K^-pp studied with Coupled-channel Complex Scaling method

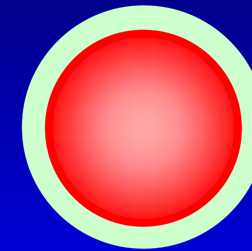
A. Doté (KEK) and T. Inoue (Tsukuba univ.)

1. Introduction
   - Result of a variational calculation with an effective *s*-wave chiral SU(3)-based $K^{bar}N$ potential
   - Recent status of K^-pp study
2. Question - *Variational calculation vs Faddeev calculation* -
3. Complex scaling method
4. Summary and Future plan

10th international conference on Hypernuclear and Strange particle physics (Hyp-X)
'09.09.17 @ Ricotti, Tokai, Japan

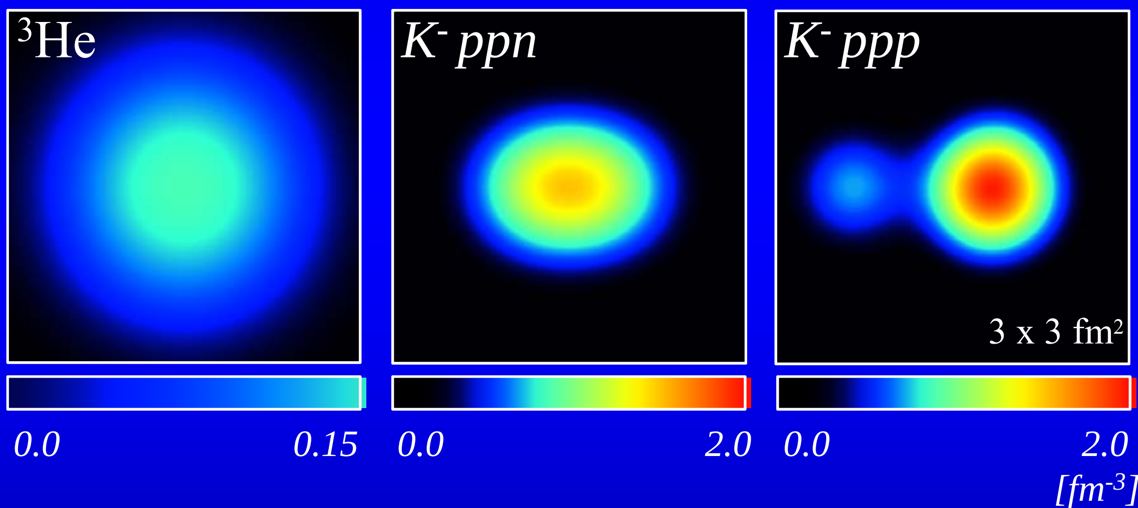
1. Introduction

K^{bar} nuclei (Nuclei with anti-koan) = *Exotic system !?*



I=0 $K^{bar}N$ potential … very attractive

→ Deeply bound (Total B.E. ~100MeV)
Highly dense state formed in a nucleus
Interesting structures that we have never seen in normal nuclei…



Calculated with
Antisymmetrized Molecular Dynamics
method employing
a phenomenological $K^{bar}N$ potential

A. Dote, H. Horiuchi, Y. Akaishi and T. Yamazaki, PRC70, 044313 (2004)

To make the situation more clear …

"K^-pp" = Prototype of K^{bar} nuclei
($K^{bar}NN$, $J^p=1/2^-$, $T=1/2$)

1. Introduction

A. Dote, T. Hyodo and W. Weise,
Nucl. Phys. A804, 197 (2008)
Phys. Rev. C79, 014003 (2009)

Variational calculation of $K^{-}pp$ with a chiral SU(3)-based $K^{bar}N$ potential

✓ *Av18 NN potential* … *a realistic NN potential with strong repulsive core.*

✓ *Effective $K^{bar}N$ potential based on Chiral SU(3) theory*

… *reproduce the original $K^{bar}N$ scattering amplitude obtained with coupled channel chiral dynamics.*

Single channel, Energy dependent, Complex, Gaussian-shape potential

✓ *Variational method*

… *Investigate various properties with the obtained wave function.*

Four variants of chiral unitary modes × $\sqrt{s} = \begin{cases} M_N + m_K - B(K) \\ M_N + m_K - B(K)/2 \end{cases}$

Total B. E. : 20 ± 3 MeV
$\Gamma(K^{bar}N \to \pi Y)$: 40 ∼ 70 MeV

Shallow binding and large decay width

NN distance = 2.2 fm
$K^{bar}N$ distance = 2.0 fm

∼ *NN* distance in normal nuclei

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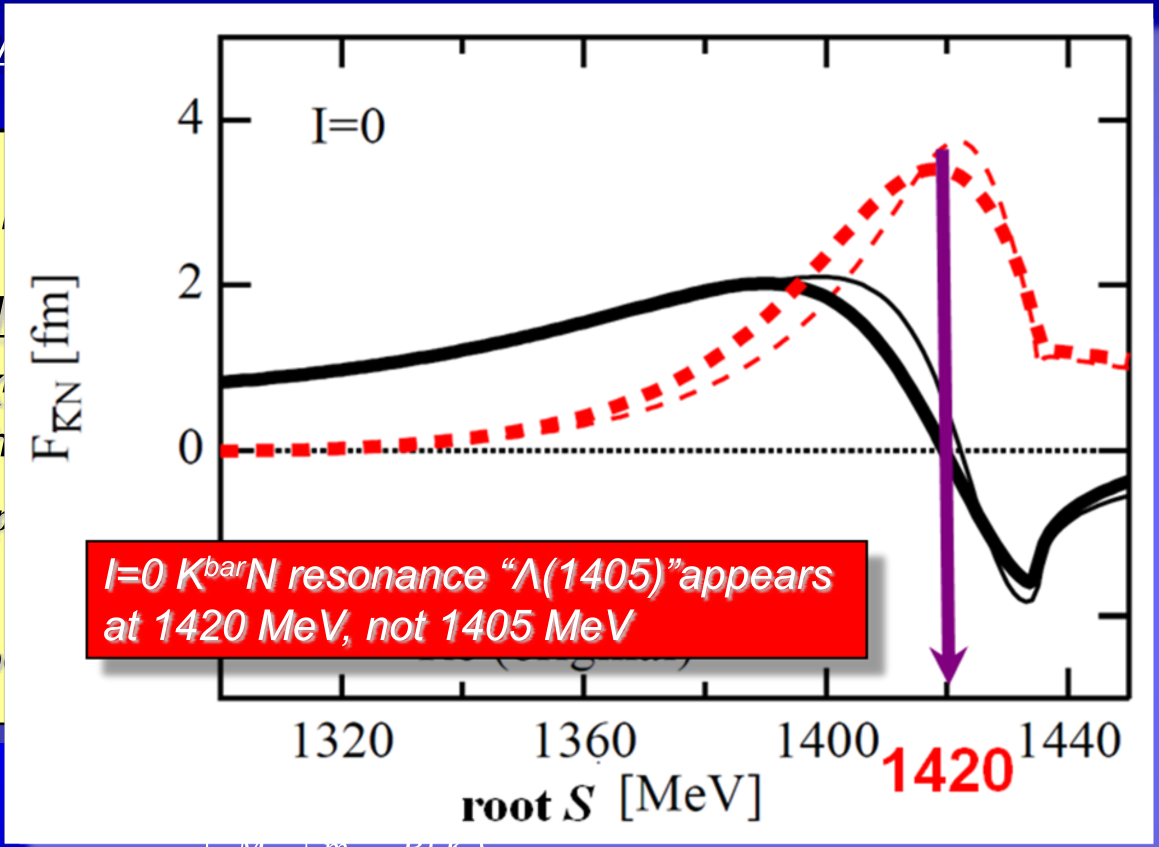

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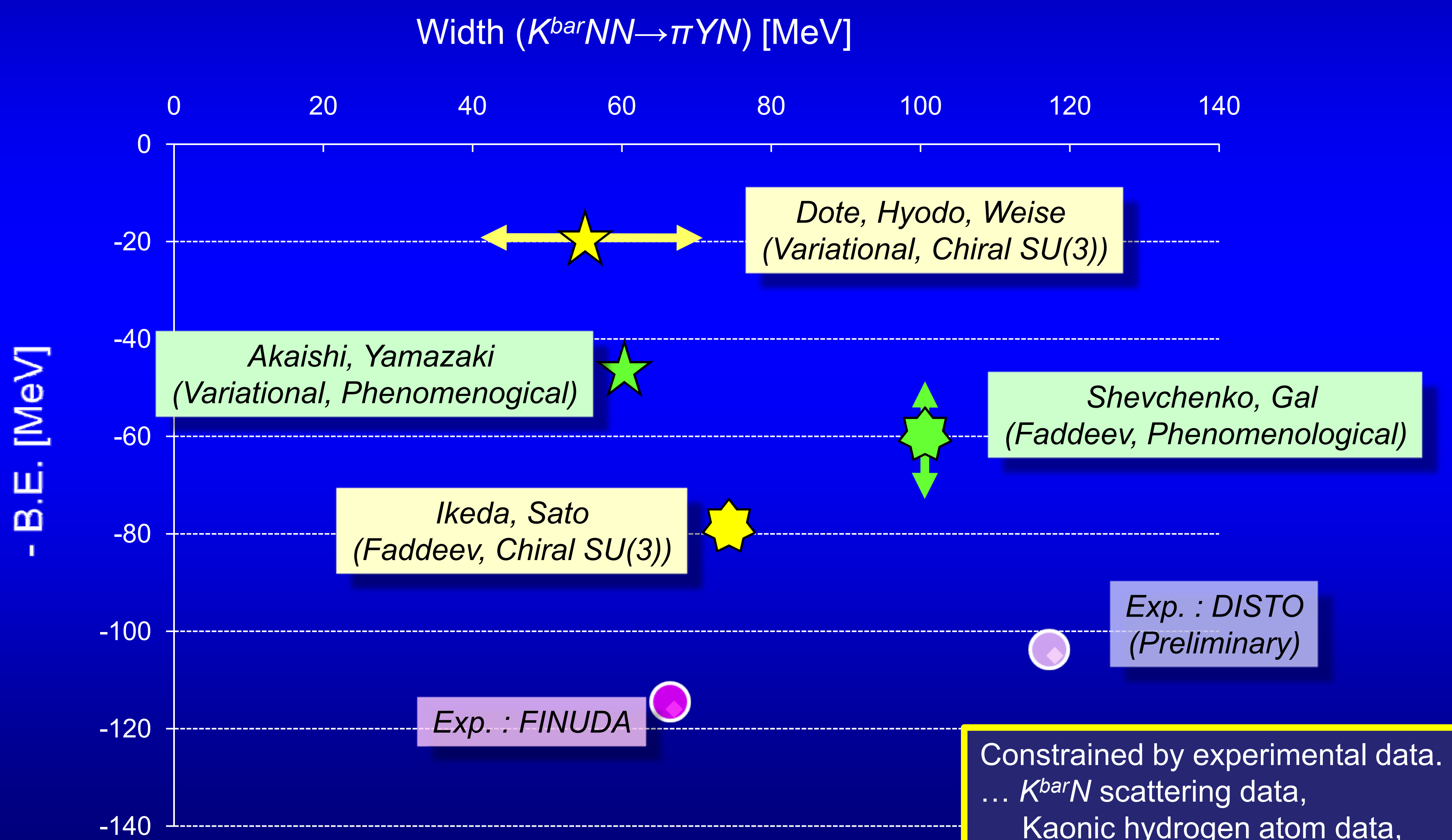
1. Introduction

Recent results of calculation of $K^{-}pp$ and related experiments



Constrained by experimental data.
… $K^{bar}N$ scattering data,
Kaonic hydrogen atom data,
"Λ(1405)" etc.

1. Introduction

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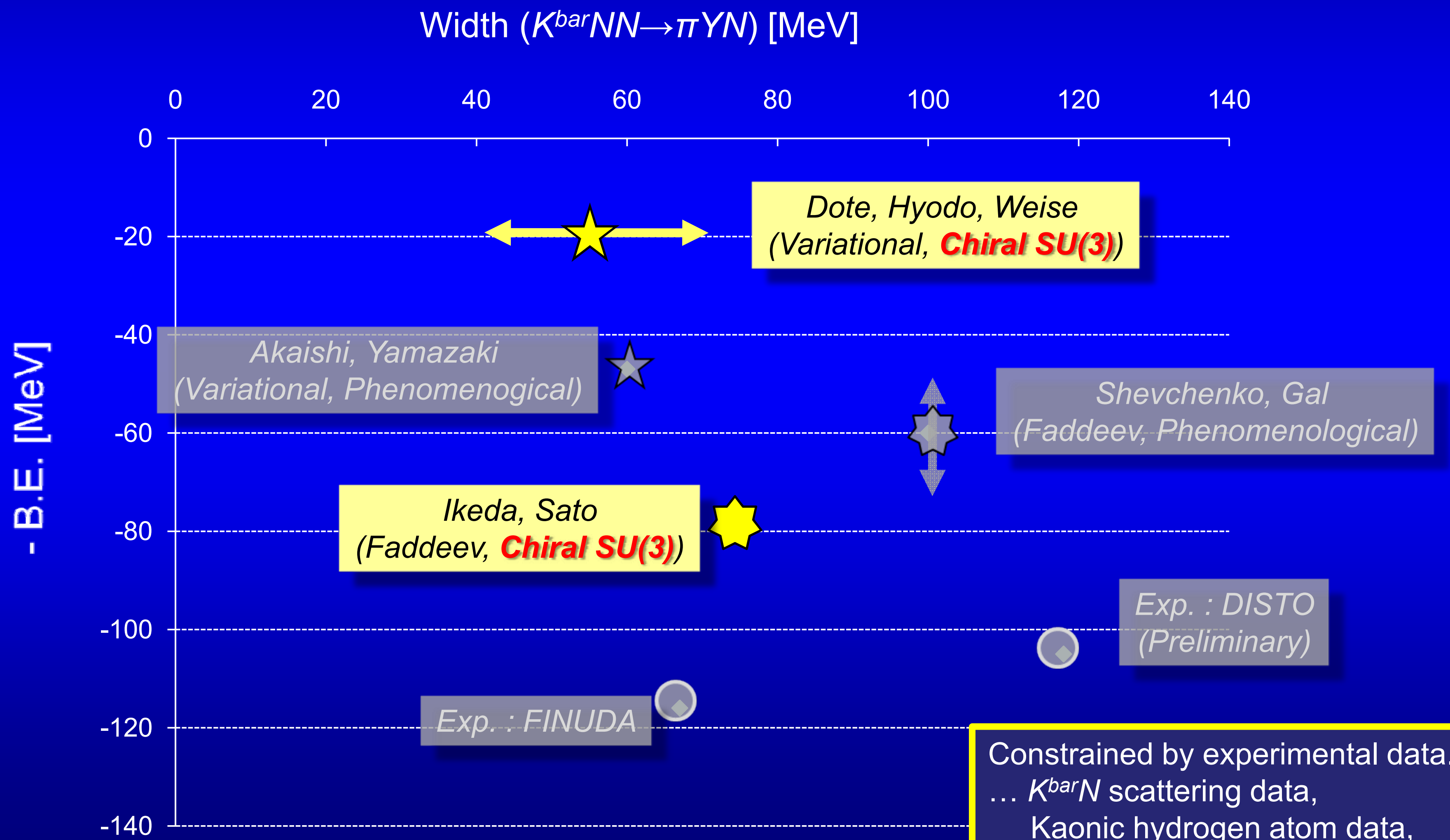

Constrained by experimental data.
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"Λ(1405)" etc.

2. Question

Variational calculation (DHW)

Total B. E. = 20±3 MeV,
Decay width = 40～70 MeV

A. Dote, T. Hyodo and W. Weise, Phys. Rev. C79, 014003 (2009)

Faddeev calculation (IS)

Total B. E. = 79 MeV,
Decay width = 74 MeV

Y. Ikeda, and T. Sato, Phys. Rev. C76, 035203 (2007)

???

Why does the result of a variational calculation (DHW) differ from that of a Faddeev calculation (IS)?

although the $K^{bar}N$ potentials used in both calculations are constrained with Chiral SU(3) theory.

- ✓ *Separable potential?*
- ✓ *Non-relativistic (semi-relativistic) vs relativistic?*
- ✓ *Energy dependence of two-body system ($K^{bar}N$) in the three-body system ($K^{bar}NN$)?*
- ✓ *Important role of πΣN thee-body dynamics?*
- ✓ *…???*

2. Question

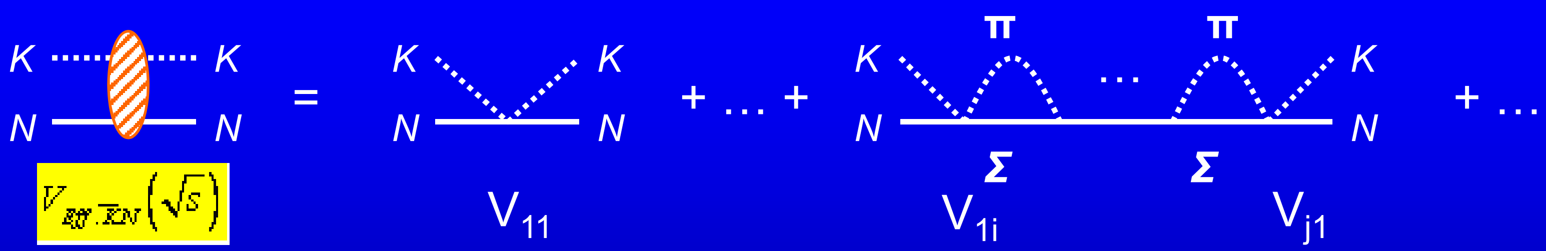
A possible reason is

→ *πΣN thee-body dynamics*

Y. Ikeda and T. Sato, arXiv:0809.1285

In the variational calculation (DHW),
πΣ channel is eliminated and incorporated into the effective $K^{bar}N$ potential.

<u>Two-body system</u>

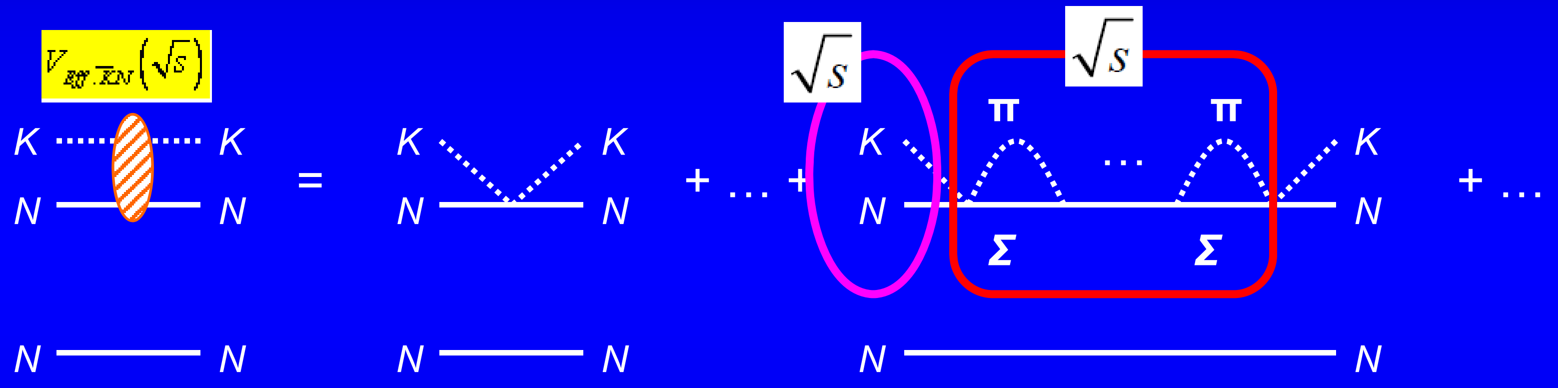


- This is correct for the two-body system.
- The effective $K^{bar}N$ potential has energy dependence due to the elimination of *πΣ* channel.
- It reproduces the original $K^{bar}N$ scattering amplitude calculated with $K^{bar}N$-*πΣ* coupled channel model.

2. Question

Apply the effective $K^{bar}N$ potential $V_{Eff.\bar{K}N}$ to the three-body system…

Three-body system calculated with the effective $K^{bar}N$ potential



The energy of the intermediate $\pi\Sigma$ state is fixed to the initial $K^{bar}N$ energy.

Not true!

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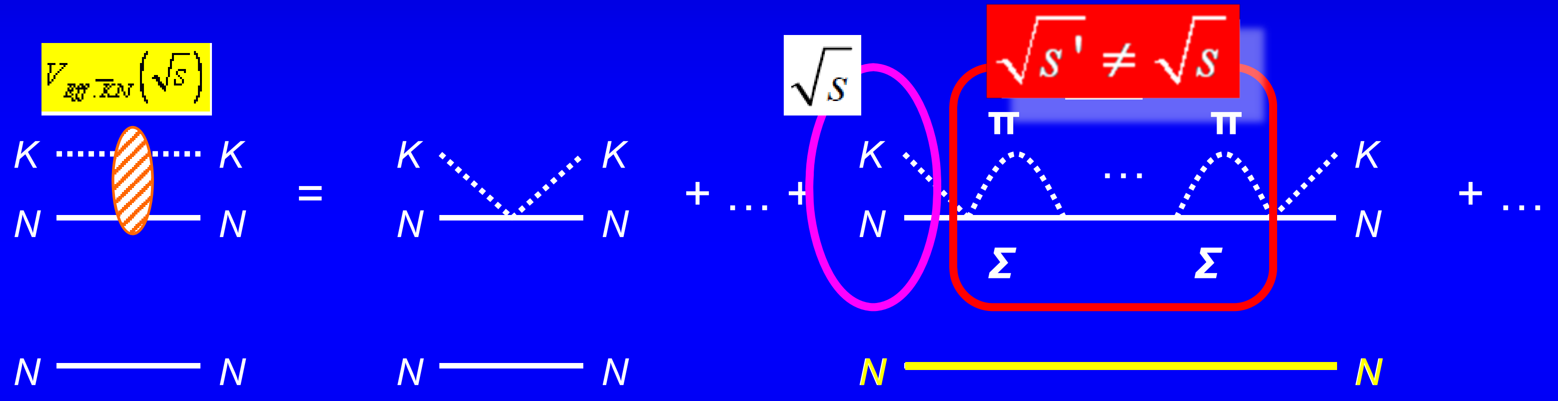

Due to the third nucleon,
the intermediate $\pi\Sigma$ energy can differ from the initial $K^{bar}N$ energy.

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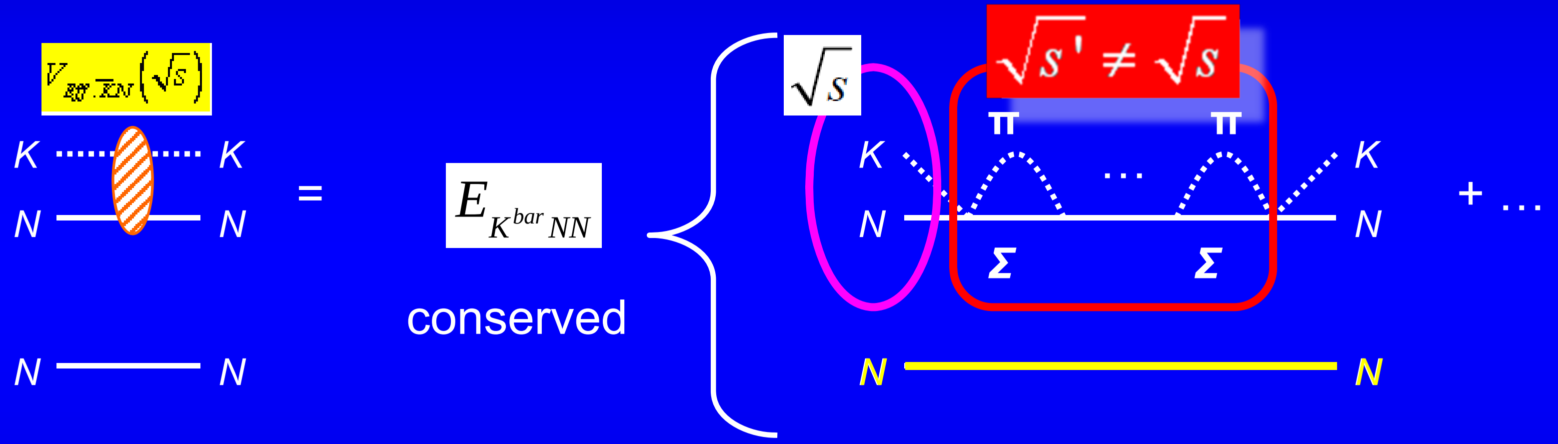


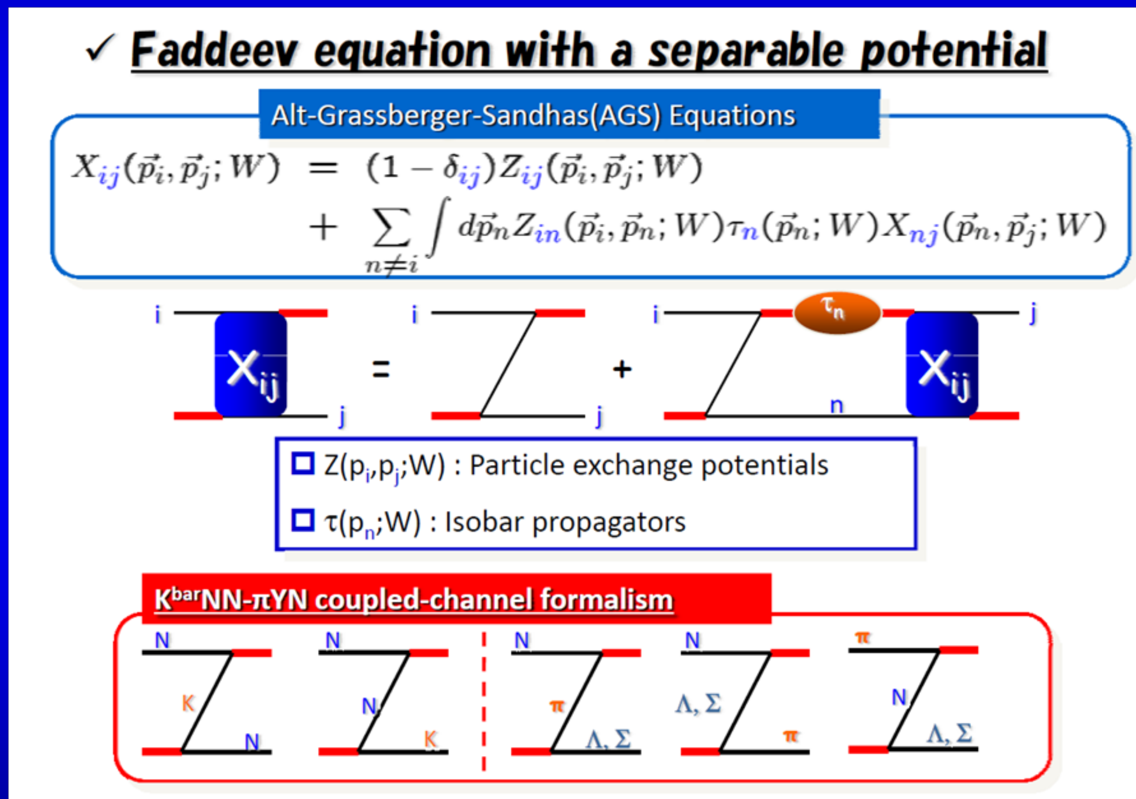
πΣN thee-body dynamics

Due to the third nucleon,
the intermediate *πΣ* energy can differ from the initial $K^{bar}N$ energy.

Only the total energy of the three-body system should be conserved.
The intermediate energy of the πΣ state can be variable,
due to the third nucleon.

2. Question

On the other hand, such a πΣN three-body dynamics is taken into account in the Faddeev calculation, because the πΣ channel is directly treated in their calculation.



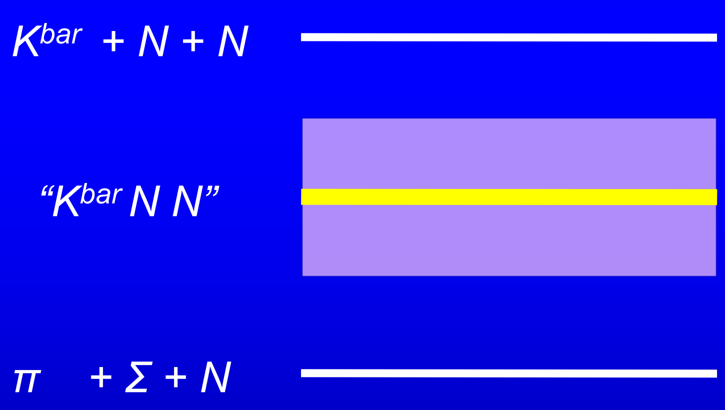
Dr. Ikeda's talk in KEK theory center workshop "Nuclear and Hadron physics" (KEK, 11-13 Aug. '09)

3. Complex Scaling method

To consider the πΣN three-body dynamics,
we should perform the coupled channel calculation also in our scheme.

➡ *The "πΣ" degree of freedom is directly treated.*

Coupled channel calculation



A bound state for K^{bar}NN channel,

but

<u>a resonant state</u> for πΣN channel

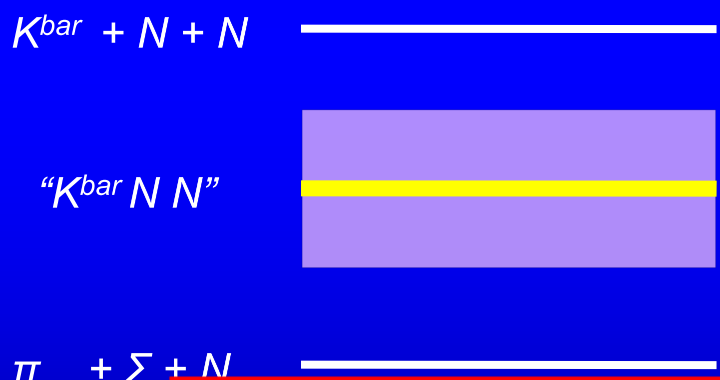
- The actual calculation is done in multi channels such as $K^{bar}N$ and πΣ.
- The obtained state is possibly to appear below $K^{bar}NN$ threshold, but above πΣN threshold.

3. Complex Scaling method

To consider the πΣN three-body dynamics,
we should perform the coupled channel calculation also in our scheme.

The "πΣ" degree of freedom is directly treated.

Coupled channel calculation



A bound state for K^{bar}NN channel,

but

<u>a resonant state</u> for πΣN channel

Resonant state can't be treated with a variational method.
Here, we employ

"Complex Scaling method"

to deal with resonant state.

- The a
- The c ... shold.

3. Complex Scaling method

Advised by
Dr. Myo (RCNP)

Complex rotation of coordinate

$$U(\theta): \quad \mathbf{r} \to \mathbf{r}\, e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}\, e^{-i\theta}$$

$$H_\theta \equiv U(\theta) H U^{-1}(\theta), \quad |\Phi_\theta\rangle \equiv U(\theta)|\Phi\rangle$$

$$E = \langle \Phi | H | \Phi \rangle = \langle \widetilde{\Phi}_\theta | H_\theta | \Phi_\theta \rangle$$

Wave function of a resonant state (two-body system)

$$\Phi_R \sim e^{ik_R r} = e^{i(\kappa - i\gamma) r}$$

$$U(\theta)\Phi_R \sim \exp\{(\gamma + i\kappa) r e^{i\theta}\}$$
$$= \exp[(\gamma\cos\theta - \kappa\sin\theta) r] \cdot \exp[i(\gamma\sin\theta + \kappa\cos\theta) r]$$

Negative!

When $\tan\theta > \frac{\gamma}{\kappa}$, the wave function of a resonant state changes to a dumping function.

→ Boundary condition is the same as that for a bound state.

The resonant state can be obtained by ***diagonalizing*** H_θ ***with Gaussian basis***, quite in the same way as calculating bound state.

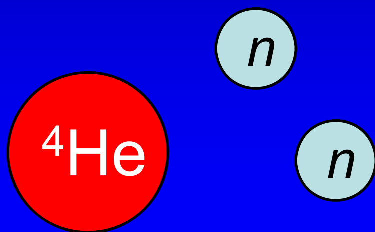
3. Complex Scaling method

Advised by Dr. Myo (RCNP)

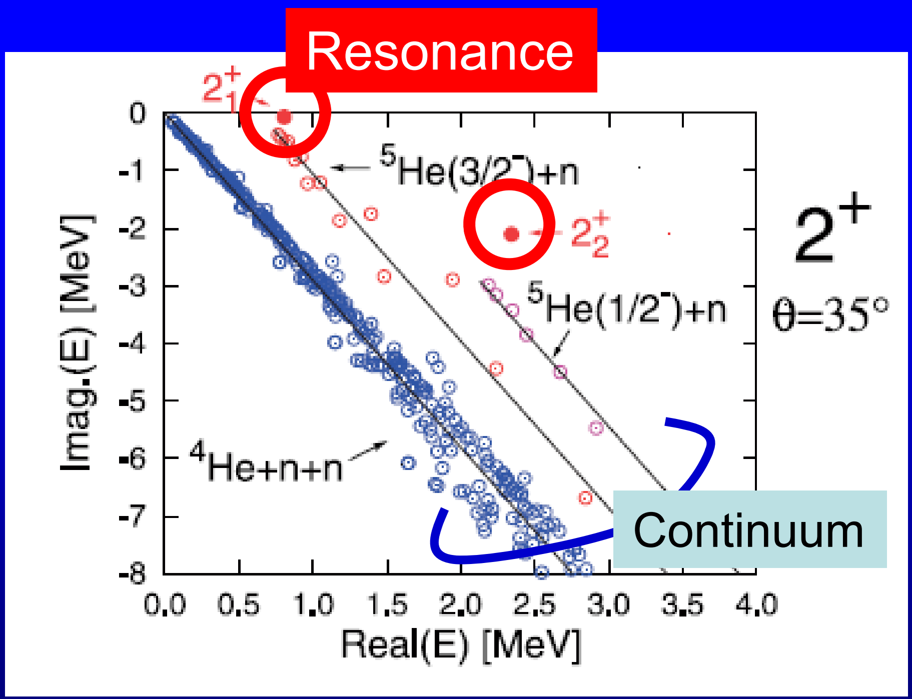
Succeeded in nuclear physics

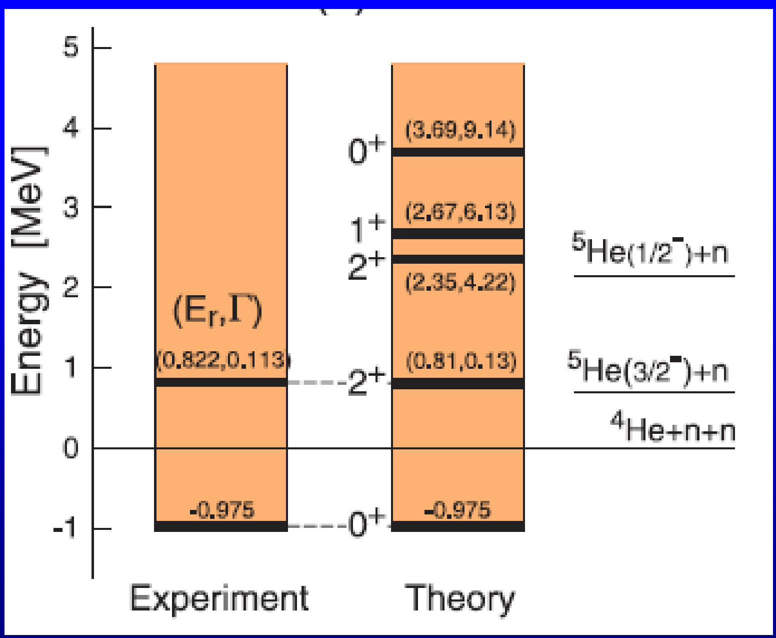
… Especially used in the study of unstable nuclei.

<u>Resonant state of ^{6}He</u>



S. Aoyama, T. Myo, K. Kato and K. Ikeda, Prog. Theor. Phys. 116, 1 (2006)





4. Summary and Future plan

Current status of the study of K^{bar} nuclei

The most essential K^{bar} nuclei "$K^{-}pp$" ($K^{bar}NN$, $J^{p}=1/2^{-}$, $T=0$) has been investigated in various ways. But the situation is still controversial…

Theory

Variational	+	*Phenom. $K^{bar}N$*	*B.E. = 47MeV,*	*Γ= 61MeV*	*PRC76, 045201(2007)*
Variational	+	*Chiral-based $K^{bar}N$*	*B.E. = 20±3MeV,*	*Γ= 40-70MeV*	*PRC79, 014003(2009)*
Faddeev	+	*Phenom. $K^{bar}N$*	*B.E. = 50～70MeV,*	*Γ=～100MeV*	*PRC76, 044004(2007)*
Faddeev	+	*Chiral-based $K^{bar}N$*	*B.E. = 79MeV,*	*Γ= 74MeV*	*PRC76, 035203(2007)*

Experiment (Unknown object which seems related to $K^{-}pp$)

FINUDA	*B.E. = 116MeV,*	*Γ= 67MeV*	*PRL94, 212303(2005)*
DISTO	*B.E. = 105MeV,*	*Γ= 118MeV*	*arXiv:0810.5182 [nucl-ex]*

? Difference between a variational calculation (DHW) and a Faddeev calculation (IS) might be caused by *πΣN three-body dynamics*.

Y. Ikeda and T. Sato, arXiv:0809.1285

Coupled channel Complex Scaling

- Resonant states can be calculated quite in similar way to treat bound states.

Diagonalizing the complex-rotated Hamiltonian, resonant states are obtained.

4. Summary and Future plan

Future plan

Do calculations!

1. Calulate two-body system … $K^{bar}N$-$\pi\Sigma$ system corresponding to $\Lambda(1405)$

- For a test calculation, a phenomenological $K^{bar}N$ potential (AY potential, energy-independent) will be employed.

Y. Akaishi and T. Yamazaki, PRC 52 (2002) 044005

- See what happen if we use a chiral SU(3)-based $K^{bar}N$ potential (HW potential, energy-dependent).

T. Hyodo and W. Weise, PRC77, 035204(2008)

2. Calulate three-body system … $K^{bar}NN$-$\pi\Sigma N$ system corresponding to "$K^{-}pp$"

Thank you very much!

I’m sorry
because I can’t show any new result…