# Shell model of light hypernuclei with $\Lambda N$ and $\Lambda NN$ forces derived from hypernuclear $\gamma$-line data

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B.F. Gibson et al. (1973)
J. Dabrowski (1975)
Y. Akaishi et al. (2000)
D.J. Millener (2004)
and more later papers

$\Sigma$ – nucleus states

usually 0h$\omega$ –states are used explicitly

Open questions:

What is a role in $\Lambda$–$\Sigma$ coupling of nh$\omega$–$\Sigma$A excitations and the two-shell structure of light $\Lambda$–hypernuclei?

80 MeV

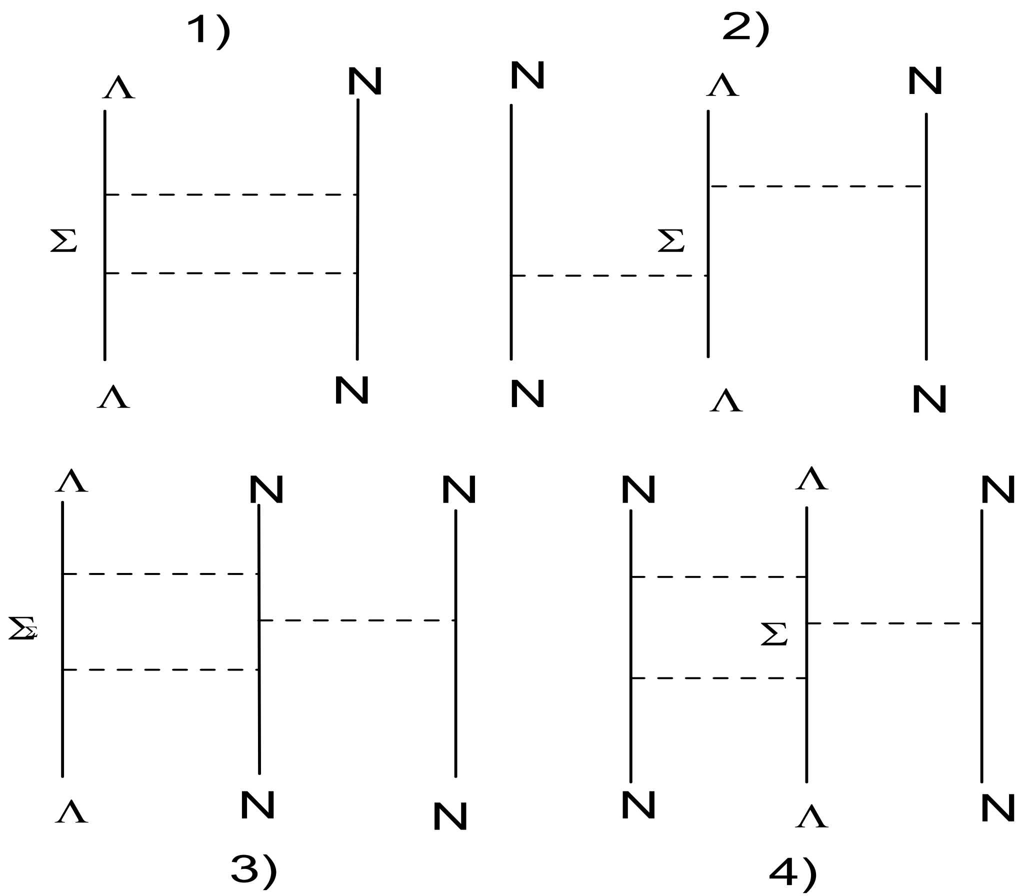
$\Lambda$ – nucleus states

In the $\Lambda\Sigma$N - configuration space

$$\Psi(\Lambda A) = \Psi(\Lambda\text{–nucl.}) + \sum \alpha_k \Phi^k(\Sigma\text{-nucl.})$$

$$V(\Lambda N \longleftrightarrow \Sigma N)$$

# ΛN configuration space




The three-body diagrams 2), 3), 4) are responsible for Λ–Σ coupling in the ΛN configuration space. Their role was considered in s-shell hpernuclei by A.R. Bodmer, Q.N. Usmani (1986), M.Shoeb(2000) with collaborators.

A. Gal (1990) found the corresponding ΛNN-potentials for s-shell using a closure approximation over all intermediate Σ–nucleus states.

In zero-range approximation the three-body $\Lambda NN$ potentials are reduced to the form

$$V = \delta(\mathbf{r}_\Lambda - \mathbf{r}_1)\delta\mathbf{r}_\Lambda - \mathbf{r}_2)(t + t^s \boldsymbol{\sigma}_\Lambda(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2))$$

Two-body (Λ N) potential parameters:

Parameter of spin-spin $\Lambda N$ interaction : $\Delta =< s_\Lambda N|V_\sigma|s_\Lambda N >$

Two spin-orbit parameters: $S_\Lambda$, $S_N$

Parameter of tenzor interaction: $T$

and two radial integrals of three-body interaction:

$$I(\Lambda pp) = \int_0^\infty R_\Lambda^2 R_p^4 r^2 dr$$

$$I(\Lambda sp) = \int_0^\infty R_\Lambda^2 R_s^2 R_p^2 r^2 dr$$

For the oscillator wave functions

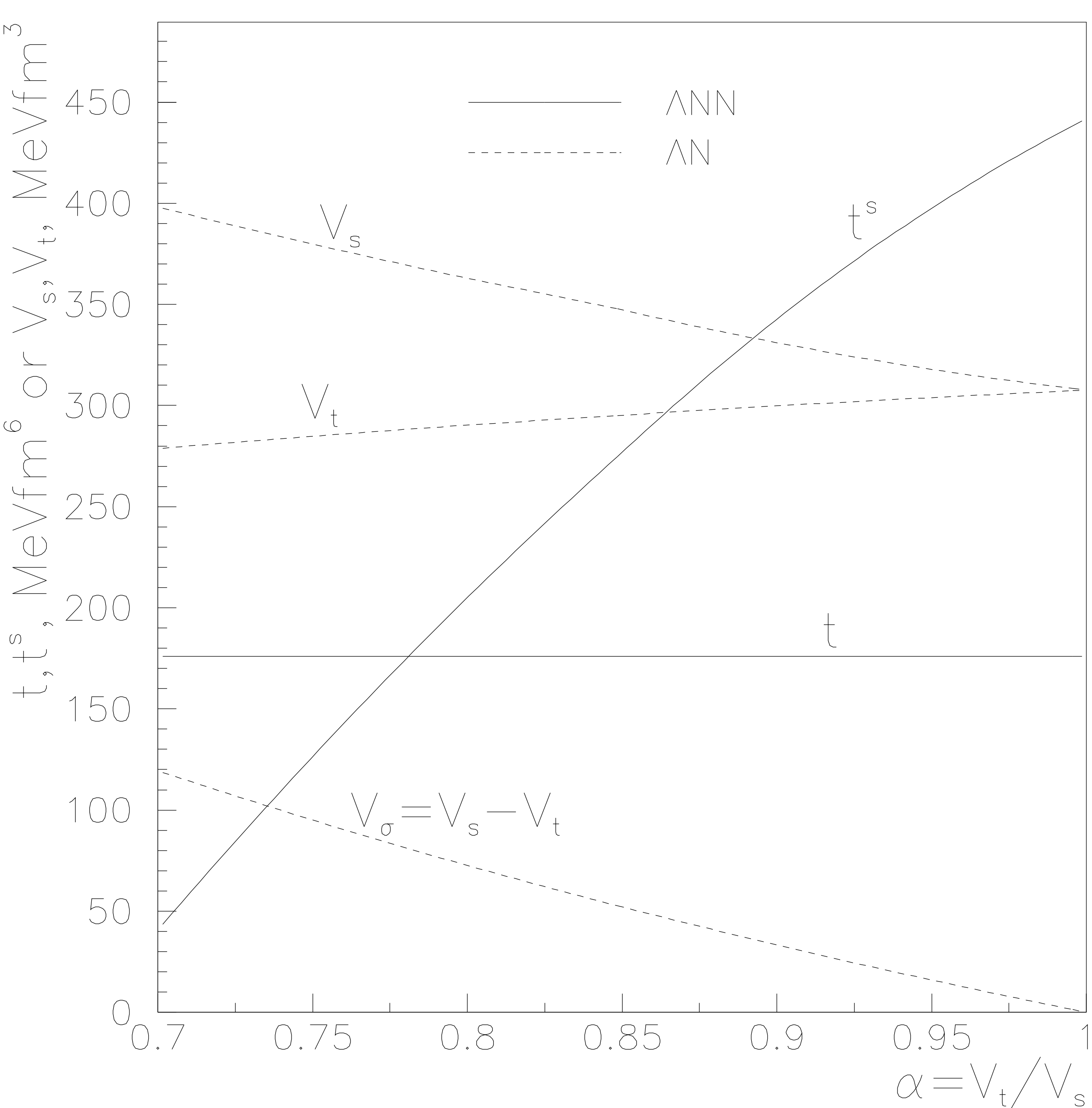
$I(\Lambda sp) = 16/3^{5/2}\pi b^6$, $I(\Lambda pp) = (5/9) I(\Lambda sp)$

The hypernuclear matrix elements for the basis states $< s_\Lambda p^n|$ are expressed in terms of the constants

$G = \frac{t}{3^{5/2} 4\pi^3 b^6}$ and $G_s = \frac{t^s}{3^{5/2} 4\pi^3 b^6}$

The radial integrals of $\Lambda NN$ interaction $I(\Lambda sp)$, $I(\Lambda pp)$ and the parameter of the spin-spin interaction $\Delta$ were calculated with the Woods-Saxon wave functions for $\Lambda$-hyperon and with the h.o. wave functions for nucleons. The ocsillator parameter $b$ was taken using data on the nuclear RMS radii.

The link between the phenomenological $\Lambda N$-interaction potential (Gaussian form) and the three-body constants $t$ and $t^s$ was found early from the phenomenological description of properties of s-shell hypernuclei (A=4,5) with the $\Lambda NN$ $\delta$-forces given above (V.N. Fetisov, JETP letters 70 (1999) 233) and it (the basic assumption) is used in the 1p-shell hypernuclei.



ΛNN
ΛN
t, t^s, MeVfm^6 or V_s, V_t, MeVfm^3
450
400
350
300
250
200
150
100
50
0
0.7
0.75
0.8
0.85
0.9
0.95
1
α=V_t/V_s
V_s
V_t
t^s
t
V_σ=V_s−V_t

Following to results of calculations (Y. Akaishi, T. Harada, S. Shinmura, and K.S. Myint, Phys. Rev. Lett. 84 (2000) 3539): only about 1/2 part of the $1^+ - 0^+$ doublet splitting in the A=4 s-shell hypernuclei is described the spin-spin $\Lambda N$ interaction we accept by the link (above) for our phenomenological approach the values of constants:

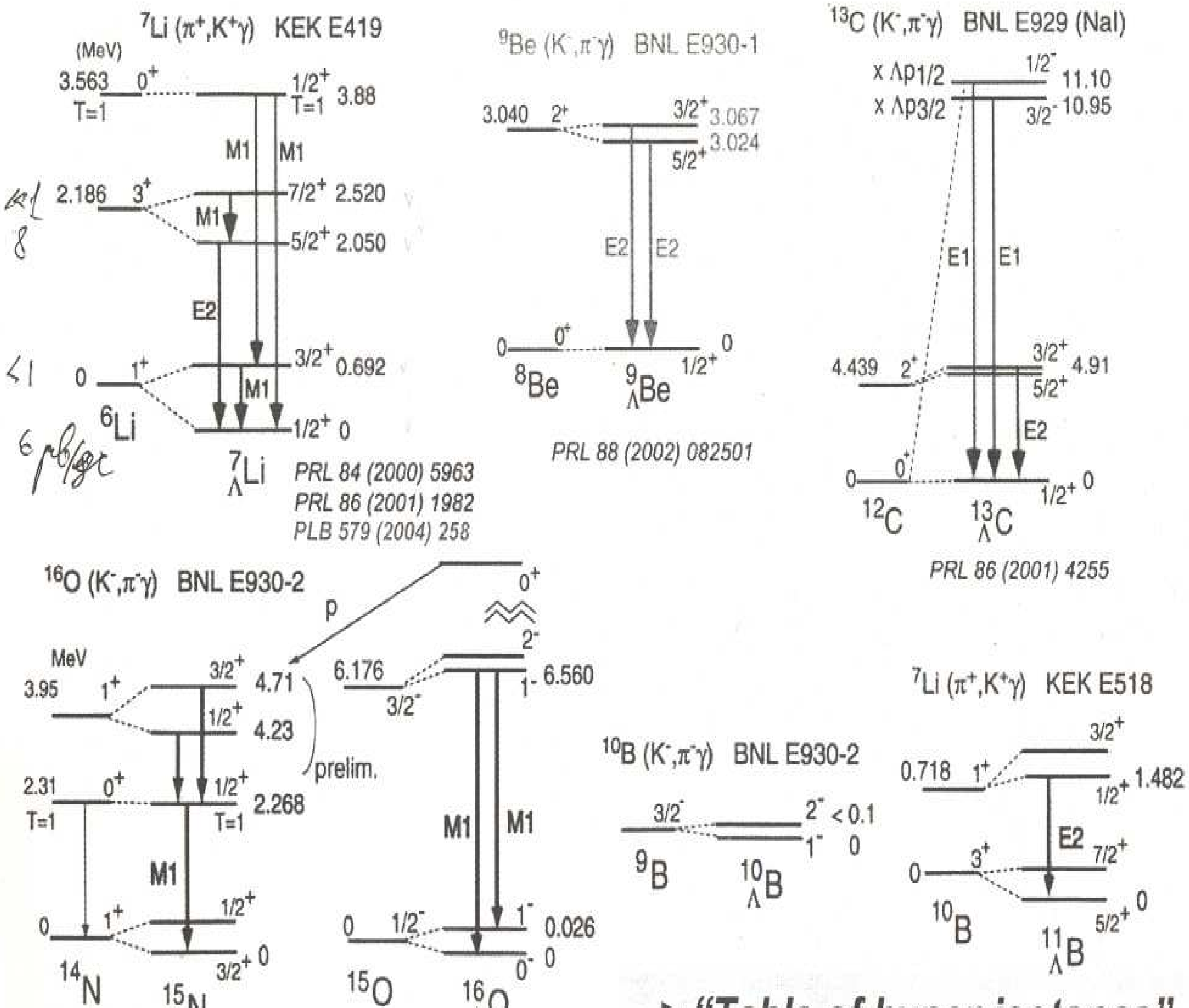
$V_\sigma = V_s - V_t = 60$ MeVfm$^6$ (for the calculation of the spin-spin constant $\Delta$),

and two three-body constants:

$t = 176.0$ MeVfm$^6$,

$t^s = 248.78$ MeVfm$^6$.

# 1. Present status




-> *"Table of hyper-isotopes"*

$^{16}_{\Lambda}\mathbf{O}$

The explicit form of the $1^-$-level matrix $C_{i,j}$ and the energies of the $E(0^-)$- and $E(2^-)$- states for $^{16}_{\Lambda}$O:

Two $1^-$-basis states: 1) ¡$s_\Lambda, p^{-1}_{1/2}; 1^-$—, 2) ¡$s_\Lambda, p^{-1}_{3/2}; 1^-$—

$$C_{1,1} = -eps - \frac{1}{12}\Delta - \frac{1}{3}(S_\Lambda - S_N) + \frac{2}{3}(S_\Lambda + S_N) + 2T - \frac{8}{3}mG_s - \frac{40}{9}G_s;$$

$$C_{1,2} = (-\frac{1}{3}\Delta + \frac{1}{3}S_\Lambda - T - \frac{32}{3}mG_s - \frac{160}{9}G_s)\sqrt{2}; \qquad C_{2,1} = C_{1,2};$$

$$C_{2,2} = -\frac{5}{12}\Delta - \frac{2}{3}(S_\Lambda + S_N) - \frac{1}{6}(S_\Lambda - S_N) + T - \frac{40}{3}mG_s - \frac{200}{9}G_s;$$

$$E(0^-) = -eps + \frac{1}{4}\Delta - S_\Lambda + S_N - 6T + 8mGs + \frac{40}{3}G_s;$$

$$E(2^-) = \frac{1}{4}\Delta + \frac{1}{2}(S_\Lambda - S_N) - \frac{3}{5}T + 8mGs + \frac{40}{3}G_s;$$

The excitation energy of the $2^-$-state in $^{16}_{\Lambda}$O is determined in this model only by the two-body potential parameters

$$E^*(2^-) = E(2^-) - E(0^-) = eps + \frac{3}{2}(S_\Lambda - S_N) + \frac{27}{5}T,$$

where $eps = 6.176$ MeV for $^{16}_{\Lambda}$O.

The energies e1=6.562 MeV and e2=0.024 MeV of two $1^-$-states in ${}^{16}_{\Lambda}$O determined by two $\gamma$-lines observed are connected with four potential parameters $\Delta$, $S_\Lambda$, $S_N$, T and $Gs$ by two equations

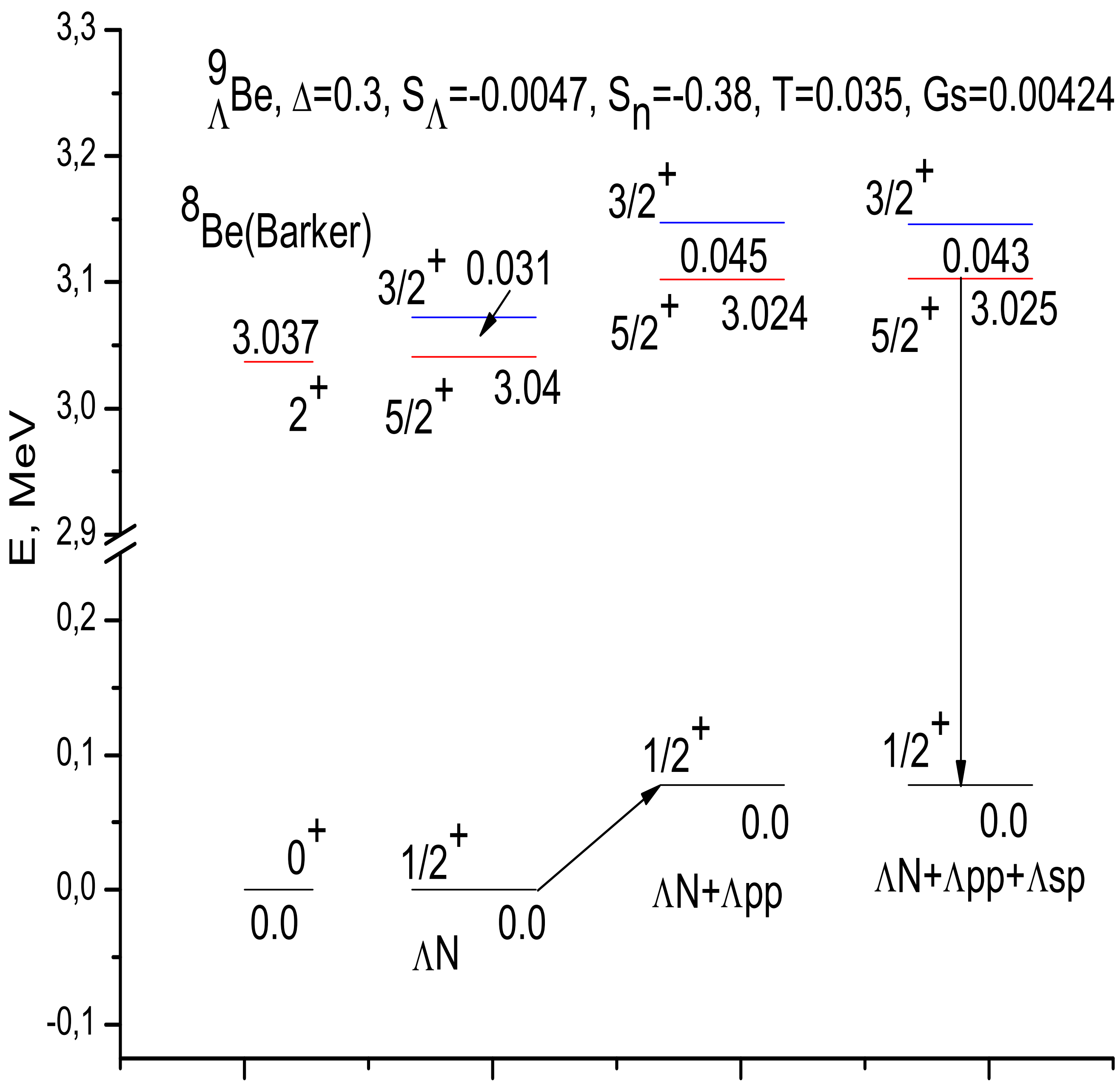
(e1-e2)$^2$ =(C$_{1,1} - C_{2,2})^2 + 4C_{2,2}^2$  (1)

e1+e2-6.176=-$\Delta + \frac{3}{2}(S_\Lambda - S_N) + 15T - 32mGs - \frac{160}{3}Gs$  (2)

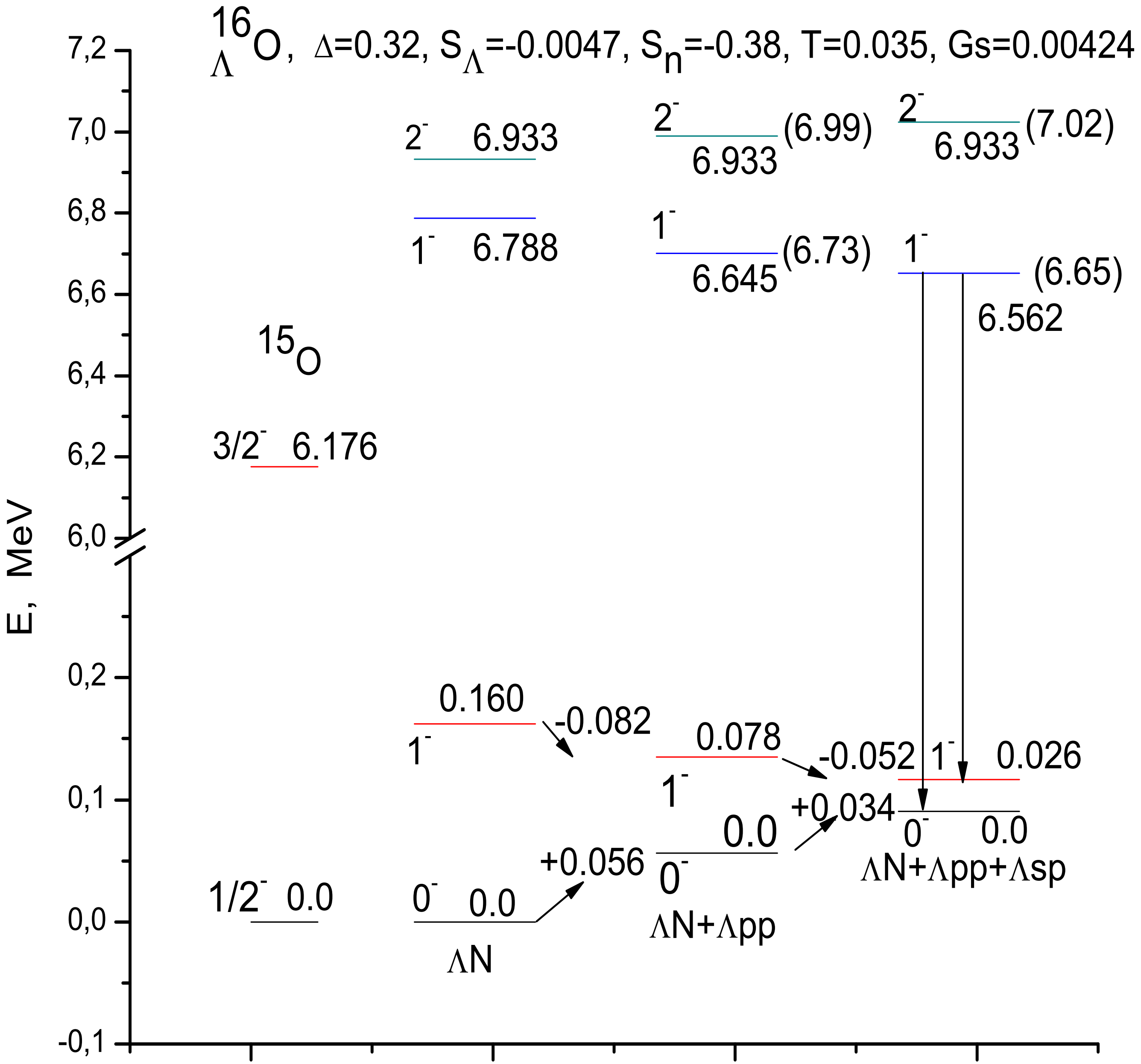
**${}^{9}_{\Lambda}$Be**

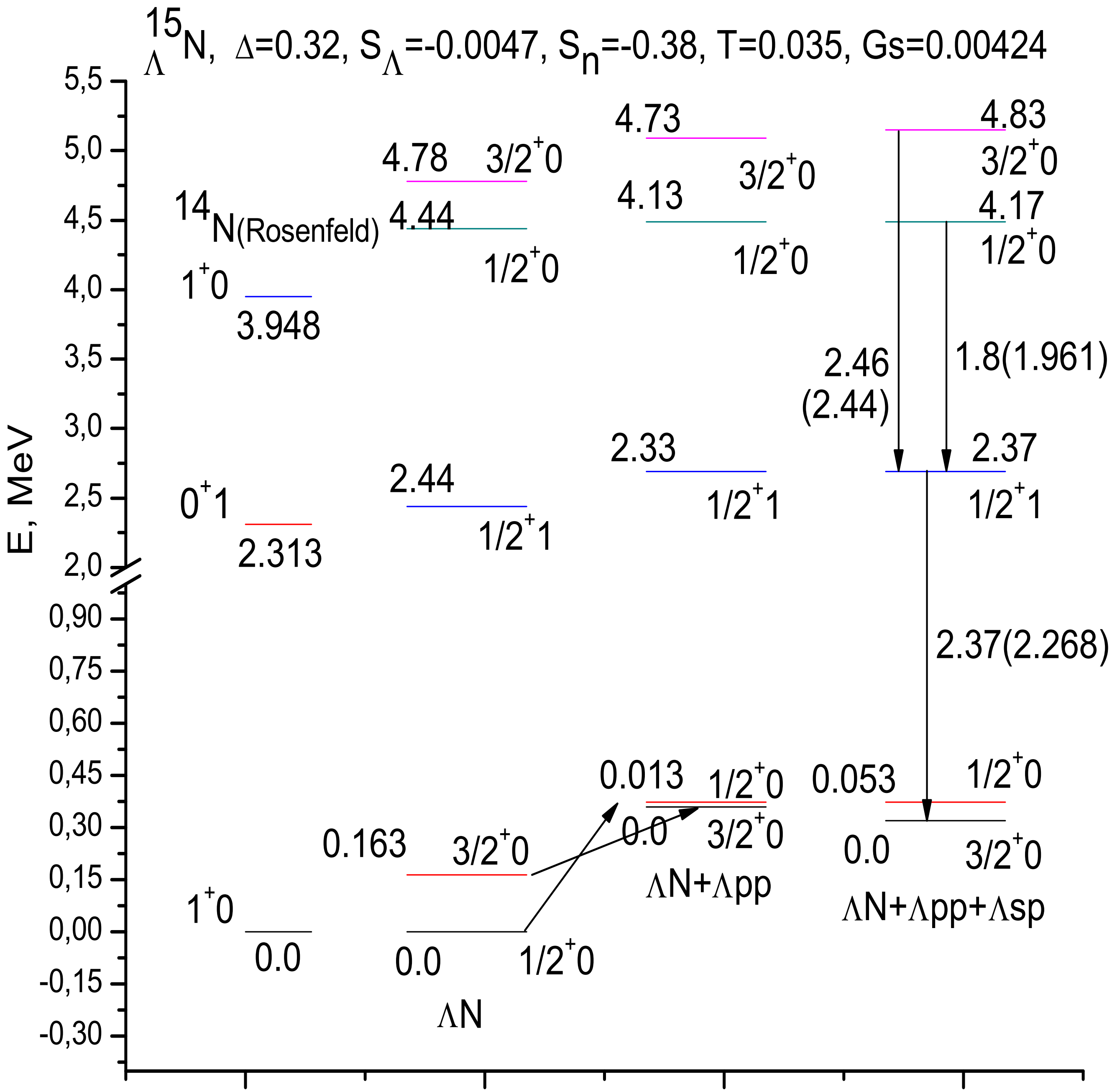
The doublet splitting d($3/2^+, 5/2^+$)=0.046 MeV in ${}^{9}_{\Lambda}Be$ found with the Barker nuclear forces for the shell model of the core nucleus $^8$Be gives the third equation for parameters

d=$-0.0130\Delta - 2.487S_\Lambda + 0.602T + 2.772Gs$  (3)

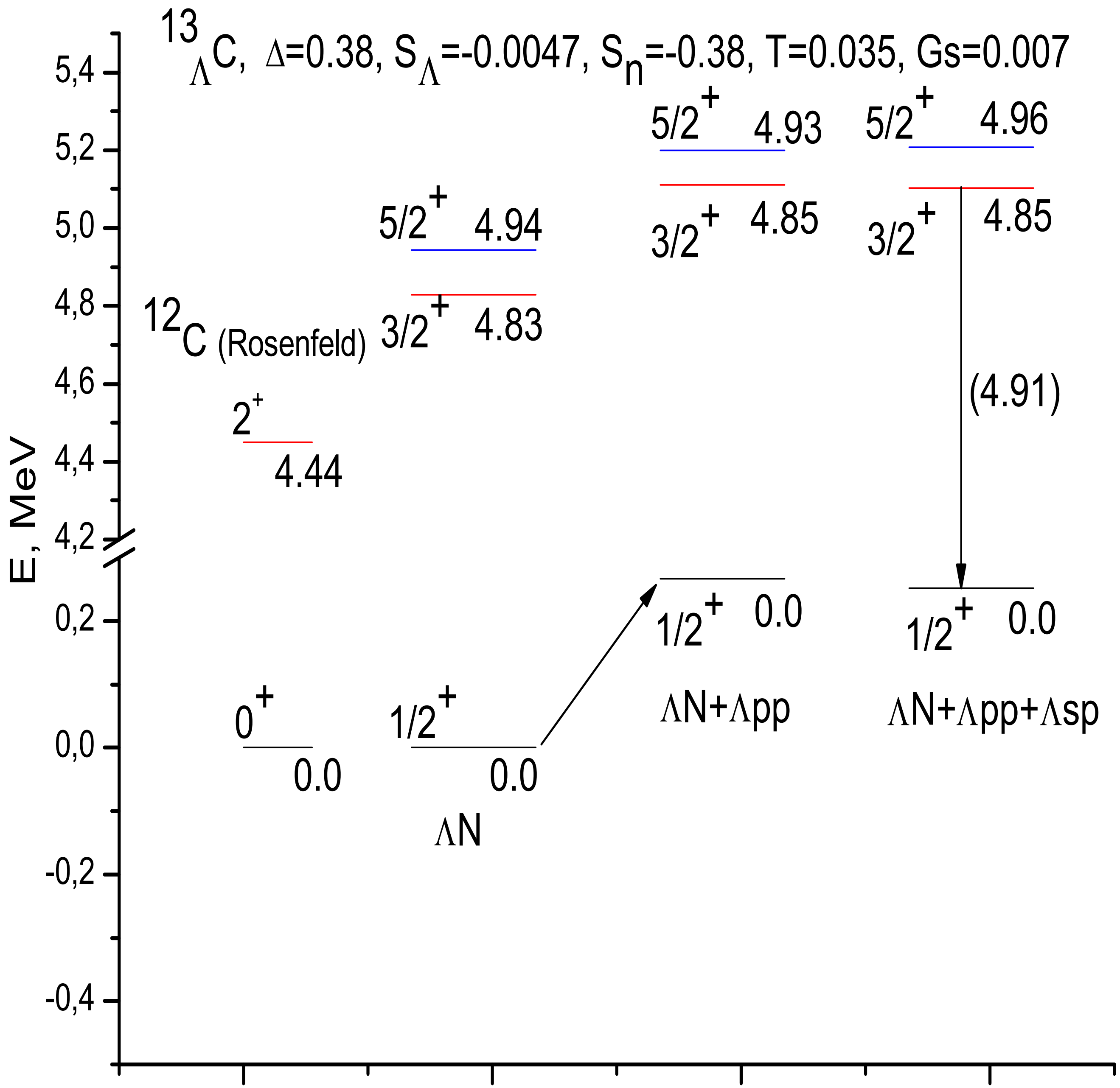


$^{9}_{\Lambda}$Be, Δ=0.3, $S_\Lambda$=-0.0047, $S_n$=-0.38, T=0.035, Gs=0.00424
$^{8}$Be(Barker)
E, MeV
3,3
3,2
3,1
3,0
2,9
0,2
0,1
0,0
-0,1
3.037
$2^+$
$0^+$
0.0
$3/2^+$ 0.031
$5/2^+$ 3.04
$1/2^+$
0.0
ΛN
$3/2^+$
0.045
$5/2^+$ 3.024
$1/2^+$
0.0
ΛN+Λpp
$3/2^+$
0.043
$5/2^+$ 3.025
$1/2^+$
0.0
ΛN+Λpp+Λsp


$^{16}_{\Lambda}O$, Δ=0.32, $S_\Lambda$=-0.0047, $S_n$=-0.38, T=0.035, Gs=0.00424
E, MeV
$^{15}O$
3/2⁻ 6.176
1/2⁻ 0.0
2⁻ 6.933
1⁻ 6.788
0.160
1⁻
0⁻ 0.0
ΛN
-0.082
+0.056
2⁻ 6.933 (6.99)
1⁻ 6.645 (6.73)
0.078
1⁻
0.0
0⁻
ΛN+Λpp
-0.052
+0.034
2⁻ 6.933 (7.02)
1⁻ (6.65) 6.562
1⁻ 0.026
0⁻ 0.0
ΛN+Λpp+Λsp



$^{15}_{\Lambda}N$, Δ=0.32, $S_\Lambda$=-0.0047, $S_n$=-0.38, T=0.035, Gs=0.00424
E, MeV
5,5
5,0
4,5
4,0
3,5
3,0
2,5
2,0
0,90
0,75
0,60
0,45
0,30
0,15
0,00
-0,15
-0,30
$^{14}$N(Rosenfeld)
$1^+0$
3.948
$0^+1$
2.313
$1^+0$
0.0
4.78
$3/2^+0$
4.44
$1/2^+0$
2.44
$1/2^+1$
0.163
$3/2^+0$
0.0
$1/2^+0$
ΛN
4.73
$3/2^+0$
4.13
$1/2^+0$
2.33
$1/2^+1$
0.013
$1/2^+0$
0.0
$3/2^+0$
ΛN+Λpp
4.83
$3/2^+0$
4.17
$1/2^+0$
2.46
(2.44)
1.8(1.961)
2.37
$1/2^+1$
2.37(2.268)
0.053
$1/2^+0$
0.0
$3/2^+0$
ΛN+Λpp+Λsp



13 Λ C, Δ=0.38, SΛ=-0.0047, Sn=-0.38, T=0.035, Gs=0.007
E, MeV
5,4
5,2
5,0
4,8
4,6
4,4
4,2
0,2
0,0
-0,2
-0,4
12C (Rosenfeld)
2+
4.44
0+
0.0
5/2+ 4.94
3/2+ 4.83
1/2+
0.0
ΛN
5/2+ 4.93
3/2+ 4.85
1/2+ 0.0
ΛN+Λpp
5/2+ 4.96
3/2+ 4.85
(4.91)
1/2+ 0.0
ΛN+Λpp+Λsp

The $b$-dependence of $\Lambda N$ and $\Lambda NN$ parameters (MeV) for $^{7}_{\Lambda}$Li calculated with
$V_\sigma = 60.0$ MeVfm$^3$ ($b = 1.92$ fm for the free core-nucleus $^6$Li).
$\Lambda N$, $\Lambda pp$ and $\Lambda sp$ (MeV) are the separate contributions of $\Lambda N$ and $\Lambda NN$ interactions involving $p$- and $s$- nucleons to the doublet splitting $E_{exp.}(\frac{3}{2}^+, \frac{1}{2}^+) = 0.692$ MeV.

| $b$, fm | $\Delta$ | $S_\Lambda$ | $S_n$ | $T$ | $\Lambda N$ | $\Lambda pp$ | $\Lambda sp$ | $E(\frac{3}{2}^+, \frac{1}{2}^+$ |
|---|---|---|---|---|---|---|---|---|
| 1.800 | 0.278 | -0.130 | -0.426 | -0.0890 | 0.367 | 0.147 | 0.178 | 0.692 |
| 1.810 | 0.274 | -0.0913 | -0.414 | -0.0595 | 0.377 | 0.142 | 0.173 | 0.692 |
| 1.820 | 0.270 | -0.0439 | -0.403 | -0.0222 | 0.388 | 0.138 | 0.166 | 0.692 |
| 1.824 | 0.268 | -0.0189 | -0.400 | -0.00183 | 0.392 | 0.136 | 0.164 | 0.692 |
| 1.828 | 0.267 | 0.0553 | -0.406 | 0.0627 | 0.397 | 0.134 | 0.161 | 0.692 |
| 1.830 | 0.266 | 0.0127 | -0.396 | 0.0236 | 0.394 | 0.133 | 0.161 | E≤0.687 |
| 1.840 | 0.262 | 0.0556 | -0.390 | 0.0576 | 0.390 | 0.129 | 0.155 | E≤0.674 |

Potential parameters: line ($\Lambda N + \Lambda NN$) - this work, line ($\Lambda\Sigma N$)- D.J. Millener, Lect. Notes Phys. **724**, 31-79 (2007)

| | | $\Delta$ | $S_\Lambda$ | $S_n$ | $T$ | $Gs$ |
|---|---|---|---|---|---|---|
| $^{7}_{\Lambda}$Li | $\Lambda N + \Lambda NN$ | 0.27 | -0.0052 | -0.40 | 0.01 | 0.0035 |
| | $\Lambda\Sigma N$ | 0.43 | -0.015 | -0.39 | 0.030 | $(\Lambda\Sigma)_i$ |
| $^{9}_{\Lambda}$Be | $\Lambda N + \Lambda NN$ | 0.30 | -0.0047 | -0.38 | 0.035 | 0.0042 |
| | $\Lambda\Sigma N$ | 0.43 | -0.015 | -0.39 | 0.030 | $(\Lambda\Sigma)_i$ |
| $^{16}_{\Lambda}$O | $\Lambda N + \Lambda NN$ | 0.32 | -0.0047 | -0.38 | 0.035 | 0.0042 |
| | $\Lambda\Sigma N$ | 0.315 | -0.015 | -0.35 | 0.0232 | $(\Lambda\Sigma)_i$ |
| $^{15}_{\Lambda}$N | $\Lambda N + \Lambda NN$ | 0.32 | -0.0047 | -0.38 | 0.035 | 0.0042 |
| | $\Lambda\Sigma N$ | 0.315 | -0.015 | -0.35 | 0.0232 | $(\Lambda\Sigma)_i$ |
| $^{13}_{\Lambda}$C | $\Lambda N + \Lambda NN$ | 0.38 | -0.0047 | -0.38 | 0.035 | 0.0070 |
| | $\Lambda\Sigma N$ | - | - | - | - | - |
| $^{10}_{\Lambda}$B | $\Lambda N + \Lambda NN$ | 0.30 | -0.0047 | -0.38 | 0.035 | 0.0042 |
| | $\Lambda\Sigma N$ | 0.33 | -0.015 | -0.35 | 0.0239 | $(\Lambda\Sigma)_i$ |

Potential parameters of the effective interactions (MeV) with (6-16)2BME CK:no link with s-shell hyp.;

$S_\Lambda$ was taken near the value accepted by Millener; $\Delta$,$S_n$,$T$ and $Gs$ derived by solving four equations for spectra ${}^{9}_{\Lambda}$Be&${}^{16}_{\Lambda}$O including probably observed E*($2^-$,${}^{16}_{\Lambda}$O)=6.784 MeV

| $\Delta$ | $S_\Lambda$ | $S_n$ | $T$ | $Gs$ | $E(2^- - 1^-, {}^{10}_{\Lambda}\mathrm{B})$ |
|---|---|---|---|---|---|
| 0.336 | -0.013 | -0.320 | 0.0223 | 0.000561 | 0.140 |
| 0.275 | -0.012 | -0.320 | 0.0221 | 0.00124 | 0.121 |
| 0.215 | -0.011 | -0.320 | 0.0217 | 0.00191 | 0.104 |
| 0.1536 | -0.010 | -0.321 | 0.0214 | 0.00259 | 0.087 |