On the resonance energy of the $K^{bar}NN$-πYN system

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In collaboration with

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- ✓ Strange dibaryon (Recent theoretical progress)
- ✓ Model of KN interaction
- ✓ Faddeev approach and variational approach
- ✓ Numerical Results
- ✓ Summary

Strange dibaryon resonance

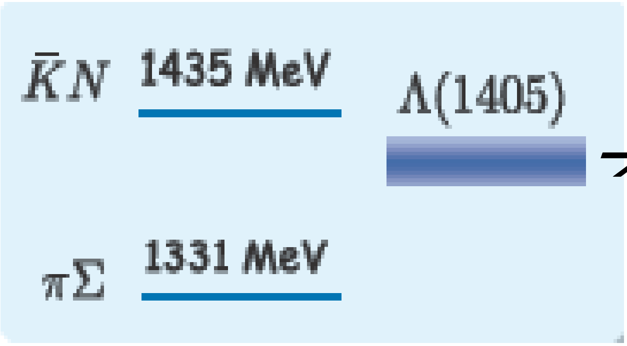
- Our motivation -

✓ Attractive KN interaction, Λ(1405), Strange dibaryon

- Analysis of Kaonic hydrogen atom and presence of Λ(1405)

  → K^{bar}N interaction is strongly attractive.

✓ Λ(1405) resonance



→ *S-wave resonance*

in the K^{bar}N-πΣ coupled channel system

✓ ***It is very important***

to take into account the "full dynamics" of KN-πΣ system

in order to investigate energy of strange dibaryon resonance.

✓ Strange dibaryon – Our motivation -

KN interaction / 3-body Method	Phenomenological	Weinberg-Tomozawa
Variational Method (B, Γ)	Akaishi, Yamazaki (48, 60)MeV	Dote, Hyodo, Weise (17-23, 40-70)MeV
Faddeev equation (B, Γ)	Shevchenko, Gal, Mares (55-70, 90-110)MeV	Ikeda, Sato (45-80, 45-75)MeV

Many theoretical studies suggest possible existence of strange dibaryon, but…

predicted binding energy and width widely spread.

✓ Major uncertainty in theoretically estimating energy of strange dibaryon is that an accurate description of the KN interaction including its off-shell behavior is still missing.

✓ Taking a reverse viewpoint, we may hope that there is a possibility to constrain KN dynamics from the study of strange dibaryon.

✓ It is crucial to treat the three-body dynamics as accurately as possible in a theoretical calculation for a given KN model.

✓ Strange dibaryon – Our motivation -

KN interaction / 3-body Method	Phenomenological	Weinberg-Tomozawa
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Many theoretical studies suggest possible existence of strange dibaryon, but…

predicted binding energy and width widely spread.

✓ In this work, we compare Faddeev approach with variational approach within our KN interaction model.

✓ From this analysis, we extract the “explicit” πYN coupled-channel effect.

…re is a possibility to constrain KN dynamics …ange dibaryon.

✓ It is crucial to treat the three-body dynamics as accurately as possible in a theoretical calculation for a given KN model.

Model of K^{bar} N interaction

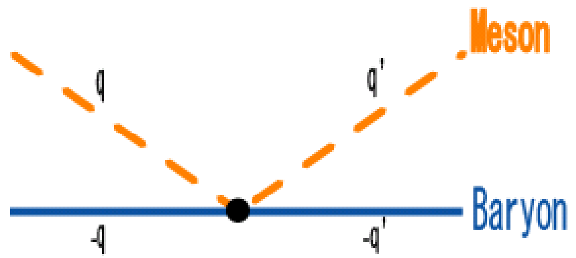
✓ K^{bar} N potential model

We start from leading order term of effective chiral Lagrangian.

Weinberg-Tomozawa interaction

$$\mathcal{L}_I = \frac{i}{8F_\pi^2} Tr[\bar{B}\gamma^\mu[\Gamma_\mu^2, B]]$$

$$\Gamma_\mu^2 = \Phi(\partial_\mu\Phi) - (\partial_\mu\Phi)\Phi$$



Φ: *Meson field* , B : *Baryon field*

S-wave separable potential

$$V_{MB}(q',q) = -4\pi\lambda_{\alpha\beta}^{(I)} \frac{1}{(2\pi)^3} \frac{1}{2F_\pi^2} \frac{1}{\sqrt{\omega'\omega}} \frac{m'+m}{2} \times \left(\frac{\Lambda^2}{\vec{q}'^2+\Lambda^2}\right)^2 \left(\frac{\Lambda^2}{\vec{q}^2+\Lambda^2}\right)^2$$

✓The strength of the potential is determined by pion decay constant.

✓The relative couplings are fixed by chiral SU(3).

✓In order to regularize loop integrals, dipole form factors are introduced.

✓ K^{bar} N potential model (parameter fit)

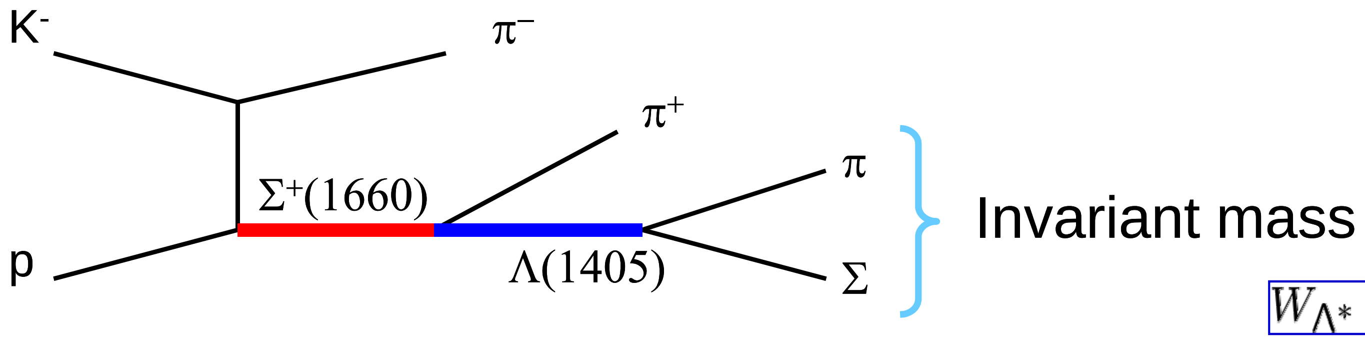
Our parameters -> cut-off of dipole form factor

Fit 1 : Λ(1405) pole position given by Dalitz (Model Dalitz)

$$W_{\Lambda^*} = 1406 - i25(MeV)$$

Fit 2 : πΣ invariant mass spectrum (Model Hemingway)

Hemingway, NPB253(1985).



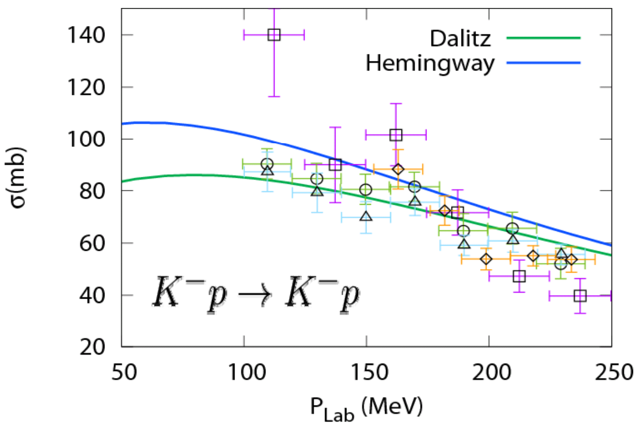
$$W_{\Lambda^*} = 1414 - i19(MeV)$$

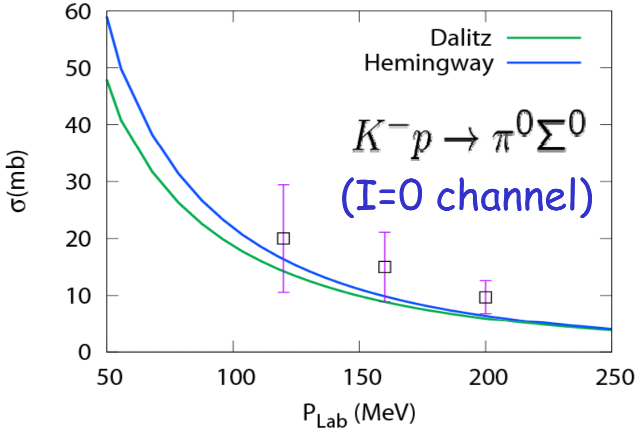
✓ We assume πΣ scattering dominates the resonance region, and the cross section formula would give the invariant mass distribution.

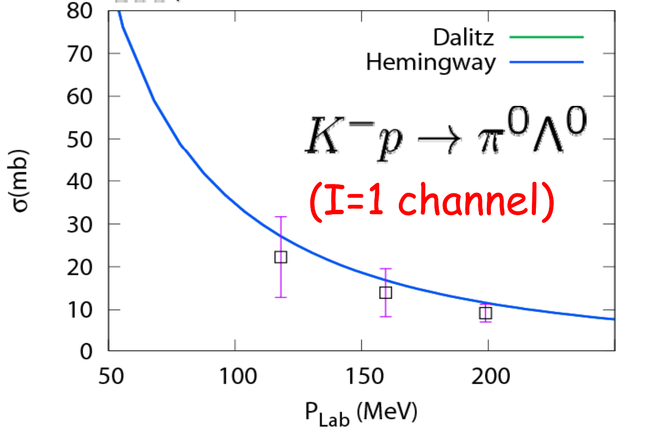
$$\frac{d\sigma}{dm} \propto |t_{\pi\Sigma-\pi\Sigma}|^2 p_{CM}$$

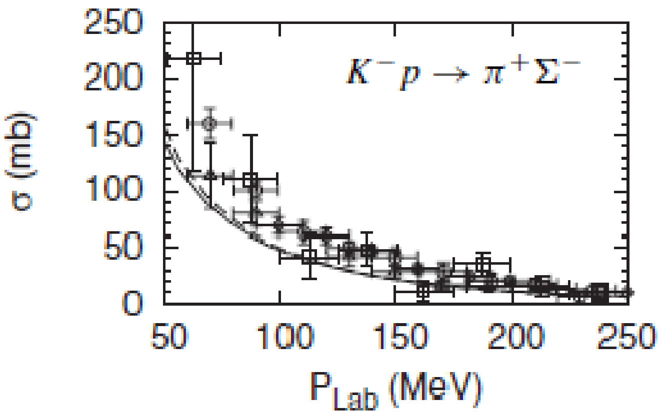
✓ K^{bar} N potential model (total cross sections)

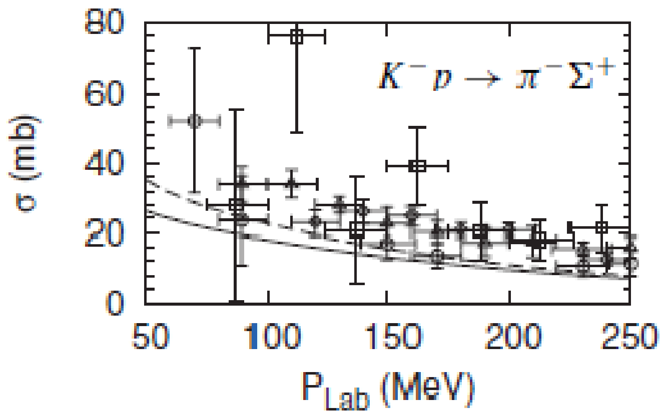
$$a^{I=1}_{\bar{K}N} = 0.68 + i0.60 fm$$











Faddeev approach

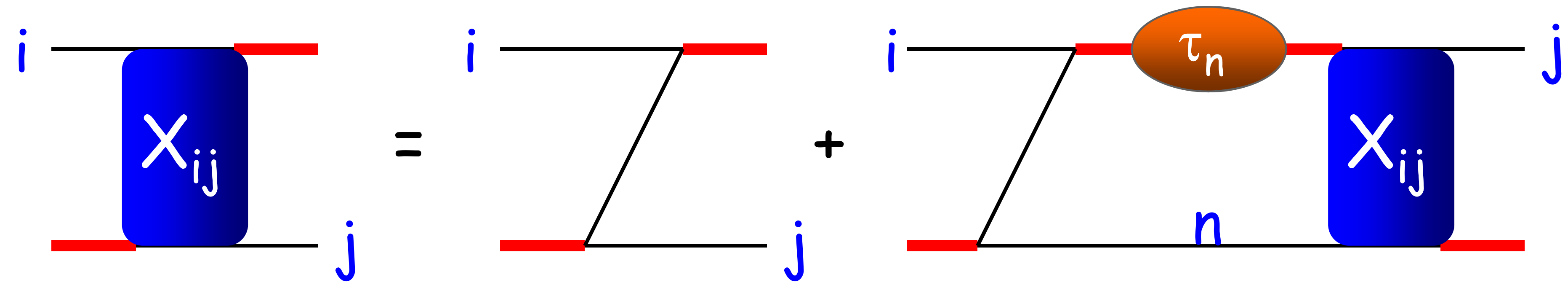
and

Variational approach

✓ Faddeev equations with separable potentials

Alt-Grassberger-Sandhas(AGS) Equations

$$X_{ij}(\vec{p}_i,\vec{p}_j;W) = (1-\delta_{ij})Z_{ij}(\vec{p}_i,\vec{p}_j;W) + \sum_{n\neq i}\int \vec{p}_n Z_{in}(\vec{p}_i,\vec{p}_n;W)\tau_n(\vec{p}_n;W)X_{nj}(\vec{p}_n,\vec{p}_j;W)$$



- W : 3-body scattering energy
- i(j) = 1, 2, 3 (Spectator particles)
- $Z(p_i,p_j;W)$: Particle exchange potentials
- $\tau(p_n;W)$: Isobar propagators (2-body amplitude)

✓ Effective $K^{bar}N$ potential

✓ In most of the existing theoretical works, the resonance energy of strange dibaryon is predicted to lie below the KNN threshold and above the πΣN threshold.

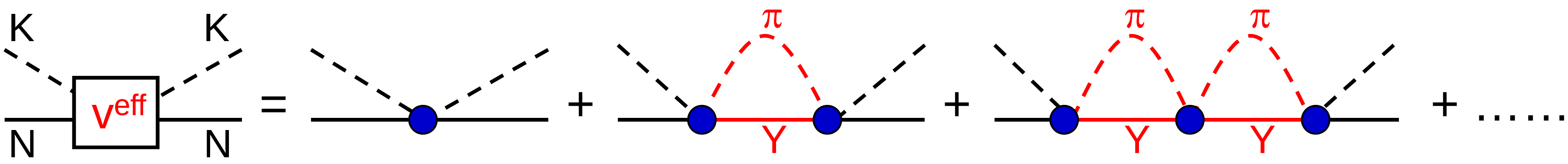
✓ The relevant states are continuum state in the πΣN and πΛN Fock spaces and localized state in the KNN Fock space.

✓ It might be useful to construct the effective KN interaction by truncating the πY channels for studies of the resonance energy of strange dibaryon.

Formula of effective KN potential Hyodo, Weise PRC77 (08)

KN - πΣ – πΛ coupled-channel → effective KN single-channel

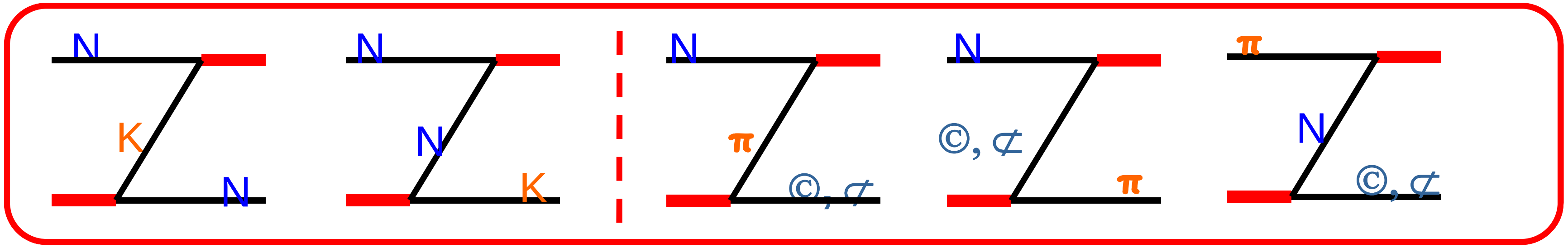
$$v^{eff}_{\bar{K}N-\bar{K}N} = v_{\bar{K}N-\bar{K}N} + v_{\bar{K}N-\pi Y} G_0^{\pi Y} (1 + t^{single}_{\pi Y-\pi Y} G_0^{\pi Y}) v_{\pi Y-\bar{K}N}$$



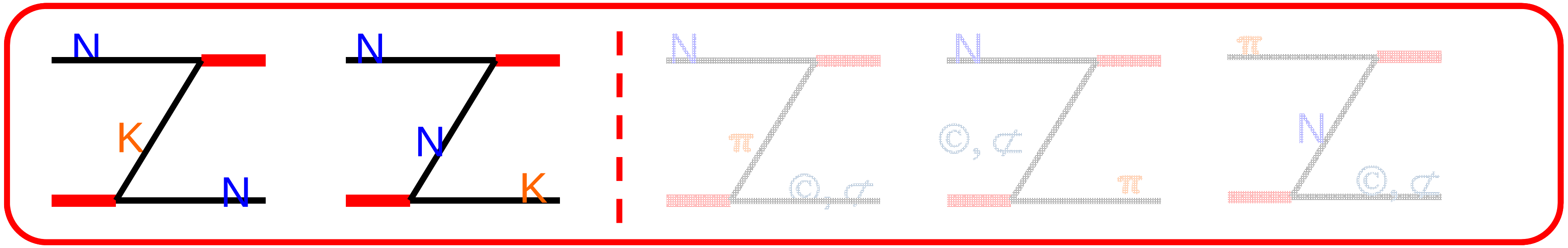
Effective KN potentials are used in variational approach.

✓ Effective $K^{bar}N$ interaction in three-body system

- In Faddeev approach, all of particle echange potentitals in KNN-πYN coupled-channel can be taken into account in AGS equations.



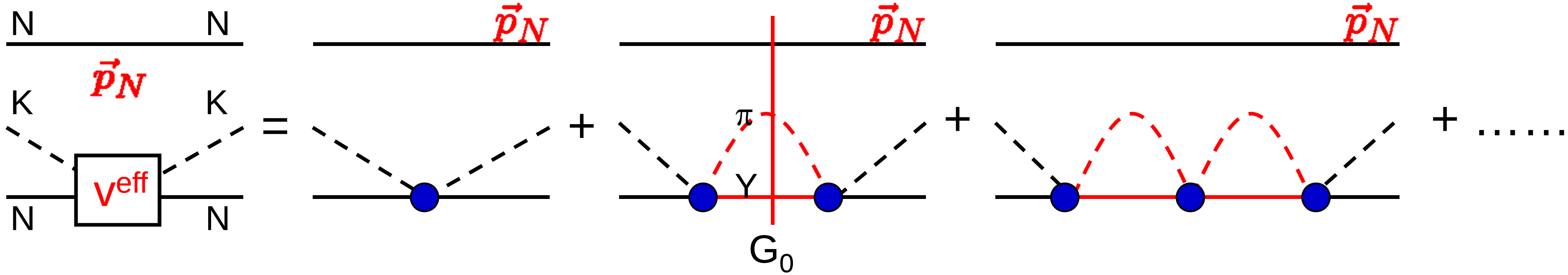
- In order to simulate the variational approach with AGS equations, we neglect π-, Σ-, Λ-, and N-exchange potentials in πYN channel.



✓ Effective K^{bar}N interaction in three-body system

✓In Faddeev approach, three-body dynamics are incorporated through the momentum of spectator particle in three-body free Green's function.

→ two-body scattering amplitude depends on the momentum of spectator particle.



$$\text{Exact}: \quad v^{eff}(W, \vec{p}_N) \longrightarrow G_0(W) = \frac{1}{W - E_N - E_\pi - E_Y + i\epsilon}$$

✓In variational approach, three-body dynamics through the momentum of spectator particle are neglected in πYN channel.

$$\text{Approximate}: \quad v^{eff}_{app}(W) \longrightarrow G_0(W) \sim \frac{1}{W - M_N - E_\pi - E_Y + i\epsilon}$$

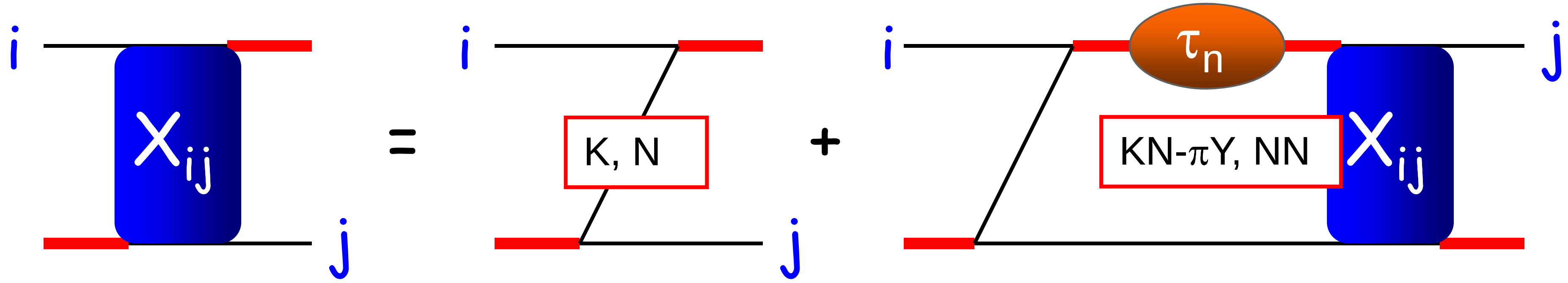
✓We can simulate variational approach in AGS formalism using approximate two-body scattering term where the spectator particle's momentum is neglected.

$$\tau_{\pi Y}(\vec{p}_N; W) : exact \ (Faddeev)$$
$$\tau_{\pi Y}(\vec{p}_N = \vec{0}; W) : approximate$$

✓ Summary of our framework

Alt-Grassberger-Sandhas(AGS) Equations

$$X_{ij}(W) = (1-\delta_{ij})Z_{ij}(W) + \sum_{n\neq i} Z_{in}(W)\tau_n(W)X_{nj}(W)$$



$$\tau_{\pi Y}(W,\vec{p}_N) : exact, \tau_{\pi Y}(W,\vec{p}_N = \vec{0}) : approximate$$

✓ Eigenvalue equation for Fredholm kernel

$$Z(W)\tau(W)|\phi_n(W)> = \eta_n(W)|\phi_n(W)>$$

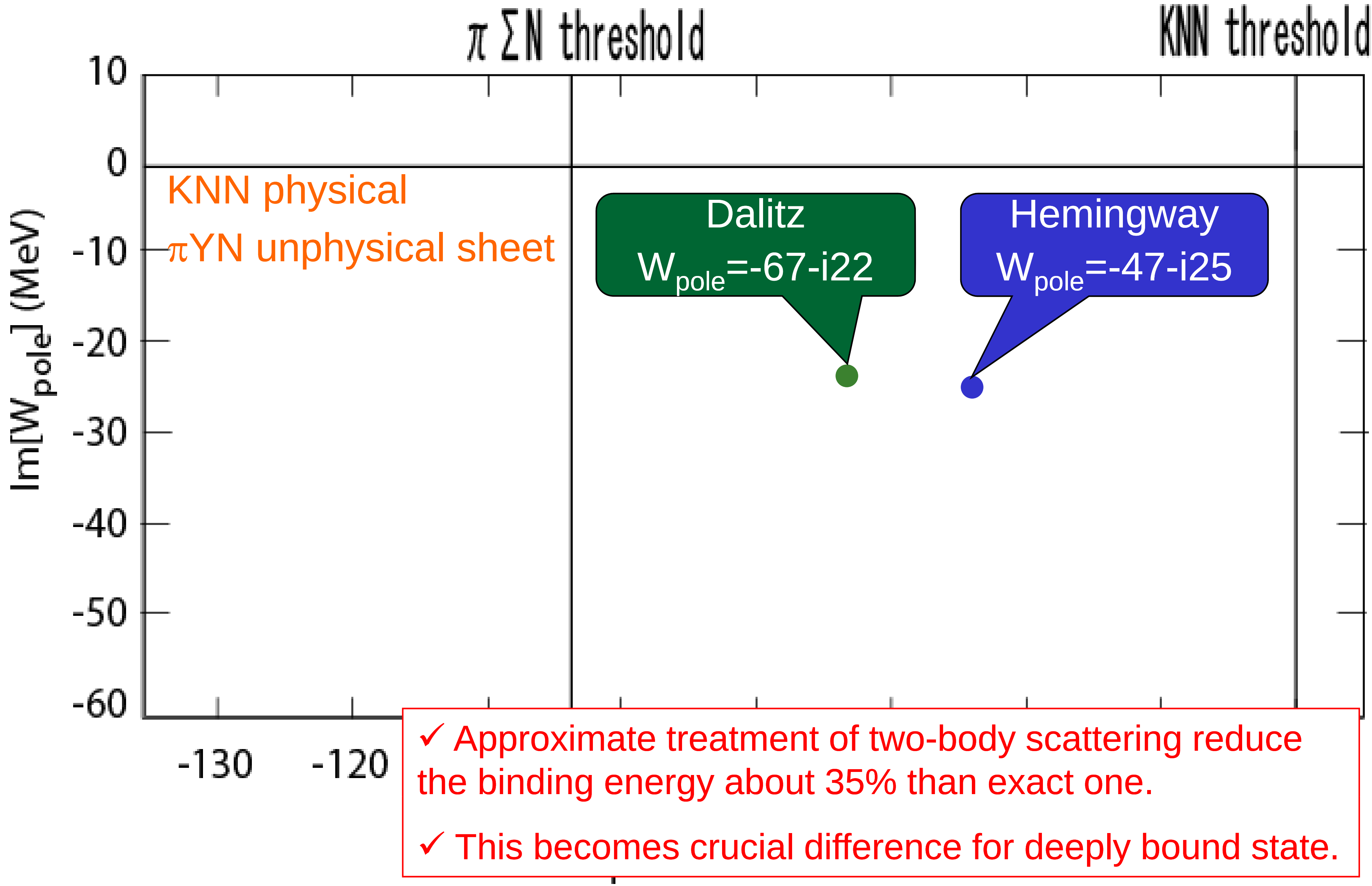
$\eta_n(W_{pole}) = 1$ ➡ 3-body resonance pole at W_{pole}

$W_{pole} = -B - i\Gamma/2$

Similar to πNN, ηNN, $K^{-}d$ studies. (Matsuyama, Yazaki, ……)

Numerical results

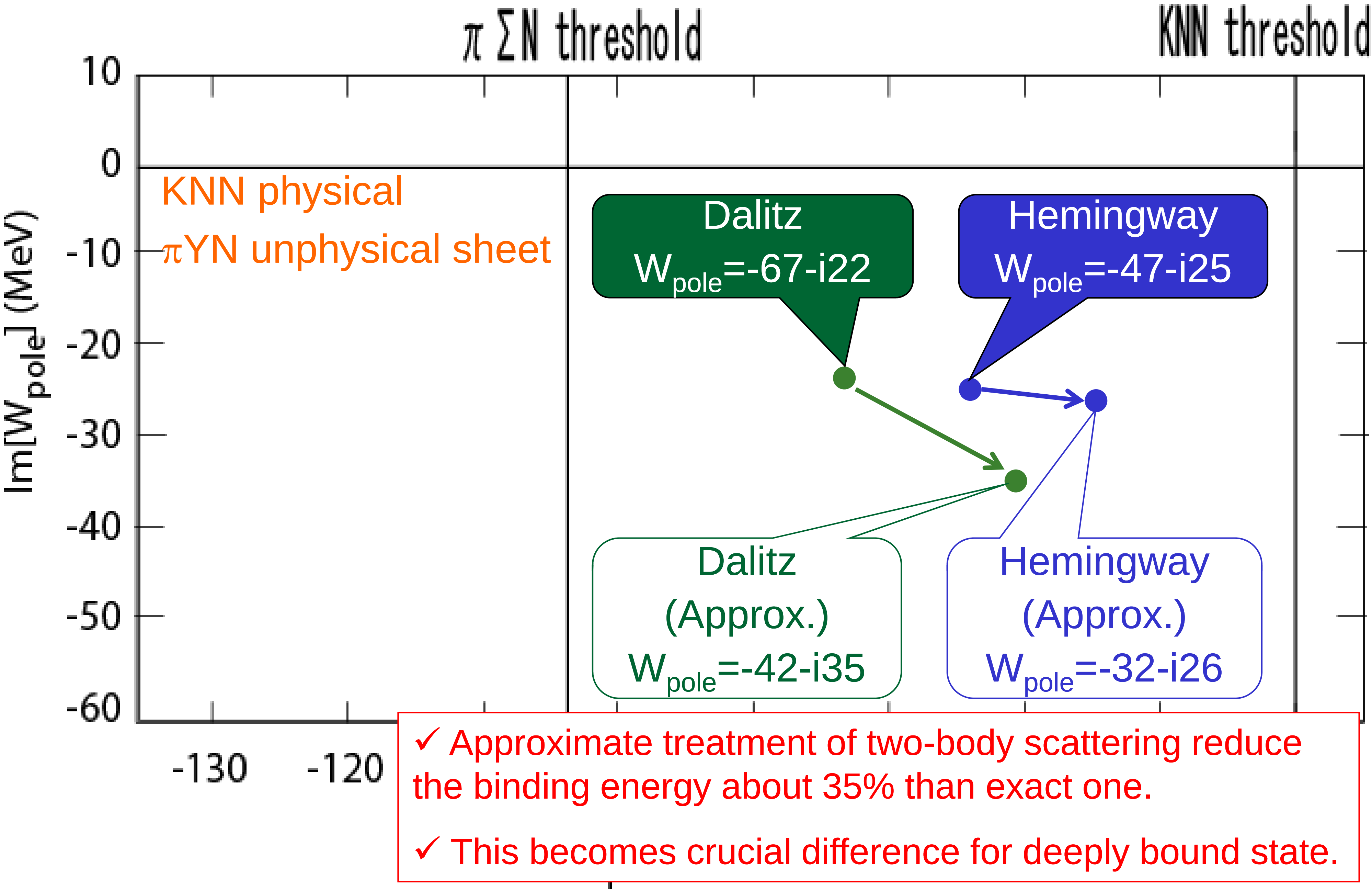
✓ Resonance poles of three-body K^{bar}NN system



✓ Approximate treatment of two-body scattering reduce the binding energy about 35% than exact one.

✓ This becomes crucial difference for deeply bound state.

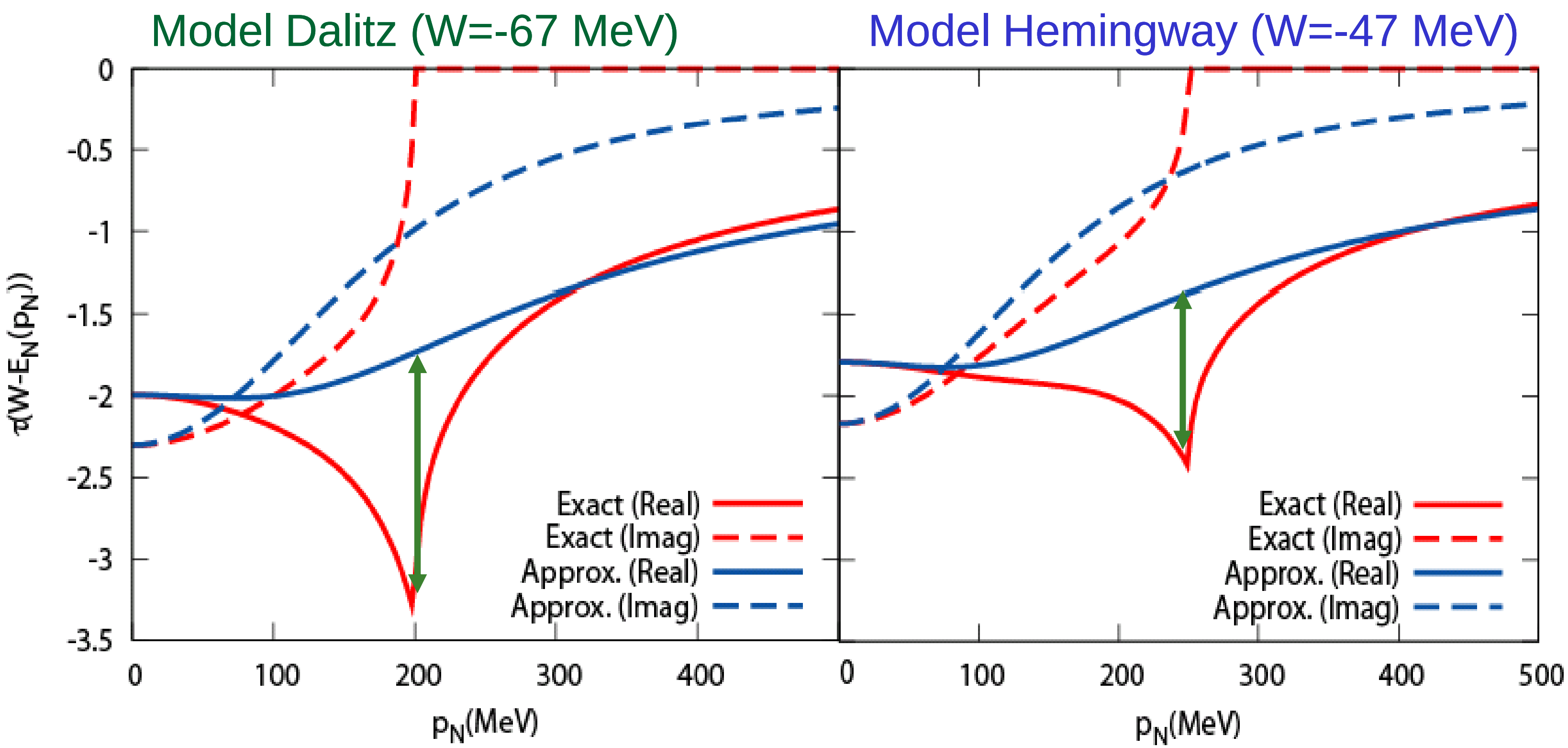
✓ Resonance poles of three-body K^{bar}NN system



✓ Approximate treatment of two-body scattering reduce the binding energy about 35% than exact one.

✓ This becomes crucial difference for deeply bound state.

✓ Reason of less binding energies

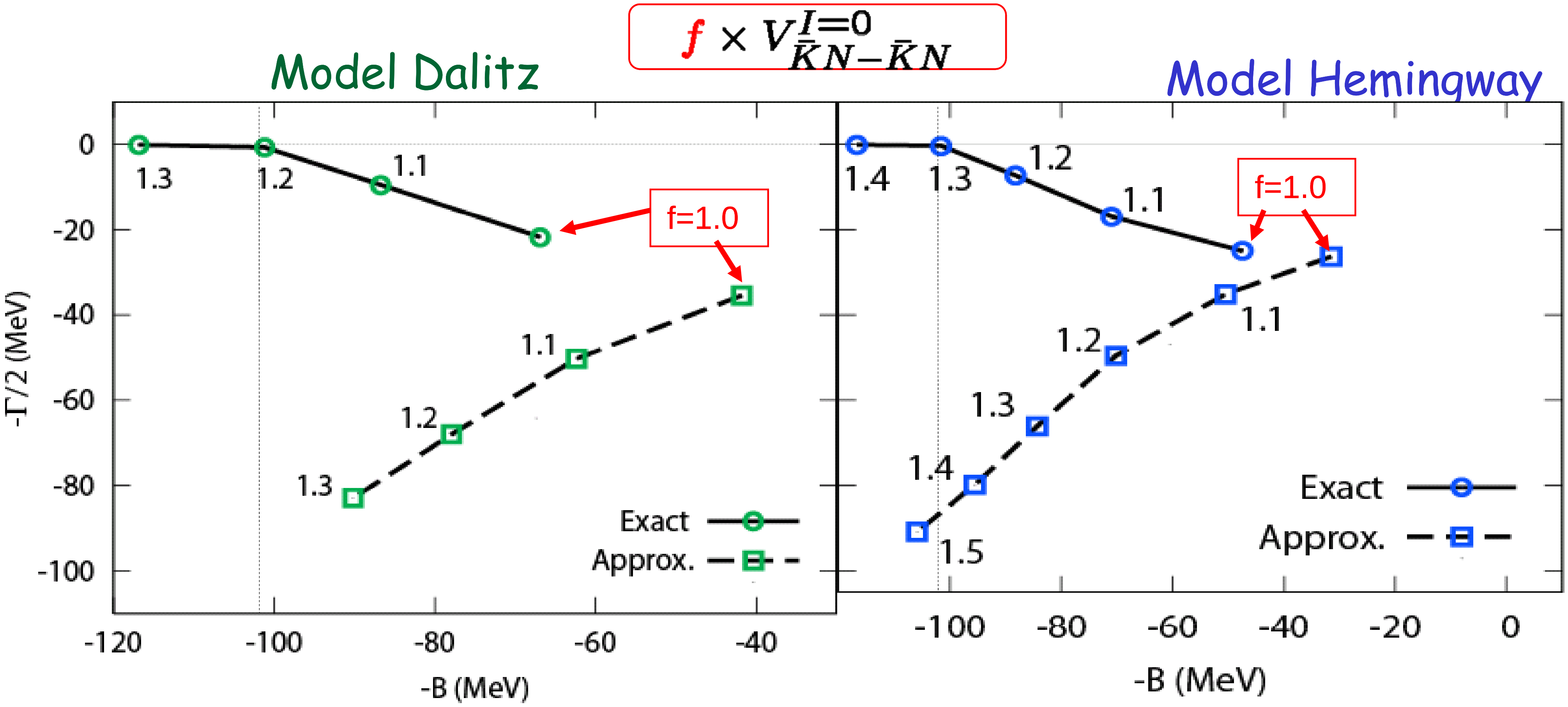


✓ When three-body dynamics is fully handled in AGS equations, cusp structure of the two-body amplitude appears at $\pi\Sigma N$ threshold.

✓ In approximate treatment of two-body amplitude like variational approach, since the $\pi\Sigma$ Fock space is truncated into the KN Fock space, the threshold behavior of the exact amplitude is missing.

✓ For deeply bound state, the threshold behavior is enhanced.

✓ KN interaction dependences of KNN poles



✓ We artificially vary the strength of the KN-KN potential.

✓ In Faddeev approach, KNN quasi-bound state becomes bound state.

✓ In variational approach where the momentum of spectator is neglected, the KNN quasi-bound state becomes virtual state.

✓ Summary

✓ We compare variational approach with Faddeev approach by using the approximate treatment of two-body KN amplitude.

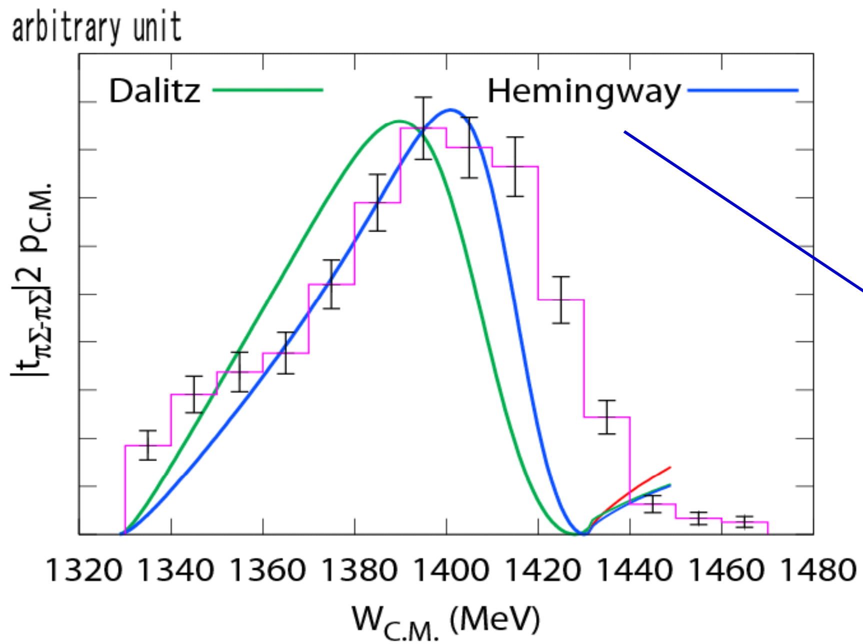
✓ We find the different pole energies corresponding to KNN quasi-bound state for each approach.

✓ KNN state becomes the bound state as increasing the strength of KN interaction in Faddeev approach, meanwhile KNN state becomes virtual state in variational approach.

✓ Full treatment of three-body dynamics plays an essential role in understanding the KNN-πYN coupled-channel deeply quasi-bound state.

Thank you very much for your attention.

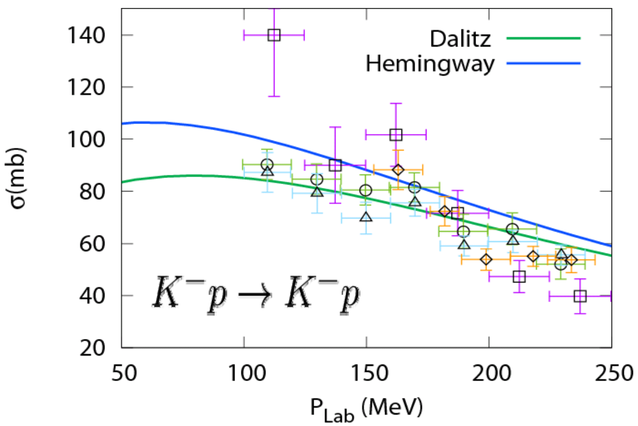
✓ K^{bar} N potential model (parameter fit)

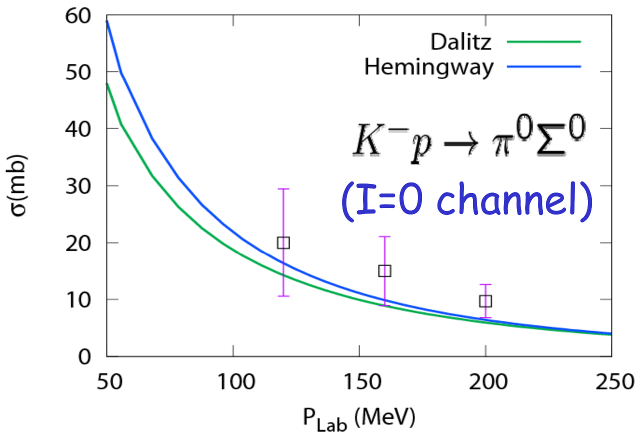


with assumption

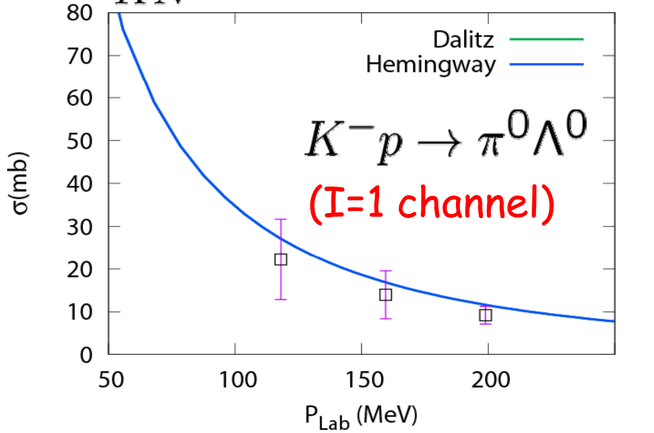
$$\frac{d\sigma}{dm} \propto |t_{\pi\Sigma-\pi\Sigma}|^2 p_{CM}$$

$$W_{\Lambda^*} = 1414 - i19(MeV)$$





$$a^{I=1}_{\bar{K}N} = 0.68 + i0.60fm$$



✓ Strange dibaryon in chiral unitary approach

Energy-dependent potential (E-dep.)

$$V_{ij}(q,q') \rightarrow V_{ij}(E) = -\frac{C_{ij}}{2F_\pi^2}\left(2E - M_i - M_j\right)$$

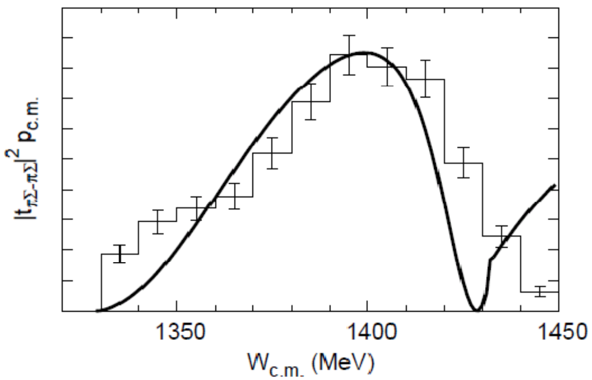
e.g., Oset, Ramos, NPA635, 99 (98)

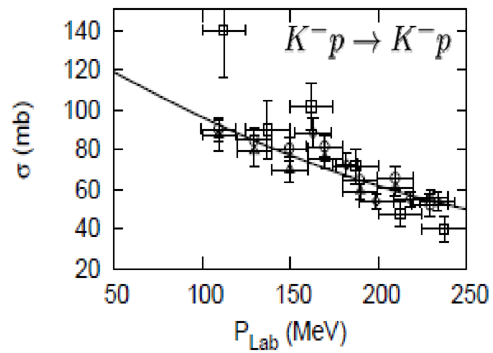
$$V_{ij}(q,q') \rightarrow V_{ij}(q,q') \times g_i(q) g_j(q')$$

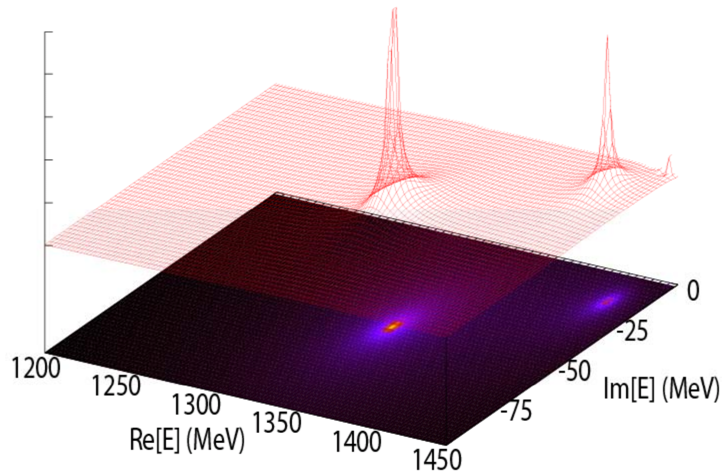
$$g_i(q) = \left(\frac{\Lambda_i^2}{\Lambda_i^2 + q^2}\right)^2$$

K^barN-sY scattering

$$\frac{dN}{dE} \propto |t^{I=0}_{\pi\Sigma-\pi\Sigma}|^2 p_{c.m.}$$



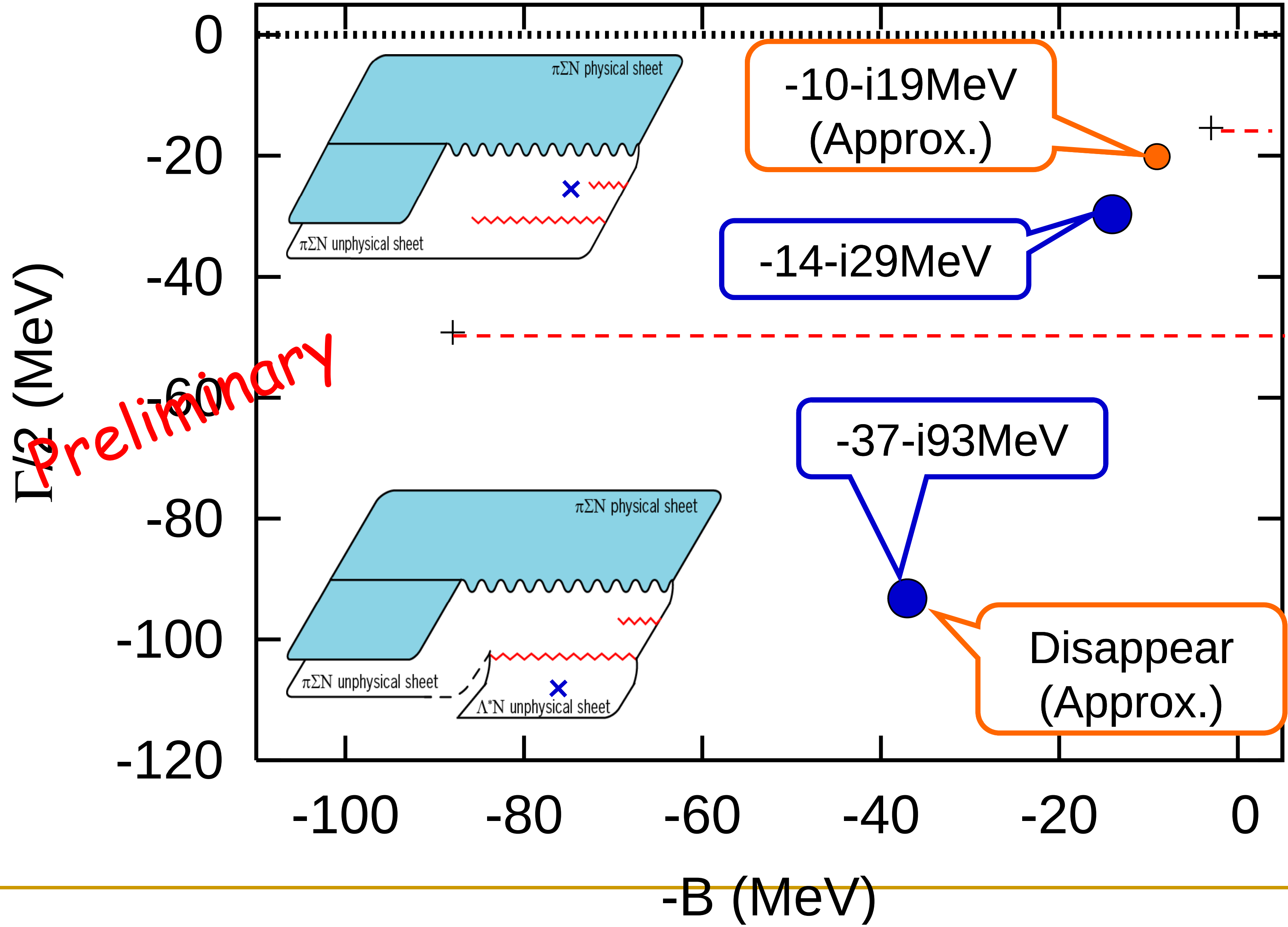




1344.0-i49.0(MeV) (sV resonance)

1428.8-i15.3(MeV) (KN bound state)

✓ Strange dibaryon in chiral unitary approach



Formal solution of AGS equation

AGS Equation

$$X(W) = Z(W) + \underline{Z(W)\tau(W)}X(W)$$

Fredholm kernel

- ✓ Eigenvalue equation for Fredholm kernel

$$Z(W)\tau(W)|\phi_n(W)> = \eta_n(W)|\phi_n(W)>$$

- ✓ Formal solution for 3-boby amplitude

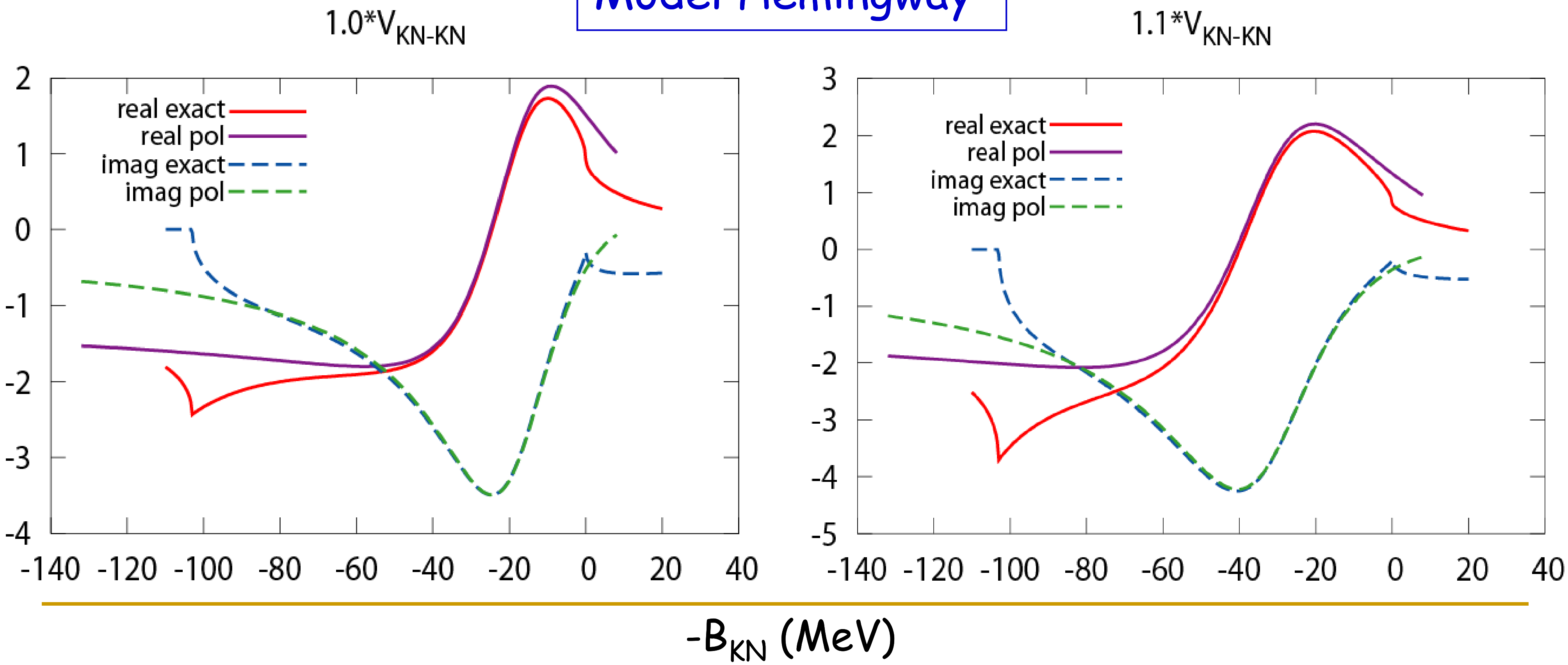
$$X(W) = \sum_n \frac{|\phi_n(W)><\tilde{\phi}_n(W)|Z(W)}{1-\eta_n(W)}$$

$\eta_n(W_{pole}) = 1$ ➡ 3-body resonance pole at W_{pole}

✓ Pole approximation of KN amplitude

$$\tau(E) = \frac{N(E)}{D(E)} \sim \underbrace{\frac{N(z_R)}{D'(z_R)}\frac{1}{E - z_R}}_{\text{Pole term}} \underbrace{- \frac{N(z_R)D''(z_R)}{2D'(z_R)^2} + \frac{N'(z_R)}{D'(z_R)}}_{\text{Constant term}} \qquad (D(z_R) = 0)$$

Model Hemingway

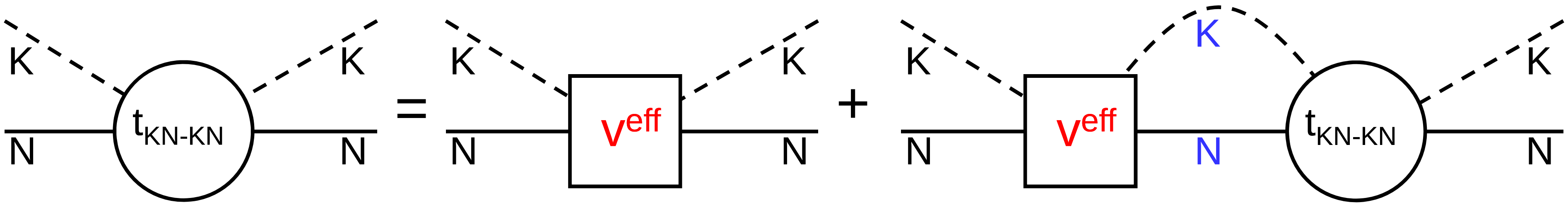


✓ Effective $K^{bar}N$ potential

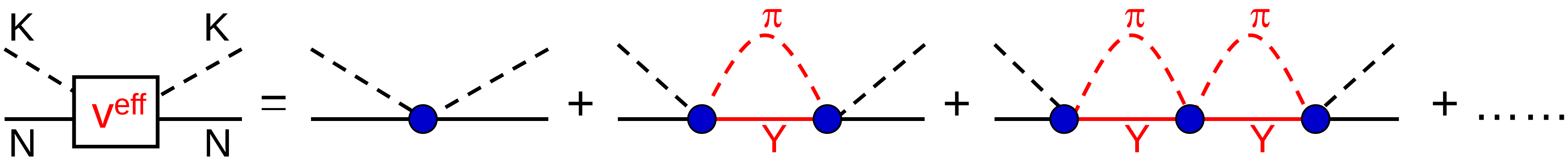
Hyodo, Weise PRC77 (08)

$K^{bar}N$ - $\pi\Sigma$ – $\pi\Lambda$ coupled-channel → $K^{bar}N$ single-channel

$$
\begin{aligned}
t_{\bar{K}N-\bar{K}N} &= v_{\bar{K}N-\bar{K}N} + v_{\bar{K}N-\bar{K}N} G_0^{\bar{K}N} t_{\bar{K}N-\bar{K}N} + v_{\bar{K}N-\pi Y} G_0^{\pi Y} t_{\pi Y-\bar{K}N} \\
&= v^{eff}_{\bar{K}N-\bar{K}N} + v^{eff}_{\bar{K}N-\bar{K}N} G_0^{\bar{K}N} t_{\bar{K}N-\bar{K}N}
\end{aligned}
$$



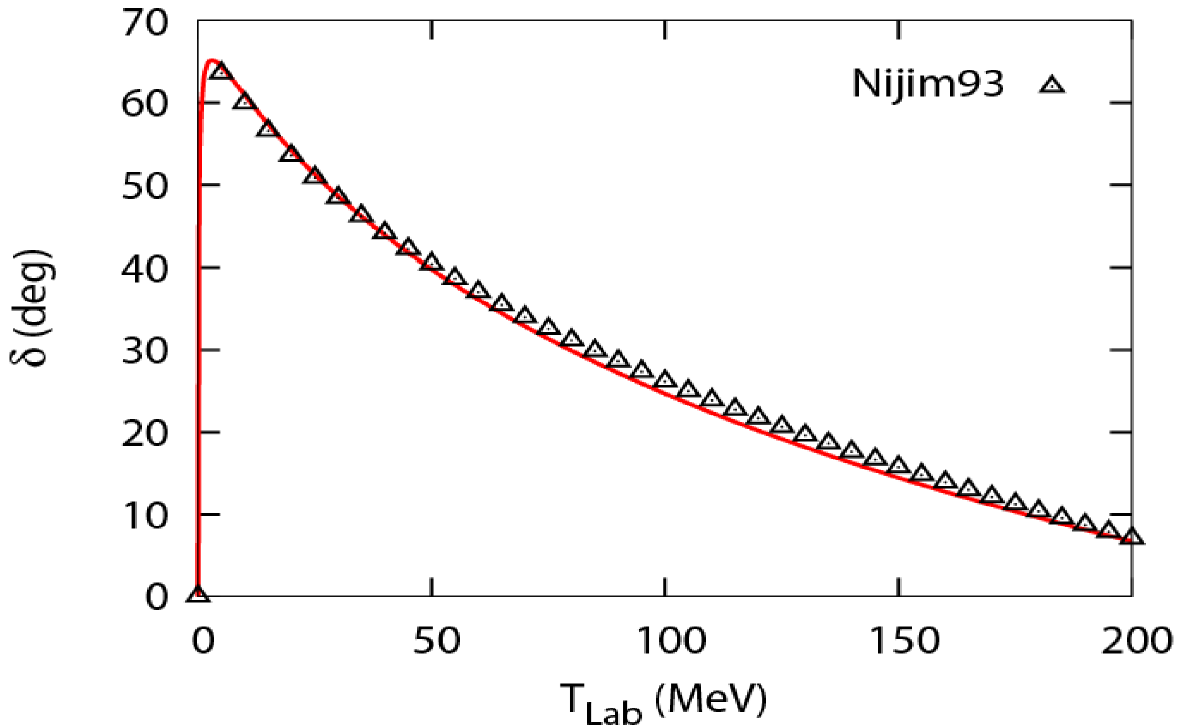
$$
v^{eff}_{\bar{K}N-\bar{K}N} = v_{\bar{K}N-\bar{K}N} + v_{\bar{K}N-\pi Y} G_0^{\pi Y} (1 + t^{single}_{\pi Y-\pi Y} G_0^{\pi Y}) v_{\pi Y-\bar{K}N}
$$



NN potential -> 2-term Yamaguchi type

$$V_{^1S_0}(\vec{p}',\vec{p}) = -C_A g_A(\vec{p}')g_A(\vec{p}) + C_R g_R(\vec{p}')g_R(\vec{p})$$

Attractive — Repulsive core



$$g(\vec{p}) = \frac{\Lambda^2}{\vec{p}^2+\Lambda^2}$$

	$\Lambda_R(MeV)$	$\Lambda_A(MeV)$	$C_R(MeV\,fm^3)$	$C_A(MeV\,fm^3)$
Relativistic	1144	333	5.33	5.61