Baryon-Baryon Interactions in Lattice QCD

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for HAL QCD Collaboration
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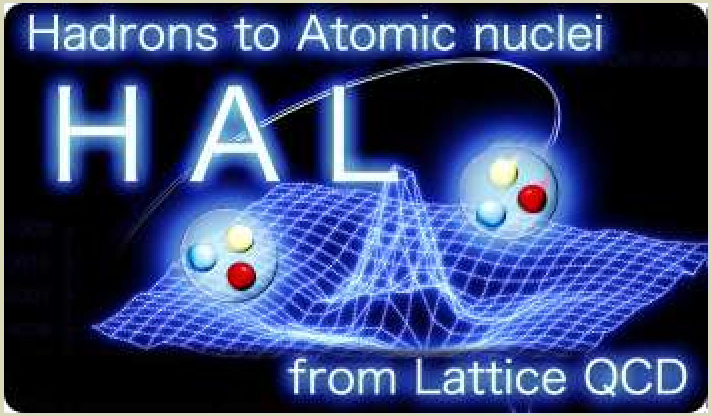
N. Ishii[3], K. Murano[2], and K. Sasaki[2],



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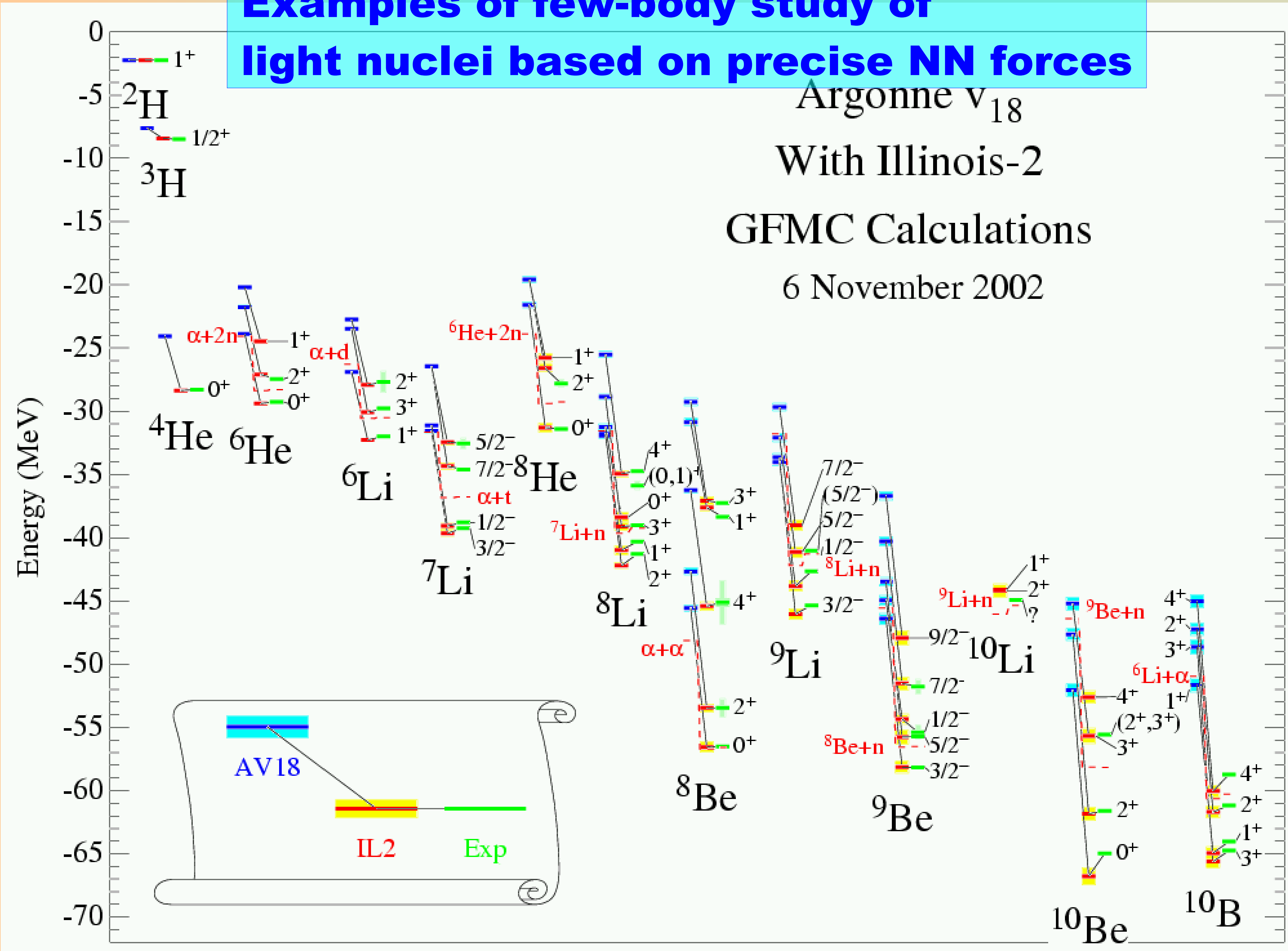
[3]*Department of Physics, University of Tokyo, Japan*

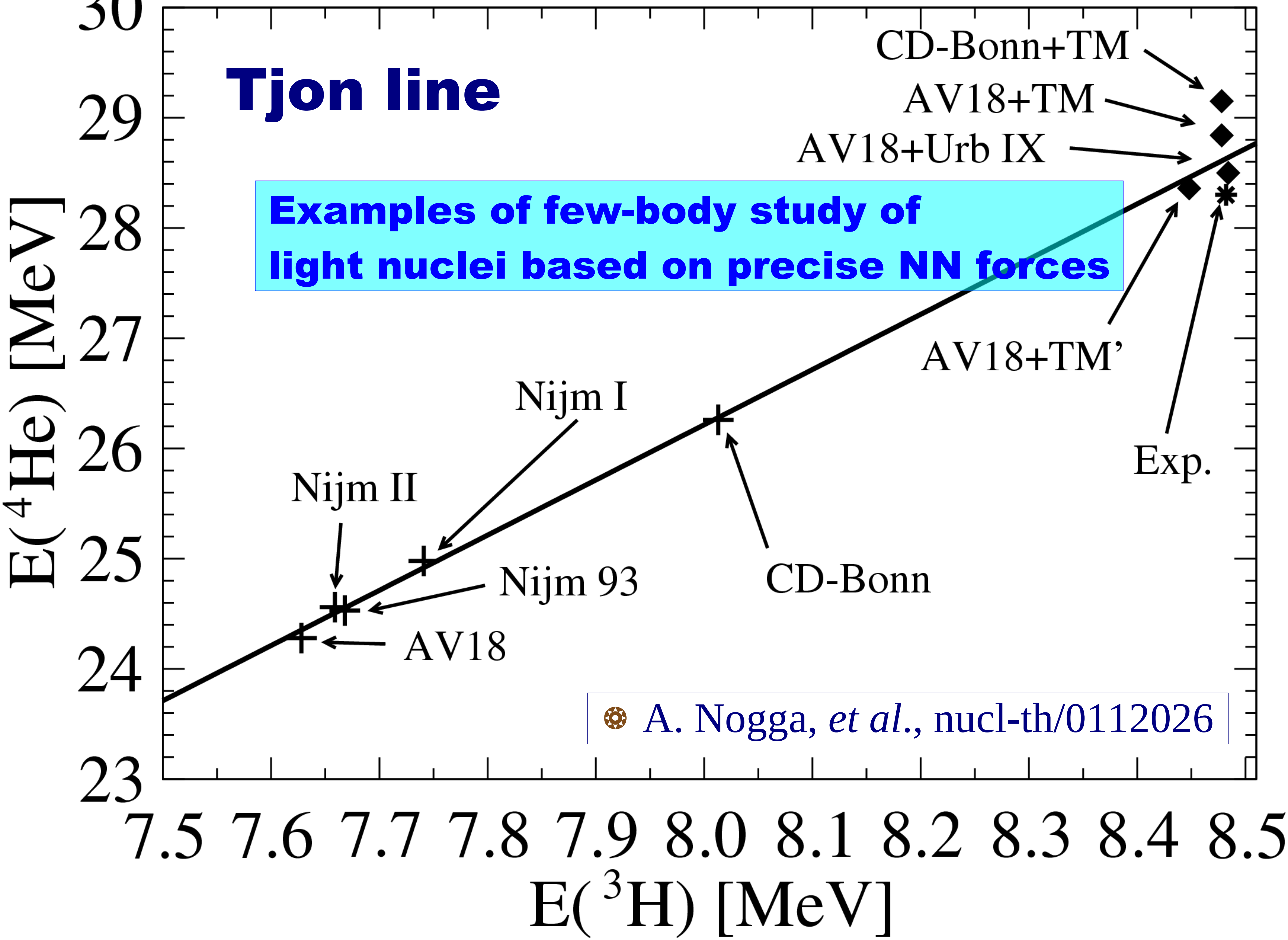
Outline

- Introduction
- Formulation --- potential (central + tensor)
- Numerical results:
  - pn force ($V_C + V_T$)
    - Energy dependence of V at $E_{CM} \approx 46\text{MeV}$
  - $p\Lambda$ force ($V_C + V_T$)
  - $p\Xi^0$ force ($V_C^{(eff)}$)
- Summary and outlook

Examples of few-body study of light nuclei based on precise NN forces



Tjon line



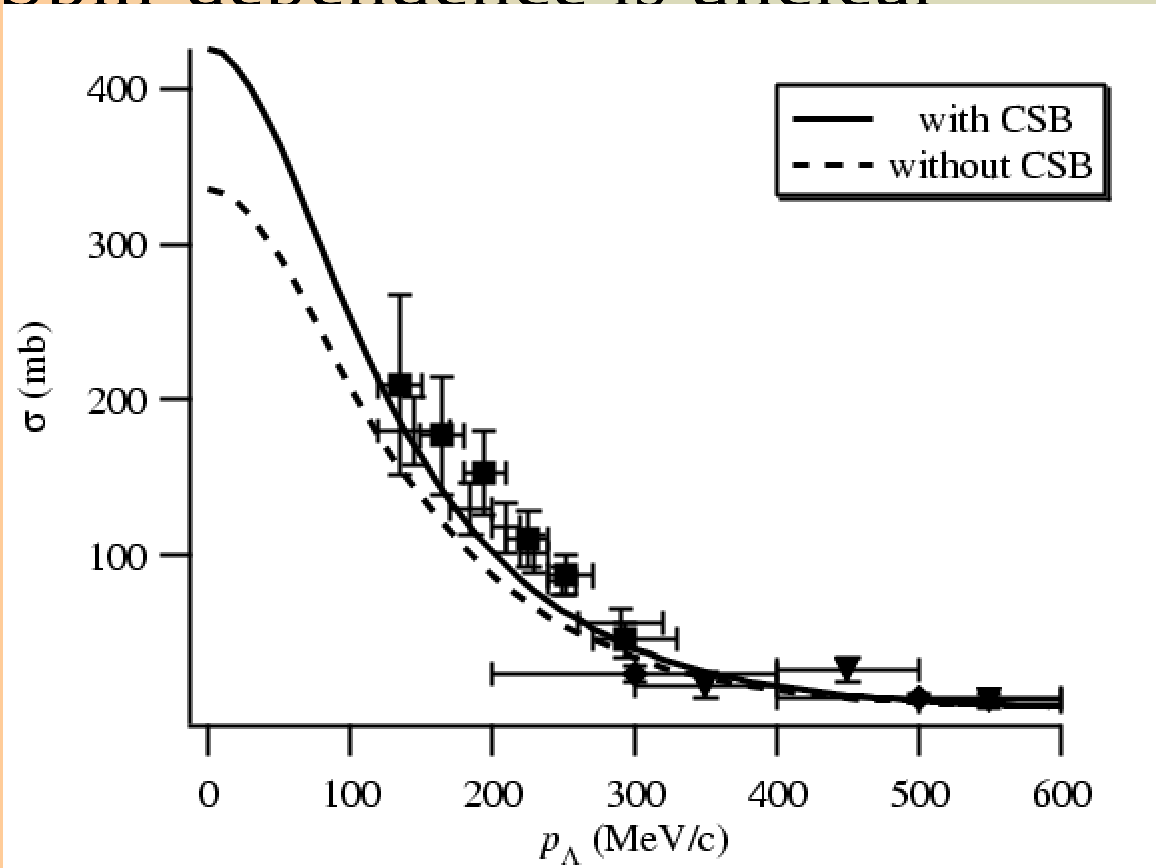
Examples of few-body study of
light nuclei based on precise NN forces
CD-Bonn+TM
AV18+TM
AV18+Urb IX
AV18+TM'
Exp.
Nijm I
Nijm II
Nijm 93
AV18
CD-Bonn
A. Nogga, *et al.*, nucl-th/0112026
30
29
28
27
26
25
24
23
7.5
7.6
7.7
7.8
7.9
8.0
8.1
8.2
8.3
8.4
8.5
E(^{4}He) [MeV]
E(^{3}H) [MeV]

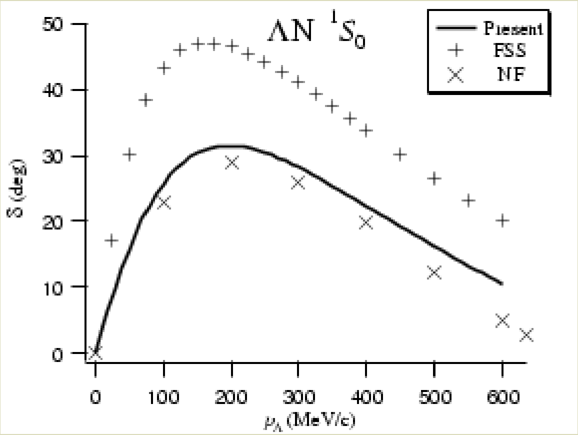
Introduction:

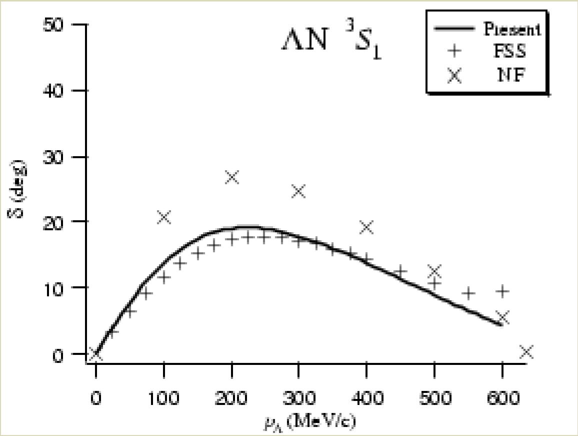
- Study of hyperon-nucleon (*YN*) and hyperon-hyperon (*YY*) interactions is one of the important subjects in the nuclear physics.
    - Structure of the neutron-star core,
        - Hyperon mixing, softning of EOS, inevitable strong repulsive force,
    - H-dibaryon problem,
        - To be, or not to be,
- The project at J-PARC:
    - Explore the multistrange world,
- However, the phenomenological description of *YN* and *YY* interactions has large uncertainties, which is in sharp contrast to the nice description of phenomenological *NN* potential.

Experimental data for ΛN interaction:

- Only total corss section.
- No phase shift analysis is avairable.
- Spin-dependence is unclear







Few-body calculations of s-shell Λ hypernuclei

	A=3	*A*=4	*A*=5	Tensor	Σ	
Dalitz, *et al.*	☑	☑	☑	☐	☑ (blue)	NP**B47**, 109 (1972).
Shinmura, *et al*.,	☑	☑	☑	☑	☐	PTP**71**, 546 (1984).
Gibson, *et al*.,	☐	☑	☐	☐	☑	PRC**37**, 679 (1988).
Carlson,	☐	☑	☑	☑	☑	AIP Conf. Proc. No. 224 (1991).
Miyagawa, *et al*,	☑	☐	☐	☑	☑	PRC**51**, 2905 (1995).
Hiyama, *et al*.,	☑	☑	☐	☑	☑	PRC**65**, 011301 (2002).
Sinha, *et al*,	☐	☑	☑	☑	☑ (blue)	PRC**66**, 024006 (2002).
Nogga, *et al*.,	☑	☑	☐	☑	☑	PRL**88**, 172501 (2002).
HN, *et al*.,	☑	☑	☑	☑	☑	PRL**89**, 142504 (2002).

☑ (blue): *ΛNN* 3BF

Pioneering works from lattice QCD:

- S. Aoki, *et al.*, PRD**71**, 094504 (2005);

  π-π scattering length from the wave function.

- N. Ishii, *et al.*, PRL**99**, 022001 (2007); nucl-th/0611096;

  NN potential from the wave function.

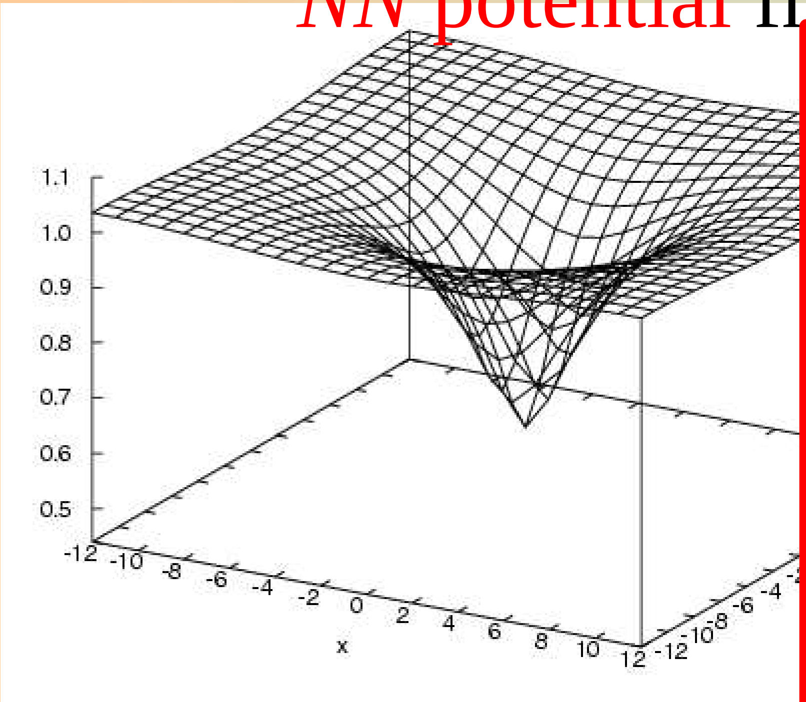
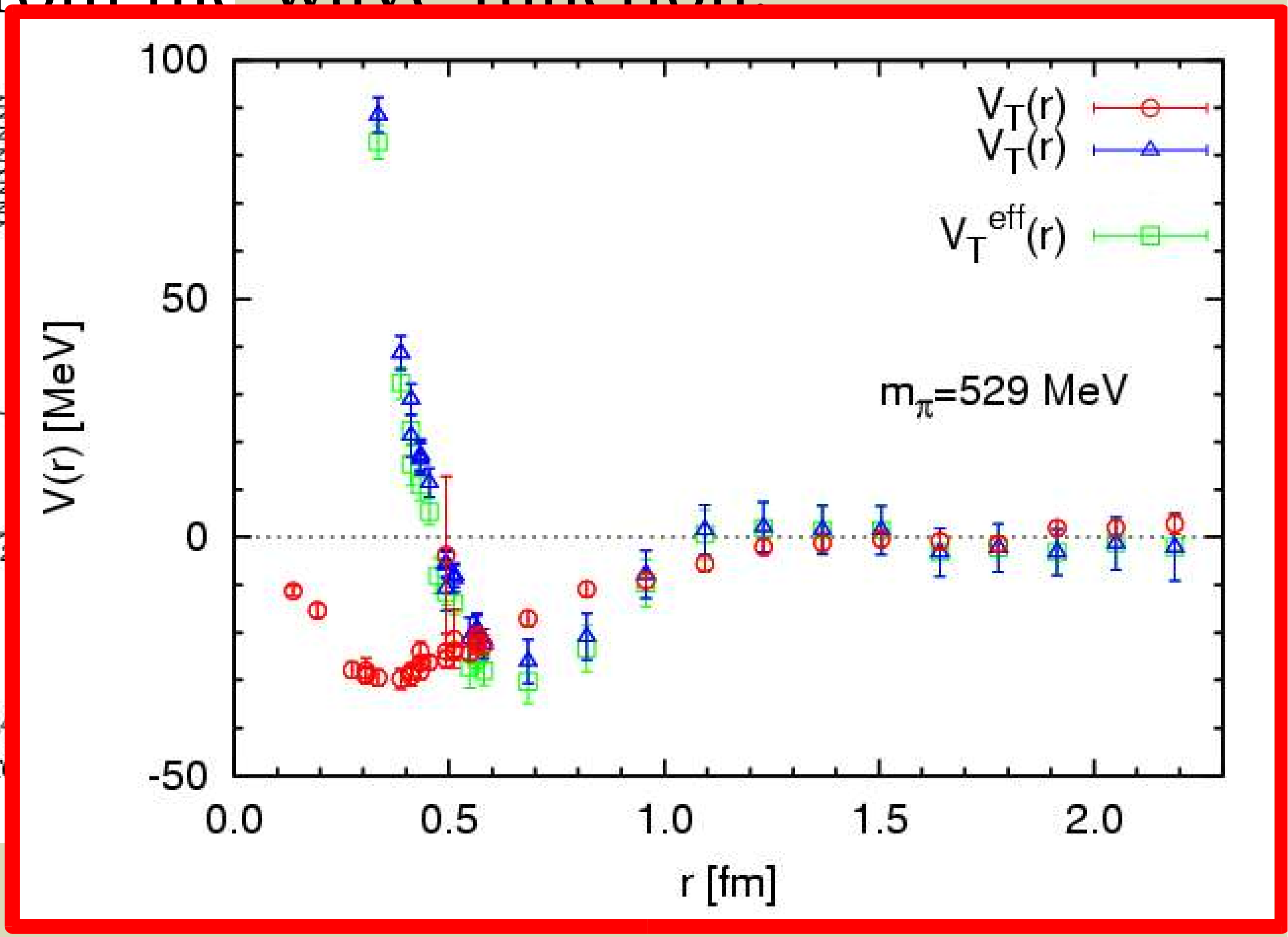


FIG. 1. Two-pion wave function $\phi(\vec{x};k)$ on 2 $(t,z) = (52,0)$ plane for $m_\pi^2 = 0.273$ GeV^2. The r tor is set at $\vec{x}_0 = (7,5,2)$ $(x_0 = |\vec{x}_0| = 8.832)$.



- ➜Comprehensive study of baryon-baryon forces.

PARTICLE PHYSICS

Hard-core revelations

Frank Wilczek

Our description of how the atomic nucleus holds together has up to now been entirely empirical. Arduous calculations starting from the theory of the strong nuclear force provide a new way into matter's hard core.

Our quest to understand the force that holds atomic nuclei together has turned out to be a glorious adventure. Along the way we have found quarks, the coloured gluons that mediate the strong nuclear force, and a wonderful theory — quantum chromodynamics, or QCD. This theory has guided experimental research at the high-energy frontier, inspired dreams of 'unified field theories' that would embrace all nature's forces, and allowed theoretical physics to penetrate into the cosmology of the early Universe. In all this, the original problem of understanding nuclear forces has rather fallen by the wayside. That changes with what may come to be seen as a landmark paper by Ishii, Aoki and Hatsuda that has recently appeared on the arXiv preprint server[1].

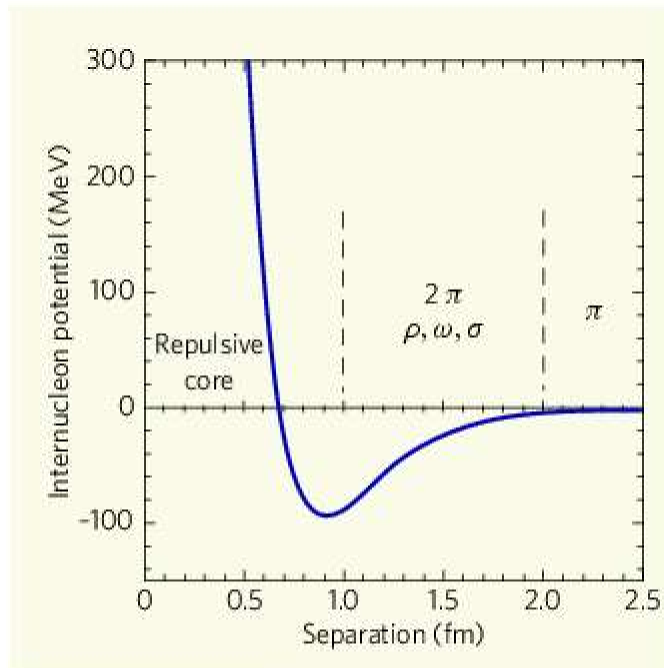


Figure 1 | The nucleon–nucleon potential. At

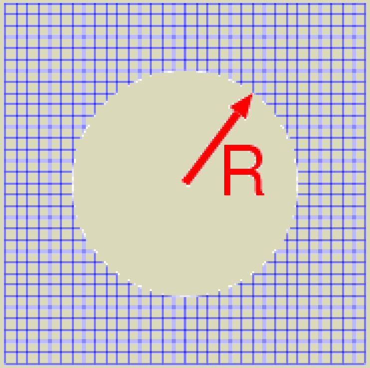
This empirical 'ansv
more questions. Doe
ory produce a force l
originally proposed l
longest-range part of
force can be attribu
lightest strongly inte
as π-mesons or pion
exchanges of heavier
tant. As we approach h
ever, this meson-exc
both unwieldy and d
relevant mesons grow
ture becomes resolve
the hard core, which i
structure of matter as
brute fact, opaque to t

In principle, the eq
all the physics of stro
But in practice, it is ex
the equations and cal
and colleagues' brea
required sophisticat
on the biggest and fa
computers currently a

Why are the calcul
main reason is simply
plicated objects. It is

Nature **445** (2007).

Formulation



i) basic procedure:

asymptotic region

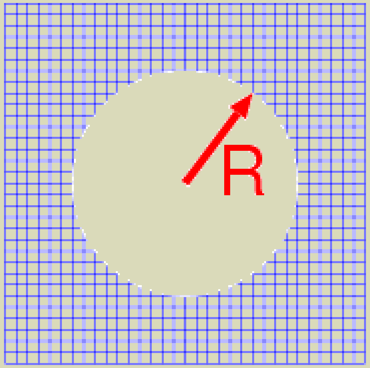
--> phase shift

ii) advanced (HAL's) procedure: interacting region

--> potential

Formulation

i) basic procedure:



asymptotic region

(or temporal correlation)

--> scattering energy $E = \frac{k^2}{2\mu}$

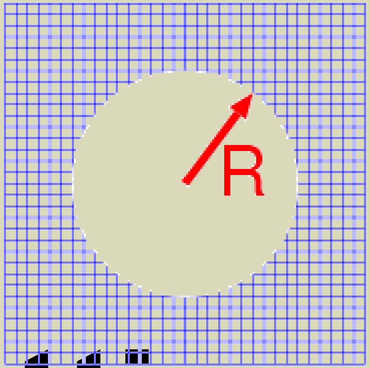
--> phase shift

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbf{Z}^3} \frac{1}{(n^2 - q^2)^s} \qquad \Re s > \frac{3}{2}$$

Luscher, NPB354, 531 (1991).
Aoki, et al., PRD71, 094504 (2005).

HAL formulation



ii) advanced procedure:

make better use of the lattice output ! (wave function)

interacting region

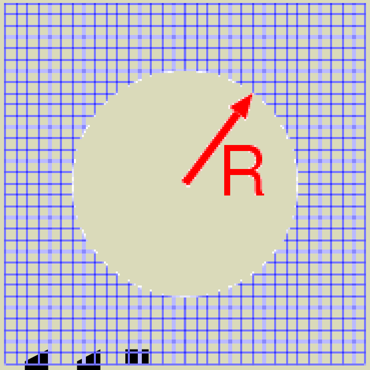
--> potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., arXiv:0805.2462[hep-ph].

NOTE:
> Potential is not a direct experimental observable.
> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

See next pages more detail....

HAL formulation



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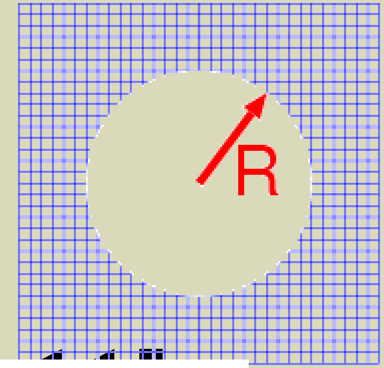
=>

> Phase shift

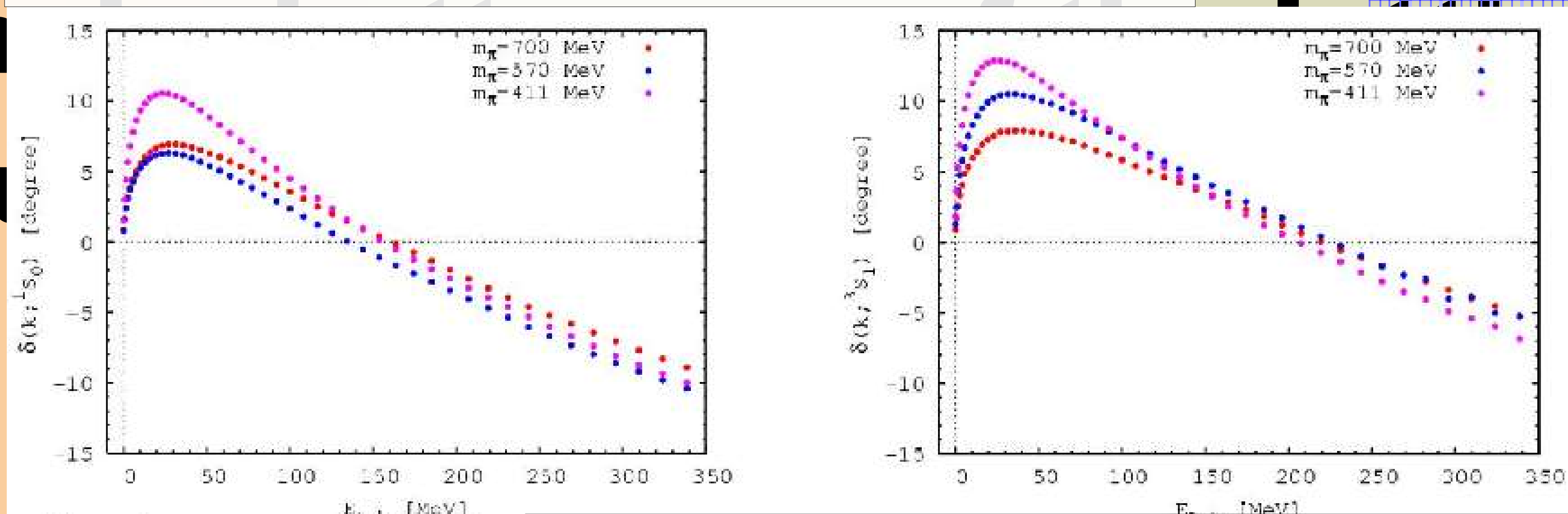
> Nuclear many-body problems

HAL formulation

ii) advanced procedure:



> m_π dependence of the phase shifts



Ishii (HAL QCD), talk at Lattice 2009.

Ibid., arXiv:0805.2462[hep-ph].

=> > Phase shift

> Nuclear many-body problems

A recipe for NY potential:

- More accurate explanation, see, e.g., arXiv:0805.2462[hep-ph].

- Start from an effective Schroedinger eq for the equal-time Bethe-Salpeter wave funciton:

$$-\frac{1}{2\mu}\nabla^2\phi(\vec{r})+\int d^3r' U(\vec{r},\vec{r}')=E\phi(\vec{r})$$

$$U(\vec{r},\vec{r}')=V_{NY}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

- A general expression of the potential:

$$V_{NY}=V_0(r)+V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_Y) +V_T(r)S_{12}+V_{LS}(r)(\vec{L}\cdot\vec{S}_+) +V_{ALS}(r)(\vec{L}\cdot\vec{S}_-)+O(\nabla^2)$$

A recipe for NΛ potential:

- .

- The equal time BS wave function with angular momentum (J,M) on the lattice,

$$\phi_{\alpha\beta}^{(JM)}(\vec{r}) = \sum_{\vec{x}} \left\langle 0 \middle| p_\alpha(\vec{r}+\vec{x}) \Lambda_\beta(\vec{x}) \middle| p\Lambda; k, JM \right\rangle$$

$$p_\alpha(x) = \varepsilon_{abc}(u_a(x) C\gamma_5 d_b(x)) u_{c\alpha}(x),$$

$$\Lambda_\alpha(x) = \varepsilon_{abc}\left\{(d_a C\gamma_5 s_b) u_{c\alpha} + (s_a C\gamma_5 u_b) d_{c\alpha} - 2(u_a C\gamma_5 d_b) s_{c\alpha}\right\}$$

- The 4-point $N\Lambda$ correlator on the lattice,

$$F_{\alpha\beta}^{(JM)}(\vec{x},\vec{y},t-t_0) = \left\langle 0 \middle| p_\alpha(\vec{x},t) \Lambda_\beta(\vec{y},t) \overline{\Theta}_{p\Lambda}^{(JM)}(t_0) \middle| 0 \right\rangle$$

$$= \sum_n A_n^{(JM)} \left\langle 0 \middle| p_\alpha(\vec{x}) \Lambda_\beta(\vec{y}) \middle| E_n \right\rangle e^{-E_n(t-t_0)}$$

$\overline{\Theta_{p\Lambda}^{(JM)}}(t_0)$ wall source at $t = t_0$

A recipe for NΛ potential:

- More accurate explanation, see, e.g., arXiv:0805.2462[hep-ph].

- Calculate the 4-point *N*Λ correlator on the lattice,

$$\phi_{N\Lambda}(x-y)e^{-E(t-t_0)} \propto \langle p_\alpha(x,t)\Lambda_\beta(y,t)\overline{\Lambda_{\beta'}}(0,t_0)\overline{p_{\alpha'}}(0,t_0)\rangle$$

  - Which has the physical meanings of,
    - Create a *N*Λ state and making imaginary time evolution, in order to have the lowest state of the *N*Λ system.
    - Take the amplitude ϕ(***x–y***), which can be understood as a wave function of the non-relativistic quantum mechanics.

- Obtain the effective central potential from the effective Schroedinger equation.

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2+V(r)\right)\phi(r)=E\phi(r)$$

$$\rightarrow \quad V(r)=E+\frac{\hbar^2}{2\mu}\frac{\nabla^2\phi(r)}{\phi(r)}$$

A recipe for NΛ potential: (contd.)

- .

- For $J = 1$, ϕ comprises *S*-wave and *D*-wave,

$$|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle$$

where,

$$|\phi_S\rangle = \mathcal{P}\,|\phi\rangle = (1/24)\sum_{\mathcal{R}\in O}\mathcal{R}|\phi\rangle$$

$$|\phi_D\rangle = Q|\phi\rangle = (1-\mathcal{P})\,|\phi\rangle$$

- Therefore, we have 2-component Schrödinger eq.

S-wave:

$$\mathcal{P}(T + V_C + V_T S_{12})\,|\phi\rangle = E\,\mathcal{P}\,|\phi\rangle$$

D-wave:

$$Q(T + V_C + V_T S_{12})\,|\phi\rangle = E\,Q\,|\phi\rangle$$

- Obtain the $V_C(r)$ and the $V_T(r)$ simultaneously.

Numerical results:

Quenched calculation with larger spatial volume:

- Plaquette gauge action and Wilson fermion action
- Gauge coupling β=5.7
- Volume: $32^3 \times 48$ (L ~ 4.5 fm).
- Lattice spacing: a ~ 0.14 fm. ($1/a$ ~ 1.4 GeV.)
- The lattice calculations were performed by using KEK Blue Gene/L supercomputer.

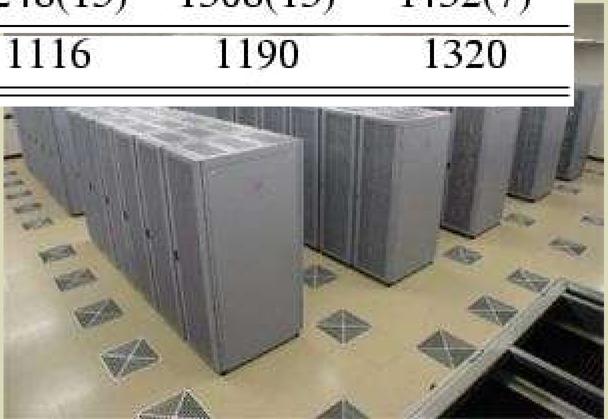
- The main results are obtained with
  - κ_{ud} = 0.1665 (or 0.1670) for the u and d quarks, and
  - κ_s = 0.1643 for s quark.

☞ Meson masses:
m_π ~ 0.511.2(6) GeV
m_ρ ~ 0.861(2) GeV
m_K ~ 0.605.3(5) GeV
m_{K^*} ~ 0.904(2) GeV

Full QCD calculations by using N_F=2+1 PACS-CS gauge configurations:

- S. Aoki, et al., (PACS-CS Collaboration), PRD**79**, 034503 (2009), arXiv:0807.1661 [hep-lat].
- Iwasaki gauge action at β=1.90 on $32^3 \times 64$ lattice
- O(a) improved Wilson quark action
- $1/a = 2.17$ GeV ($a = 0.0907$ fm)

$(\kappa_{ud})_{N_{\rm conf}}$	m_π	m_ρ	m_K	m_{K^*}	m_N	m_Λ	m_Σ	m_Ξ
2+1 flavor QCD by PACS-CS with $\kappa_s = 0.13640$ @present calc (Dirichlet BC along T)								
$(0.13700)_{609}$	700.0(4)	1108(3)	785.8(3)	1159(2)	1573(4)	1632(4)	1650(5)	1700(4)
$(0.13727)_{481}$	567.9(6)	1000(4)	723.7(7)	1081(3)	1396(6)	1491(4)	1519(5)	1599(4)
$(0.13754)_{260\times4}$	415(1)	903(5)	639.7(8)	1024(4)	1232(10)	1354(6)	1415(7)	1512(4)
$(0.13770)_{422}$	301(3)	845(10)	592(1)	980(6)	1079(12)	1248(15)	1308(13)	1432(7)
Exp.	135	770	494	892	940	1116	1190	1320

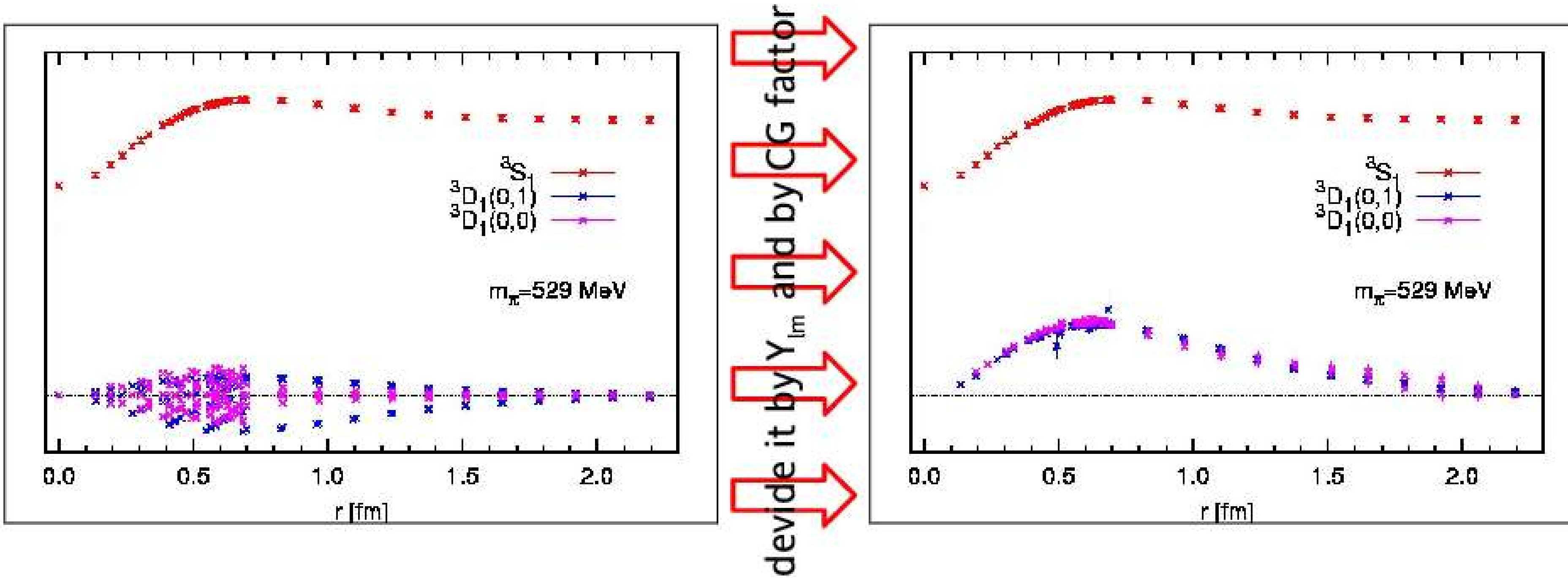


Results of NN force

> pn; multi-valued wave function of $^3D_1$

d-wave BS wave function

$J^P=1^+$, M=0



Angular dependence ➔ Multi-valued

d-wave ∝ "spinor harmonics"

$$\begin{bmatrix} \psi^{(D)}_{\Uparrow\Uparrow}(\vec r) & \psi^{(D)}_{\Uparrow\Downarrow}(\vec r) \\ \psi^{(D)}_{\Downarrow\Uparrow}(\vec r) & \psi^{(D)}_{\Downarrow\Downarrow}(\vec r) \end{bmatrix} \propto \begin{bmatrix} Y_{2,-1}(\hat r) & -\frac{2}{\sqrt{6}} Y_{2,0}(\hat r) \\ -\frac{2}{\sqrt{6}} Y_{2,0}(\hat r) & Y_{2,+1}(\hat r) \end{bmatrix}$$

Almost Single-valued

➔ $\psi^{(D)}$ is dominated by d-wave.

NOTE:

(0,1) [blue] ←→ E-representation

(0,0) [purple] ←→ T_2-representation

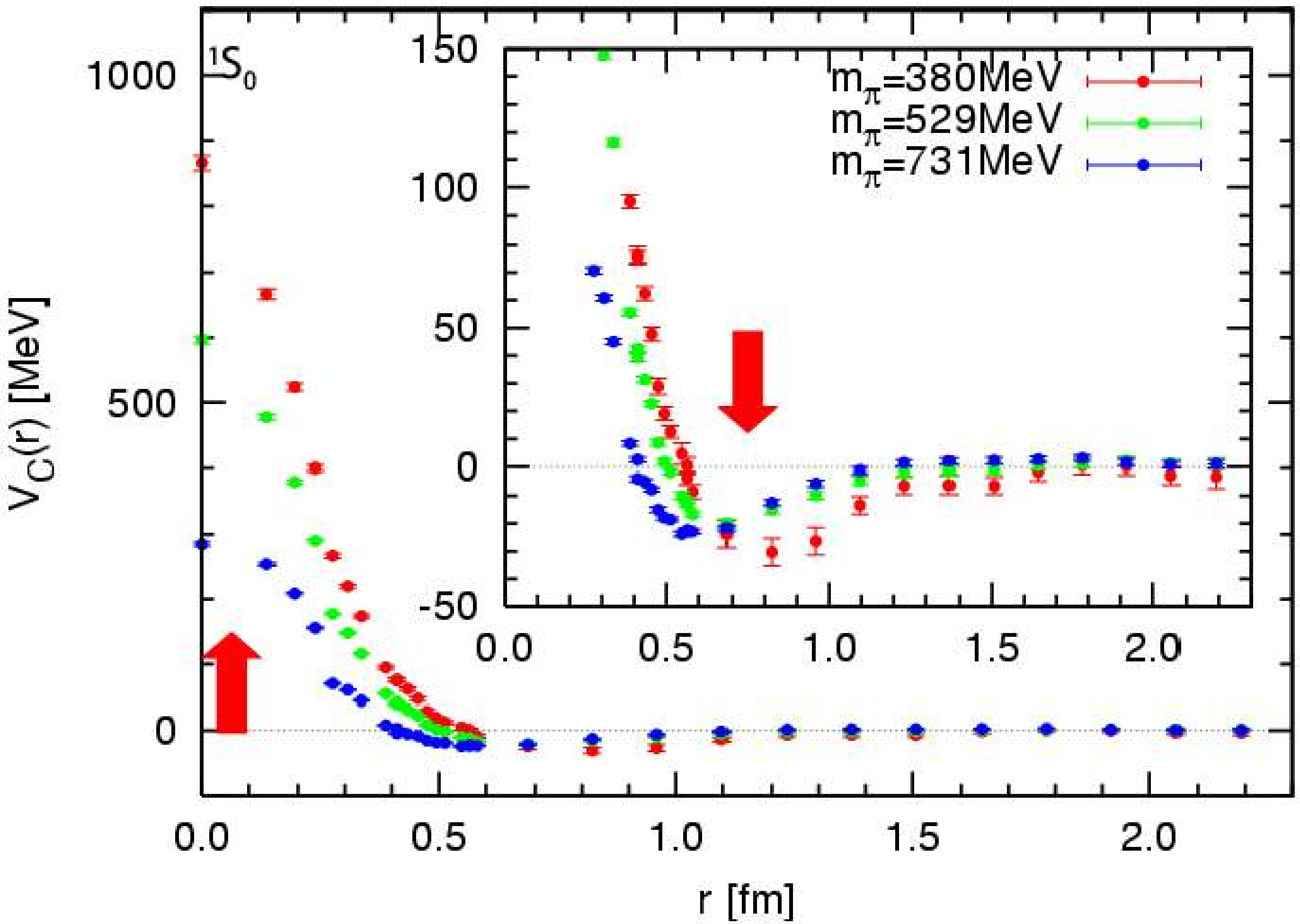
Difference of these two →violation of SO(3)

Ishii (HAL QCD), talk at Lattice 2009.

Quark mass dependence

> pn $V_c(^1S_0)$

Aoki,Hatsuda,Ishii, CSD1,015009(2008).



In the light quark mass region,

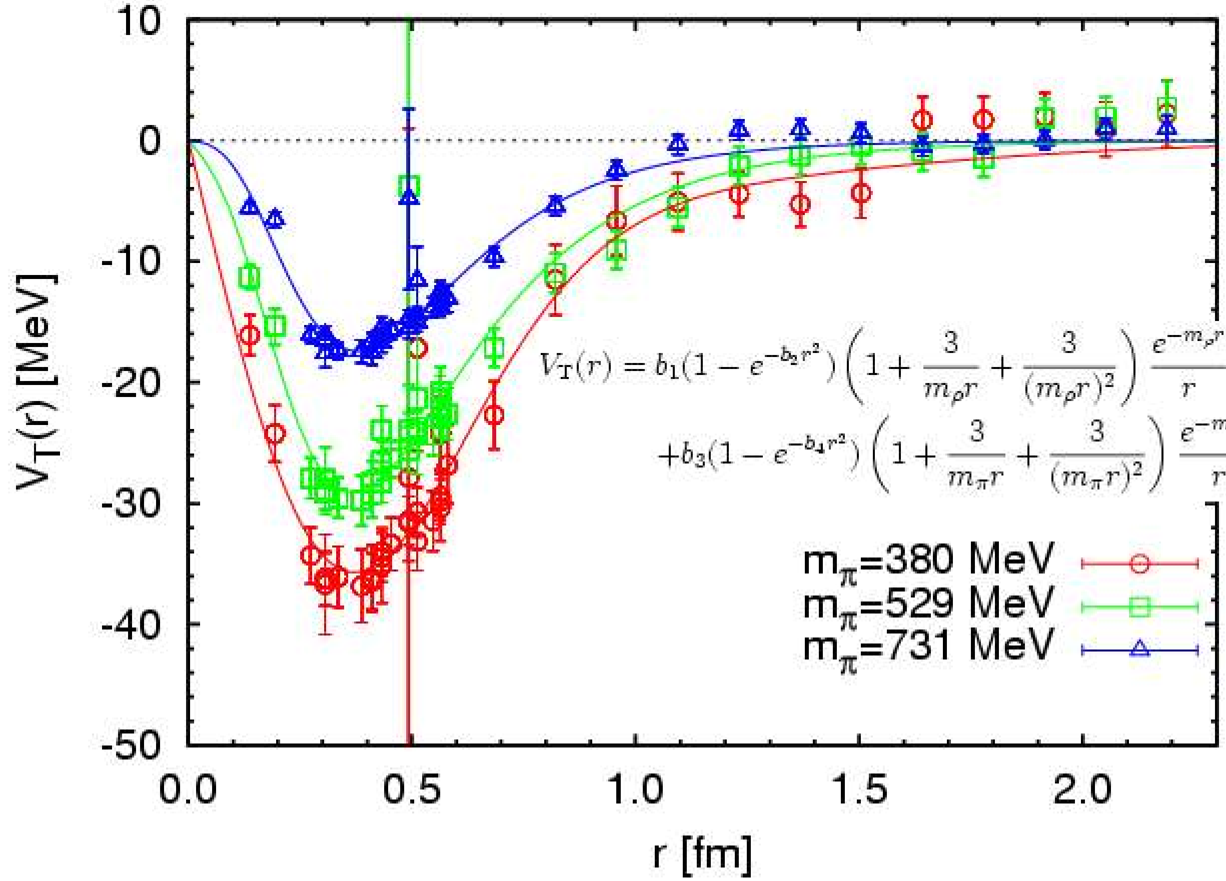
✓ The repulsive core grows rapidly.

✓ Attraction gets stronger.

Ishii (HAL QCD), talk at Lattice 2009.

> pn $V_T(^3S_1-^3D_1)$

Tensor potential (quark mass dependence)



$$V_T(r) = b_1(1 - e^{-b_2 r^2})\left(1 + \frac{3}{m_\rho r} + \frac{3}{(m_\rho r)^2}\right)\frac{e^{-m_\rho r}}{r}$$
$$+ b_3(1 - e^{-b_4 r^2})\left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r},$$

Tensor potential is enhanced
in the light quark mass region

Aoki, Hatsuda, Ishii (2009).

> pn; E-dependence of V_C and V_T

How good is the derivative expansion ?

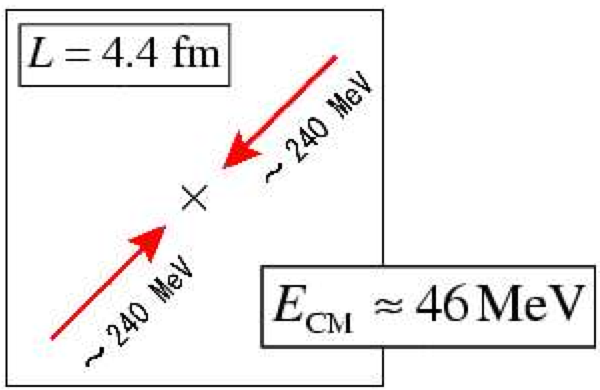
At leading order:

$$U(x,y) = \left(V_C(r) + V_T(r)\cdot S_{12} + V_{LS}(r)\cdot L\cdot S + \{V_D(r), \nabla^2\} + \cdots \right) \delta(x-y)$$

Strategy:

- Calculate two potentials at leading order at different energies

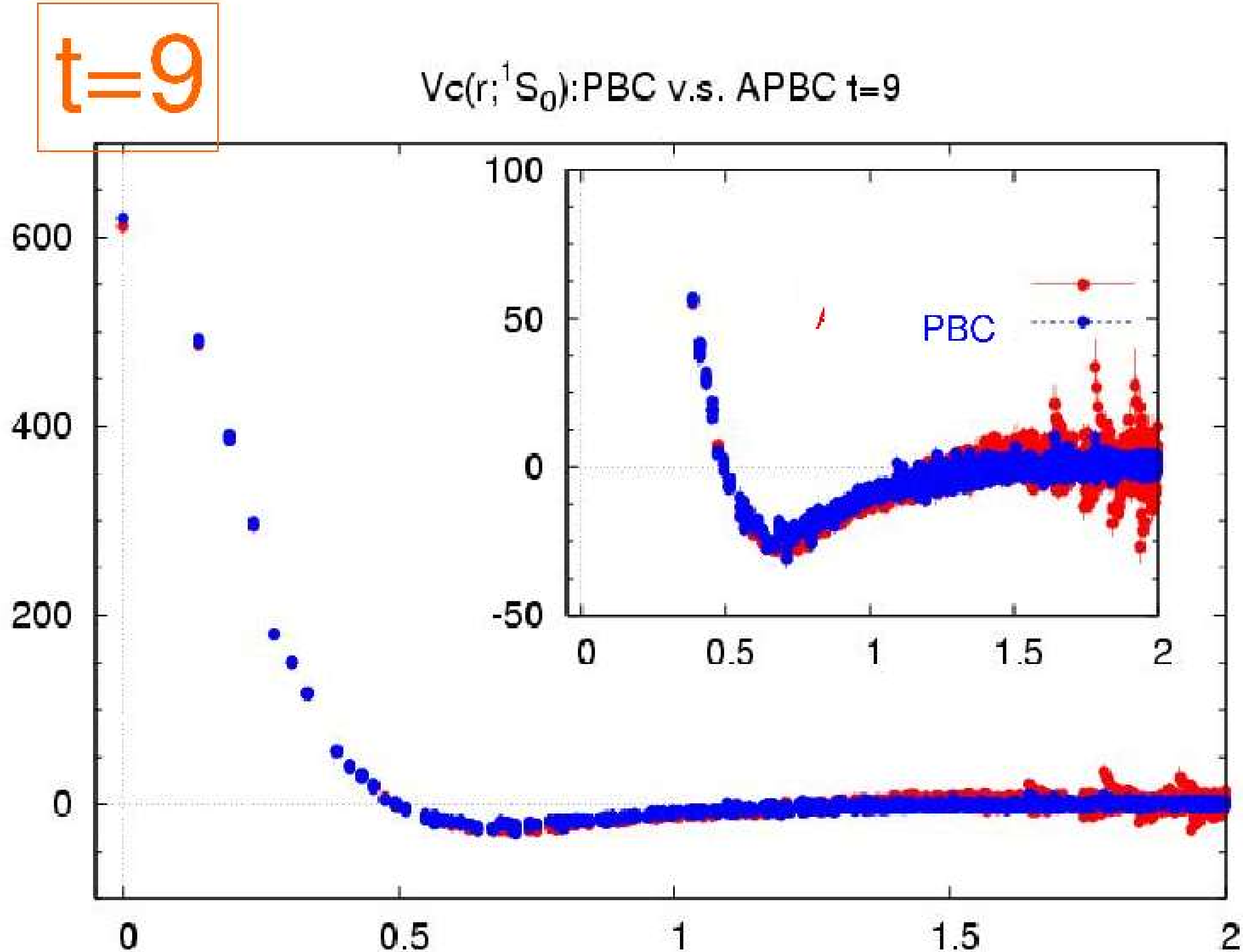
  Potential at $E \neq 0$ is constructed by anti-periodic BC



- Difference ←→ size of higher order effects
- Small difference
  - ➡ small higher order effects
  - ➡ leading order potential at E~0 MeV serves as a good starting point for the E-independent non-local potential U(x,y)

Murano and Ishii (HAL QCD), talk at Lattice 2009.

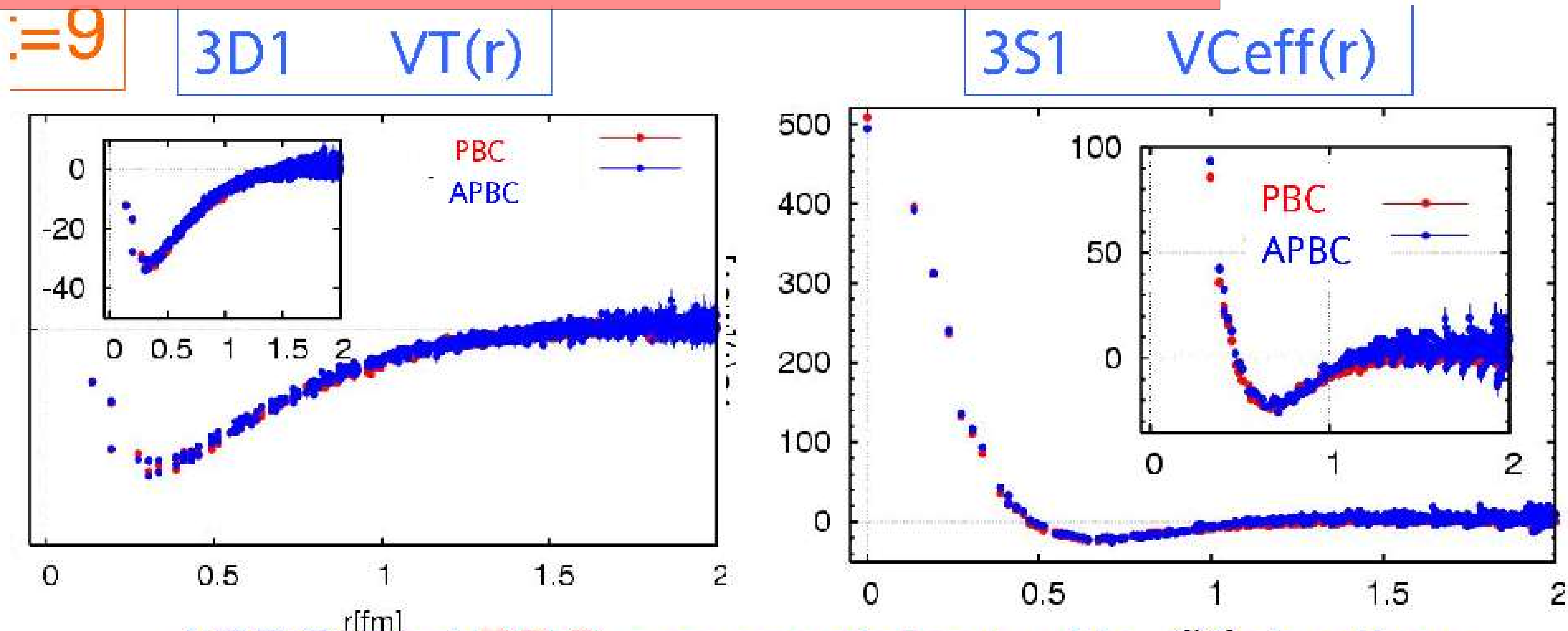
> pn; E-dependence of V_c and V_T



APBC and PBC are consistent with each other.

Murano (HAL QCD), talk at Lattice 2009.

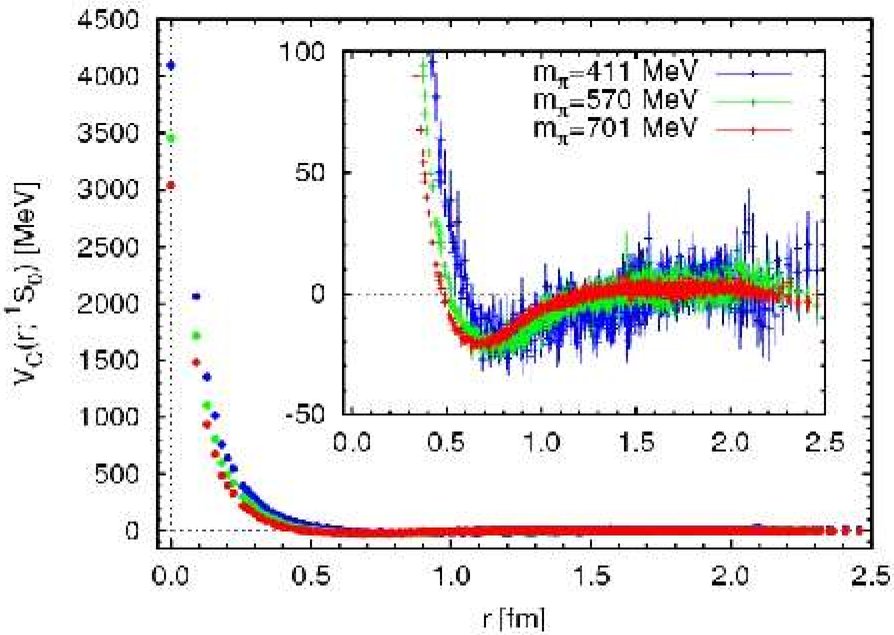
> pn; E-dependence of V_c and V_T

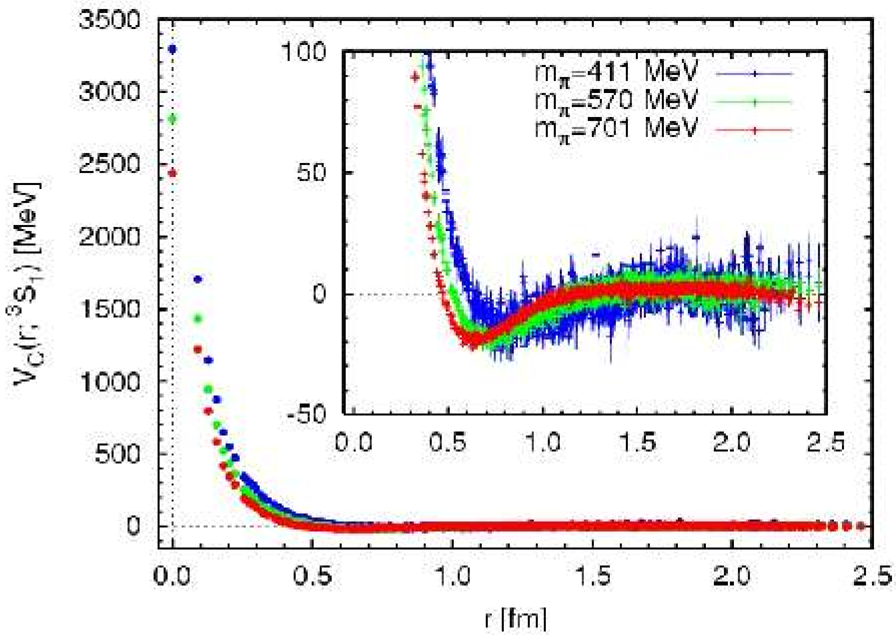


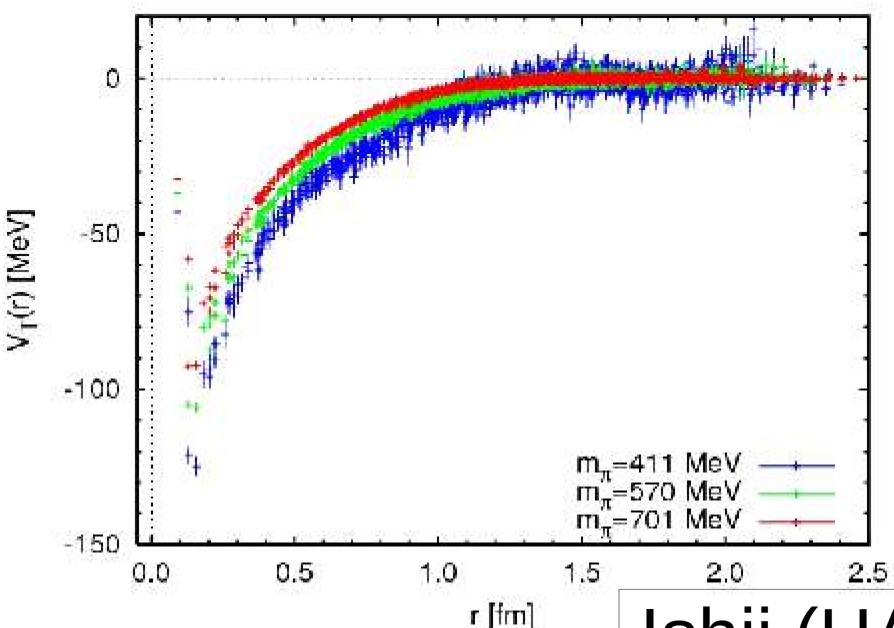
APBC and PBC are consistent with each other.

Murano (HAL QCD), talk at Lattice 2009.

NN potentials (quark mass dependence)







In the light quark mass region,

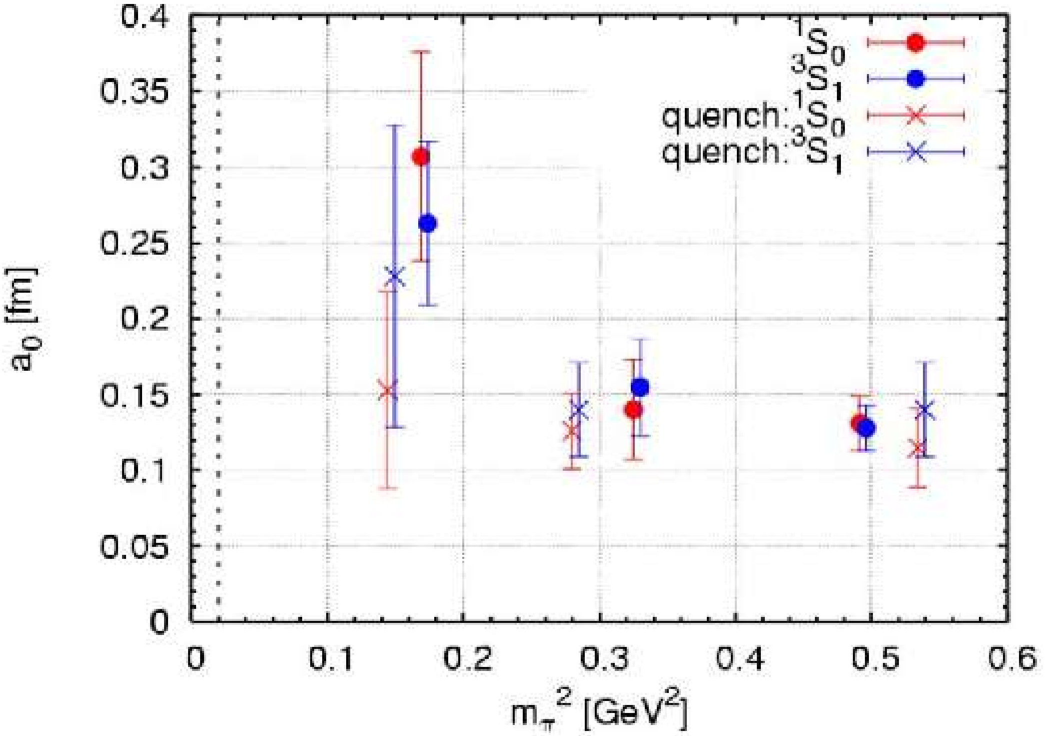
- Repulsive core grows.
- Attraction becomes stronger

Ishii (HAL QCD), talk at Lattice 2009.

Scattering length of NN (quark mass dependence)

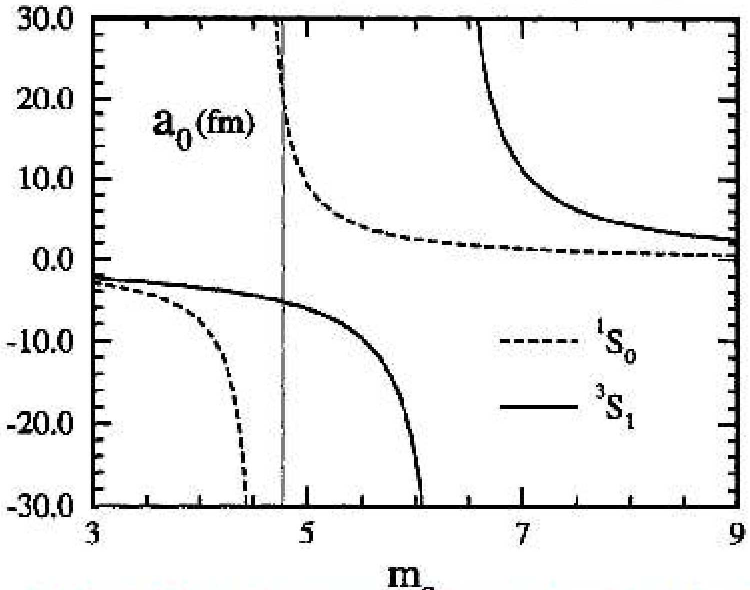
> pn

wave function → k^2 → Luscher's formula



- Attractive scattering length
- Attraction is enhanced as the quark mass decreases.
- The behavior is similar to the model below

OBE potential + lattice hadron mass
Kuramashi, PTP122,153(1996)

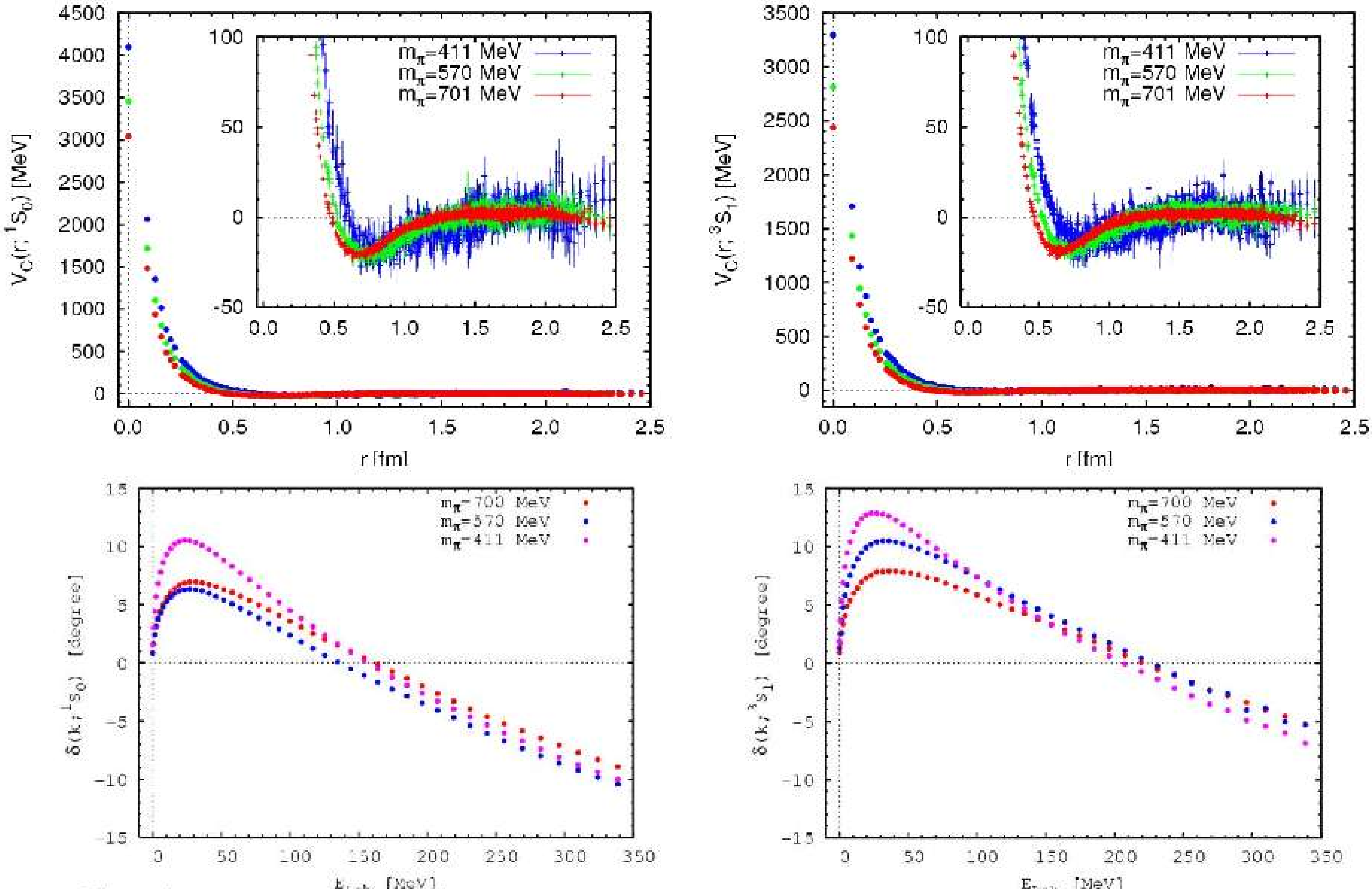


Drastic change near physical m_q.

Ishii (HAL QCD), talk at Lattice 2009.

NN (phase shift from potentials)

> pn



They have reasonable shapes.

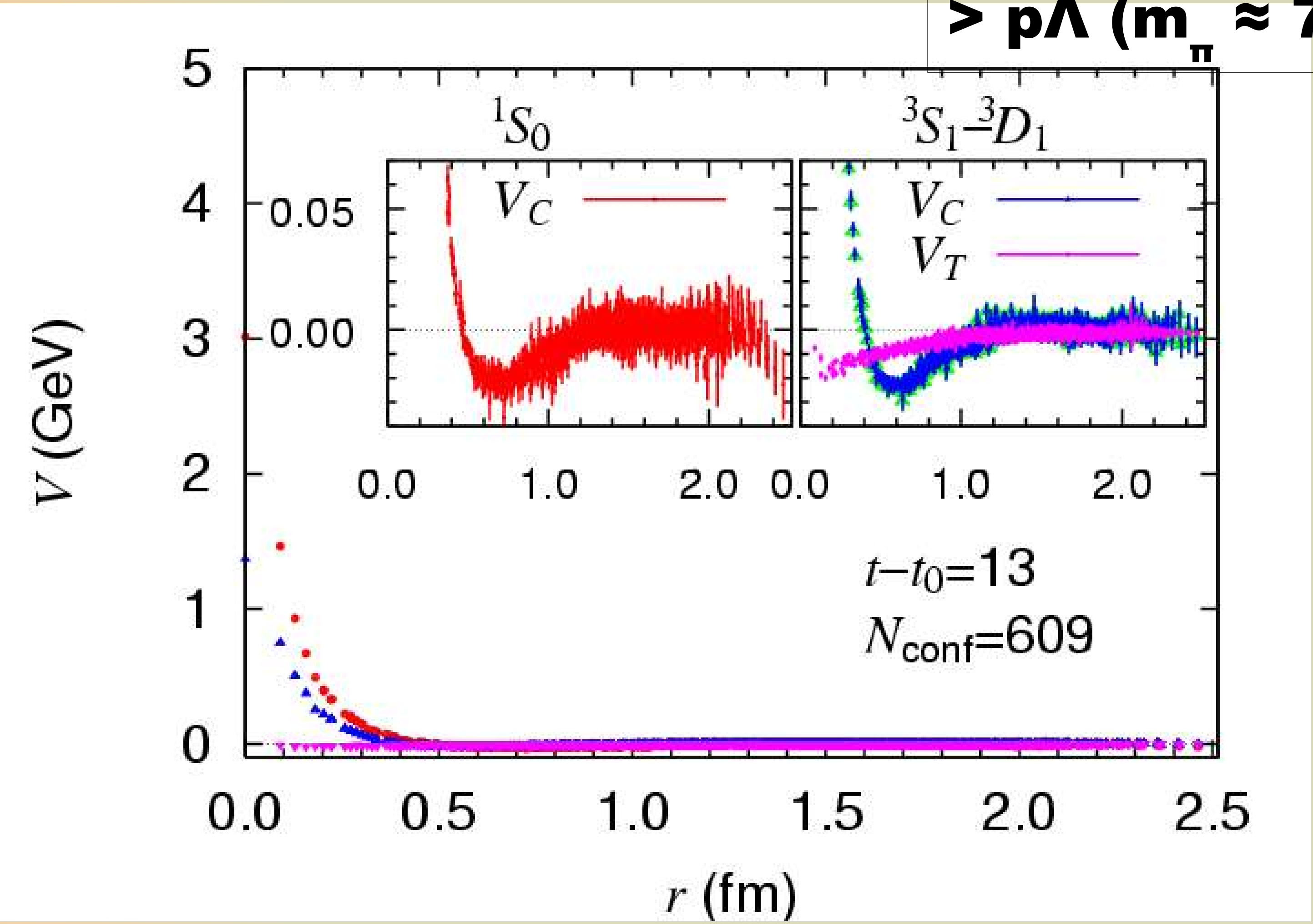
Ishii (HAL QCD), talk at Lattice 2009.

Results of N Λ force

Results —— central + tensor potential

- NΛ potential, from lattice QCD for the first time.

> pΛ (m_π ≈ 700 MeV)

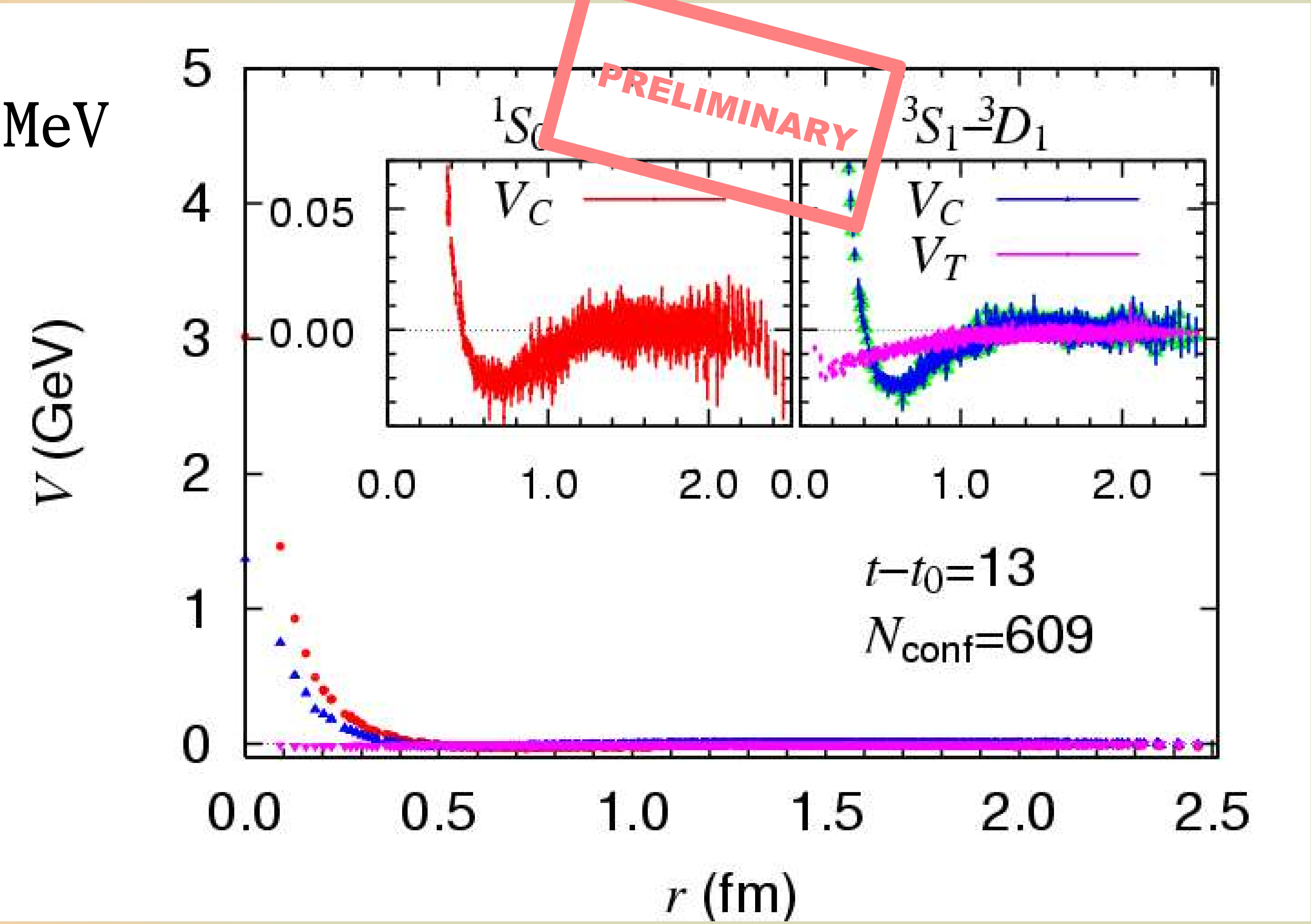


- Strong repulsive core in spin S=0 channel.
- Weak tensor force.

Results —— central + tensor potential

- NΛ potential, quark mass dependence.

$m_\pi \approx 700\,\mathrm{MeV}$

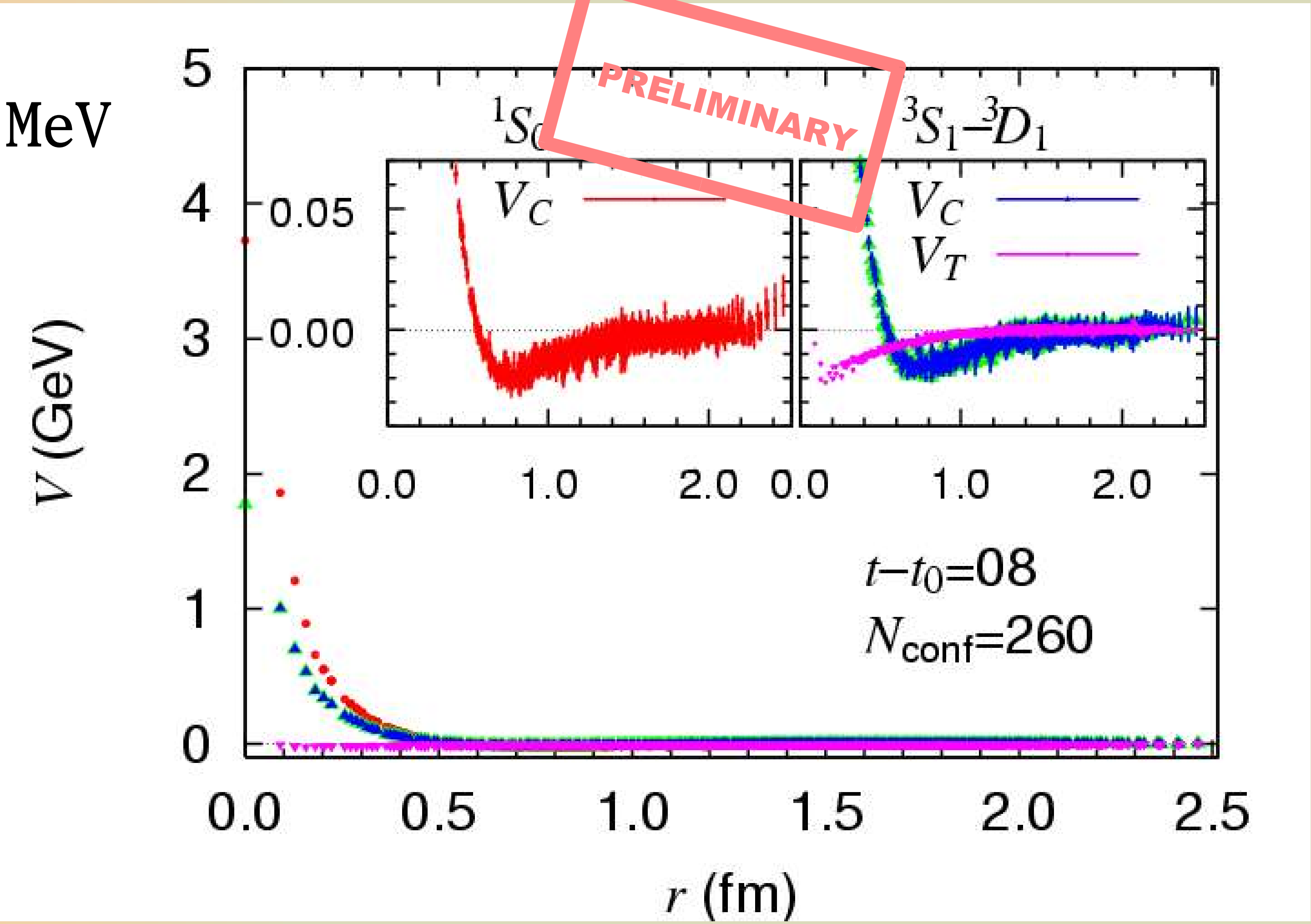


- Repulsive core increases.
- Interaction range increases.

Results —— central + tensor potential

- NΛ potential, quark mass dependence.

$m_\pi \approx 400$ MeV

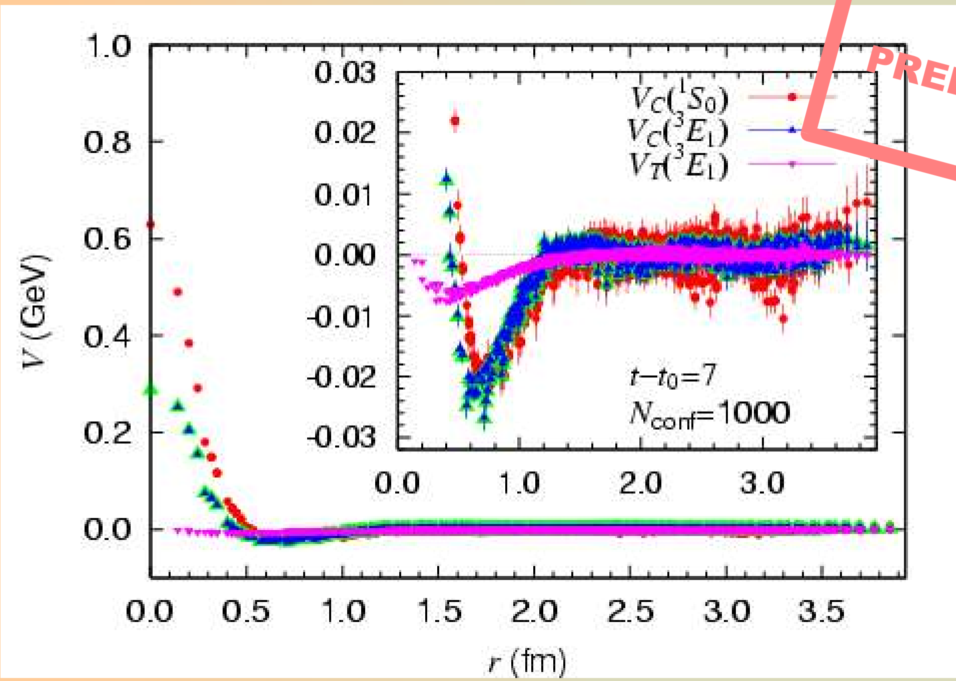


- Repulsive core increases.
- Interaction range increases.

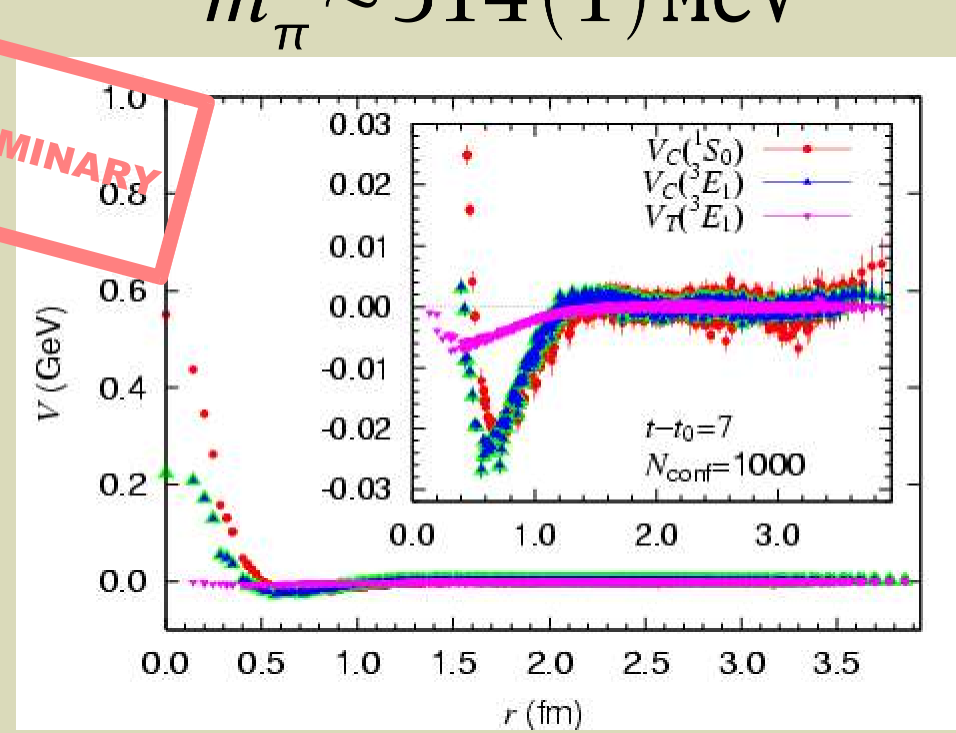
Results —— central + tensor potential

- NΛ potential, from quenched QCD.

$m_\pi \approx 465(1)\,\mathrm{MeV}$



$m_\pi \approx 514(1)\,\mathrm{MeV}$

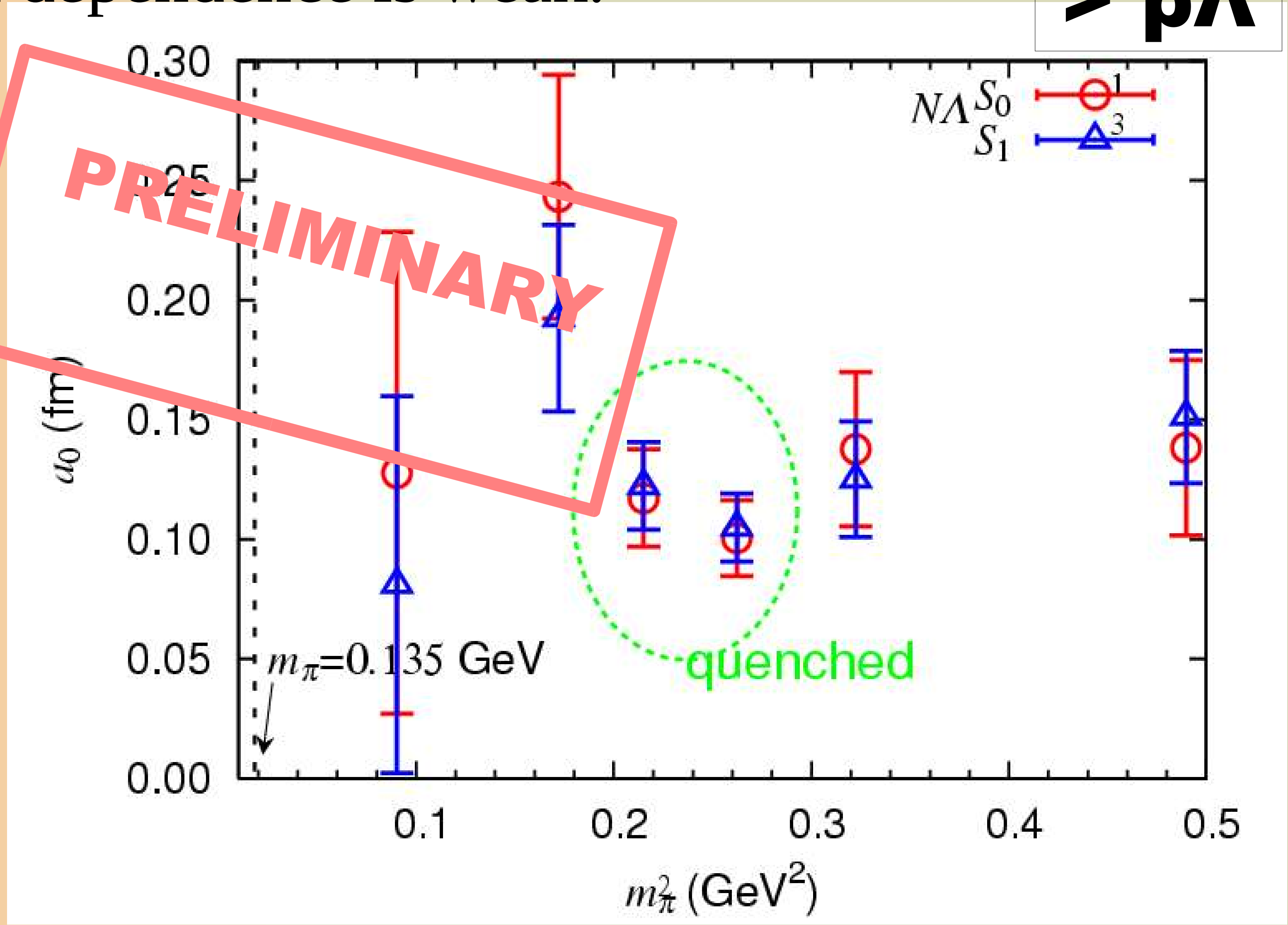


- Qualitatively similar results to those by full QCD.
  - Strong repulsive core in spin S=0 channel.
    (but relatively weaker than that from the full QCD)
  - Spin dependence.
  - Weak tensor.

Results —— scattering length

- Attractive scattering lenghts are obtained at several m_π^2 in both spin S=0 and 1.
- Spin dependence is weak.

> pΛ

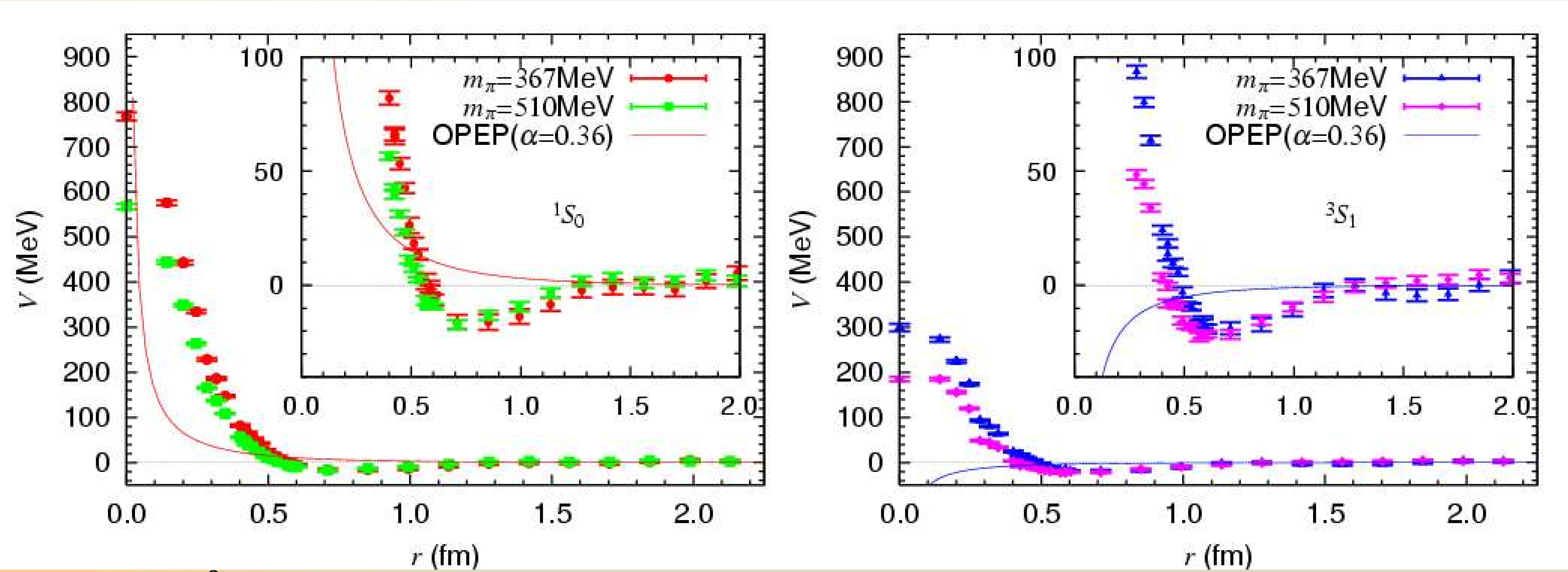


Results of N Ξ force

Results —— potential

- Compare with OPEP .

> $p\Xi^0$ (m_π = 370, 510 MeV)



0

$$V_C^\pi = -(1-2\alpha)\frac{g_{\pi NN}^2}{4\pi}\frac{(\vec{\tau}_N\cdot\vec{\tau}_\Xi)(\vec{\sigma}_N\cdot\vec{\sigma}_\Xi)}{3}\left(\frac{m_\pi}{2m_N}\right)^2\frac{\mathrm{e}^{-m_\pi r}}{r},$$

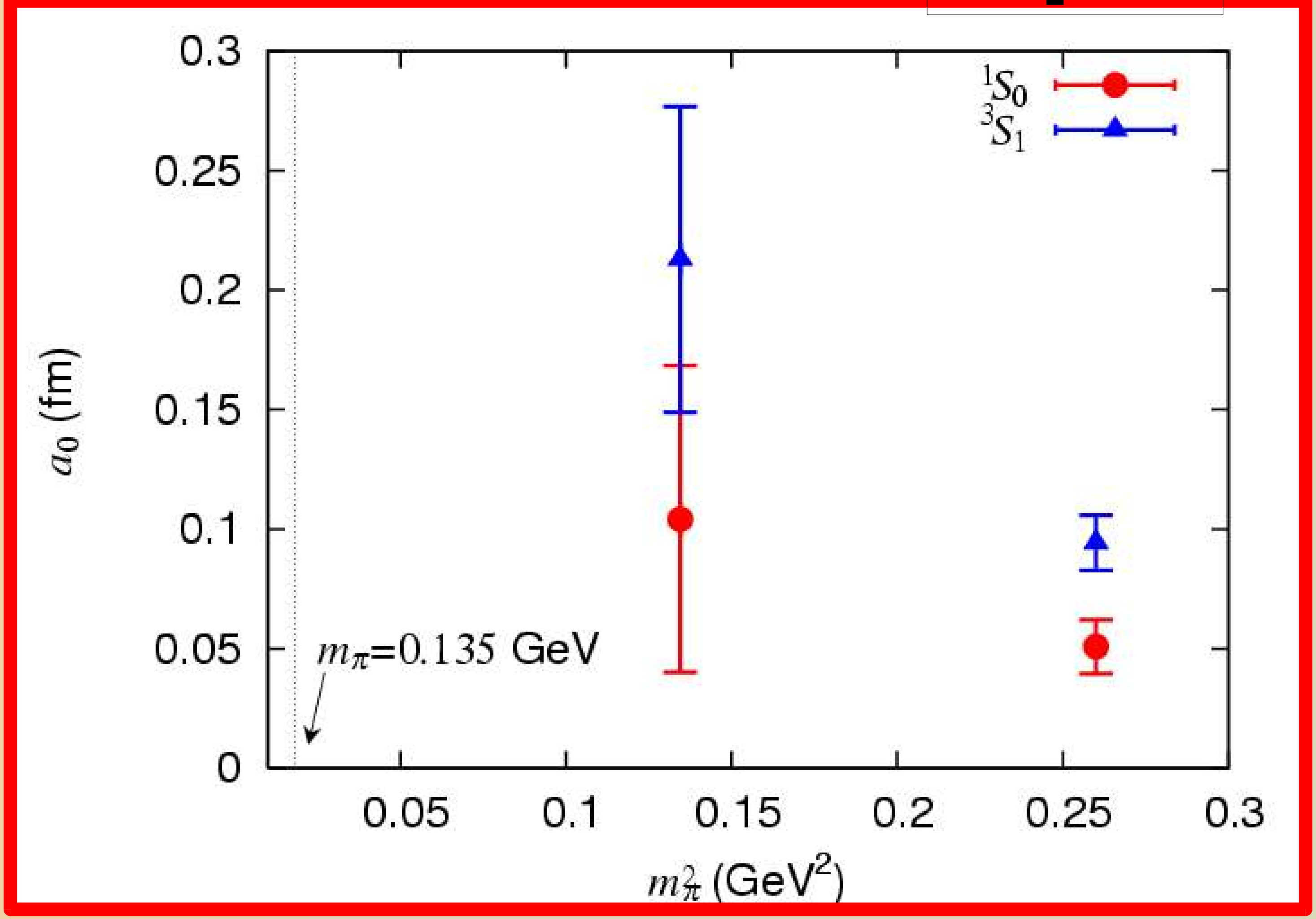
The pseudo-vector πNN coupling $f_{\pi NN}$ and the $\pi\Xi\Xi$ coupling $f_{\pi\Xi\Xi}$ are related as $f_{\pi\Xi\Xi} = -f_{\pi NN}(1-2\alpha)$ with the parameter $\alpha = F/(F+D)$ ratio[3]. Also we define $g_{\pi NN} \equiv f_{\pi NN}\frac{m_\pi}{2m_N}$.

The solid lines in Fig.3 is the one pion exchange potential (OPEP) obtained from Eq.(13) with $m_\pi \simeq 368$ MeV, $m_N \simeq 1167$ MeV (corresponding to $\kappa_{ud} = 0.1678$) and the empirical values, $\alpha \simeq 0.36$ [24] and $g_{\pi NN}^2/(4\pi) \simeq 14.0$ [1]. Unlike the case of the NN potential in the S-wave, the OPEP in the present case has

The pΞ^0 interaction from lattice QCD:

- H. N., *et al.*, PLB**673**, 136 (2009), [arXiv:0806.1094[nucl-th]].
  - The scattering lengths indicate attractive forces in both of $^1S_0$ and $^3S_1$ channels.

> pΞ^0



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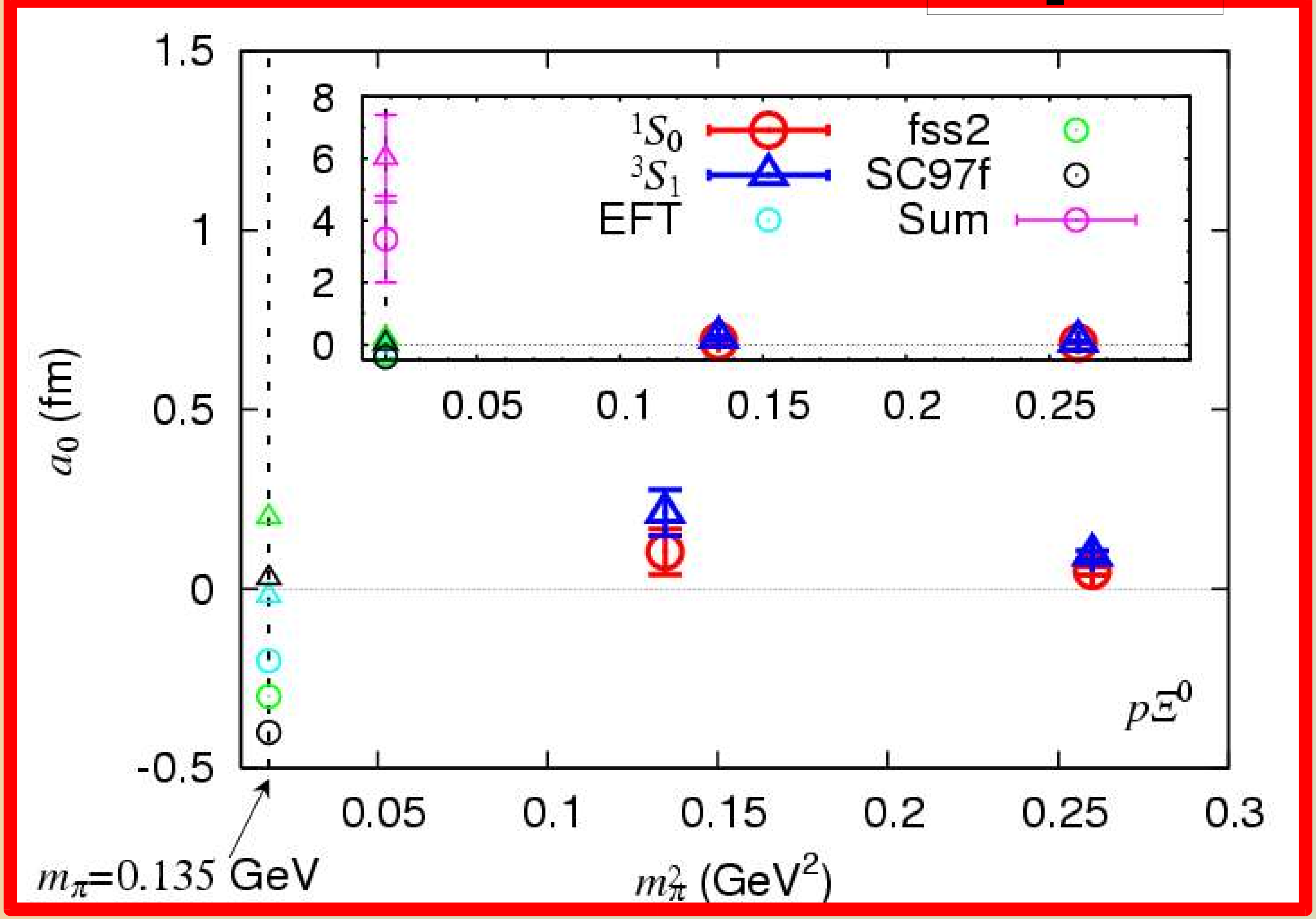
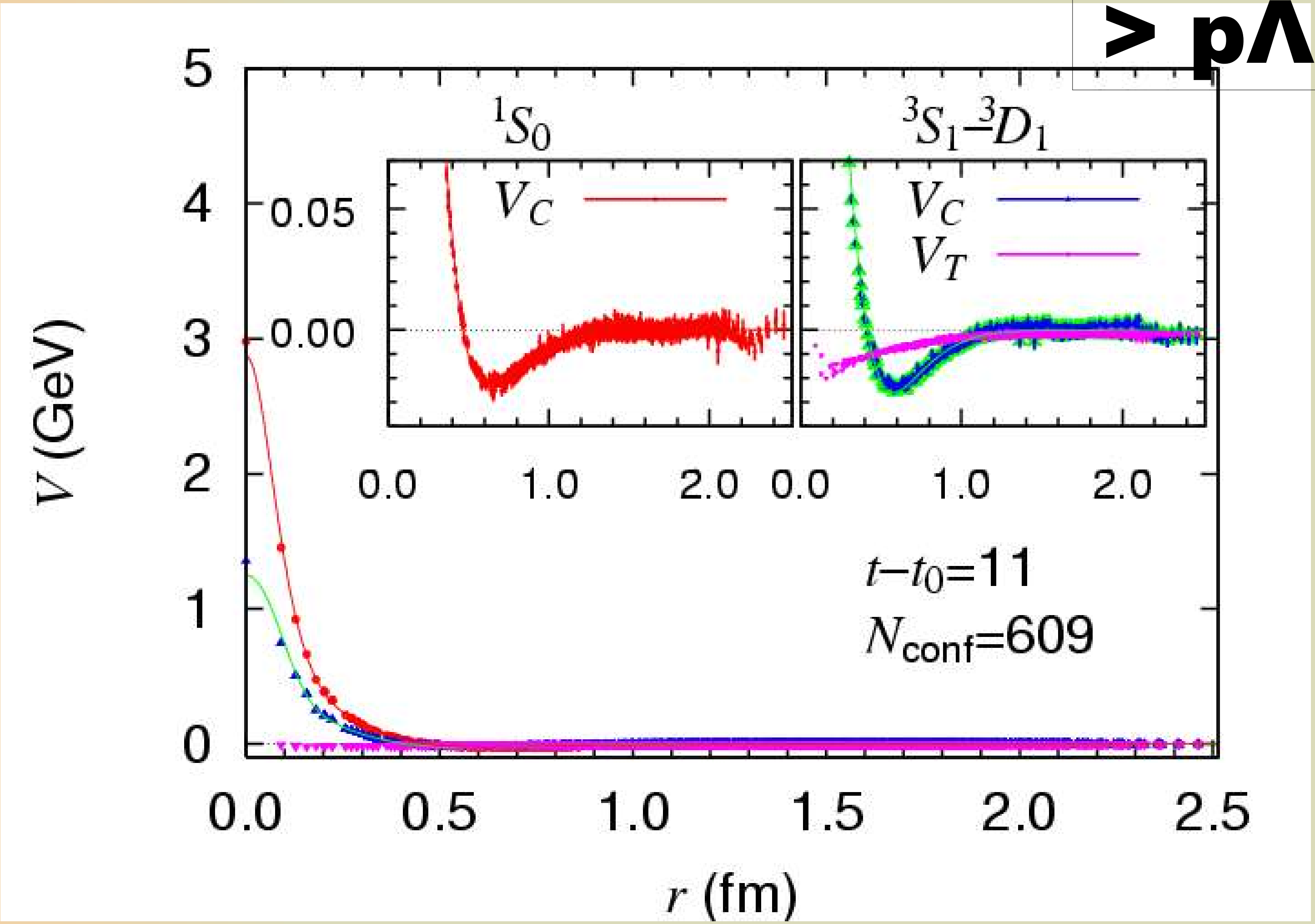


Summary:

- The lattice QCD studies for baryon-baryon interactions.
- NN:
  - Central, tensor, energy dependence. (full and quenched QCD)
- YN:
  - $p\Xi^0$:
    - Effective central. (quenched QCD)
  - $p\Lambda$:
    - Central, tensor. (full and quenched QCD)
- Qualitatively similar to well-known nuclear forces.
  - Repulsive at short distance.
  - Attractive well at medium to long distance.
- Quark mass dependence.
- Scattering lengths.

Future prospects —— hypernuclei

- Provide the lattice potential, to study the hypernuclei.

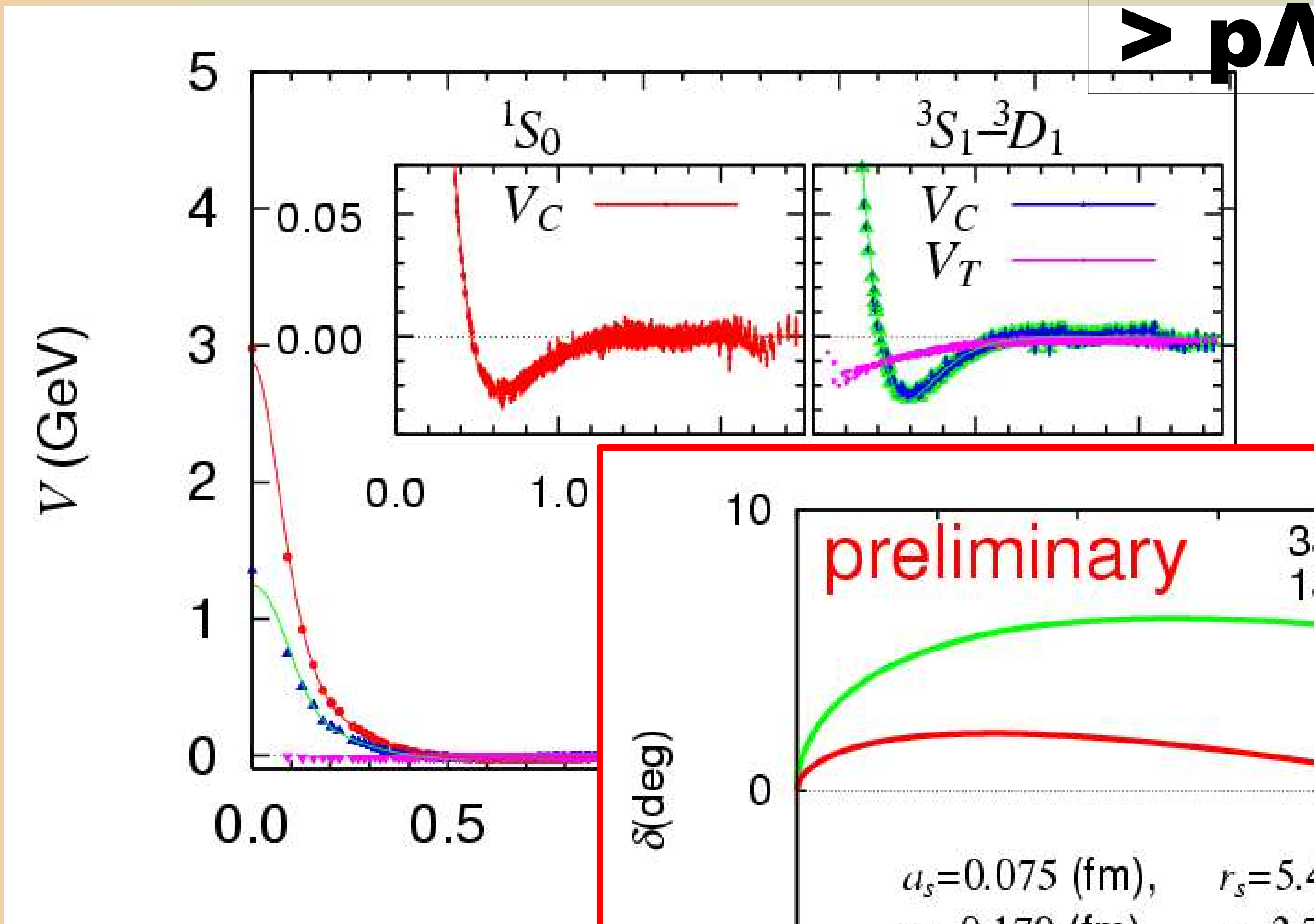


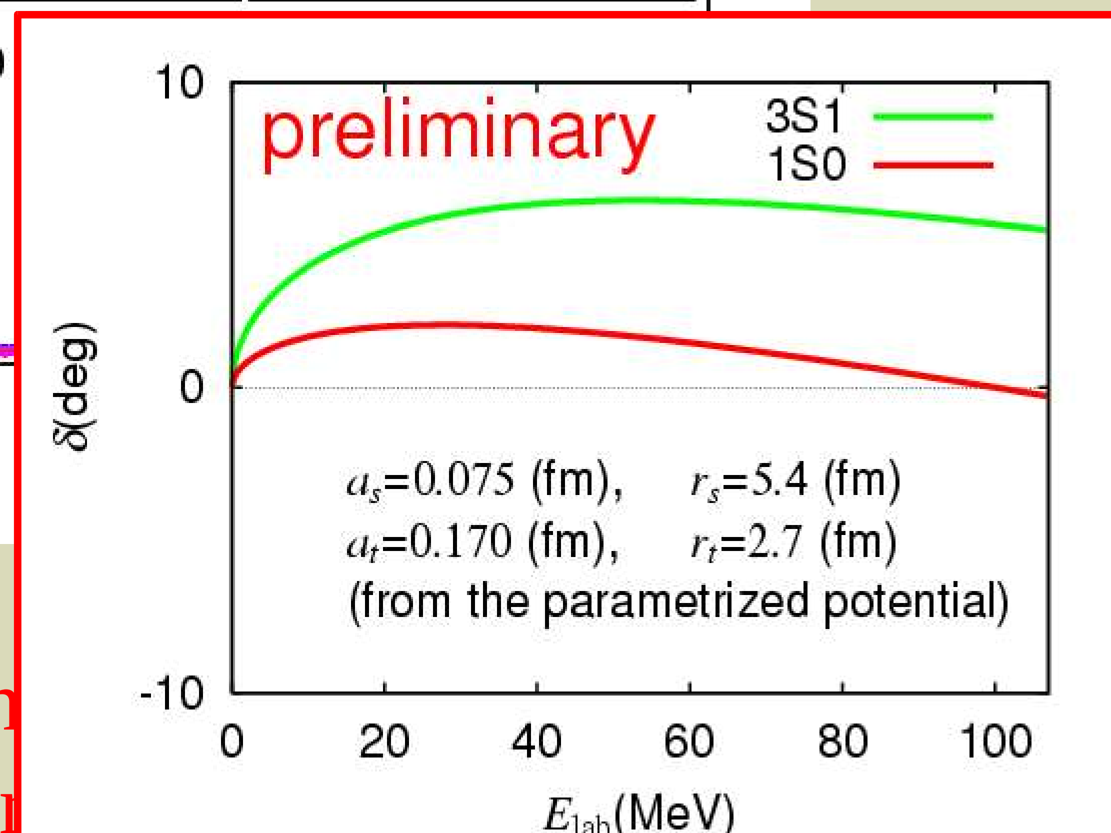
- Search for better functional form.
- Compare the meson theory.

Future prospects —— hypernuclei

- Provide the lattice potential, to study the hypernuclei.

> pΛ

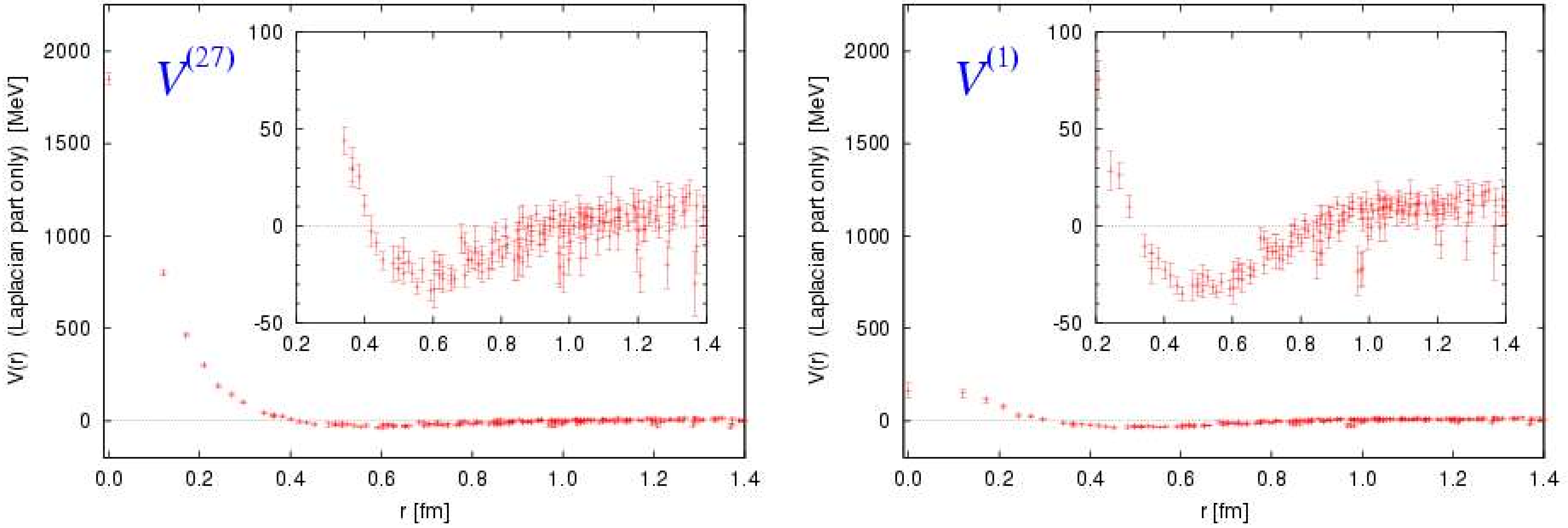




- Search for better function
- Compare the meson theor

> potentials at flavor SU(3) limit

Flavor sym ^1S potential



Tiny shift by energy is omitted here. Fixing energy is difficult since the box $(L=2[fm])^3$ is not large enough.

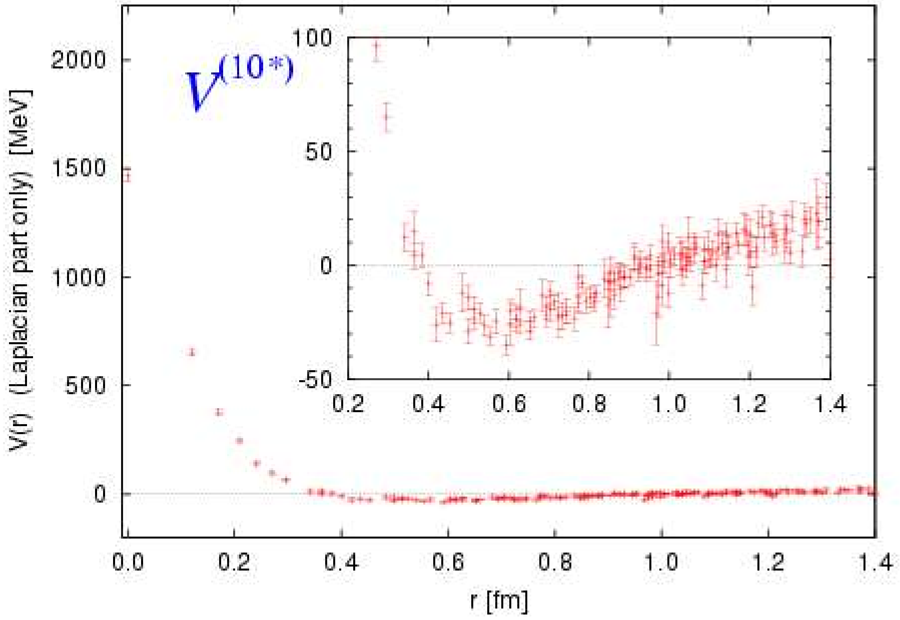
We see $V^{(27)} >> V^{(1)}$ at $r < 0.2$ [fm].

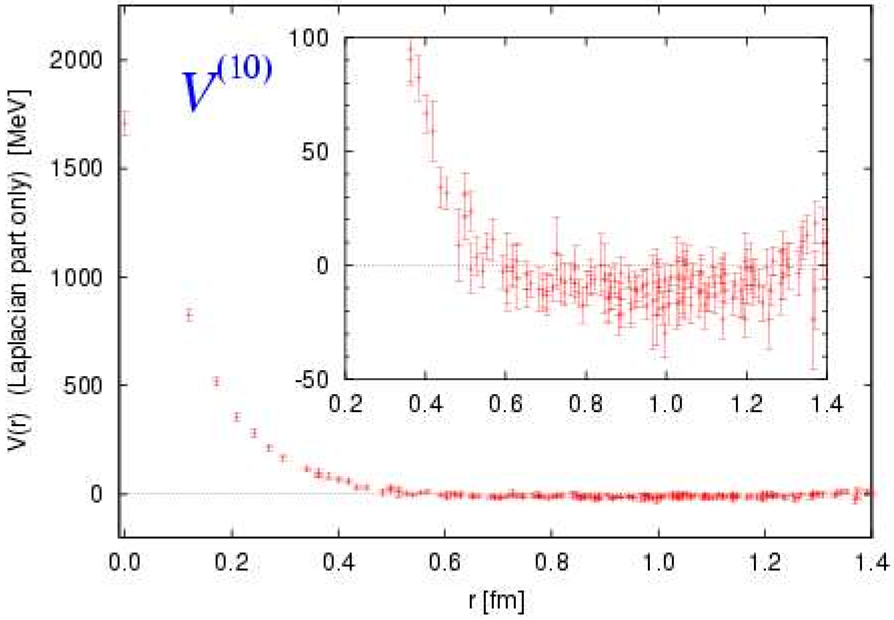
Both have attractive pocket around r=0.6 [fm].

Inoue (HAL QCD), talk at Lattice 2009.

> potentials at flavor SU(3) limit

Flavor anti-sym. ^3S potential





We see $V^{(10*)} < V^{(10)}$ at r < 0.2 [fm].

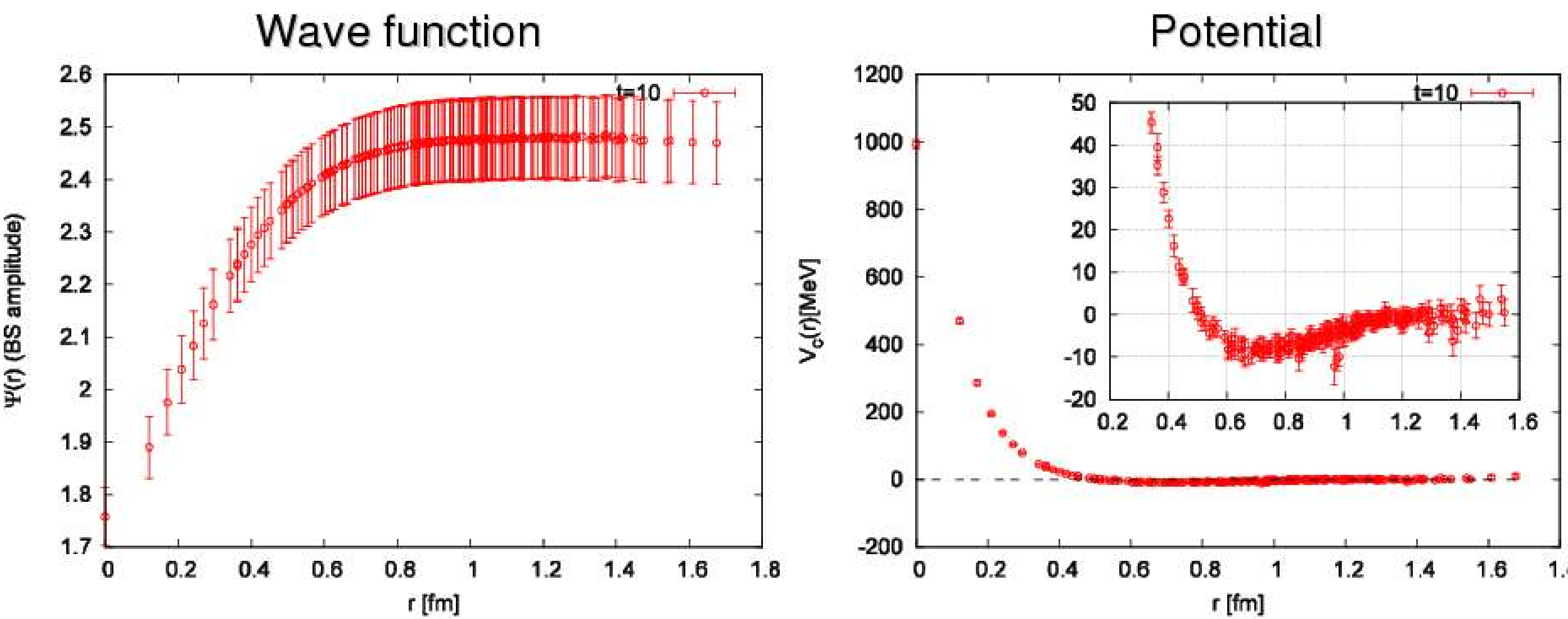
$V^{(10)}$ repulsive core is strongest among the rep.

$V^{(10)}$ has no vivid attractive pocket, very shallow.

Inoue (HAL QCD), talk at Lattice 2009.

> meson-baryon potential due to HAL

✓ S-wave nK^+ BS wave function and potential



- Repulsive core at short distance (r<0.5[fm])
- Attractive pocket in middle range (0.5<r<1.2 [fm])

Ikeda (HAL QCD), talk at FewBody19.