The order of the deconfinement phase transition in a heavy quark region - Dependence on the number of flavors -

## WHOT-QCD Collaboration:

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## Phase diagram

The order of the deconfinement phase transition is a fundamental topic. The order of the transition depends on the quark mass.
We investigate the end point of 1st order transition in the ( $m_{\mathrm{ud}}, m_{\mathrm{s}}$ ) plane of $N_{\mathrm{f}}=2+1$ by using lattice QCD simulation. In this study, we discuss it in a heavy quark mass region.


## Probability distribution function

Probability distribution function is useful to identify the order of a phase transition.

* Probability distribution function:

$$
w\left(P^{\prime}\right)=\int \mathcal{D} U \mathcal{D} \psi \mathcal{D} \bar{\psi} \delta\left(P(U)-P^{\prime}\right) e^{-S(\beta, \kappa)}
$$

An effective potential:

$$
V(P)=-\ln w(P)
$$



## * Reweighting method

(i) $\beta$-Direction : To get $d V / d P$ in a wide range


(ii) $\boldsymbol{k}$-Direction : To evaluate the end point $V(P, \kappa, \beta)=-\ln R(P, \kappa)+V_{0}(P, \beta)$
$\left\{\begin{array}{c}\text { Reweighting term }: R\left(P^{\prime}, \kappa\right)=\frac{\int \mathcal{D} U \delta\left(P(U)-P^{\prime}\right)[\operatorname{det} M(U, \kappa)]}{\int \mathcal{D} U \delta\left(P(U)-P^{\prime}\right)} \\ \begin{array}{l}\text { The change of the order is caused by dynamical quark } \\ \text { effect which is included by the quark determinant factor. }\end{array} \\ V_{0}\left(P^{\prime}, \beta\right)=-\ln \left[\int \mathcal{D} U \delta\left(P(U)-P^{\prime}\right) e^{-S_{g}(\beta)}\right] \text { HS talk in Lattice2010 }\end{array}\right.$

## $N_{\mathrm{f}}=2+1$ phase diagram

The $N_{\mathrm{f}}$ - dependence appears in the quark determinant:
$R\left(P^{\prime}, \kappa\right)=\frac{\int \mathcal{D} U \delta\left(P(U)-P^{\prime}\right)\left[\left\{\operatorname{det} M\left(U, \kappa_{\text {ud }}\right)\right\}^{2} \operatorname{det} M\left(U, \kappa_{\mathrm{s}}\right)\right]}{\int \mathcal{D} U \delta\left(P(U)-P^{\prime}\right)}$
We can consider the many flavors case by replacing into $[\operatorname{det} M(U, \kappa)]^{N_{f}}$.

Lattice setup

- Action : Standard wilson fermion
( hopping parameter $\kappa \approx m_{q}^{-1}$ ) and Plaquette action
$\bullet$ HeatBath / • Simulation Setup $\leadsto$
- The $\operatorname{det} M(\kappa)$ is evaluated by using a hopping parameter expansion.
The LO of the quark determinant factor has a coefficient $N_{\mathrm{f}} \kappa_{\mathrm{ep}}$. The shape of the potential unchanged when the coefficient is constant.

| $\boldsymbol{\beta}$ | Confs |
| :---: | :---: |
| 5.68 | 100,000 |
| 5.685 | 430,000 |
| 5.69 | 500,000 |
| 5.6925 | 670,000 |
| 5.7 | 100,000 |

## Result

## * The order of the deconfinement phase transition



* The $N_{\mathrm{f}}=2+1$ phase diagram

The result is consistent with the result of $Z(3)$ model reported by Alexandrou et al(1990).



## Summary \& Future prospect

i. The $\kappa_{\mathrm{ep}}$ value decreases as $N_{\mathrm{f}}$ increases.
ii. The method can be applied to the case of nonzero $\mu$. s. Ejiri's talk
iii. The method of probability distribution function to identify the order of the transition works well.

