Flavour-singlet meson and multi-hadron spectroscopy using a new hadron correlator algorithm

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Motivation

The Hadron Spectrum Collaboration has made considerable progress in computing stationary-state energies in isovector-meson, kaon, and light-baryon sectors using carefully-chosen sets of single-hadron interpolators. These interpolator sets consist of operators constructed from smeared quark fields and incorporating different types of spatial displacements.

However, to date, little progress has been made in the $I = 0$ meson sector, and studies have not incorporated the multi-hadron operators needed to extract resonance parameters. These states are a particular challenge because their two-point functions are not amenable to conventional point-to-all quark-propagator techniques.

Exact all-to-all quark propagation

Since the quark fields in hadron interpolators are smeared ($\psi \to \tilde{\psi} = S\psi$), we require an estimate for the smeared quark propagator $\tilde{M}^{-1} = SM^{-1}S$, only.

With this in mind, we define Laplacian-Heaviside, or LapH, quark smearing :

$$S_{ab}(x,y) = \delta_{x_4,y_4} \sum_{k=1}^{N_{ev}} v_a^{(k)}(x)\, v_b^{(k)}(y)^*, \quad (1)$$

with

$$\tilde{\Delta}_{ab}(x,y)\, v_b^{(i)}(y) = -\lambda_i(x_4)\, v_a^{(i)}(x) \quad (0 < \lambda_i < \lambda_{i+1}), \quad (2)$$

where $\tilde{\Delta}$ is a gauge-covariant lattice Laplace operator constructed from stout-smeared link variables. The level of smearing is varied by changing N_{ev} in Eq. 1; reducing N_{ev} excludes more high-momentum quark modes. Using LapH smearing, the total number of fermion-matrix inversions needed to compute $\tilde{M}^{-1}$ on a single configuration is $N_{ev} \times N_t \times N_{spin}$. Therefore, with a high-enough level of quark smearing, $\tilde{M}^{-1}$ can be computed exactly.

Stochastic LapH

The LapH smearing scheme outlined above works well on small lattices. However, the density of Laplacian eigenmodes scales linearly with the volume, making this method impractical on larger lattices. To mitigate the volume dependence, we form a dilute stochastic estimate for the smeared quark propagator:

$$\tilde{M}^{-1} = SM^{-1}|v\rangle \sum_d E\left(\eta^{[d]}\eta^{[d]\dagger}\right)\langle v|. \quad (3)$$

The noise vector components satisfy

$$\sum_d E\left(\eta_{i\alpha}^{[d]}(t)\,\eta_{j\beta}^{[d]*}(t')\right) = \delta_{ij}\delta_{\alpha\beta}\delta_{tt'}, \quad E\left(\eta_{i\alpha}^{[d]}(t)\right) = 0,$$

$$\left|\eta_{i\alpha}^{[d]}(t)\right| = 1 \text{ if } (i,\alpha,t) \in d, \quad (= 0,\ \text{otherwise}), \quad (4)$$

where $i\,(j)$, $\alpha\,(\beta)$, and $t\,(t')$ denote eigenmode, spin, and time indices respectively.

First tests of the method

Preliminary tests of the Stochastic LapH algorithm were performed on $N_f = 2+1$ ensembles generated on small lattice volumes, with light sea quark masses corresponding to a pion mass of 390 Mev.

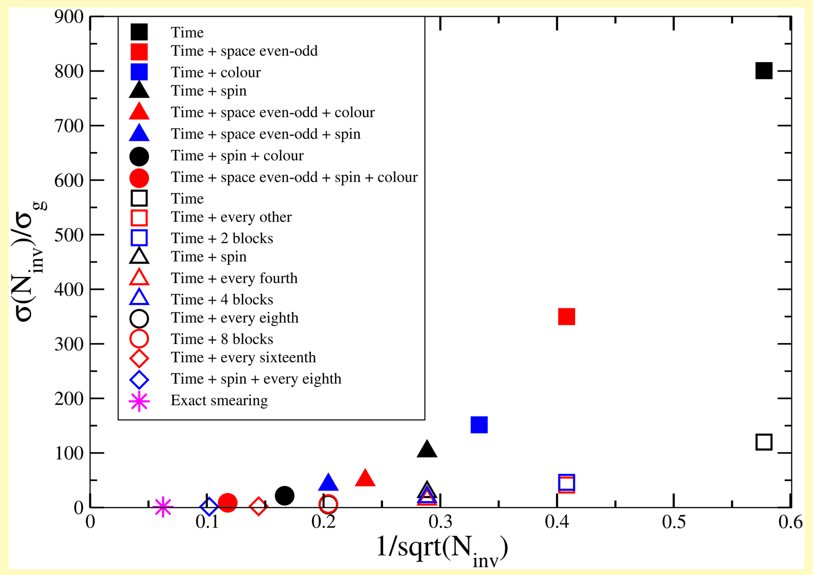


Figure 1).

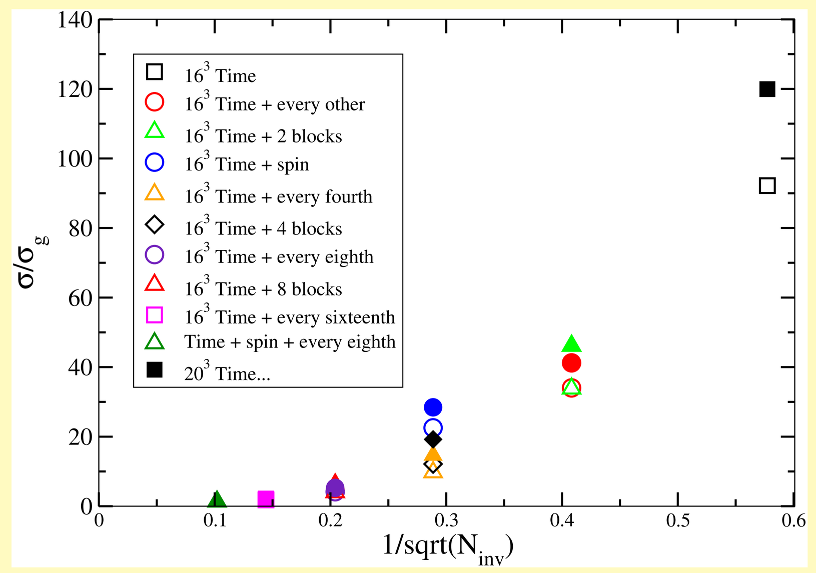


Figure 2).

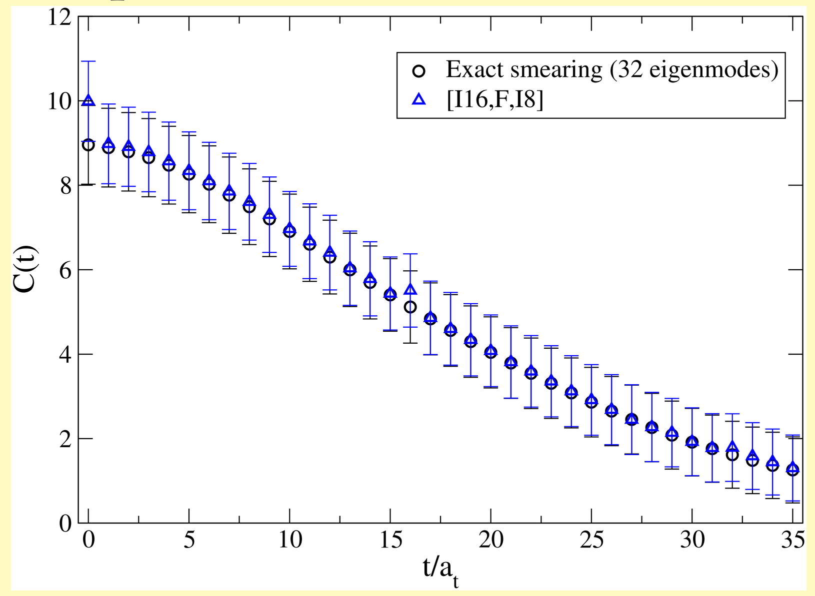


Figure 3).

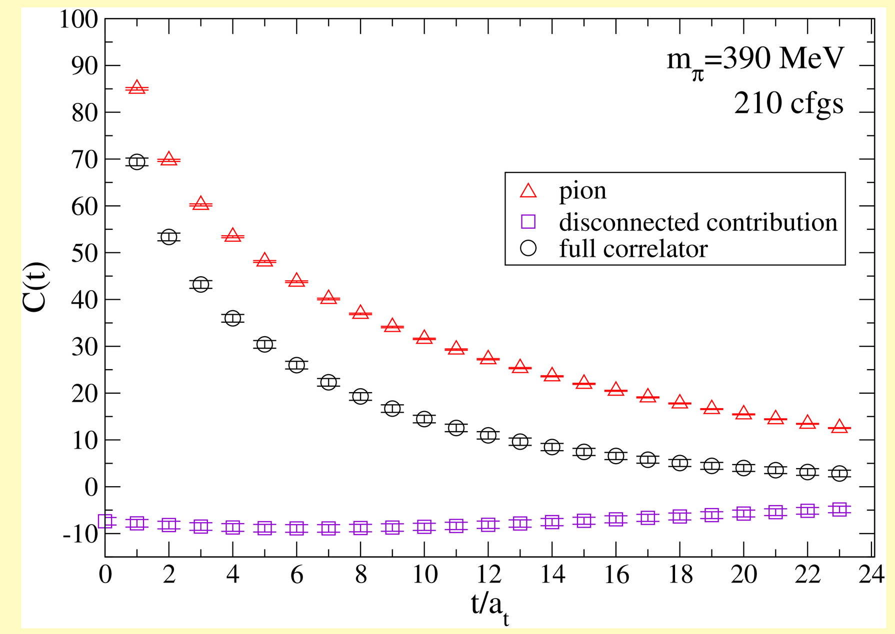
Fig. 1 is a comparison of error estimates on a LapH-smeared nucleon correlator at a single-time separation (t=5). The open symbols are Stochastic LapH results, and the closed data points were obtained using dilute stochastic estimates for the unsmeared quark propagator. For each dilution scheme considered, Stochastic LapH significantly outperforms conventional quark-propagator dilution.

Fig. 2 demonstrates the effect of changing the spatial volume on nucleon error estimates. Stochastic LapH error estimates on 16^3 and 20^3 spatial lattices are shown. The Laplacian eigenvalue cutoff is approximately constant across both lattices. Compared to the exact smearing scheme, Stochastic LapH exhibits a mild volume dependence.

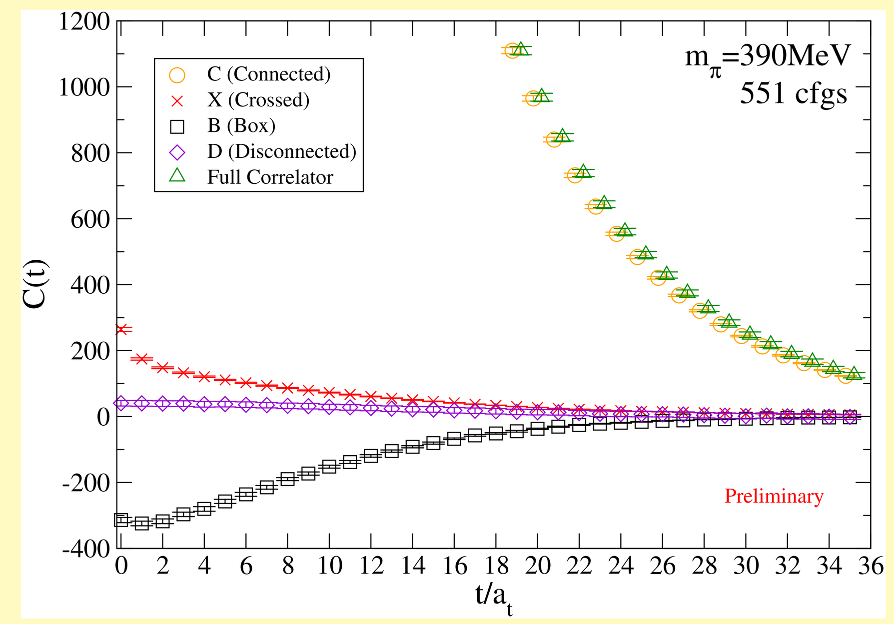
Fig. 3 contains results for the disconnected contribution to an isosinglet pseudoscalar correlator. These measurements were performed on a $16^3 \times 128$ lattice, and used 32 Laplacian eigenmodes. The dilution scheme for the stochastic quark-line estimates appears in the legend: $[I16, F, I8]$ indicates that interlace-16 ($I16$) dilution is applied in time, which means the diluted noise vectors have support on every 16^{th} lattice time slice; full spin dilution is implemented, as well as interlace-8 dilution in the eigenmode indices. In this example, the stochastic estimate uses a single (diluted) noise vector. The stochastic method is comparable to exact smearing, but requires far fewer quark-matrix inversions.

Work in progress

- Two $N_f = 2+1$ ensembles on $(2.9\text{ fm})^3$ spatial volumes ($\sim$ 580 configs per ensemble).
- Anisotropic clover and Symanzik-improved glue ($a_s = 0.12$ fm, $a_s/a_t = 3.5$) .
- m_π = 390 MeV, 220 MeV.
- 112 Laplace eigenmodes.
- Quark-line estimates for all two-meson and meson-baryon two-point functions.
- $[F, F, I8]$-dilution for quark-lines connecting different time slices.
- $[I16, F, I8]$-dilution for same-time quark lines.



First results for an $I = 0$ pseudoscalar on the heavier-pion ensemble. In this case, the interpolating operator is a light-quark bilinear.



$I = 0$ S-wave two-pion correlator. The four quark-line diagram contributions are plotted separately.

References

[1] M. Peardon et al. Phys. Rev. D 80 (2009) 054506 [arXiv:0905.2160 [hep-lat]].

[2] J. Foley et al. [arXiv:1011.0481 [hep-lat]].