# Closed Orbit Correction in CELSIUS 

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#### Abstract

CELSIUS is a storage ring for intermediate energy nuclear and particle physics. The ring consists of four $90^{\circ}$ arcs and four straight sections and has a circumference of 81 m . The closed orbit is measured by a Beam Position Monitor (BPM) system at 10 positions in the ring and the data is fed into the orbit correction program. A measured response matrix is used for calculating the required changes of correctors' currents (up to 12 in the horizontal plane and up to 8 in the vertical plane). The singular value decomposition (SVD) algorithm is used for finding the best solution in the sense of minimizing the corrector excitation or minimizing the mean square error. The BPM hardware layout and the main features of the orbit correction program are presented. automatic correction procedure for the development of acceleration cycles is outlined.


## 1 Introduction

Delivering the accelerated beams to the users is a twostep process:

- storing the beam in the ring
- creation of the acceleration cycle

In the first phase all magnet power supplies are static and the orbit correction is done by applying the calculated set values to the correctors. On the other hand during the acceleration cycle the power supplies are no longer controlled directly from the control system but from the autonomous function generators. The tables modified in one cycle will be executed in the following cycle. Due to the fact that CELSIUS main dipoles are solid-core magnets the eddy currents prevent us to do quick ramping. Therefore the operational cycles in CELSIUS are between 90 and 300 seconds long. A typical cycle has an 8 s long flat bottom at which injection takes place, 22 s acceleration ramp, 60-240 s flat top and 30 s ramp to the flat bottom.
The change of the closed orbit due to the changes of correctors can be described by a system of linear equations

$$
\begin{equation*}
[\mathbf{d x}]=[\mathbf{A}] \cdot[\mathbf{d I}] \tag{1}
\end{equation*}
$$

where:
[dx] - vector of closed orbit changes at given positions in the ring (M)
[A] - response matrix ( $\mathrm{M} \times \mathrm{N}$ )
[dI] - vector of corrector magnet changes ( N )
Correcting the orbit requires finding corrector excitations that minimize the difference of measured BPM positions to their desired values. In this process the matrix [A] needs to be inverted (in the least squares sense). Even though normally the system of equations is over-determined, badly placed BPMs or correctors may cause [A] to be nearly
degenerate, which is the case for the vertical orbit correction in CELSIUS. In order to overcome this problem we utilize Singular Value Decomposition (SVD), a method used for the SLC final focus [1].

### 1.1 Singular value decomposition (SVD)

The SVD method [2] is based on the following theorem of linear algebra:
Any M x N matrix [A] whose number of rows M is greater than or equal to its number of columns N , can be written as the product of an $\mathrm{M} \times \mathrm{N}$ columnorthogonal matrix [U], an $\mathrm{N} x \mathrm{~N}$ diagonal matrix [W] with positive or zero elements, and the transpose of an Nx N orthogonal matrix [V]

$$
\begin{equation*}
[\mathbf{A}]=[\mathbf{U}] \cdot[\mathbf{W}] \cdot[\mathbf{V}]^{\mathrm{T}} \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
[\mathbf{U}]^{\mathrm{T}} \cdot[\mathbf{U}]=[\mathbf{V}]^{\mathrm{T}} \cdot[\mathbf{V}]=[\mathbf{1}] . \tag{3}
\end{equation*}
$$

The decomposition (2) can always be done, no matter how singular the matrix is. If the matrix $[\mathbf{A}]$ is square, then $[\mathbf{U}],[\mathbf{V}]$ and $[\mathbf{W}]$ are all square matrices of the same size. Since [U] and [V] are orthogonal, and [W] is diagonal the inverse of $[\mathbf{A}]$ is

$$
\begin{equation*}
[\mathbf{A}]^{-1}=[\mathbf{V}] \cdot\left[\operatorname{diag}\left(\mathbf{1} / \mathbf{w}_{\mathbf{j}}\right)\right] \cdot[\mathbf{U}]^{\mathrm{T}} \tag{4}
\end{equation*}
$$

The solution of (1) can be written as
$[d I]=[A]^{-1} \cdot[d x]=[V] \cdot\left[\operatorname{diag}\left(1 / w_{j}\right)\right] \cdot[U]^{T} \cdot[d x]$
By zeroing those $1 / w_{j}$ for which $w_{j}$ is small, we can improve the solution (5) in a sense that we throw away linear combination of the set of equations (the one which "pulls" the solution vector towards big values) and find the solution which minimizes the sum of $\left(\mathrm{dII}_{\mathrm{j}}\right)^{2}$. It can be proved that using equation (5) for overdetermined set of equations we find the solution that minimizes

$$
\begin{equation*}
|[\mathbf{A}] \cdot[\mathrm{dII}]-[\mathrm{dx}]| \tag{6}
\end{equation*}
$$

i.e. solves the linear least-squares problem.

Using the SVD algorithm we can easily find the solution of (1) which minimizes the weighted error $\chi^{2}$

$$
\begin{equation*}
\chi^{2}=\Sigma\left(\left(\left(\mathrm{dx}_{\mathrm{i}}-\mathrm{A}_{\mathrm{i}} \cdot[\mathrm{dI}]\right)\right) / \delta_{\mathrm{i}}\right)^{2}(\mathrm{i}=1, \ldots \mathrm{M}) \tag{7}
\end{equation*}
$$

By dividing each element of the $i$-th row of the response matrix $\mathbf{A}_{\mathrm{i}}$ by $\delta_{\mathrm{i}}$ and by substituting the $\mathrm{dx}_{\mathrm{i}}$ in equation (1) with $\left(\mathrm{dx}_{\mathrm{i}} / \mathrm{\delta}_{\mathrm{i}}\right)$ we reduce the problem to the "standard" least squares case.

## 2 Hardware

The hardware used for the closed orbit measurements is schematically shown in fig. 1.
The signals from the electrostatic pickups mounted in the
ring are amplified, the sum and delta signals are created for each pickup and these signals are fed into a multiplexer (MUX). The output from the MUX is then used by the beam position detector (BPM) which produces a signal proportional to the closed orbit deviation from the center of the vacuum chamber. The position signal is digitized by a 12 bit ADC. The ADC and MUX are controlled by a microprocessor in a G64 crate. The microprocessor is synchronized with the acceleration cycle by an external pulse from the timing system. The BPM system can work in two modes:

- asynchronous mode in which the beam positions are measured all the time
- synchronous mode in which every measurement is taken at a well known moment in the acceleration cycle
The input channel of the multiplexer is changed every 50 ms , thus a full scan for 20 pickups takes one second. The closed orbit correction program can compensate for the time delay between the measurements from different pickups in the ring.
The BPM controller is connected to the control system via an RS-232 serial line. A simple communication protocol on this link supports the mode selection, specification of the times when the data is taken and measurement data transfer from the BPM to the control system.


Figure 1 Beam position monitor hardware

## 3 Software

There are two groups of programs dealing with the closed orbit correction:

- programs for obtaining the response matrices
- programs for the orbit correction


### 3.1 Measurement of the response matrix

A special program has been written in order to quickly get the response matrix used by the static and dynamic closed orbit correction programs. The program can be used after storing the beam in CELSIUS with enough intensity to get valid data from the BPM system. The program
changes a given corrector in CELSIUS and observes the changes of the closed orbit. After making several changes (typically data is taken for 3 different corrector settings) one column of the response matrix [A] in equation (1) is calculated. The procedure is repeated for every corrector used for the closed orbit correction ( 12 horizontal +8 vertical). The result is saved in a format that can be directly used by the closed orbit correction programs.
A new response matrix needs to be measured only after some changes of the characteristics of the corrector magnets or change of the working point of CELSIUS. Typically the process of obtaining a new response matrix takes about 5 minutes. Additional time is needed for checking the possible range of correctors changes for which the beam's position readouts are reliable.

### 3.2 Orbit correction

From the operators point of view the purpose of the orbit correction program is a change of the closed orbit (relative mode) by a specified amount or to set the orbit to a given position (absolute mode). In the latter case the program needs to read the current beam position and calculate required changes in the closed orbit in order to find the necessary changes of the correctors. The programs use the SVD method to find the solution of the system of equations (1). The threshold for substituting $1 / \mathrm{w}_{\mathrm{j}}$ with 0 (SVD cut-off) in the equation (5) is set to $10^{-4}$ but can be changed by the operator.
The main features of the programs (static and dynamic) are:

- "relative" and "absolute" modes are available
- possibility to select which correctors to use
- possibility to select positions at which the orbit should be corrected
- different weight can be applied to each corrector
- possibility to define the weights which describe how well the orbit should be corrected are specific places ( $\delta_{\mathrm{i}}$ in equation (7))
- maximum allowed correction can be set


### 3.2.1 Static correction

There are two flavours of the program: one which has a graphical user interface and the operator can change the closed orbit using the sliders and another which sets the closed orbit to a specified position. The first program can be used as a quick try and error tool whereas the latter is best suited for setting the closed orbit to beforehand known position. The program can be set up to achieve the requested orbit in a given number of iterations. Usually, during iterations, only a fraction of the calculated corrector changes (for example $50 \%$ ) is applied in order to improve stability.

### 3.2.2 Dynamic correction

The dynamic closed orbit correction program xdyncor is used when the acceleration cycle is developed. The
calculation engine is the same, and the same response matrix is used as in the case of static correction. The corrections are scaled with the main magnetic field which is calculated from the main dipole current using a 3rd order polynomial. The corrections are calculated only at the times of the vector ends. The beam positions are measured synchronously with the cycle and there is one sample per second from each of the 20 pickups. Due to the fact that the input to the BPM detector is multiplexed, the positions at all pickups are not measured at the same time. The program, however, can compensate for it by using interpolated values. Such interpolation is justified at the flat top when the changes in the closed orbit are slow whereas it is not always appropriate for the corrections during the acceleration ramp. An example of traces from the BPM system is shown in fig. 2 (acceleration ramp from 8 to 30 s ).


Figure 2 BPM traces

While developing the acceleration cycle the operator specifies the time when the correction should start and end.

These times must lie in the period when valid BPM data is available. It is possible to keep the last applied correction also for a specified time outside the current correction window. As the development of the acceleration cycle proceeds, and the beam survives longer, the time of the last correction is moved towards the end of the cycle. As soon as BPM data is available for the whole acceleration cycle, the final adjustments can be done in an iterative manner.
We also included a "stickiness parameter" $s$ in the dynamic orbit correction program that prevents the corrector values from deviating too much from the previous values. It is implemented by generalizing the least squares problem to minimizing

$$
\chi_{\mathrm{g}}^{2}=\chi^{2}+\Sigma\left(\left(\mathrm{dI}_{\mathrm{j}}[\mathrm{t}]-\mathrm{dI}_{\mathrm{j}}[\mathrm{t}-1]\right) / \mathrm{s}\right)^{2} \quad(\mathrm{j}=1, \ldots \mathrm{~N})
$$

where $\chi^{2}$ is defined by equation (7).
This feature has, however, not been tested yet.

## 4 Conclusion

The closed orbit correction programs used previously at CELSIUS used a Gauss-Jordan elimination method for solving the system of equations (1). It worked reasonably well for horizontal corrections but very poorly for vertical. The use of the SVD algorithm improved the situation a lot and made the correction programs much more robust and flexible.

## Acknowledgments

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## References

[1] V. Ziemann, Corrector Ironing in the SLC Final Focus, presented at the 1993 Particle Accelerator Conference, Washington, DC, 1993, p. 498.
[2] Numerical Recipes in FORTRAN: the art of scientific computing, William H. Press, et al., 2nd ed., Cambridge University Press.

