QCD THERMODYNAMICS WITH WILSON-TYPE QUARKS

SUMMARY OF THE RESULTS FROM THE WHOT-QCD COLLABORATION

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Members

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Objectives

QCD thermodynamics with improved Wilson quarks

- theoretically clean
- less expensive than chiral quarks ⇨ chance to touch experiment

Wilson more expensive ⇨ need improvements / tricks.

What's WHOT?

Wilson + hot qcd => what happens?

by Tetsuo Hatsuda in 2006. First used at QM2006.

Originally [hwɔ́t], but we don't mind to pronounce as [dʌ́bəljùː hɔ́t].

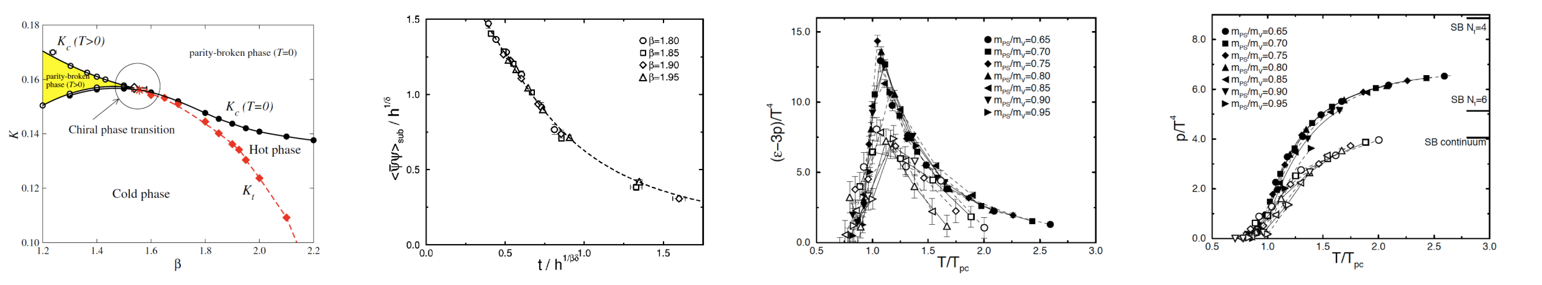
Prehistory — hot Wilson quarks at Tsukuba

QCDPAX (standard Wilson quarks + RG-improved Iwasaki gauge) 1989--1998

N_F=2 O(4) scaling; N_F=0 FSS, interface tension; many flavor QCD

CP-PACS (clover-improved Wilson quarks + RG-improved Iwasaki gauge) 1998--2007

systematic study of N_F=2 QCD at T>0 => O(4) scaling, EOS, c_s, ...



anisotropic lattices => EOS, charmonia at T>0

WHOT-QCD 2006--

N_F =2 Wilson quarks at μ≠0

PRD82, 014508 (2010)

N_F =2; Nt=4; Iwasaki glue + Clover-improved Wilson; m_{PS}/m_V=0.65, 0.8

First μ≠0 study with Wilson-type quarks.

Taylor expansion up to n=2

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_q}{T}\right)^n$$

Results:

$$\frac{\chi_q}{T^2} = \left(\frac{\partial}{\partial(\mu_u/T)} + \frac{\partial}{\partial(\mu_d/T)}\right)\frac{n_u+n_d}{T^3}$$

$$\frac{\chi_I}{T^2} = \left(\frac{\partial}{\partial(\mu_u/T)} - \frac{\partial}{\partial(\mu_d/T)}\right)\frac{n_u-n_d}{T^3}$$

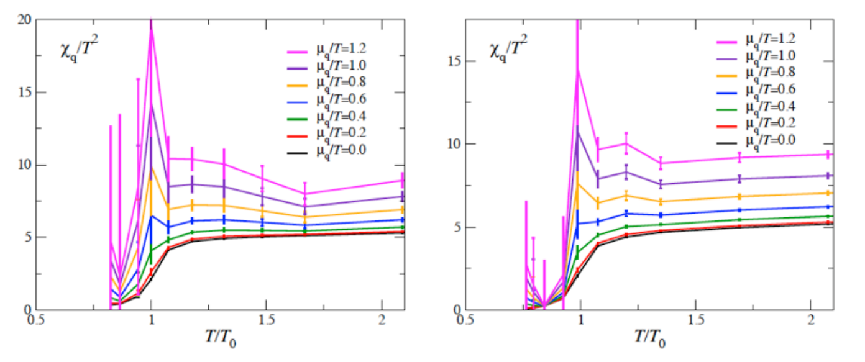


FIG. 14: Quark number susceptibility at finite μ_q for $m_{PS}/m_V = 0.65$ (left) and 0.80 (right).

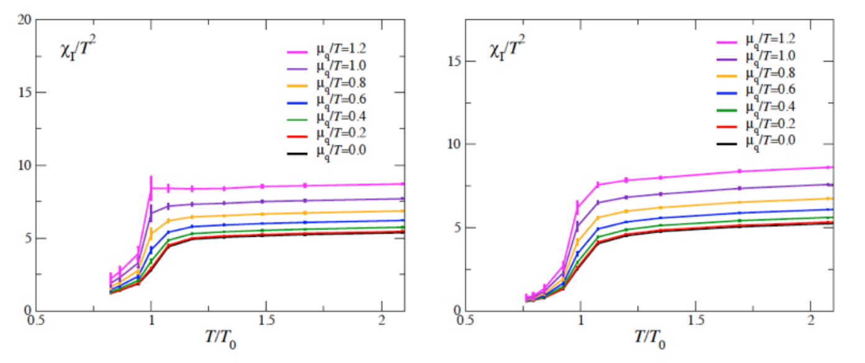


FIG. 15: Isospin susceptibility at finite μ_q for $m_{PS}/m_V = 0.65$ (left) and 0.80 (right).

* Critical point at finite μ

* Isospin suscept. insensitive

* Consistent with the results of staggered quarks.

Improvement by a Gaussian method:

a hybrid Taylor+reweighting method

Allton et al. PRD66(02); Ejiri PRD77(08)

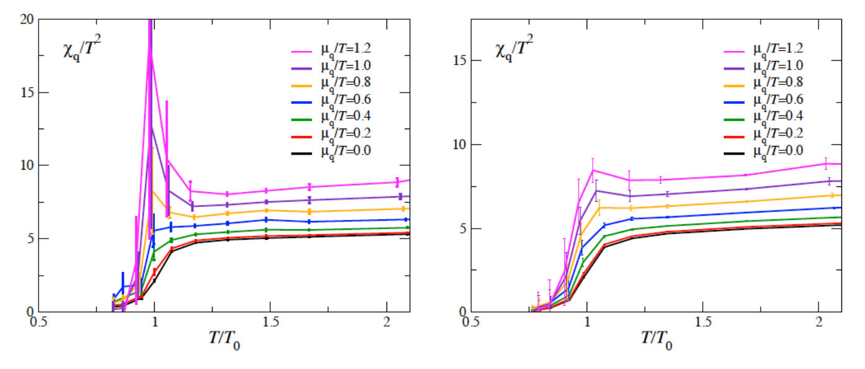


FIG. 24: Quark number susceptibility for each μ_q/T at $m_{PS}/m_V = 0.65$ (left) and 0.80 (right).

* A part of higher orders incorporated

* Reduce the sign problem using the empirical Gaussian distribution of the phase of det$M$

$$\theta(\mu) = N_f \mathrm{Im}\left[\ln\det M(\mu)\right]$$

$$= N_f \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} \mathrm{Im}\left[\frac{\partial^{2n+1}(\ln\det M(\mu))}{\partial\mu^{2n+1}}\right]_{(\mu=0)} \mu^{2n+1} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} \mathrm{Im}\mathcal{D}_{2n+1}\mu^{2n+1}$$

Heavy quark potential & screening masses

N_F =2, μ=0

Color channel dependence

PRD75,074501(2007)

$$\left.\begin{aligned} e^{-F_1(r,T)/T} &= \tfrac{1}{3}\langle \mathrm{Tr}\Omega^\dagger(\mathbf{x})\Omega(\mathbf{y})\rangle, \\ e^{-F_8(r,T)/T} &= \tfrac{1}{8}\langle \mathrm{Tr}\Omega^\dagger(\mathbf{x})\,\mathrm{Tr}\Omega(\mathbf{y})\rangle - \tfrac{1}{24}\langle \mathrm{Tr}\Omega^\dagger(\mathbf{x})\Omega(\mathbf{y})\rangle, \end{aligned}\right\} Q\overline{Q}$$

$$\left.\begin{aligned} e^{-F_6(r,T)/T} &= \tfrac{1}{12}\langle \mathrm{Tr}\Omega(\mathbf{x})\,\mathrm{Tr}\Omega(\mathbf{y})\rangle + \tfrac{1}{12}\langle \mathrm{Tr}\Omega(\mathbf{x})\Omega(\mathbf{y})\rangle, \\ e^{-F_{3^*}(r,T)/T} &= \tfrac{1}{6}\langle \mathrm{Tr}\Omega(\mathbf{x})\,\mathrm{Tr}\Omega(\mathbf{y})\rangle - \tfrac{1}{6}\langle \mathrm{Tr}\Omega(\mathbf{x})\Omega(\mathbf{y})\rangle, \end{aligned}\right\} QQ$$

Coulomb gauge

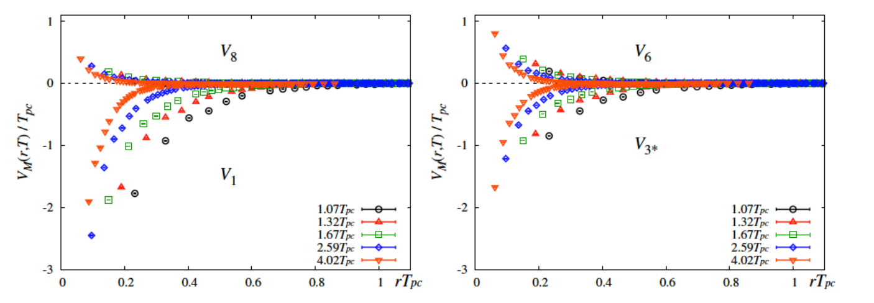


FIG. 4 (color online). Simulation results of the normalized free energies scaled by T_{pc} for color-singlet and octet $Q\bar{Q}$ channels (left) and color antitriplet and sextet QQ channels (right) at $m_{PS}/m_V = 0.65$.

Channel-dependence described by the Casimir factor a la Pert.Th

Electric / magnetic screening masses

PRD81,091501(2010)

* decomposed by Euclidian time-reflection and charge conjugation

* gauge-independent definitions

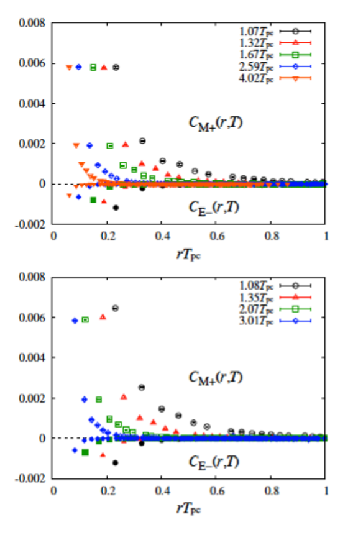


FIG. 1: Results of $C_{M+}(r,T)$ and $C_{E-}(r,T)$ for several temperatures as a function of rT_{pc} at $m_{PS}/m_V = 0.65$ (upper panel) and 0.80 (lower panel).

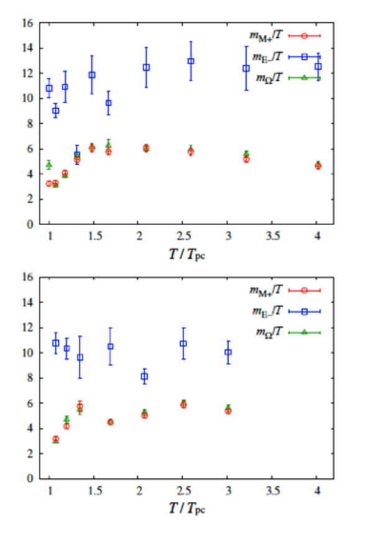


FIG. 2: Results of magnetic and electric screening mass m_{M+}/T and m_{E-}/T together with the screening mass obtained from the standard Polyakov-line correlation m_Ω/T as a function of temperature at $m_{PS}/m_V = 0.65$ (upper panel) and 0.80 (lower panel).

N_F =2, μ≠0; N_F =2+1, μ=0

Lat8 - Lat10 => papers in preparation

N_F =2+1 QCD

More improvements needed .

Fixed scale approach

PRD79,051501(2009)

Fixed scale approach: vary T by varying Nt with all coupling parameters fixed.

⇨ * one T = 0 simulation applicable for all T = 0 subtractions, * automatically on a LCP

⇨ large reduction of T=0 simulation costs

Conventional integral method inapplicable due to the integration in the coupling param. space.

⇨ ***T*-integration method**: $T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right) = \frac{\epsilon-3p}{T^4} \longrightarrow \frac{p}{T^4} = \int_{T_0}^{T} dT\, \frac{\epsilon-3p}{T^5}$

Pros and cons:

high T: lattice artifacts large due to small Nt, but the spatial volume kept

low T and near Tc: more costs due to large Nt, but a is kept small

These are just complementar to the fixed-Nt approach. Our approach has advantages near Tc.

A test in quenched QCD: promishing!

- consistent with the previous fixed-Nt results on large lattices.
- scaling well achieved around Tc.
- systematic errors due to the discreteness in T are well under control.

A big advantage of the fixed scale approach:

can borrow high statistic configurations of previous studies at T=0 which are public, e.g. on the International Lattice Data Grid ILDG

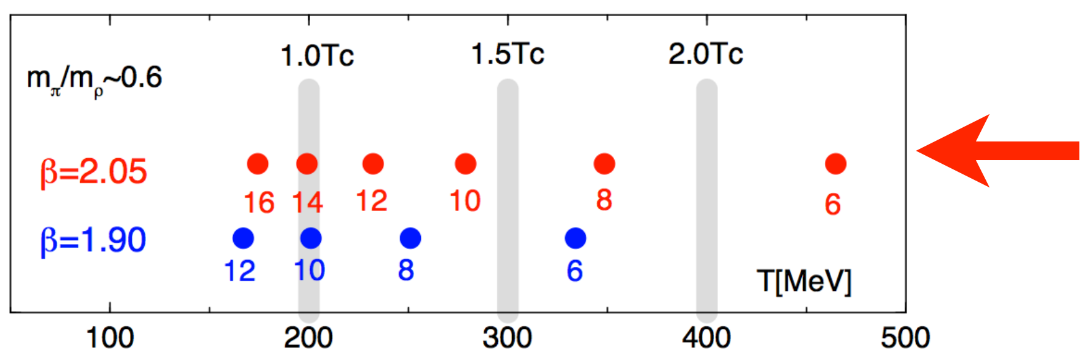
N_F=2+1 first study

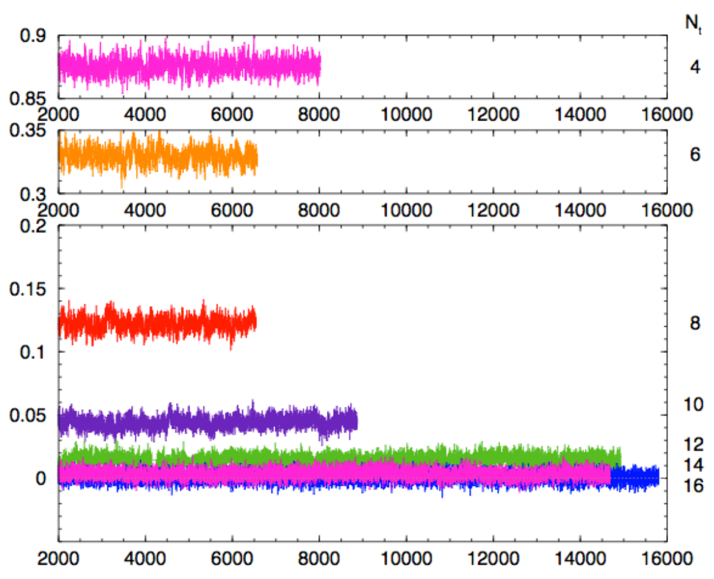
Lat09, Lat10

T=0 : CP-PACS+JLQCD N_F=2+1 config [*PRD78, 011502* ('08)] Iwasaki + clover

We borrow the finest and lightest lattice: a=0.07 fm, m_{PS}/m_V(LL)≈0.63, m_{PS}/m_V(SS)≈0.74, 28^3x56

T>0 simulations on 32^3xNt (Nt = 4, 6, ..., 16)





The simulations are still under way.

EOS for 2+1 flavor Wilson quarks

Beta function:

fit CP-PACS+JLQCD data for $am_\rho, m_\pi/m_\rho$ and $m_{\eta_{ss}}/m_\phi$ at 30 data points

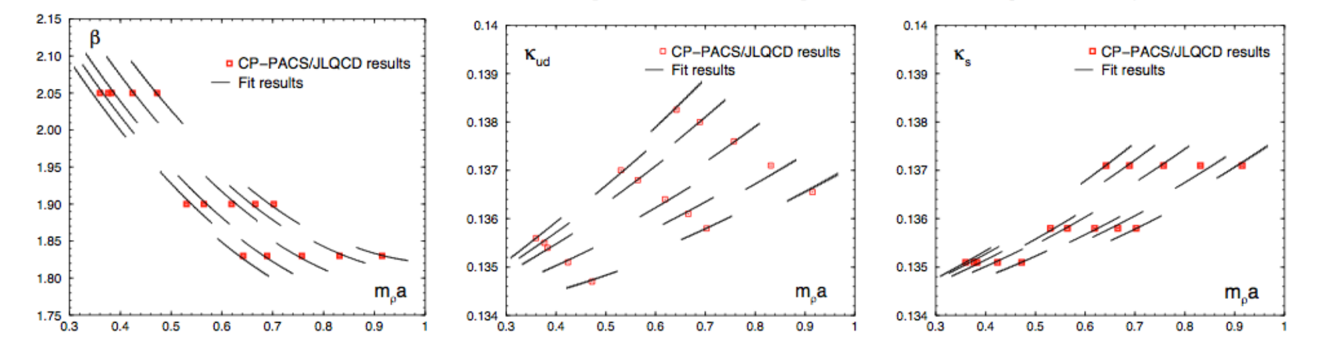


Figure 2: The global fit for coupling parameters, *Left:* β, *Center:* κ_{ud}, *Left:* κ_s, as the function of $m_\rho a$. Square symbols show coupling parameters in CP-PACS/JLQCD study. The Solid lines show the global fit results for each simulation point with corresponding m_ρ/m_π and $m_{\eta_{ss}}/m_\phi$.

then

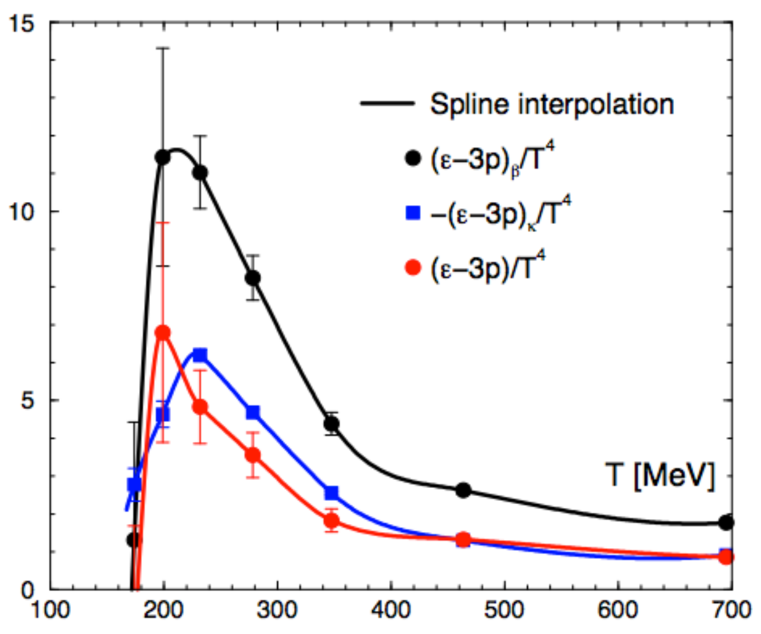
$$a\frac{\partial\beta}{\partial a} = -0.334(4),\ a\frac{\partial\kappa_{ud}}{\partial a} = 0.00289(6) \text{ and } a\frac{\partial\kappa_s}{\partial a} = 0.00203(5)$$ at our simul.pt.

Trace anomaly:

$$\frac{\varepsilon-3p}{T^4} = = \frac{N_t^3}{N_s^3}\left(a\frac{\partial\beta}{\partial a}\left\langle\frac{\partial S}{\partial\beta}\right\rangle_{sub} + a\frac{\partial\kappa_{ud}}{\partial a}\left\langle\frac{\partial S}{\partial\kappa_{ud}}\right\rangle_{sub} + a\frac{\partial\kappa_s}{\partial a}\left\langle\frac{\partial S}{\partial\kappa_s}\right\rangle_{sub}\right)$$

$$\left\langle\frac{\partial S}{\partial\beta}\right\rangle = N_s^3 N_t\left(-\left\langle\sum_{x,\mu>\nu} c_0 W_{\mu\nu}^{1\times1}(x) + \sum_{x,\mu,\nu} c_1 W_{\mu\nu}^{1\times2}(x)\right\rangle + N_f\frac{\partial c_{SW}}{\partial\beta}\kappa_f\left\langle\sum_{x,\mu>\nu}\mathrm{Tr}^{(c,s)}\sigma_{\mu\nu}F_{\mu\nu}(D^{-1})_{x,x}\right\rangle\right)$$

$$\left\langle\frac{\partial S}{\partial\kappa_f}\right\rangle = N_f N_s^3 N_t\left(\left\langle\sum_{x,\mu}\mathrm{Tr}^{(c,s)}\{(1-\gamma_\mu)U_{x,\mu}(D^{-1})_{x+\hat\mu,x} + (1+\gamma_\mu)U^\dagger_{x-\hat\mu,\mu}(D^{-1})_{x-\hat\mu,x}\}\right\rangle + c_{SW}\left\langle\sum_{x,\mu>\nu}\mathrm{Tr}^{(c,s)}\sigma_{\mu\nu}F_{\mu\nu}(D^{-1})_{x,x}\right\rangle\right)$$



* large cancellation between ß- and κ-derivatives

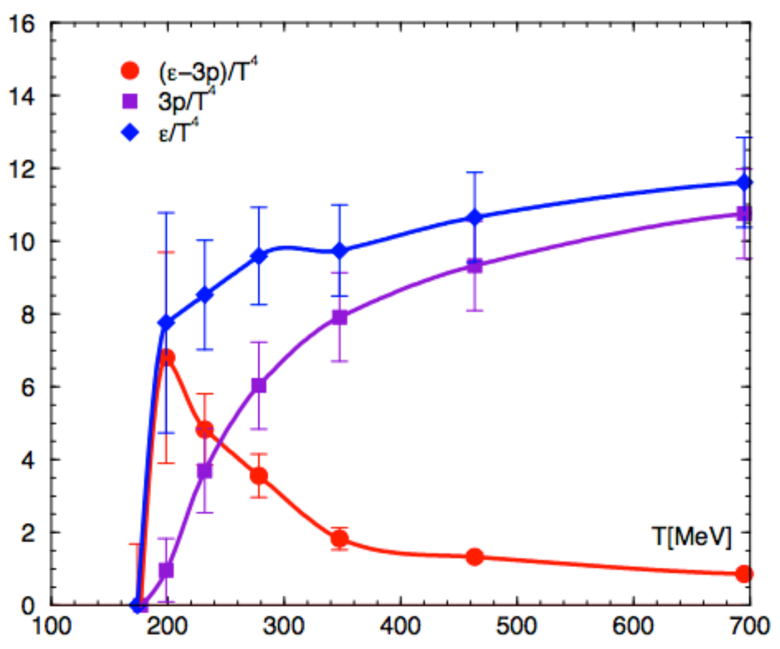
* low peak height ~ 6 [roughly consistent with recent highly improved stag. quarks]

EOS by T-integration:

using a trapezoidal interpolation

T-integral

$$\frac{p}{T^4} = \int_{T_0}^{T} dT\, \frac{\epsilon-3p}{T^5}$$



Still large errors.

But this is the first EOS in 2+1 flavor QCD with Wilson-type quarks.

Underway:

more statistics at low T (large Nt), add ß=1.90, beta funct. by reweighting, etc.

Other on-going attempts

- Charmonium spectral functions / wave functions with a variational method ==> Ohno's poster
- Phase structure of 2+1 flavor QCD ==> Saito's poster
- Explore μ ≠ 0 in 2+1 flavor QCD ==> Ejiri's talk
- Our final objective is to explore N_F = 2+1 QCD at the physical point. We are planning to extend the EOS study using the PACS-CS T = 0 configurations generated just at the physical point.