The 10th International Conference
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Shell-model study on Sigma-mixing in neutron-rich lithium Lambda hypernuclei

Atsushi UMEYA (RIKEN)

Toru HARADA (Osaka E.C. Univ.)

A. Umeya and T. Harada, Phys. Rev. C 79, 024315 (2009).
T. Harada, A. Umeya, and Y. Hirabayashi, Phys. Rev. C 79, 014603 (2009).

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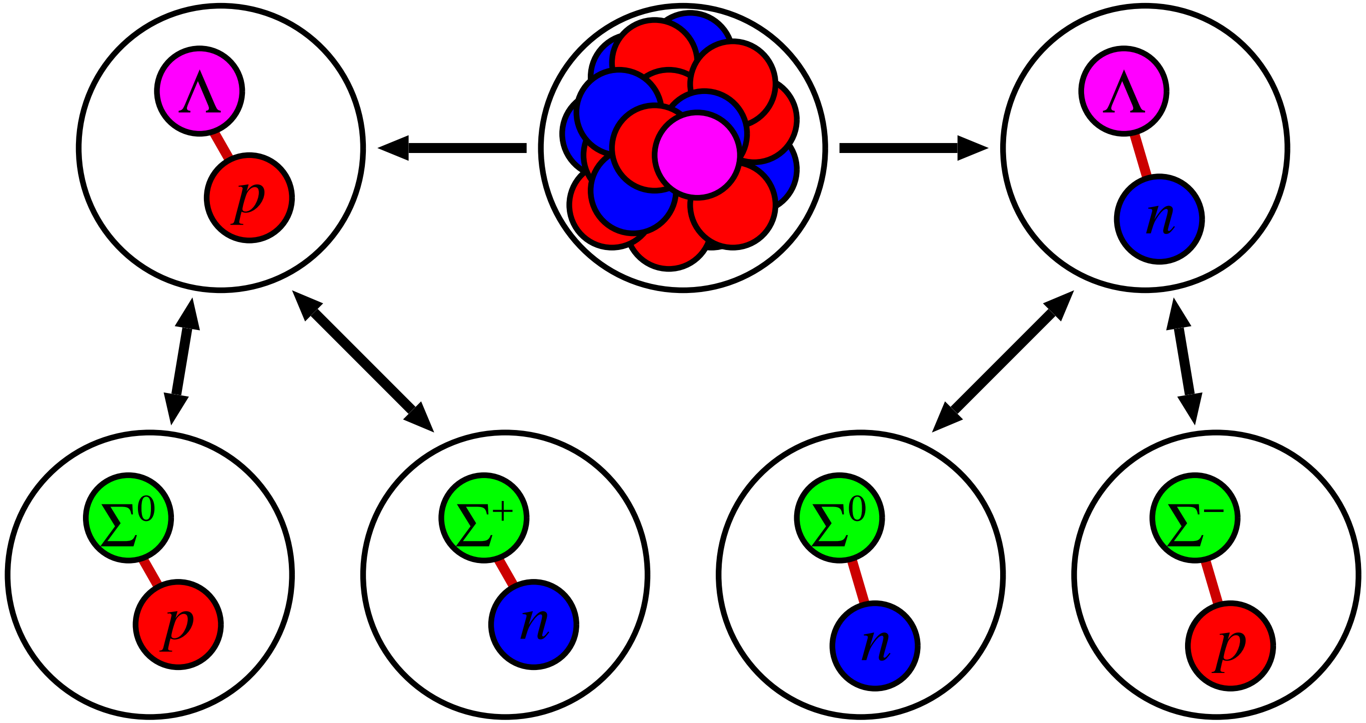
Λ-Σ coupling

The ΛN-ΣN coupling might induce a Σ-mixing in nuclei.



A Σ-mixing probability in a state of Λ hypernuclei is few percent because Σ hyperon has a larger mass than a Λ hyperon by about 80 MeV.

Σ-mixing → the production of hypernuclei by the double charge exchange reactions
→ the energy spacings of doublets

Production of neutron-rich Λ hypernuclei

talk by Harada (15th, plenary session)

Double Charge Exchange (DCX) Reactions

$^{A}Z\ (K^{-}, \pi^{+})\ ^{A}_{\Lambda}(Z{-}2)$, $^{A}Z\ (\pi^{-}, K^{+})\ ^{A}_{\Lambda}(Z{-}2)$

Double charge exchange reactions can reduce two protons from target nuclei, and suitable for productions of neutron-rich Λ hypernuclei.

Experimental attempts to produce neutron-rich Λ hypernuclei

$(K^{-}_{\mathrm{Stopped}}, \pi^{+})$ reactions

KEK ^{9}Be (K^{-}, π^{+}) $^{9}_{\Lambda}$He, ^{12}C (K^{-}, π^{+}) $^{12}_{\Lambda}$Be, ^{16}O (K^{-}, π^{+}) $^{16}_{\Lambda}$C at rest. K. Kubota *et al.*, NPA602, 323 (1996).

DAΦNE ^{6}Li (K^{-}, π^{+}) $^{6}_{\Lambda}$H, ^{7}Li (K^{-}, π^{+}) $^{7}_{\Lambda}$H at rest. M. Agnello *et al.*, PLB640, 145 (2006).

(π^{-}, K^{+}) reactions

KEK ^{10}B (π^{-}, K^{+}) $^{10}_{\Lambda}$Li at $p_{\pi} = 1.05, 1.20$ GeV$/c$. P. K. Saha *et al.*, PRL94, 052502 (2005).

J-PARC E10 ^{6}Li (π^{-}, K^{+}) $^{6}_{\Lambda}$H, ^{9}Be (π^{-}, K^{+}) $^{9}_{\Lambda}$He at $p_{\pi} = 1.20$ GeV$/c$

← posters by Yoshida & Sakaguchi (14th, poster session)

Theoretical calculation by using the coupled-channel DWIA

T. Harada, A. Umeya, Y. Hirabayashi, PRC79, 014603 (2009).

The experimental spectrum of the $^{10}_{\Lambda}$Li hypernucleus can be explained by the one-step mechanism via Σ^{-} doorways.

Production of neutron-rich Λ hypernuclei

talk by Harada (15th, plenary session)

Double Charge Exchange (DCX) Reactions $^{A}Z\ (K^-, \pi^+)\ ^{A}_{\Lambda}(Z-2)$, $^{A}Z\ (\pi^-, K^+)\ ^{A}_{\Lambda}(Z-2)$

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(π^-, K^+) reactions

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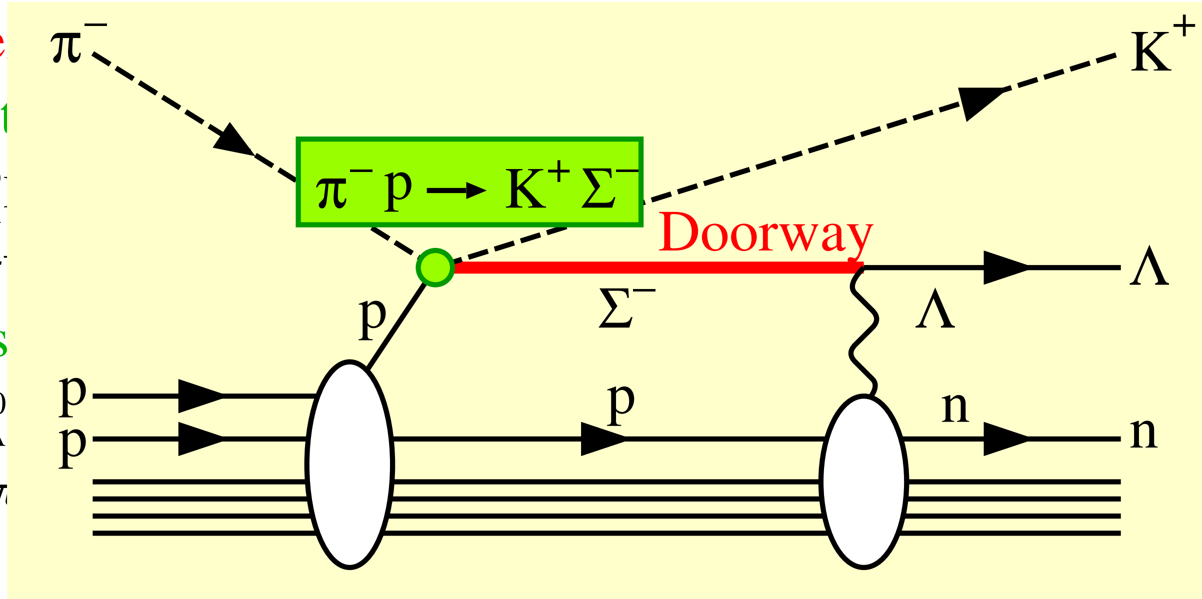
J-PARC E10 ^{6}Li $(\pi$

(14th, poster session)



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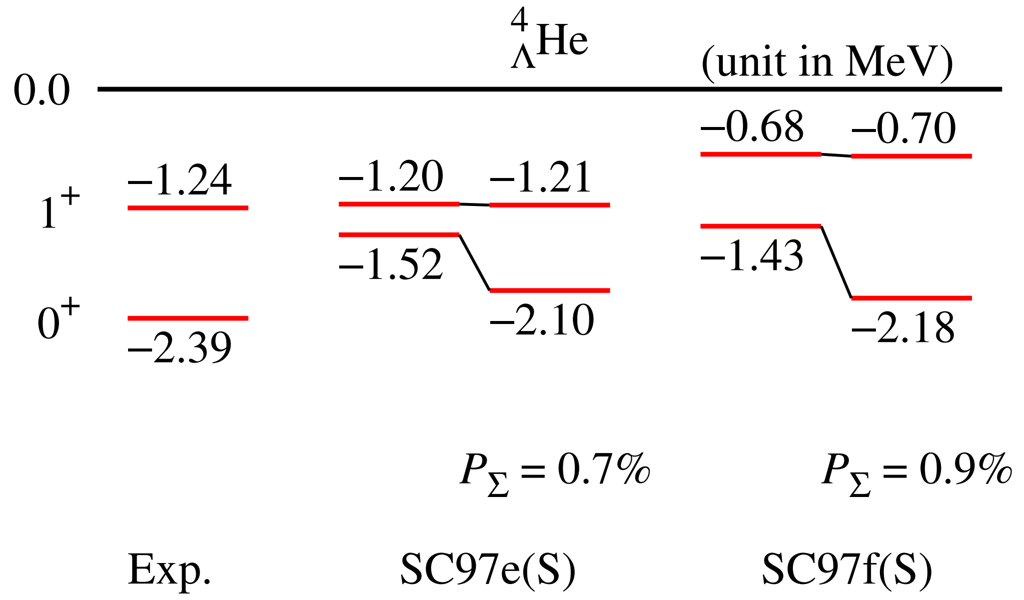
Coherent Λ-Σ coupling

talk by Harada (15th, plenary session)

Akaishi *et al.* suggested that a coherent Λ-Σ coupling is important to explain the splitting of the 1^+ and 0^+ states of ${}^{4}_{\Lambda}$He which has long been recognized as a problem.

Khin Swe Myint *et al.*, FBS. Suppl. 12 (2000) 383.

Y. Akaishi *et al.*, PRL84 (2000) 3539.



Akaishi *et al.*, PRL 84 (2000) 3539.

The problem might be solved by the Λ-Σ coupling which strongly affects the 0^+ states of the $A = 4$ hypernuclei.

It is expected that the attraction by the coherent Λ-Σ coupling enhances in a neutron-rich environment.

Purpose of the present shell-model study

To theoretically clarify the structure of the neutron-rich Λ hypernuclei

We investigate the structure of the Li hypernuclei, especially ${}^{10}_{\Lambda}$Li, in microscopic shell-model calculations considering the Λ-Σ coupling effect.

- Σ-mixing probabilities
- Energy shifts due to the Σ-mixing
- Enhancement of the coupling strengths in the neutron-rich Λ hypernuclei
- Relation to the β-transition properties of the nuclear core state

Method

A state of Λ hypernuclei in a multi-cofiguration nuclear shell model

$|({}^{A}_{\Lambda}Z)\nu TJ\rangle$

A : mass number, Z : atomic number, T : isospin, J : angular momentum

ν : quantum number to distinguish states with the same T and J

Hamiltonian in the configuration space for the Λ hypernucleus involving a Σ-mixing

$H = H_\Lambda + H_\Sigma + V_{\Lambda\Sigma} + V_{\Sigma\Lambda}$

H_Λ : Hamiltonian in the Λ configuration space

H_Σ : Hamiltonian in the Σ configuration space

$V_{\Lambda\Sigma}, V_{\Sigma\Lambda}$: two-body Λ-Σ coupling interaction

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We can write the state of H with T, J as

$$|({}^{A}_{\Lambda}Z)\nu TJ\rangle = \sum_{\mu} C_{\nu,\mu}|\psi^{\Lambda}_{\mu}; TJ\rangle + \sum_{\mu'} D_{\nu,\mu'}|\psi^{\Sigma}_{\mu'}; TJ\rangle,$$

where

$$H_\Lambda|\psi^{\Lambda}_{\mu}; TJ\rangle = E^{\Lambda}_{\mu}|\psi^{\Lambda}_{\mu}; TJ\rangle,$$
$$H_\Sigma|\psi^{\Sigma}_{\mu'}; TJ\rangle = E^{\Sigma}_{\mu'}|\psi^{\Sigma}_{\mu'}; TJ\rangle.$$

Method

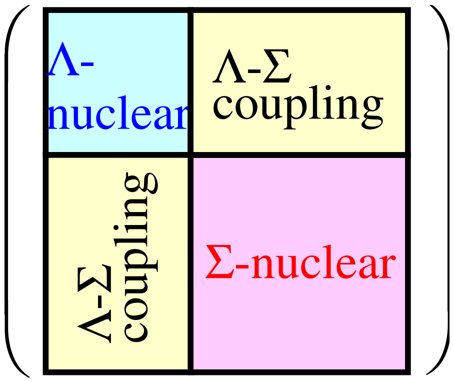
First-order perturbation

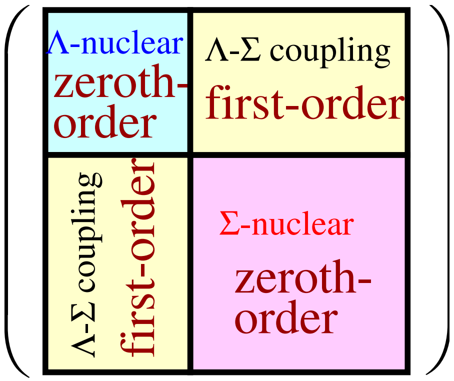
We treat $V_{\Lambda\Sigma}$ and $V_{\Sigma\Lambda}$ as perturbation because a Σ hyperon has a larger mass than a Λ hyperon by about 80 MeV.

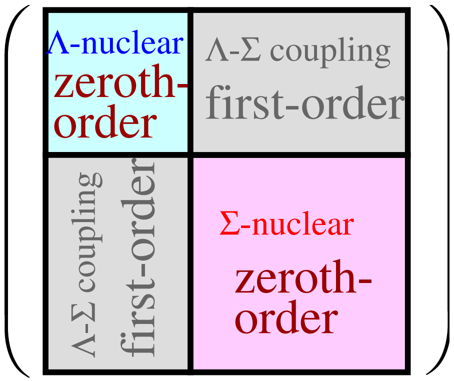
$$|({}^{A}_{\Lambda}Z)\nu TJ\rangle = \sum_{\mu} C_{\nu,\mu}|\psi^{\Lambda}_{\mu};TJ\rangle + \sum_{\mu'} D_{\nu,\mu'}|\psi^{\Sigma}_{\mu'};TJ\rangle,$$

$$C_{\nu,\mu} = \delta_{\nu\mu}, \quad D_{\nu,\mu'} = -\frac{\langle\psi^{\Lambda}_{\nu};TJ|V_{\Lambda\Sigma}|\psi^{\Sigma}_{\mu'};TJ\rangle}{E^{\Sigma}_{\mu'} - E^{\Lambda}_{\nu}}$$

Matrix of Hamiltonian







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First-order perturbation

We treat $V_{\Lambda\Sigma}$ and $V_{\Sigma\Lambda}$ as perturbation because a Σ hyperon has a larger mass than a Λ hyperon by about 80 MeV.

$$|({}^{A}_{\Lambda}Z)\nu TJ\rangle = \sum_{\mu} C_{\nu,\mu}|\psi^{\Lambda}_{\mu};TJ\rangle + \sum_{\mu'} D_{\nu,\mu'}|\psi^{\Sigma}_{\mu'};TJ\rangle,$$

$$C_{\nu,\mu} = \delta_{\nu\mu}, \quad D_{\nu,\mu'} = -\frac{\langle\psi^{\Lambda}_{\nu};TJ|V_{\Lambda\Sigma}|\psi^{\Sigma}_{\mu'};TJ\rangle}{E^{\Sigma}_{\mu'} - E^{\Lambda}_{\nu}}$$

→ Λ-Σ coupling strength $|D_{\nu,\mu'}|^2$ for each Σ eigenstate $|\psi^{\Sigma}_{\mu'};TJ\rangle$

→ Σ-mixing probability $P_{\Sigma} = \sum_{\mu'} |D_{\nu,\mu'}|^2$

→ energy shift $\Delta E = \sum_{\mu'} \left(E^{\Sigma}_{\mu'} - E^{\Lambda}_{\nu}\right) |D_{\nu,\mu'}|^2$

→ binding energy $E^{\Lambda}_{\nu} - \Delta E$

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First-order perturbation

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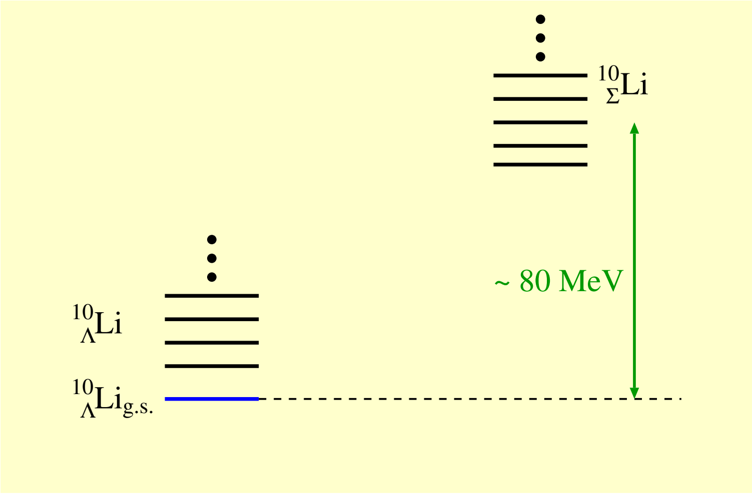
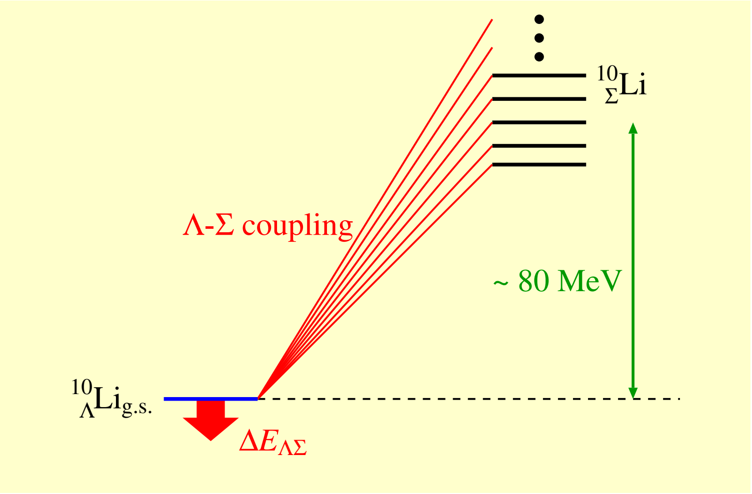
$$C_{\nu,\mu} = \delta_{\nu\mu}, \quad D_{\nu,\mu'} = -\frac{\langle\psi^{\Lambda}_{\nu};TJ|V_{\Lambda\Sigma}|\psi^{\Sigma}_{\mu'};TJ\rangle}{E^{\Sigma}_{\mu'} - E^{\Lambda}_{\nu}}$$

Energy shift due to the Λ-Σ coupling





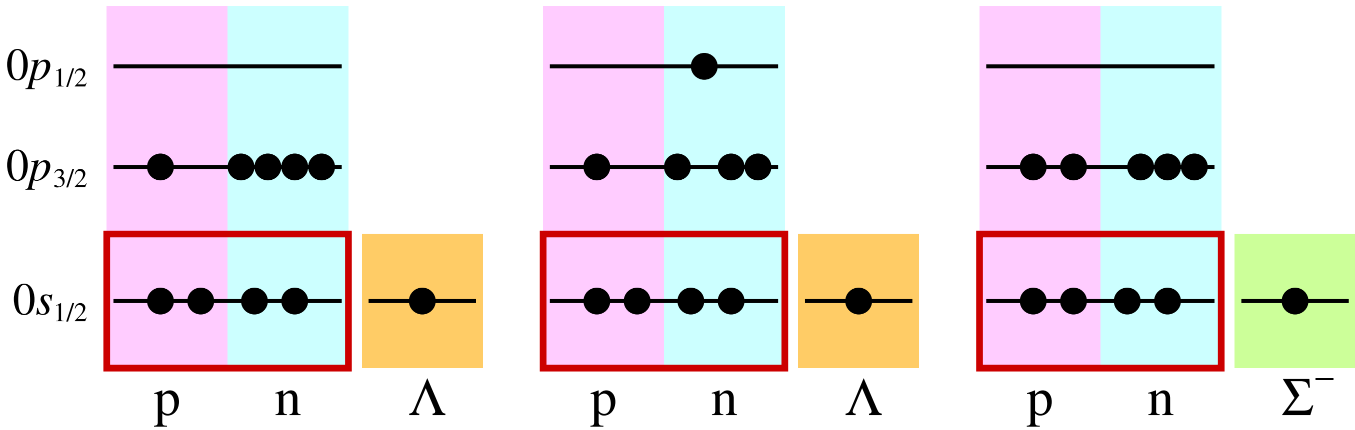
Shell-Model Setup

Model space

- Four nucleons are inert in the ^{4}He core.
- $(A-5)$ valence nucleons move in the p-shell orbits.
- The Λ or Σ hyperon is assumed to be in the lowest $0s_{1/2}$ orbit.

In the case of $^{10}_{\Lambda}$Li ($T=\frac{3}{2}$, $J^\pi=1^-$), 9-nucleon states have
$T_N=\frac{3}{2}$, $J^\pi_N=\frac{1}{2}^-,\frac{3}{2}^-$ for $|\psi^\Lambda_\mu;TJ\rangle$ and $T_N=\frac{1}{2},\frac{3}{2},\frac{5}{2}$, $J^\pi_N=\frac{1}{2}^-,\frac{3}{2}^-$ for $|\psi^\Sigma_{\mu'};TJ\rangle$.

Examples of basis states



Shell-Model Setup

Shell model Hamiltonian

NN effective interaction : Cohen-Kurath (8-16) 2BME, NP73, 1 (1965).

YN effective interaction :

$\langle N\Lambda|V_\Lambda|N\Lambda\rangle_{TJ}$ $\langle N\Sigma|V_\Sigma|N\Sigma\rangle_{TJ}$ $\langle N\Lambda|V_{\Lambda\Sigma}|N\Sigma\rangle_{TJ}$ $\langle N\Sigma|V_{\Sigma\Lambda}|N\Lambda\rangle_{TJ}$

$$V_Y = \underbrace{V_0(r)}_{\bar{V}} + \underbrace{V_\sigma(r)}_{\Delta}\, \boldsymbol{s}_N \cdot \boldsymbol{s}_Y + \underbrace{V_{\rm LS}(r)}_{S_+}\, \boldsymbol{\ell} \cdot \boldsymbol{s}_+ + \underbrace{V_{\rm ALS}(r)}_{S_-}\, \boldsymbol{\ell} \cdot \boldsymbol{s}_- + \underbrace{V_{\rm T}(r)}_{T}\, S_{12}$$

	Isospin	$\bar{V}$	Δ	S_+	S_-	T
V_Λ	$T = 1/2$	-1.2200	0.4300	-0.2025	0.1875	0.0300
V_Σ	$T = 1/2$	1.0100	-7.2150	-0.0010	0.0000	-0.3640
V_Σ	$T = 3/2$	-1.1070	2.2750	-0.2680	0.0000	0.1870
$V_{\Lambda\Sigma}, V_{\Sigma\Lambda}$	$T = 1/2$	1.4500	3.0400	-0.0850	0.0000	0.1570

← NSC97e, f(S)

$V_\Lambda, V_{\Lambda\Sigma}, V_{\Sigma\Lambda}$: D. J. Millener, Lect. Notes Phys. 724 (2007) 31.

V_Σ : D. J. Millener, private communication.

Numerical Results : Σ-mixing probabilities and energy shifts

	$(J^\pi \; ; T)$	Z	N	P_Σ [%]	$\Delta E_{\Lambda\Sigma}$ [MeV]	$\Delta E_{\Lambda\Sigma}$ [MeV] Millener's work
${}^{7}_{\Lambda}\mathrm{Li}$	$(\frac{1}{2}^+ ; 0)$	3	3	0.098	0.085	0.078
${}^{8}_{\Lambda}\mathrm{Li}$	$(1^- ; \frac{1}{2})$	3	4	0.172	0.139	
${}^{10}_{\Lambda}\mathrm{Li}$	$(\frac{3}{2}^+ ; 1)$	3	5	0.211	0.172	
${}^{10}_{\Lambda}\mathrm{Li}$	$(1^- ; \frac{3}{2})$	3	6	0.345	0.280	
${}^{11}_{\Lambda}\mathrm{B}$	$(\frac{5}{2}^+ ; 0)$	5	5	0.076	0.073	0.066

Millener's work for $Z = N$ nuclei ${}^{7}_{\Lambda}\mathrm{Li}$ and ${}^{11}_{\Lambda}\mathrm{B}$

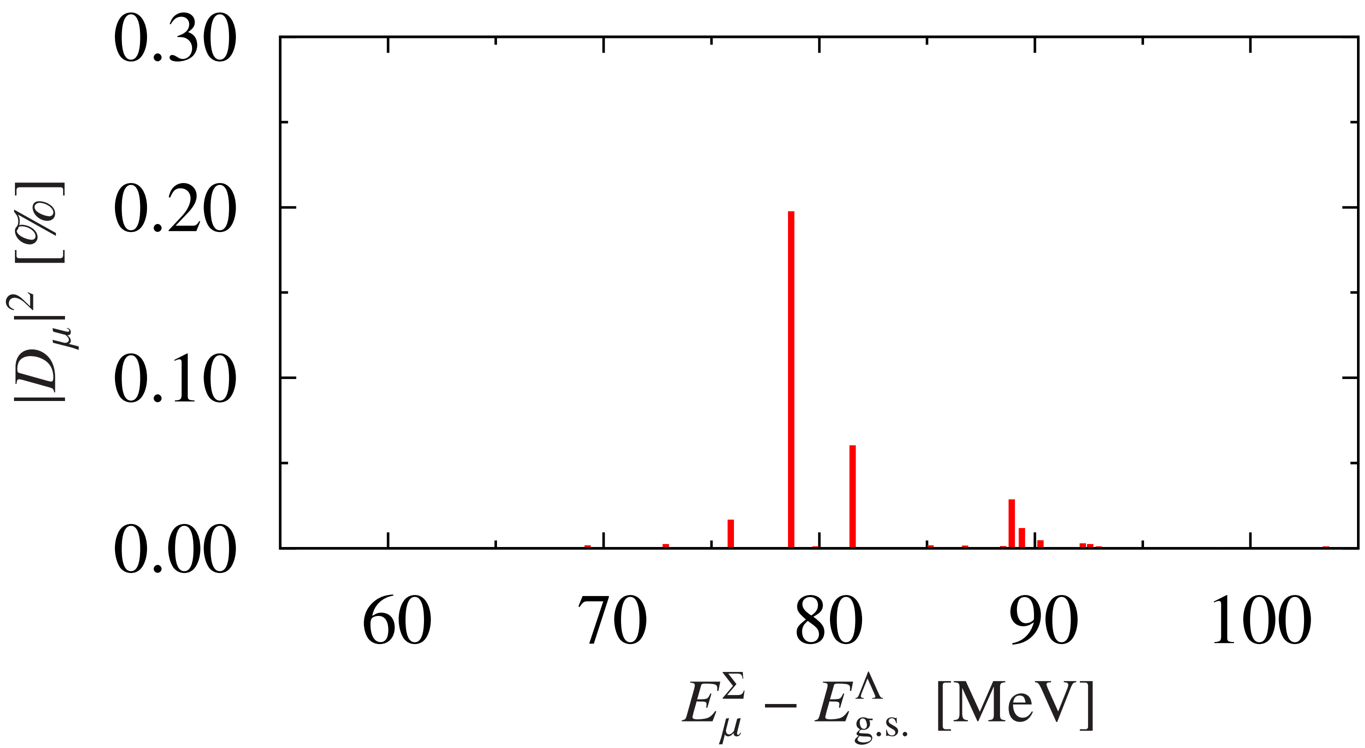
D. J. Millener, Lect. Notes Phys. 724, 31 (2007).

The Σ-mixing probabilitiy and the energy shift increase as the neutron number increases.

The energy shift ΔE for ${}^{10}_{\Lambda}\mathrm{Li}$ is about 3 times larger than that for ${}^{7}_{\Lambda}\mathrm{Li}$.

Numerical Results : Λ-Σ coupling strengths

$$|D_{\mu'}|^2 = \left| \frac{\langle \psi^{\Lambda}_{\text{g.s.}}; TJ | V_{\Lambda\Sigma} | \psi^{\Sigma}_{\mu'}; TJ \rangle}{E^{\Sigma}_{\mu'} - E^{\Lambda}_{\text{g.s.}}} \right|^2$$



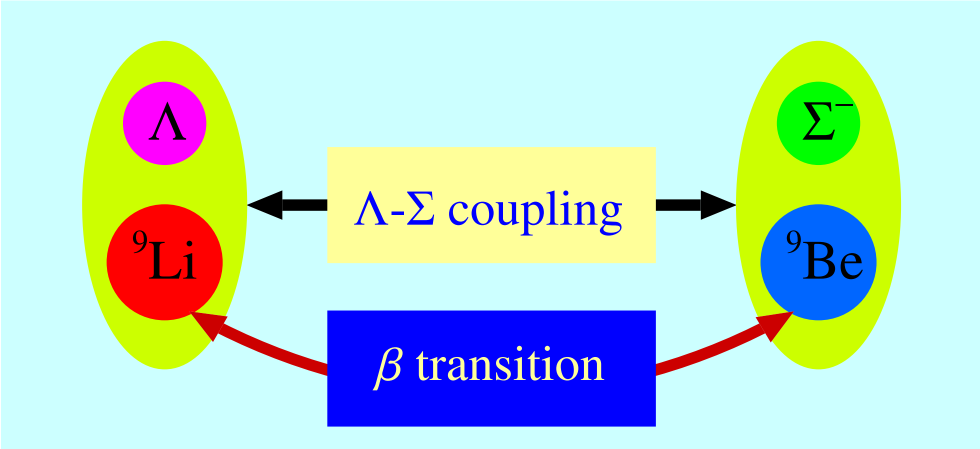
A contribution of the Σ ground state $|\psi^{\Sigma}_{\text{g.s.}}\rangle$ to the Σ-mixing is reduced to 0.002 %.

The several Σ excited states in the $E^{\Sigma}_{\mu'} - E^{\Lambda}_{\text{g.s.}} \approx 80$ MeV region considerably contribute to the Σ-mixing.

These contributions are coherently enhanced by the configuration mixing.

Discussion

A mechanism of the Λ-Σ coupling in neutron-rich hypernuclei ?



It is interesting to consider the β transitions between the nuclear-core components of Λ and Σ eigenstates to investigate the strength distribution of the Λ-Σ coupling.

Λ-Σ coupling interaction

$$V_{\Sigma\Lambda} \simeq \underbrace{V_{\rm F}(r)\boldsymbol{t}_N \cdot \boldsymbol{\phi}_{\Sigma\Lambda}}_{V^{\rm F}_{\Sigma\Lambda}} + \underbrace{V_{\rm GT}(r)\,(\boldsymbol{\sigma}_N \cdot \boldsymbol{\sigma}_{\Sigma\Lambda})\,\boldsymbol{t}_N \cdot \boldsymbol{\phi}_{\Sigma\Lambda}}_{V^{\rm GT}_{\Sigma\Lambda}},$$

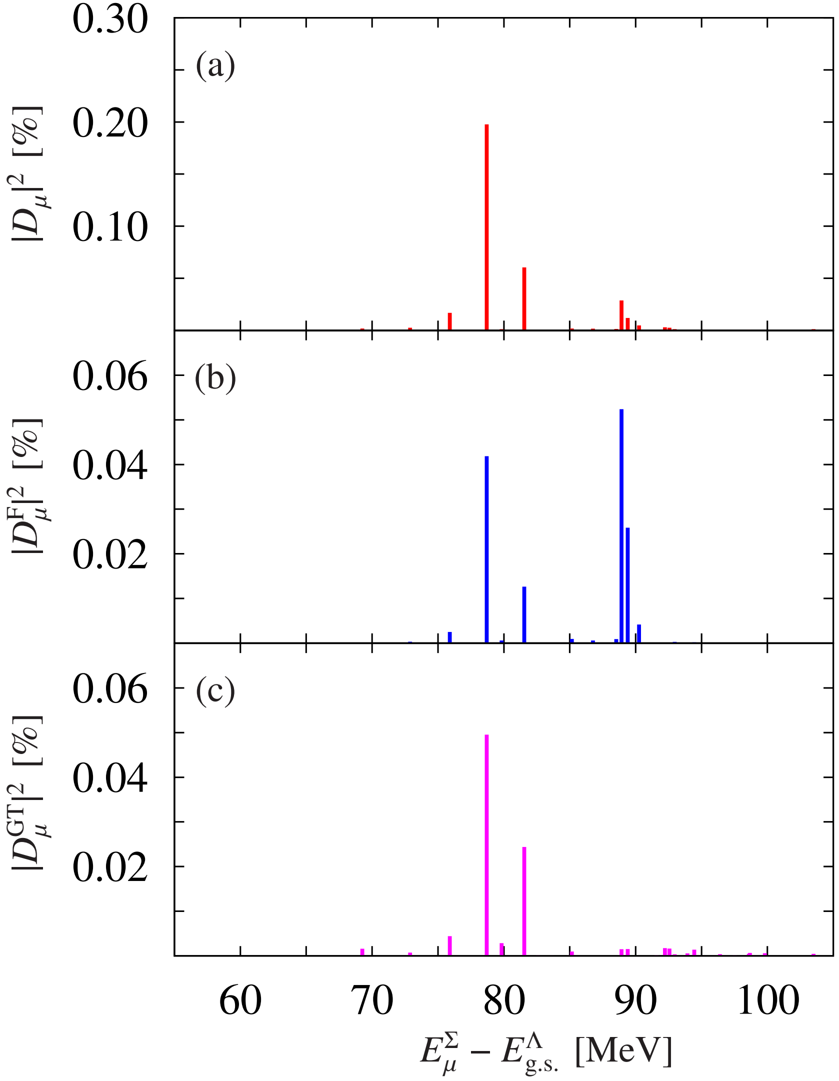
$V^{\rm F}_{\Sigma\Lambda}$: Fermi-type coupling ($\boldsymbol{t}_N$: Fermi β-transition operator for nucleons)

$V^{\rm GT}_{\Sigma\Lambda}$: Gamow-Teller-type coupling ($\boldsymbol{\sigma}_N \boldsymbol{t}_N$: Gamow-Teller β-transition operator for nucleons)

$|j_\Sigma\rangle = \boldsymbol{\phi}_{\Sigma\Lambda}|j_\Lambda\rangle$

Discussion

Fermi- and Gamow-Teller-type coupling strengths



$$|D_\mu|^2 = \left| \frac{\langle \psi_\Sigma;\mu | V_{\Sigma\Lambda} | \phi_\Lambda;\nu \rangle}{E_{\Lambda;\nu} - E_{\Sigma;\mu}} \right|^2 \rightarrow P_\Sigma = 0.345\ \%$$

$$|D_\mu^{\mathrm{F+GT}}|^2 = \left| \frac{\langle \psi_\Sigma;\mu | \left(V_{\Sigma\Lambda}^{\mathrm{F}} + V_{\Sigma\Lambda}^{\mathrm{GT}}\right) | \phi_\Lambda;\nu \rangle}{E_{\Lambda;\nu} - E_{\Sigma;\mu}} \right|^2 \rightarrow P_\Sigma^{\mathrm{F+GT}} = 0.350\ \%$$

$$|D_\mu^{\mathrm{F}}|^2 = \left| \frac{\langle \psi_\Sigma;\mu | V_{\Sigma\Lambda}^{\mathrm{F}} | \phi_\Lambda;\nu \rangle}{E_{\Lambda;\nu} - E_{\Sigma;\mu}} \right|^2 \rightarrow P_\Sigma^{\mathrm{F}} = 0.144\ \%$$

$$|D_\mu^{\mathrm{GT}}|^2 = \left| \frac{\langle \psi_\Sigma;\mu | V_{\Sigma\Lambda}^{\mathrm{GT}} | \phi_\Lambda;\nu \rangle}{E_{\Lambda;\nu} - E_{\Sigma;\mu}} \right|^2 \rightarrow P_\Sigma^{\mathrm{GT}} = 0.098\ \%$$

The Fermi and Gamow-Teller components coherently contribute to the Λ-Σ coupling strengths.

Discussion

Why is the energy shift for ${}^{10}_{\Lambda}$Li about 3 times larger than that for ${}^{7}_{\Lambda}$Li ?

The enhancement of the Σ-mixing probabilities in neutron-rich Λ hypernuclei is mainly due to the Fermi-type coupling interaction $V^{\rm F}_{\Sigma\Lambda}$.

Fermi-type coupling

We use the weak-coupling limit for simplicity.

$$\to \langle V^{\rm F}_{\Sigma\Lambda} \rangle = \langle T_N{=}T\ J_N, j_\Sigma; T J \| V^{\rm F}_{\Sigma\Lambda} \| T_N{=}T\ J_N, j_\Lambda; T J \rangle$$

$$\to \langle V^{\rm F}_{\Sigma\Lambda} \rangle = \bar{V} \sqrt{\frac{4T(T+1)}{3}}$$

$$\to |D^{\rm F}_{\mu'}|^2 \propto T(T+1)$$

	T	J^π	$P^{\rm F}_\Sigma$ [%]	$P^{\rm GT}_\Sigma$ [%]	$P^{\rm F+GT}_\Sigma$ [%]	P_Σ [%]
${}^{7}_{\Lambda}$Li	0	$1/2^+$	0.000	0.088	0.088	0.098
${}^{8}_{\Lambda}$Li	1/2	1^-	0.032	0.066	0.168	0.172
${}^{9}_{\Lambda}$Li	1	$3/2^+$	0.077	0.099	0.206	0.211
${}^{10}_{\Lambda}$Li	3/2	1^-	0.144	0.098	0.350	0.345

Summary and conclusion

We have investigated the structure of the Li hypernuclei in shell-model calculations considering the Λ-Σ coupling in the perturbation method.

We have found that the Σ-mixing probabilities and the energy shifts of $^{10}_{\Lambda}$Li eigenstates are coherently enhanced by the Λ-Σ coupling configurations.

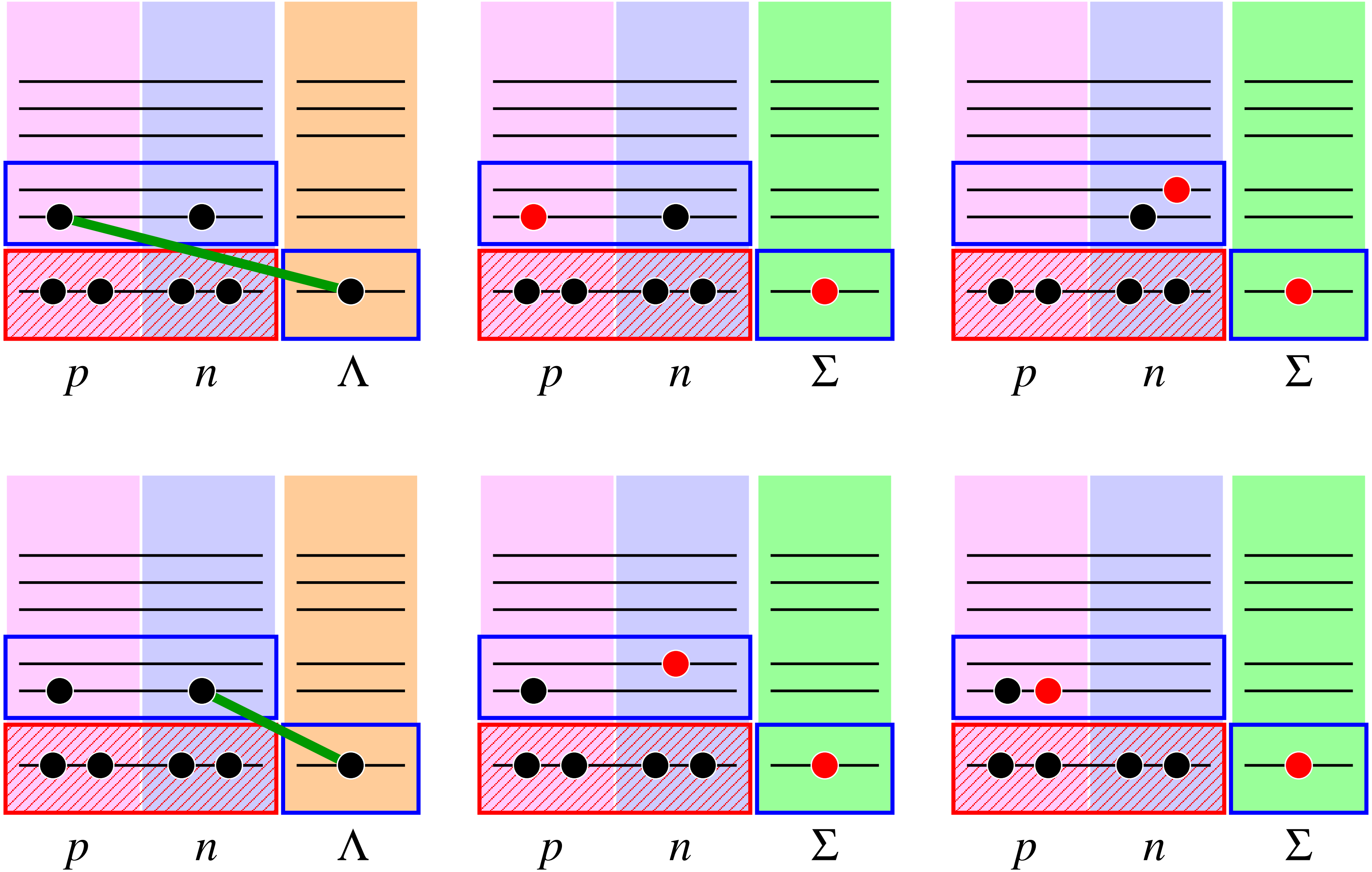
The reasons why the Σ-mixing probabilities are enhanced are summarized as follows:

1. The multi-configuration Σ excited states can be strongly coupled with the Λ ground state with the help of the ΣN interaction.
2. These strong Λ-Σ couplings are coherently enhanced by the Fermi- and Gamow-Teller-type coupling components.
3. The Fermi-type coupling becomes more effective in the neutron-rich environment increasing as $T(T+1)$.

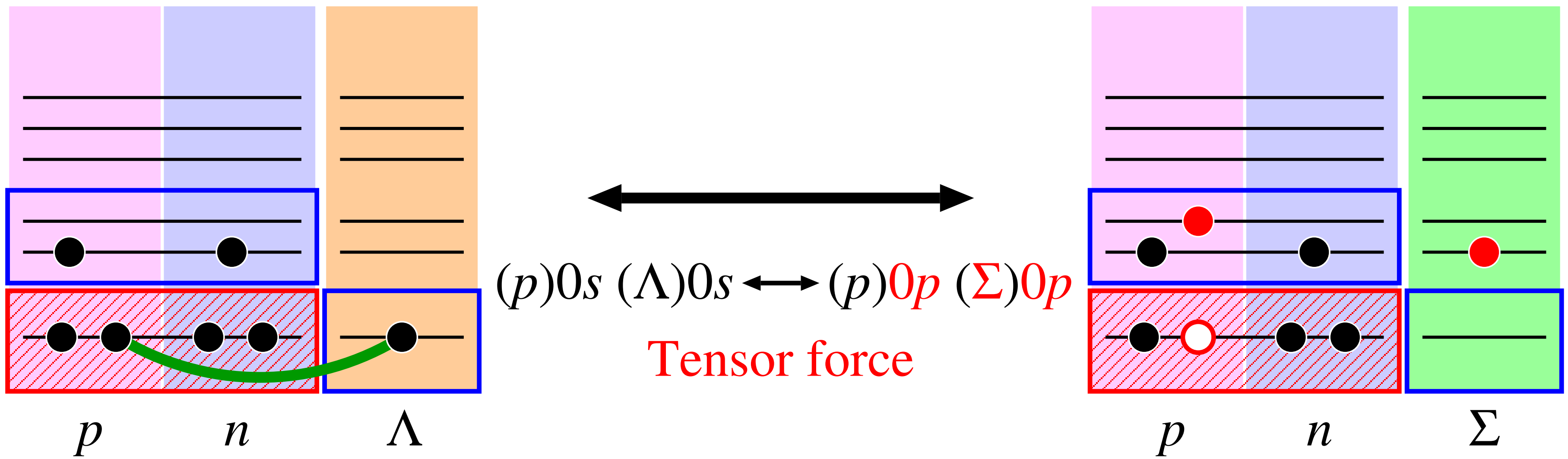
In conclusion, we have found that the Σ-mixing probability is about 0.35 % and the energy shift is about 0.28 MeV for the neutron-rich $^{10}_{\Lambda}$Li $1^-_{\rm g.s.}$ ground state, which is about 3 times larger than that for $^{7}_{\Lambda}$Li.

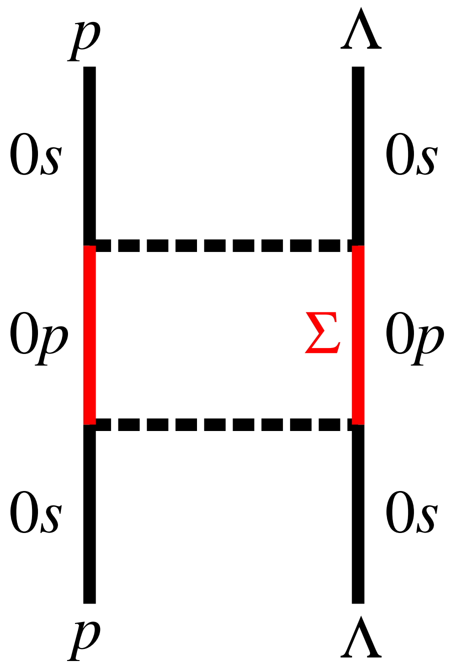
Backup Slide

Λ-Σ coupling



Λ-Σ coupling





Effective single-particle energy of Λ hyperon in 0*s* orbit

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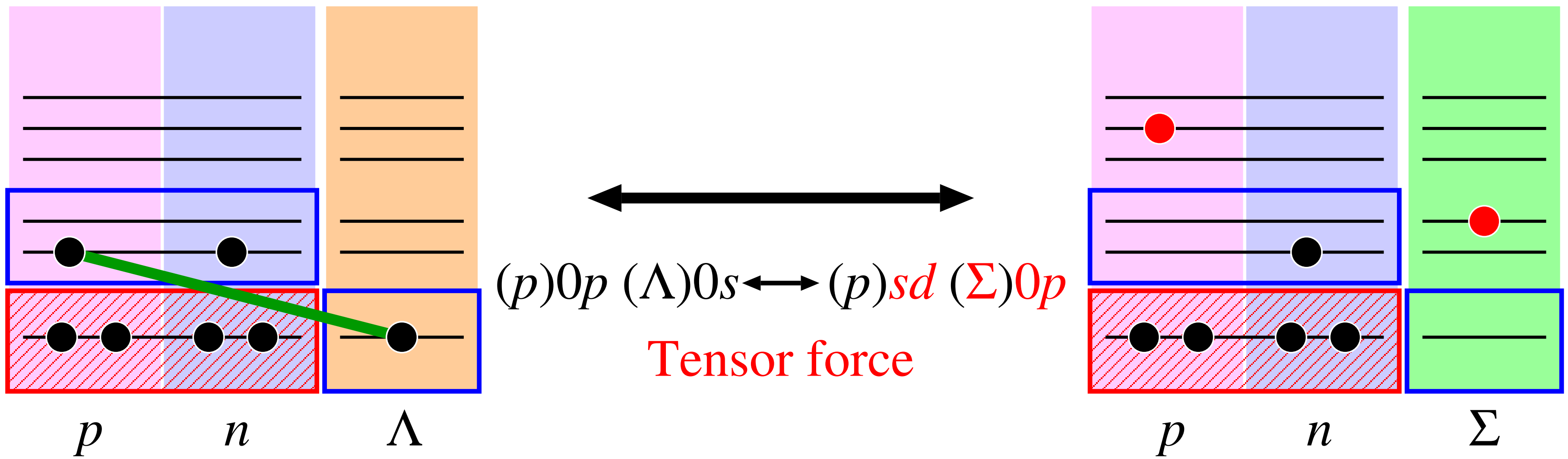
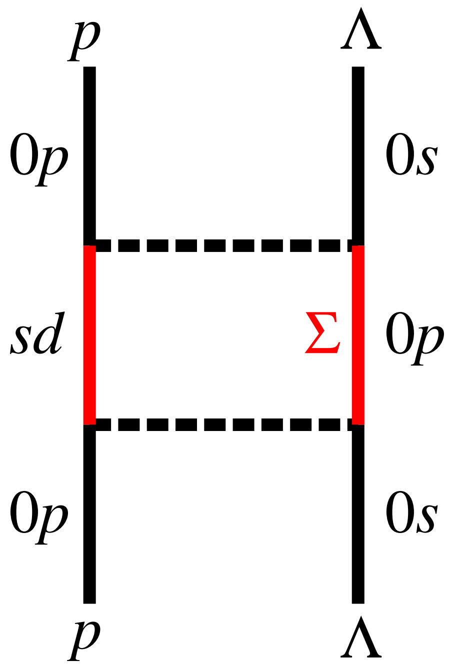
Binding energy of Λ hypernuclei

Λ-Σ coupling





Effective ΛN interaction
< (p)0p (Λ)0s | V | (p)0p (Λ)0s >

Binding energy of Λ hypernuclei