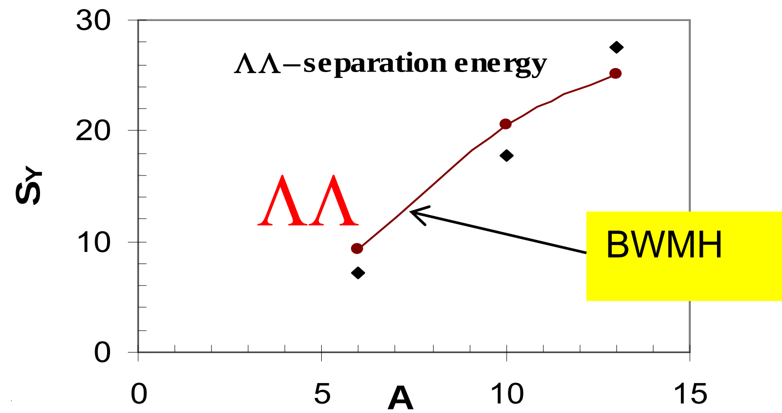
Hyperon separation energy from a Hypernucleus

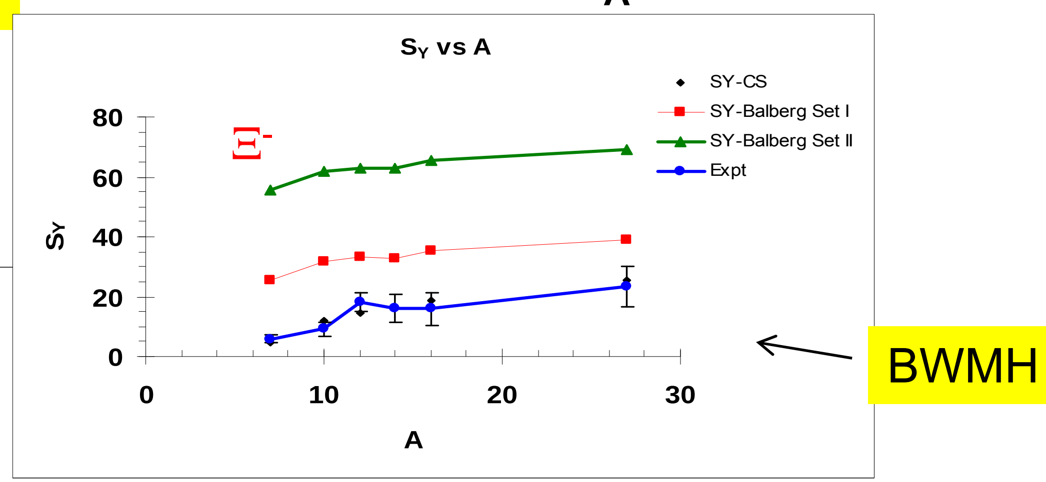
C. Samanta, P. Roy Chowdhury ,D. N. Basu , J.Phys.G:Nucl.Part.Phys.32(2006) 363-373

Hyperon-Separation Energy: $S_Y = BE(A, Z)_{hyper} - BE(A - n_Y, Z_c)_{core}$ (in MeV)

BWMH calculated Λ , ΛΛ, Ξ⁻ separation energies agree with Experimental data







PHYSICAL REVIEW C **76**, 024909 (2007)

Production of hypernuclei in multifragmentation of nuclear spectator matter

A. S. Botvina[1] and J. Pochodzalla[2]

Because of its simplicity, the Mass formulas are used for hypernuclear yield calculation in H.I. collisions as done by Botvina and Pochodzalla with the mass formulas of W. Greiner and C. Samanta.

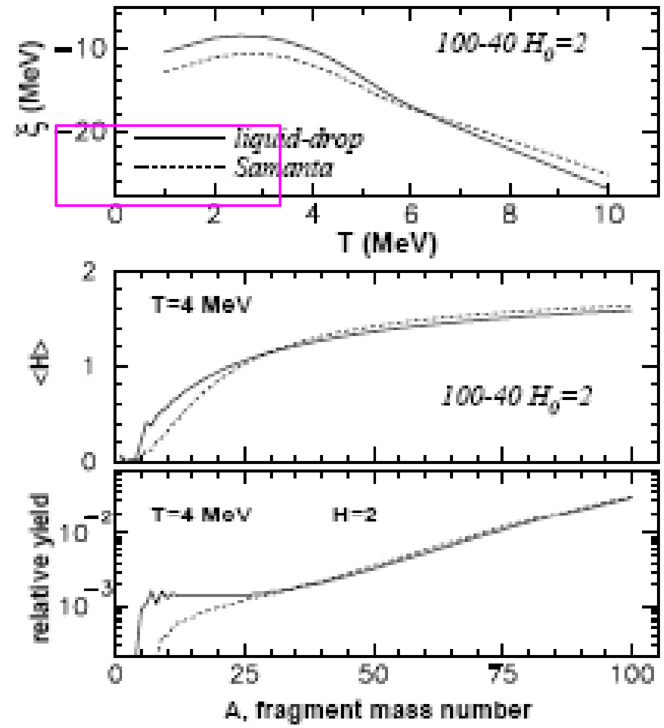


FIG. 3: Comparison of SMM calculations with the liquid-drop and Samanta descriptions of hyper terms in the mass formula, for the same sources as in Fig. 2. Top panel – the strangeness chemical potential ξ versus temperature T. Middle panel – average number of Λ hyperons in fragments, and bottom panel – yields of fragments with two Λ, at T=4 MeV.

Quark-Meson Coupling (QMC) model

P. Guichon, PLB 200, 235 (1988)

(For a review, PPNP 58, 1 (2007))

Baryons described as a system of non-overlapping spherical bags containing three valence quarks interacting by the exchange of σ, ω and ρ mesons coupled directly to the confined quarks.

Self-consistent !

Sigma, omega, rho meson mean fields couple only to the light (u and d) quarks, but not to the strange quark.

Strong Point of the model: The light quark-meson coupling constants are the same for all the light quarks in different hadrons.

Assumptions in QMC Model for Λ, Σ and Ξ hypernuclei

K. Tsushima, K.Saito, J. Haidenbauer, A.W. Thomas,Nuclear Physics A 630 (1997) 691
Pierre A.M. Guichon, Anthony W. Thomas, Kazuo Tsushima, Nucl.Phys.A814 (2008) 66

This is model assumption, based on that:

- ✓ The light quarks play dominant role for chiral symmetry (restoration) in-medium

- ✓ Also the change of the light quark condensate is larger than that of the **s**-quark.
- ✓ The coupling constants are determined by the nuclear matter saturation properties (BE/A =-15.7 MeV at normal ρ=0.15 fm^{-3})

  Thus, based on this procedure/assumption.

- ✓ The binding energy of Ξ -hyperon is obtained automatically without introducing new parameters.

Continued…

Continued…

Assumptions in QMC Model for Λ, Σ and Ξ hypernuclei

K. Tsushima, K.Saito, J. Haidenbauer, A.W. Thomas,Nuclear Physics A 630 (1997) 691
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- The (self-consistent) exchanged scalar, and vector, mesons couple only to the up and down quarks, not to the strange (**S**) quarks.

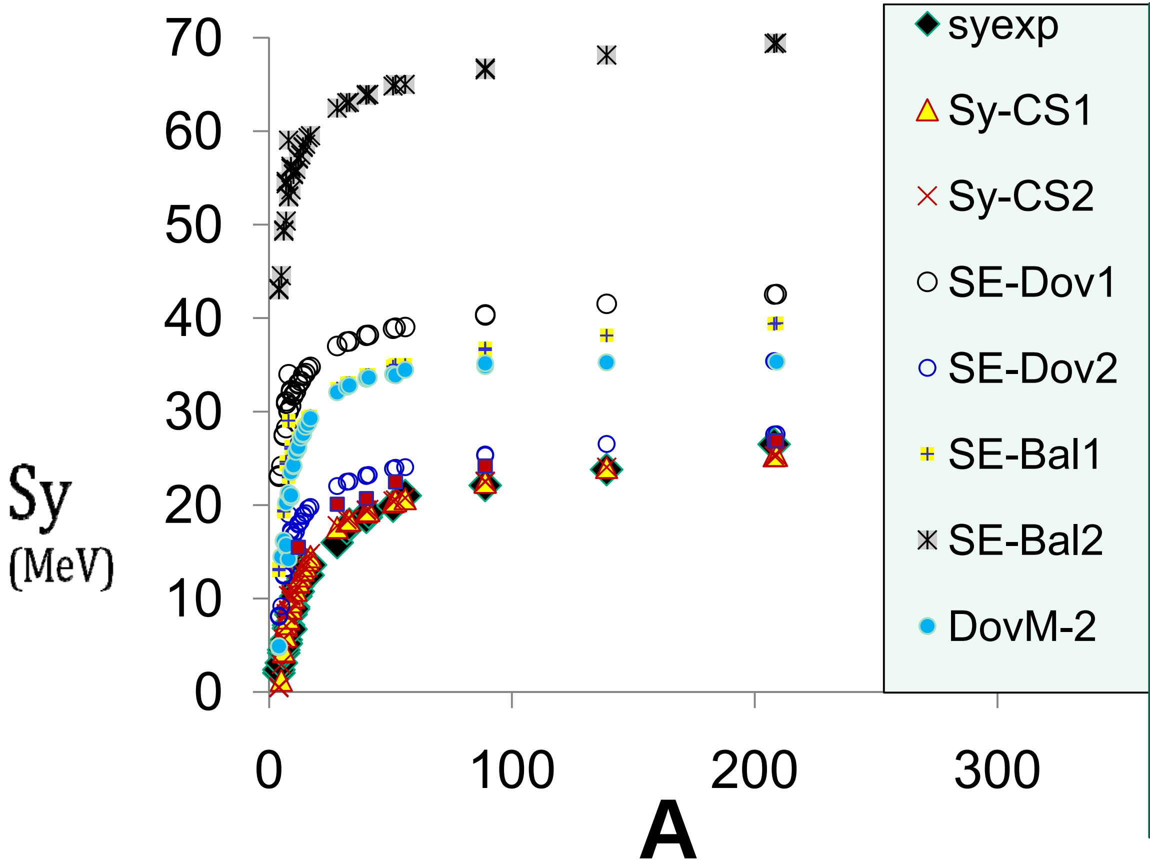
- An flavor & spin SU(6) valence quark model for the bound nucleons and hyperon. The radial part of the quark wave function is the MIT bag.

  The model automatically leads to a very weak spin-orbit interaction for the Λ in a hypernucleus.

- Effects of the Pauli blocking at the quark level, particularly in the open, coupled, ΣN-ΛN channel (strong conversion), is also taken into account in a phenomenological way.

Λ -Hyperon Separation Energy (Sy) VS A

QMC results for Λ agrees with the CS1 and CS2

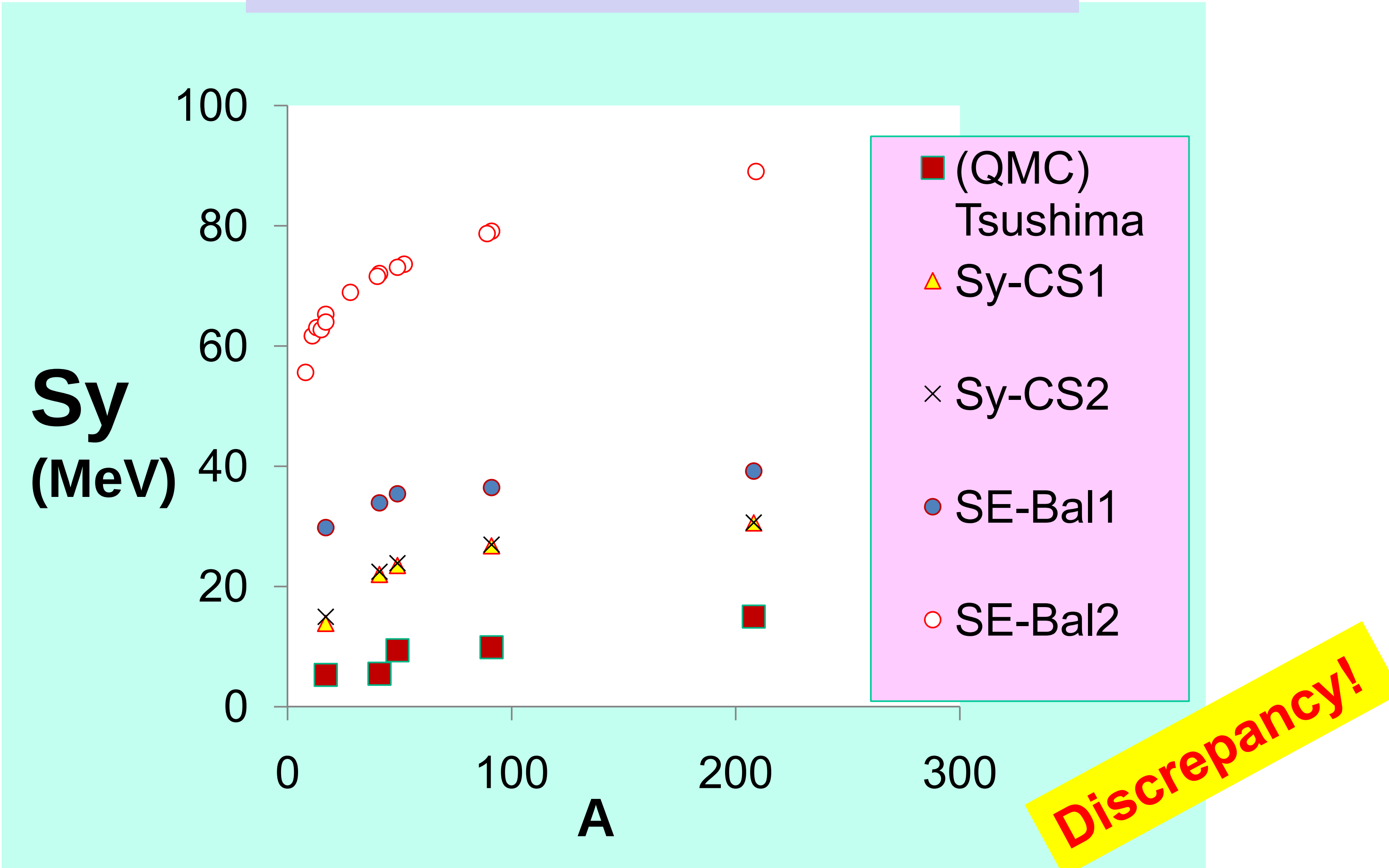


CS1, CS2: Separation energies (Sy) calculated by a recently developed single mass formula for non-strange nuclei, strange hypernuclei as well as multiply strange nuclear systems (SHM) - including pure hyperonic one . **CS1, CS2** fit the model 1 and model 2 of RMF calculations.

See Poster T21, titled: "A new mass formula for non-strange and multi-strange nuclei", C. Samanta

Note: The "new mss formula" of C. Samanta reproduces **both RMF** predictions of B.E./A vs A of SHM (by Schaffner et al) and the **experimental data** on hyperon separation energies. Hence it can be easily used to calculate SHM yield in high energy collision.

Ξ^0 Hyperon Separation Energy



QMC results for Ξ^0 hypernuclei are about the half of the predictions of C.Samanta.

TABLE I: Separation energy S_Y in the mass formula BWMH [1], and absolute single-particle energy value for the $1s_{1/2}$ state in QMC [2], in units of MeV. ${}^{A}_{Y}Z_Q$ stands for a hypernucleus with the net charge Z_Q.

${}^{A}_{Y}Z_Q$	S_Y (Expt.)	BWMH	QMC	${}^{A}_{Y}Z_Q$	S_Y (Expt.)	BWMH	QMC
${}^{12}_{\Lambda}B$	11.37	12.0	15.5	${}^{89}_{\Lambda}Sr$	—	22.5	24.2
${}^{28}_{\Lambda}Al$	—	17.6	20.1	${}^{15}_{\Lambda}N$	13.59	13.74	—
${}^{40}_{\Lambda}K$	—	19.3	20.7	${}^{33}_{\Lambda}S$	17.96	18.44	—
${}^{51}_{\Lambda}V$	19.90	20.37	—	${}^{52}_{\Lambda}V$	—	20.5	22.5
${}^{209}_{\Lambda}Pb$	—	25.31	26.9	${}^{208}_{\Lambda}Pb$	26.5	25.28	—
${}^{17}_{\Xi^o}O$	—	14.14	5.3	${}^{91}_{\Xi^o}Zr$	—	26.98	9.9
${}^{41}_{\Xi^o}Ca$	—	22.24	5.5	${}^{49}_{\Xi^o}Ca$	—	23.73	9.4
${}^{209}_{\Xi^o}Pb$	—	30.60	15.0	${}^{208}_{\Xi^o}Pb$	—	30.56	—
${}^{12}_{\Xi^-}Be$	—	12.1	5.7	${}^{17}_{\Xi^-}N$	—	18.0	8.8
${}^{28}_{\Xi^-}Mg$	—	24.8	11.4	${}^{40}_{\Xi^-}Ar$	—	29.5	16.1
${}^{41}_{\Xi^-}K$	—	30.0	14.6	${}^{49}_{\Xi^-}K$	—	31.1	11.8
${}^{52}_{\Xi^-}Ti$	—	32.4	14.3	${}^{89}_{\Xi^-}Rb$	—	38.7	18.0
${}^{91}_{\Xi^-}Y$	—	39.3	19.2	${}^{209}_{\Xi^-}Tl$	—	50.2	25.4

Z_Q = Net charge

${}^{41}_{\Xi^0}Ca = {}^{40}Ca + \Xi^0$

${}^{41}_{\Xi^-}K = {}^{40}Ca + \Xi^-$

- **QMC ≈ BWMH For Λ hypernuclei**
- **QMC ≈1/2 BWMH For Ξ hypernuclei**

[1] BWMH: C. Samanta, et al. , J.P.G.32 (2006) 363; ibid 35 (2008) 065101

[2] QMC: Pierre A.M. Guichon, Anthony W. Thomas, Kazuo Tsushima, Nucl.Phys.A814 (2008) 66.

Summary

❖ Λ and Ξ Hyperon separation energies obtained using generalized mass formulas are compared with the predictions of Quark Meson Coupling (**QMC**) model.

❖ For Λ^0 (**uds**) hyperon separation energies, both mass formulas of Samanta and QMC exhibit excellent agreement with each other and with the available experimental data.

❖ However, for Ξ^-(dss), Ξ^0 (uss) hypernuclei, the difference is striking; the QMC values being almost half of the mass formula results.

In QMC, the Sigma, omega, rho meson mean fields couple only to the light (u and d) quarks, but not to the strange quark. Possibly the treatment of **multi s-quarks** systems in nuclear medium needs a different approach. We have not done this so far.

Experimental data on Cascade nuclei with higher accuracy are needed to resolve this observed striking discrepancy.

ありがとう。

Thank you